

Association Rule Mining

- Given a set of transactions, find rules that will predict the occurrence of an item based on the occurrences of other items in the transaction

Market-Basket transactions

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Example of Association Rules

$\{\text{Diaper}\} \rightarrow \{\text{Beer}\}$,
 $\{\text{Milk, Bread}\} \rightarrow \{\text{Eggs, Coke}\}$,
 $\{\text{Beer, Bread}\} \rightarrow \{\text{Milk}\}$,

Implication means co-occurrence,
not causality!

Frequent Itemset

- **Itemset**

- A collection of one or more items
 - ◆ Example: {Milk, Bread, Diaper}
- k-itemset
 - ◆ An itemset that contains k items

- **Support count (σ)**

- Frequency of occurrence of an itemset
- E.g. $\sigma(\{\text{Milk, Bread, Diaper}\}) = 2$

- **Support**

- Fraction of transactions that contain an itemset
- E.g. $s(\{\text{Milk, Bread, Diaper}\}) = 2/5$

- **Frequent Itemset**

- An itemset whose support is greater than or equal to a $minsup$ threshold

<i>TID</i>	<i>Items</i>
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Association Rule

● Association Rule

- An implication expression of the form $X \rightarrow Y$, where X and Y are itemsets
- Example:
 $\{\text{Milk, Diaper}\} \rightarrow \{\text{Beer}\}$

● Rule Evaluation Metrics

- Support (s)
 - ◆ Fraction of transactions that contain both X and Y
- Confidence (c)
 - ◆ Measures how often items in Y appear in transactions that contain X

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Example:

$$\{\text{Milk, Diaper}\} \Rightarrow \text{Beer}$$

$$s = \frac{\sigma(\text{Milk, Diaper, Beer})}{|T|} = \frac{2}{5} = 0.4$$

$$c = \frac{\sigma(\text{Milk, Diaper, Beer})}{\sigma(\text{Milk, Diaper})} = \frac{2}{3} = 0.67$$

Association Rule Mining

- Given a set of transactions T , the goal of association rule mining is to find all rules having
 - support $\geq \text{minsup}$ threshold
 - confidence $\geq \text{minconf}$ threshold
- Brute-force approach:
 - List all possible association rules
 - Compute the support and confidence for each rule
 - Prune rules that fail the minsup and minconf thresholds

⇒ Computationally prohibitive!

Mining Association Rules

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Example of Rules:

$\{\text{Milk}, \text{Diaper}\} \rightarrow \{\text{Beer}\}$ ($s=0.4, c=0.67$)
 $\{\text{Milk}, \text{Beer}\} \rightarrow \{\text{Diaper}\}$ ($s=0.4, c=1.0$)
 $\{\text{Diaper}, \text{Beer}\} \rightarrow \{\text{Milk}\}$ ($s=0.4, c=0.67$)
 $\{\text{Beer}\} \rightarrow \{\text{Milk}, \text{Diaper}\}$ ($s=0.4, c=0.67$)
 $\{\text{Diaper}\} \rightarrow \{\text{Milk}, \text{Beer}\}$ ($s=0.4, c=0.5$)
 $\{\text{Milk}\} \rightarrow \{\text{Diaper}, \text{Beer}\}$ ($s=0.4, c=0.5$)

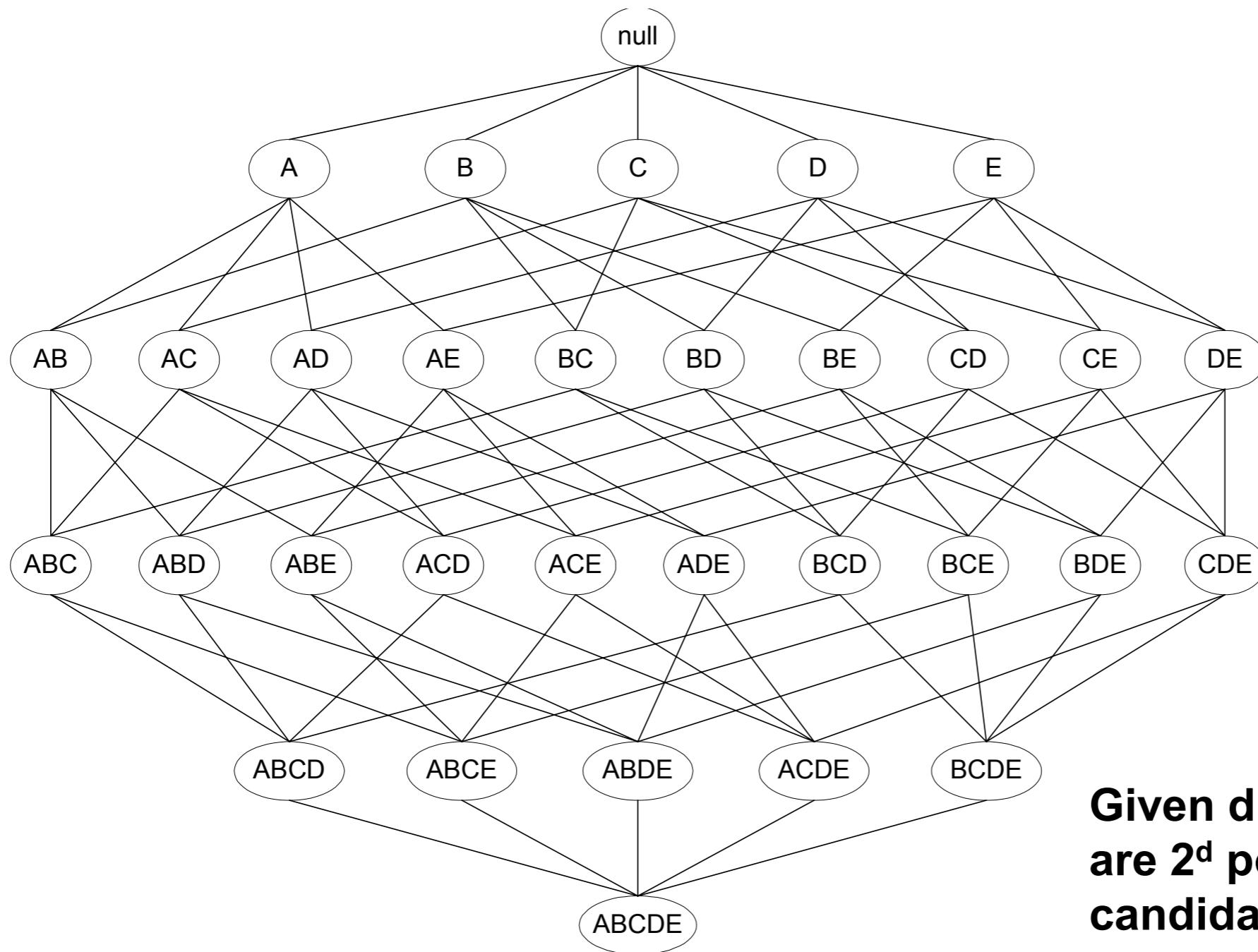
Observations:

- All the above rules are binary partitions of the same itemset:
 $\{\text{Milk}, \text{Diaper}, \text{Beer}\}$
- Rules originating from the same itemset have identical support but can have different confidence
- Thus, we may decouple the support and confidence requirements

Mining Association Rules

- Two-step approach:
 1. Frequent Itemset Generation
 - Generate all itemsets whose support $\geq \text{minsup}$
 2. Rule Generation
 - Generate high confidence rules from each frequent itemset, where each rule is a binary partitioning of a frequent itemset
- Frequent itemset generation is still computationally expensive

Frequent Itemset Generation

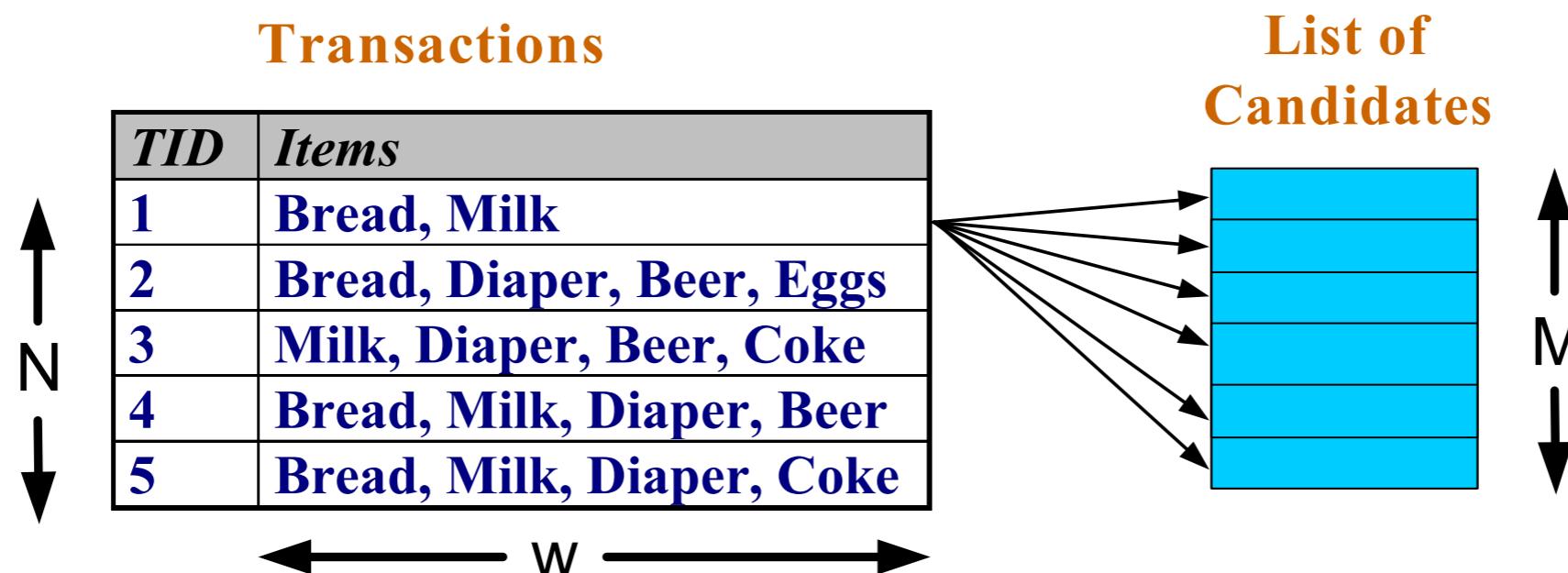


**Given d items, there
are 2^d possible
candidate itemsets**

Frequent Itemset Generation

- Brute-force approach:

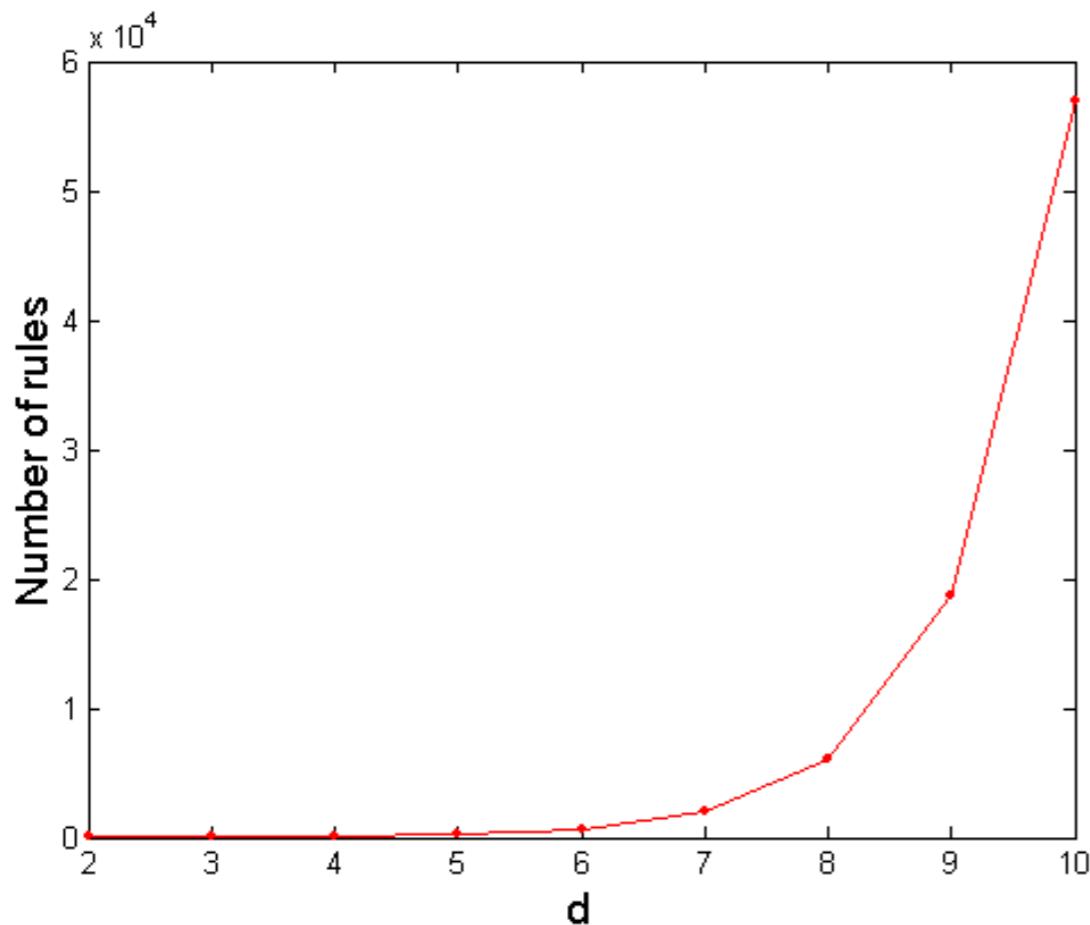
- Each itemset in the lattice is a **candidate** frequent itemset
- Count the support of each candidate by scanning the database



- Match each transaction against every candidate
- Complexity $\sim O(NMw)$ => **Expensive since $M = 2^d$!!!**

Computational Complexity

- Given d unique items:
 - Total number of itemsets = 2^d
 - Total number of possible association rules:



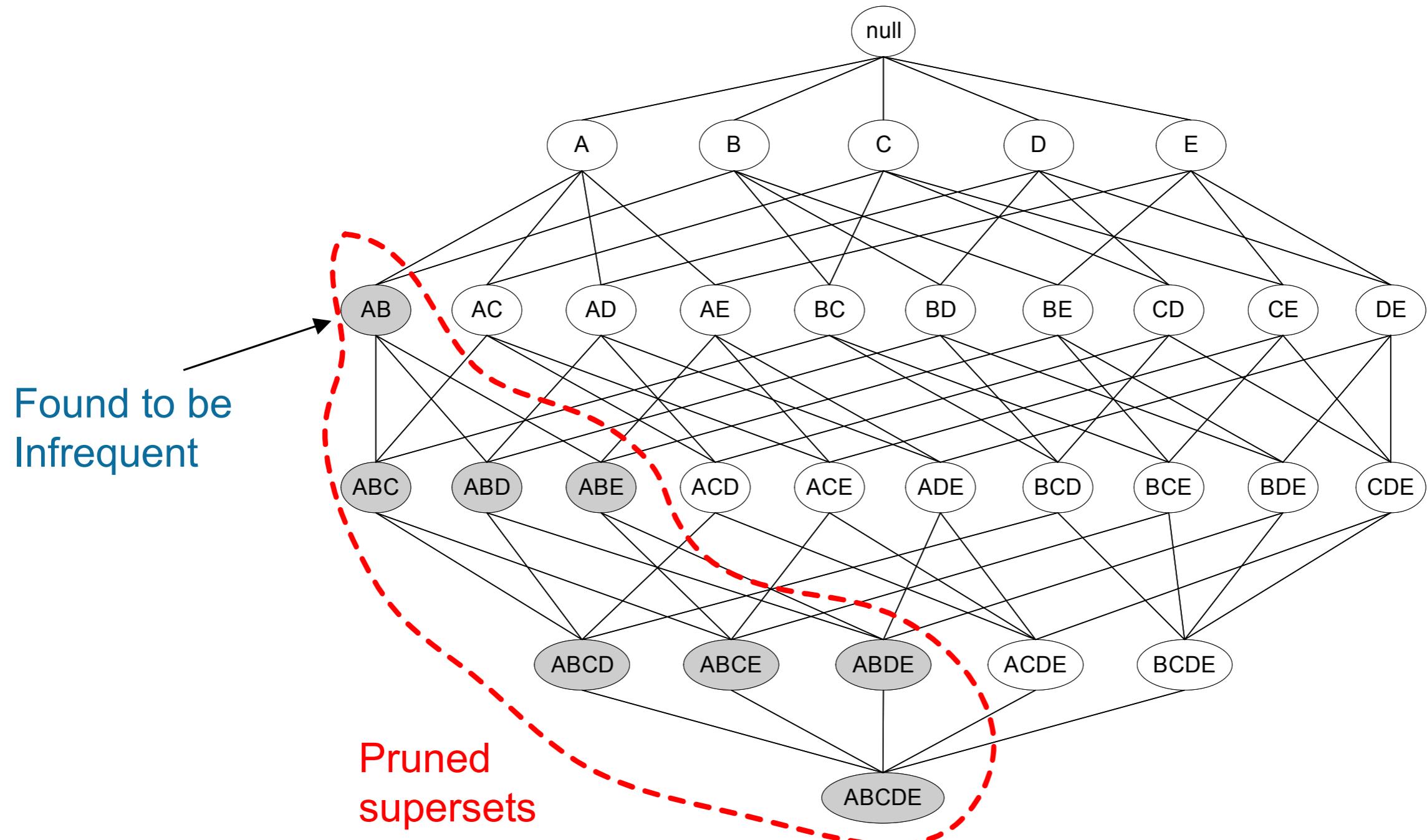
$$\begin{aligned}R &= \sum_{k=1}^{d-1} \binom{d}{k} \times \sum_{j=1}^{d-k} \binom{d-k}{j} \\&= 3^d - 2^{d+1} + 1\end{aligned}$$

If $d=6$, $R = 602$ rules

Reducing Number of Candidates

- **Apriori principle:**
 - If an itemset is frequent, then all of its subsets must also be frequent
- Apriori principle holds due to the following property of the support measure:
$$\forall X, Y : (X \subseteq Y) \Rightarrow s(X) \geq s(Y)$$
 - Support of an itemset never exceeds the support of its subsets
 - This is known as the **anti-monotone** property of support

Illustrating Apriori Principle



Illustrating Apriori Principle

Item	Count
Bread	4
Coke	2
Milk	4
Beer	3
Diaper	4
Eggs	1

Minimum Support = 3

Items (1-itemsets)



Itemset	Count
{Bread,Milk}	3
{Bread,Beer}	2
{Bread,Diaper}	3
{Milk,Beer}	2
{Milk,Diaper}	3
{Beer,Diaper}	3

Pairs (2-itemsets)

(No need to generate candidates involving Coke or Eggs)



Triplets (3-itemsets)

Item set	Count
{Bread,Milk,Diaper}	3

If every subset is considered,

$${}^6C_1 + {}^6C_2 + {}^6C_3 = 41$$

With support-based pruning,

$$6 + 6 + 1 = 13$$



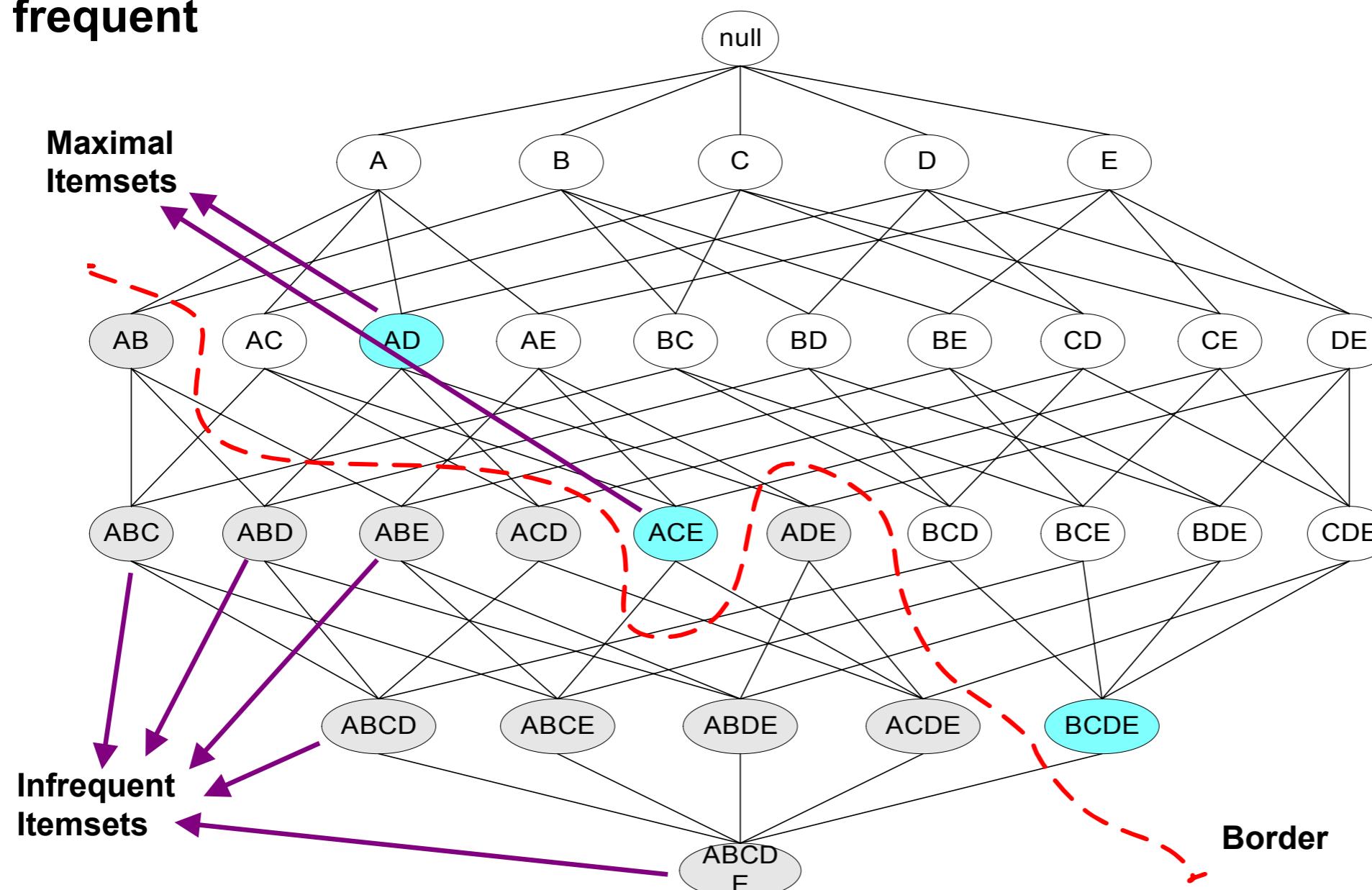
Apriori Algorithm

- Method:

- Let $k=1$
- Generate frequent itemsets of length 1
- Repeat until no new frequent itemsets are identified
 - ◆ Generate length $(k+1)$ candidate itemsets from length k frequent itemsets
 - ◆ Prune candidate itemsets containing subsets of length k that are infrequent
 - ◆ Count the support of each candidate by scanning the DB
 - ◆ Eliminate candidates that are infrequent, leaving only those that are frequent

Maximal Frequent Itemset

An itemset is maximal frequent if none of its immediate supersets is frequent



Closed Itemset

- An itemset is closed if none of its immediate supersets has the same support as the itemset

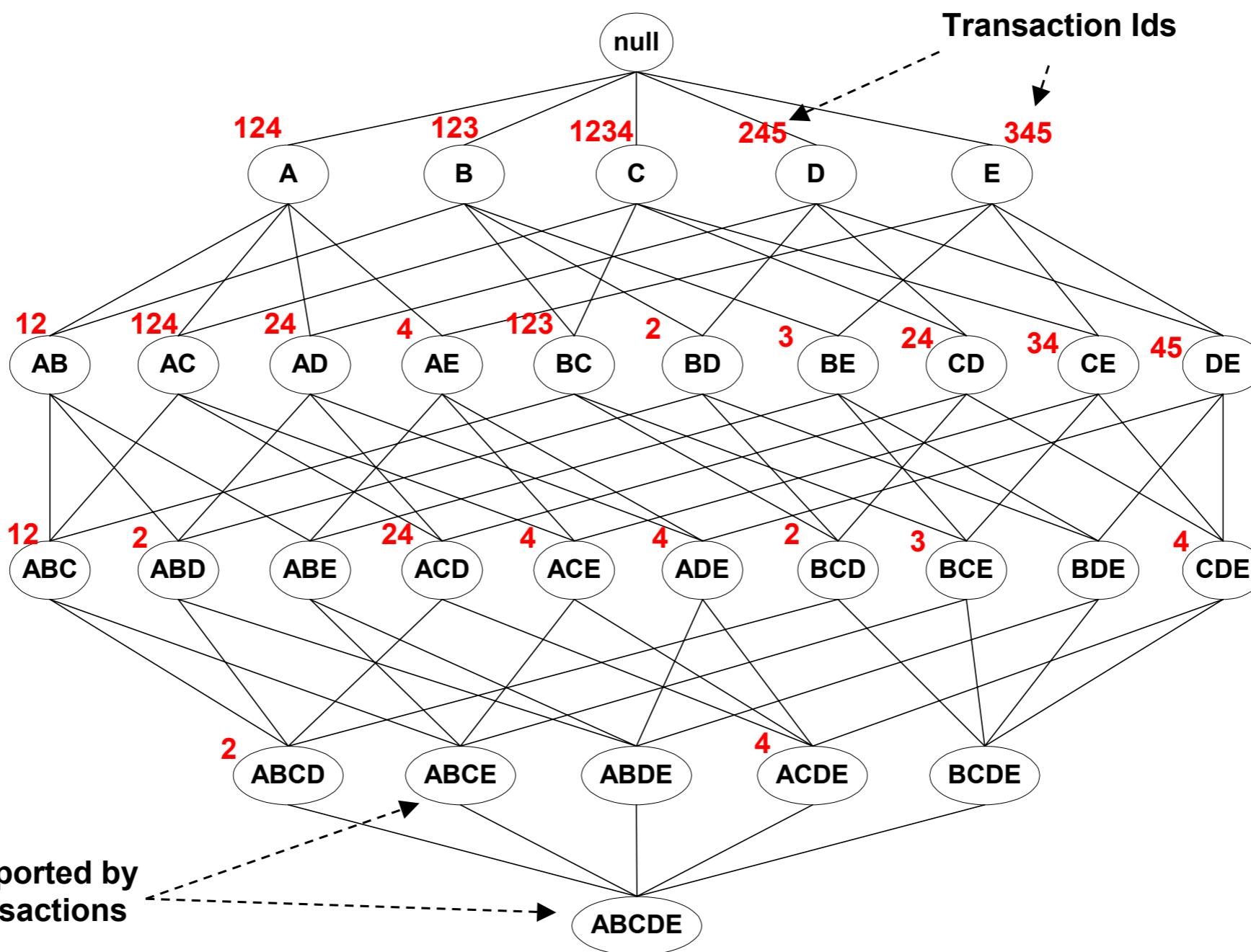
TID	Items
1	{A,B}
2	{B,C,D}
3	{A,B,C,D}
4	{A,B,D}
5	{A,B,C,D}

Itemset	Support
{A}	4
{B}	5
{C}	3
{D}	4
{A,B}	4
{A,C}	2
{A,D}	3
{B,C}	3
{B,D}	4
{C,D}	3

Itemset	Support
{A,B,C}	2
{A,B,D}	3
{A,C,D}	2
{B,C,D}	3
{A,B,C,D}	2

Example: Closed Itemsets

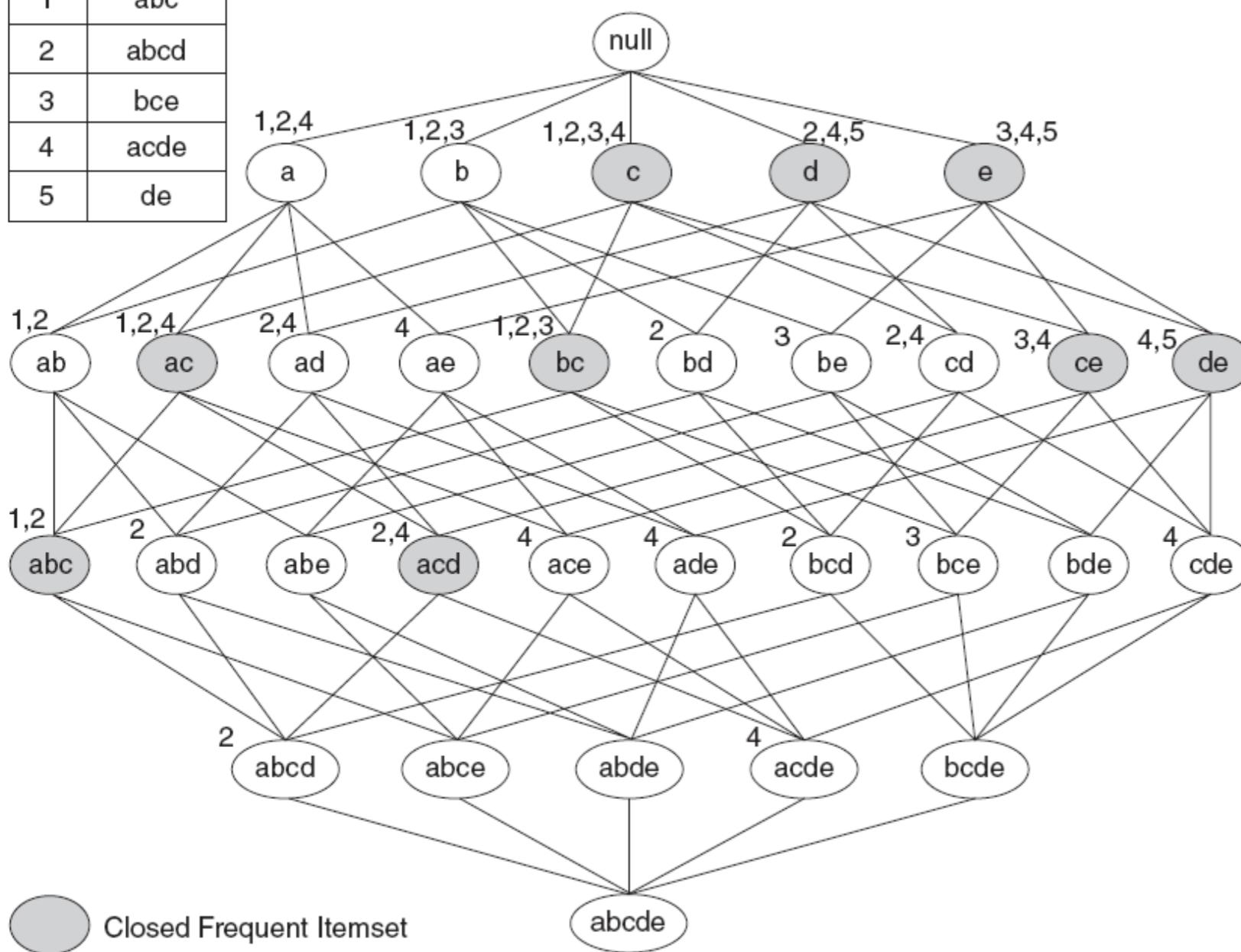
TID	Items
1	ABC
2	ABCD
3	BCE
4	ACDE
5	DE



Example: Closed Frequent Itemsets

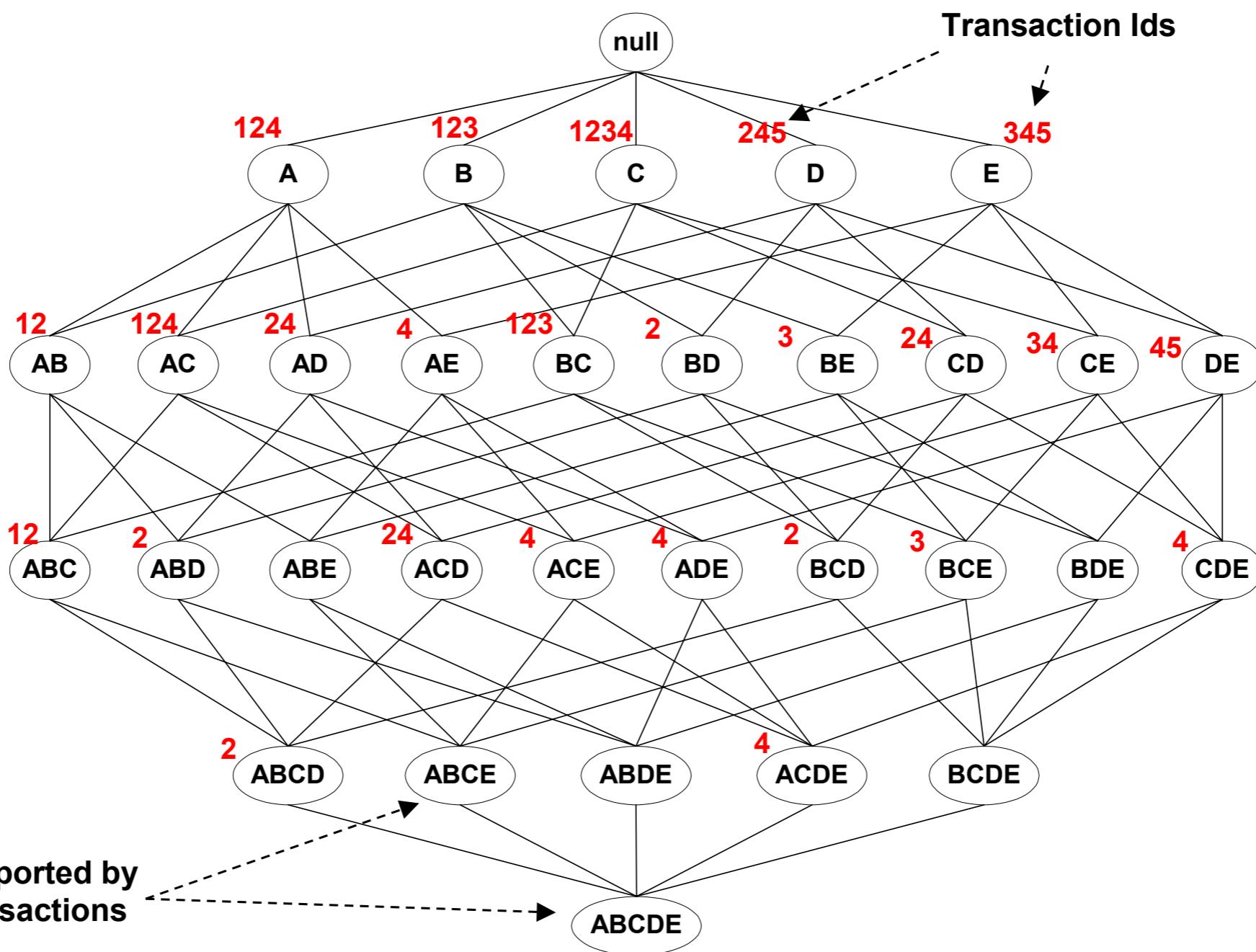
TID	Items
1	abc
2	abcd
3	bce
4	acde
5	de

minsup = 40%

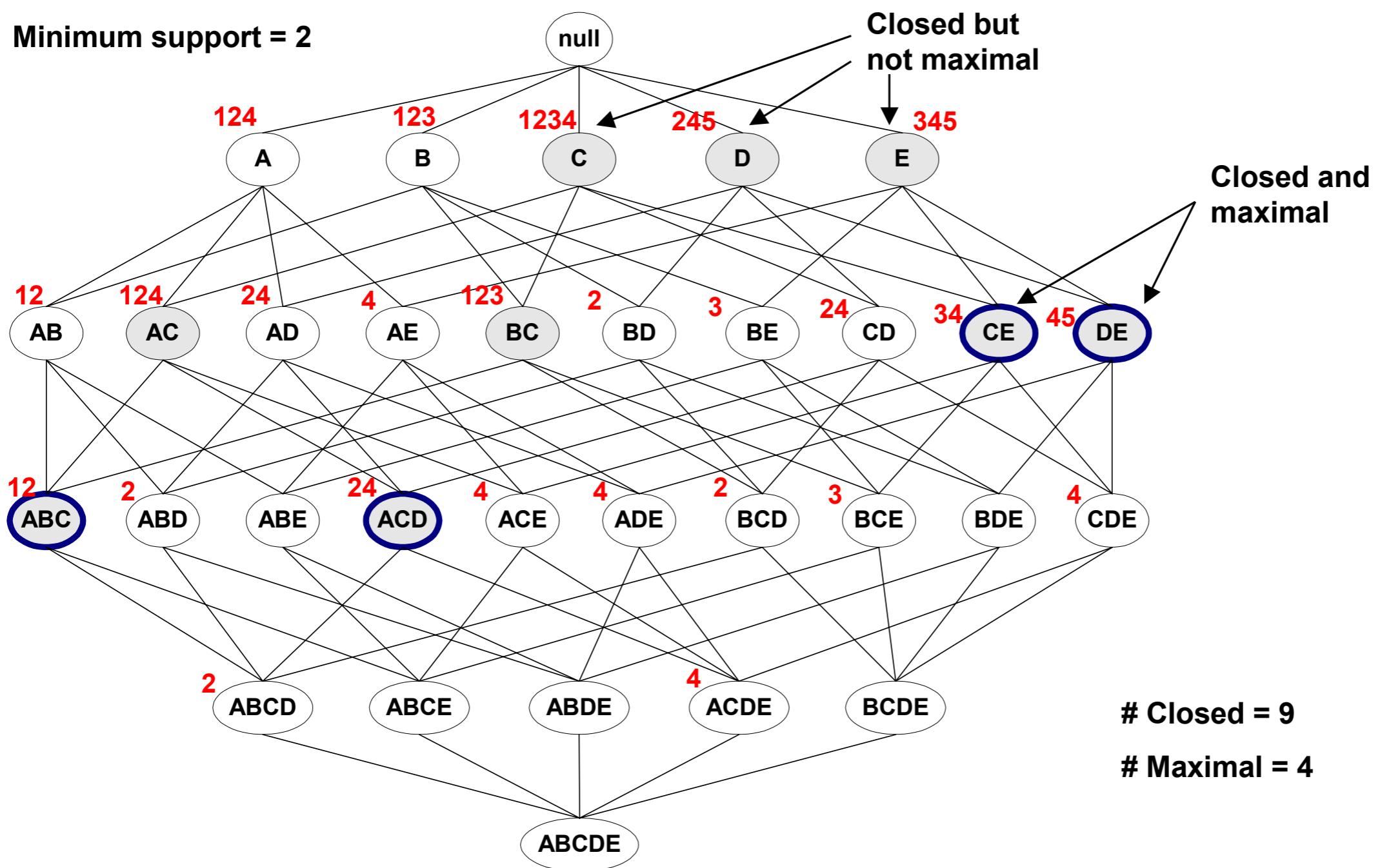


Maximal vs Closed Itemsets

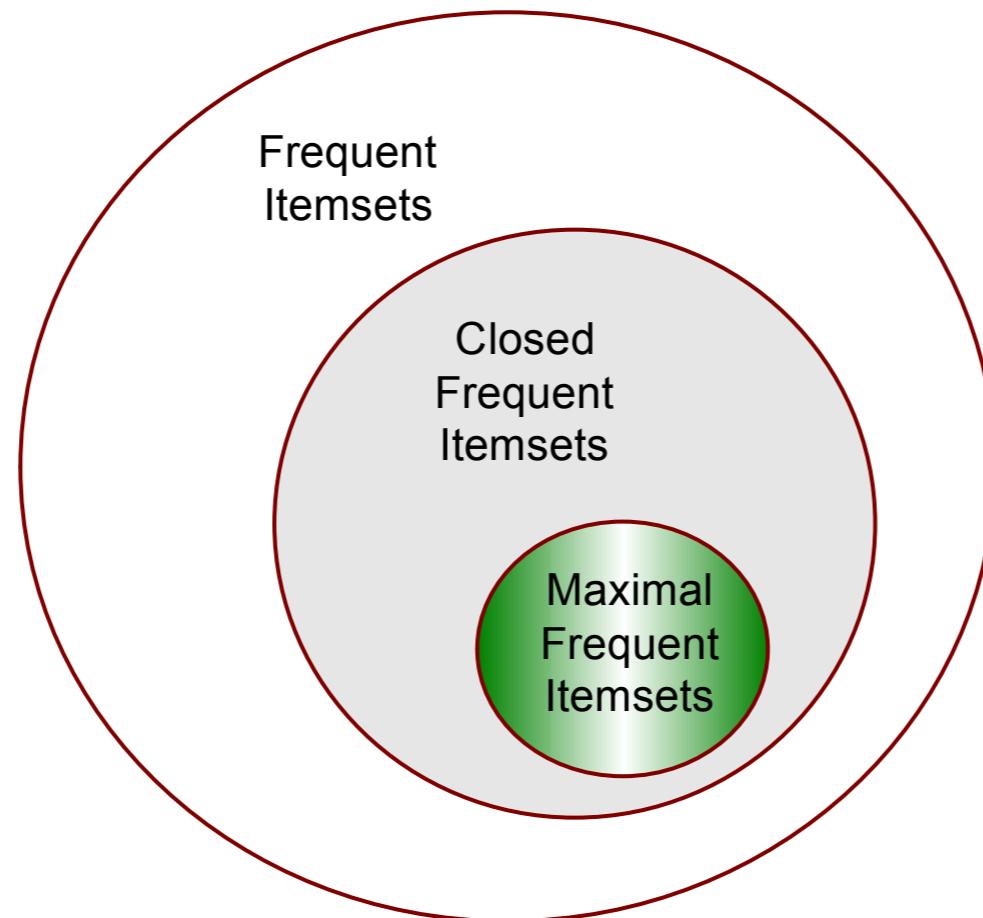
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Maximal vs Closed Itemsets



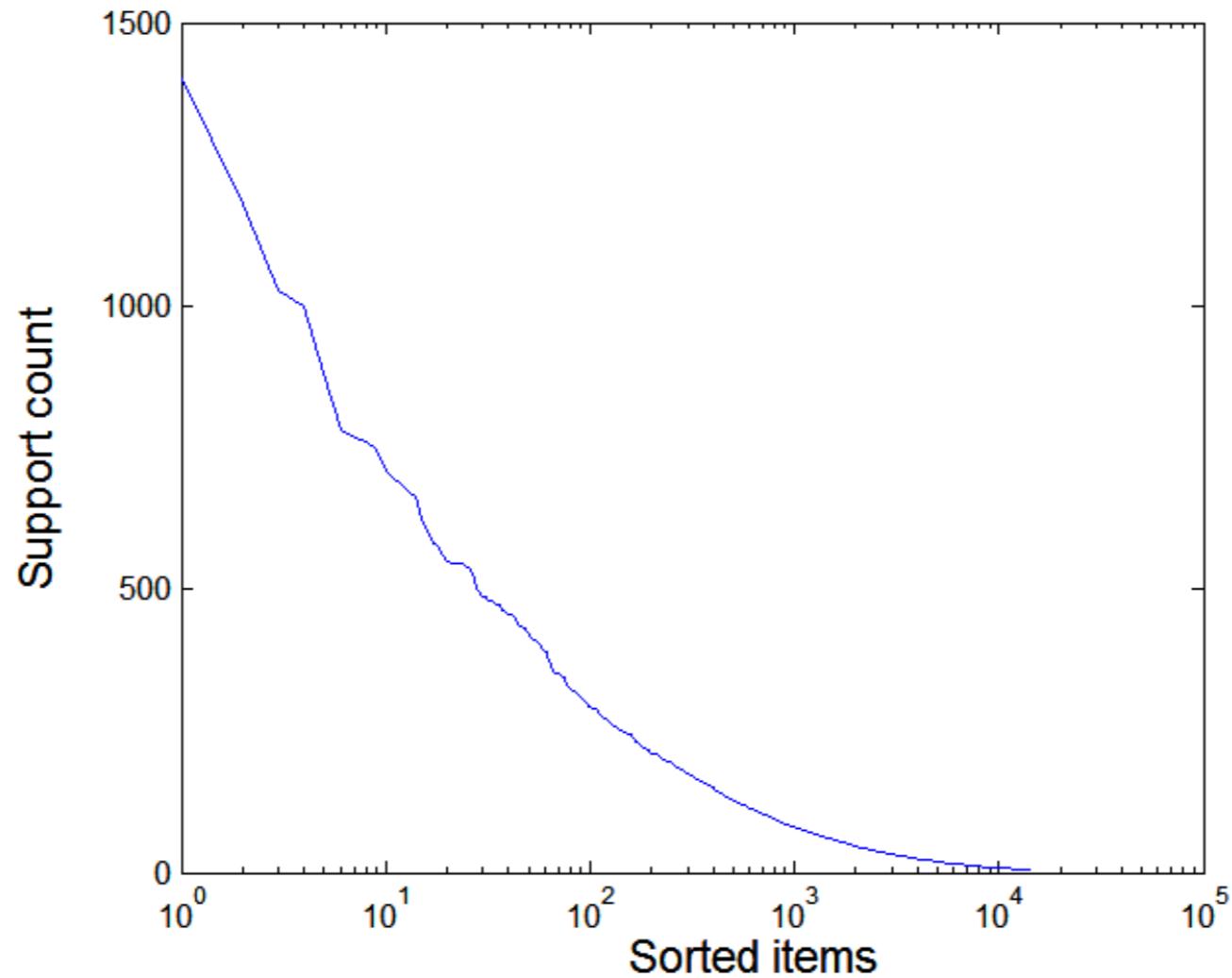
Maximal vs Closed Frequent Itemsets



Effect of Support Distribution

- Many real data sets have skewed support distribution

**Support
distribution of
a retail data set**

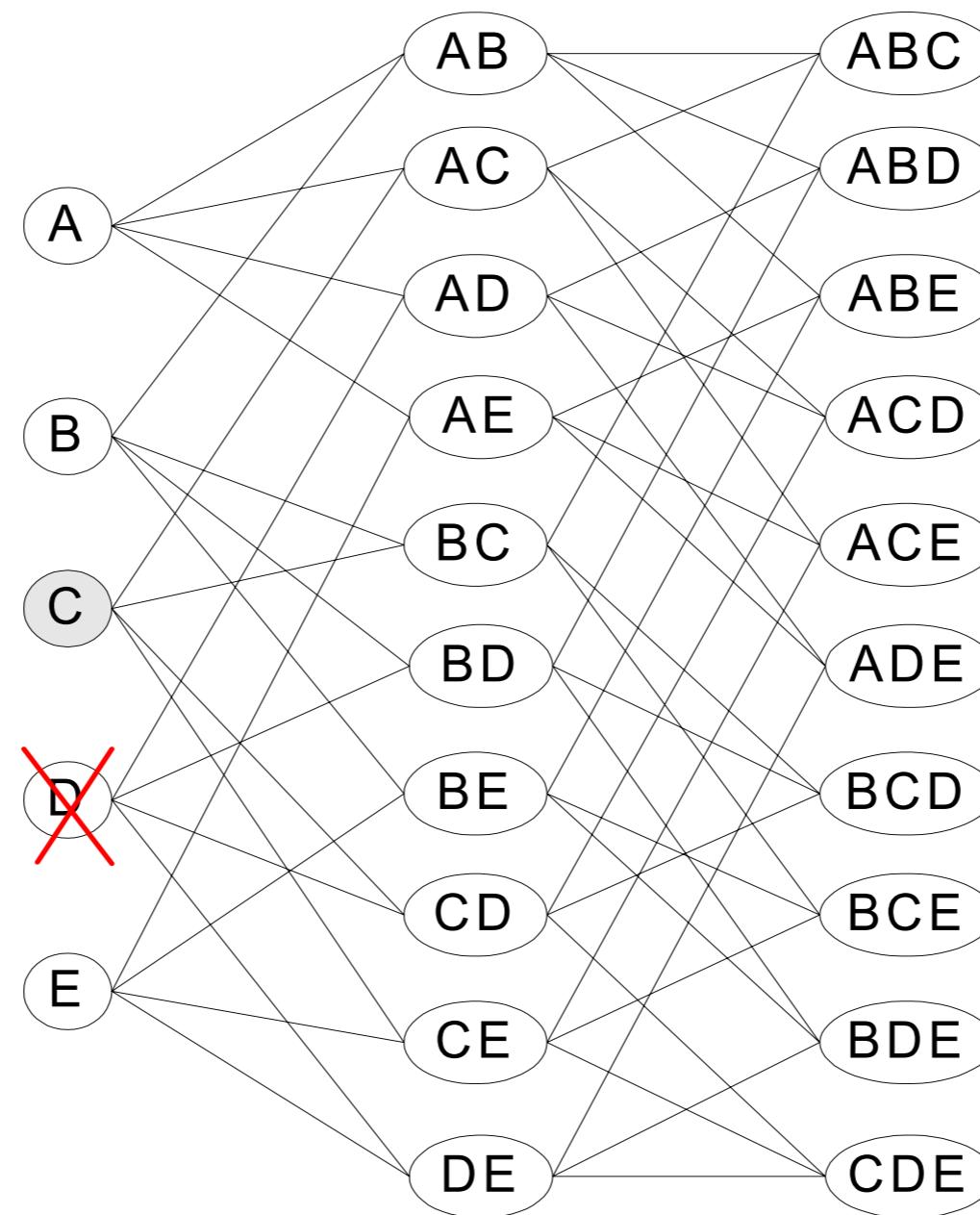


Multiple Minimum Support

- How to apply multiple minimum supports?

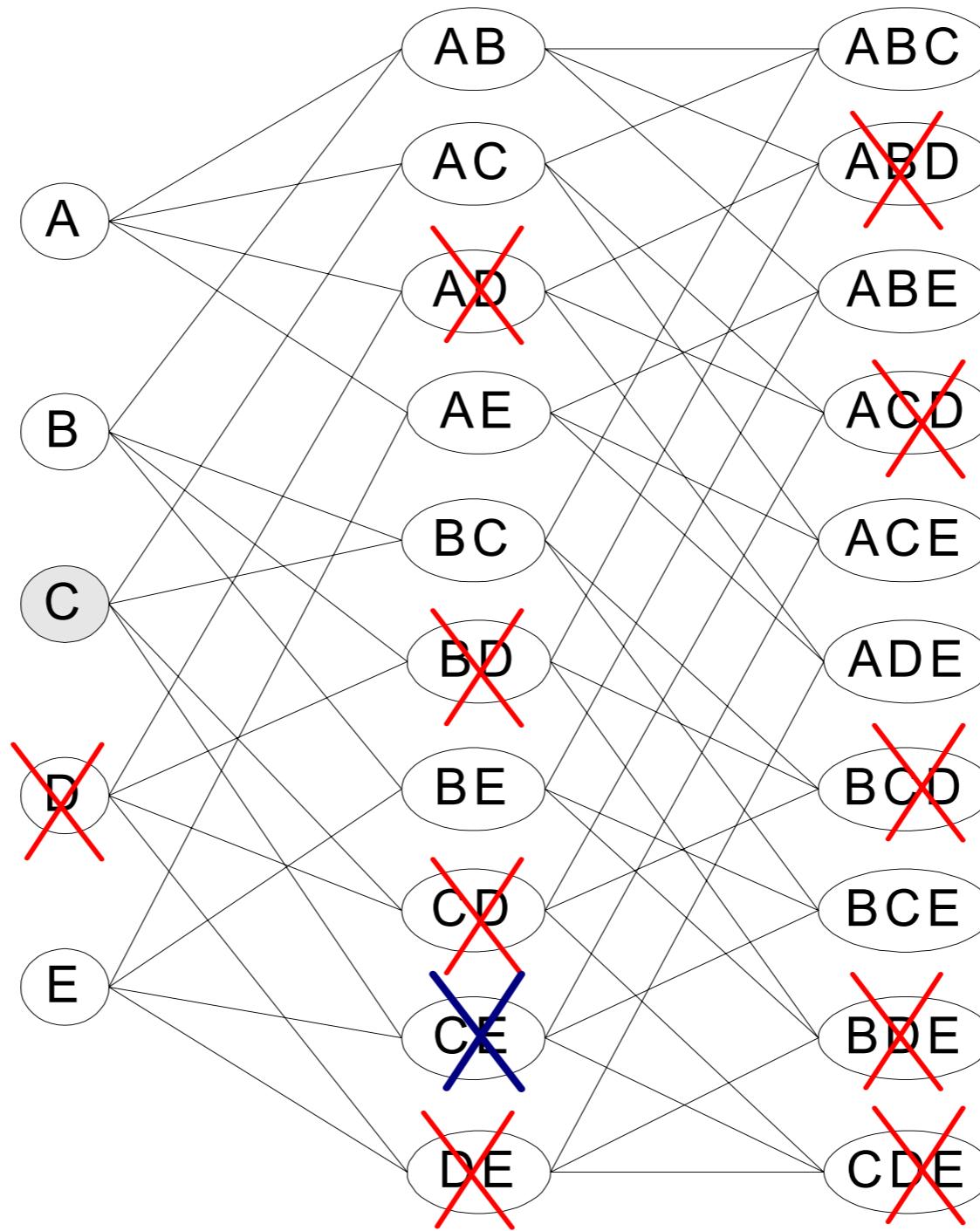
Multiple Minimum Support

Item	MS(I)	Sup(I)
A	0.10%	0.25%
B	0.20%	0.26%
C	0.30%	0.29%
D	0.50%	0.05%
E	3%	4.20%



Multiple Minimum Support

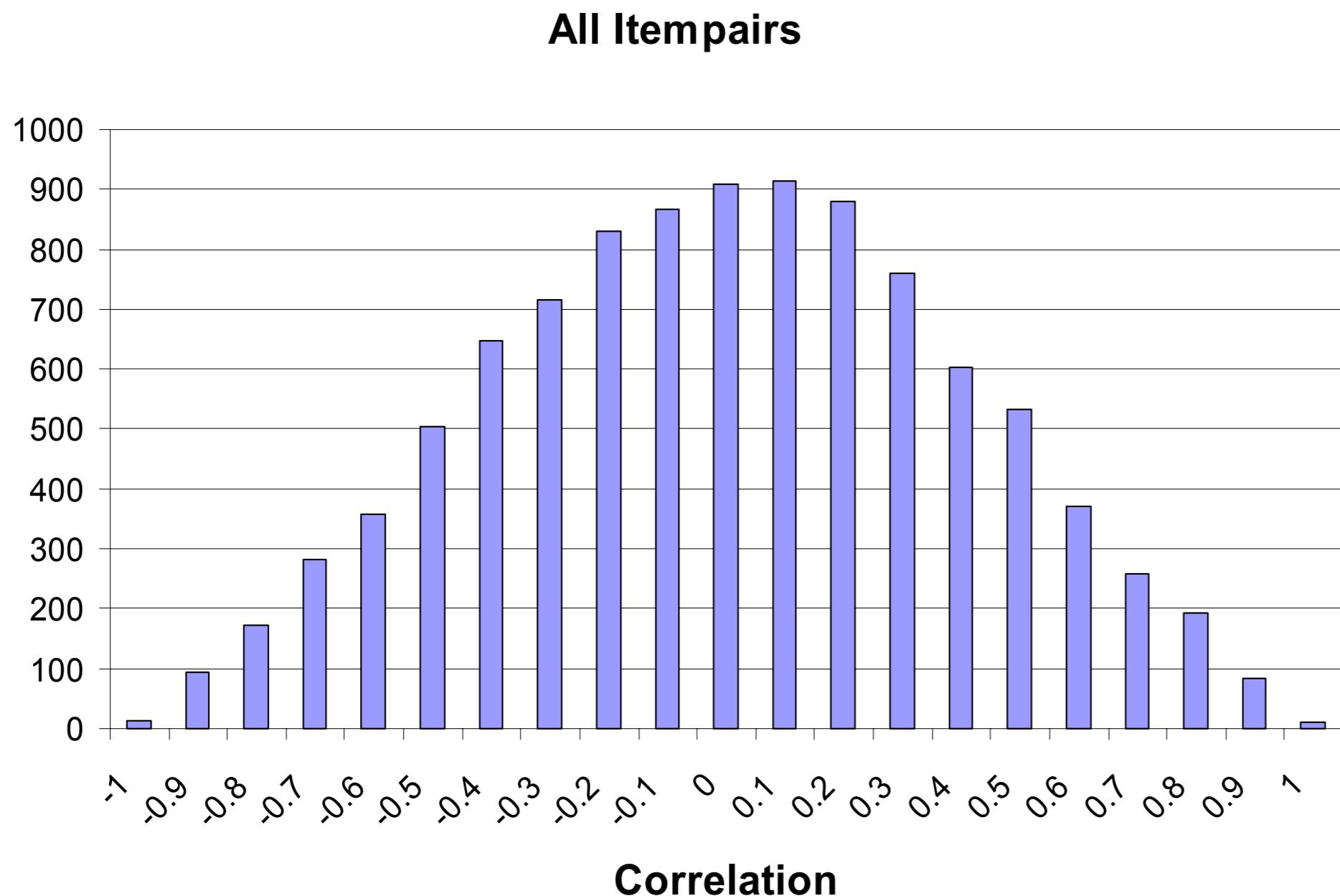
Item	MS(I)	Sup(I)
A	0.10%	0.25%
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C	0.30%	0.29%
D	0.50%	0.05%
E	3%	4.20%



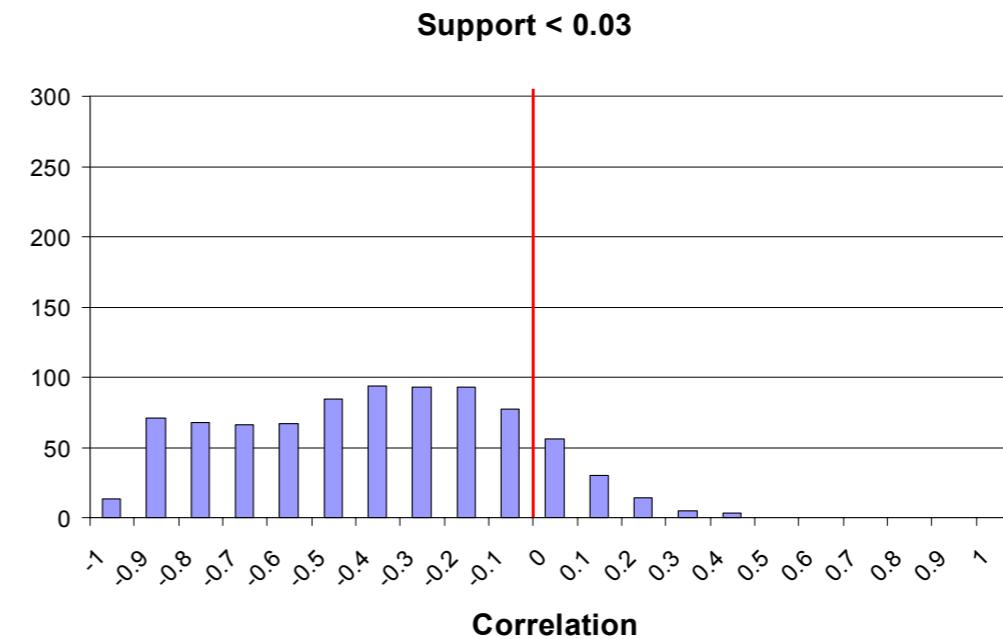
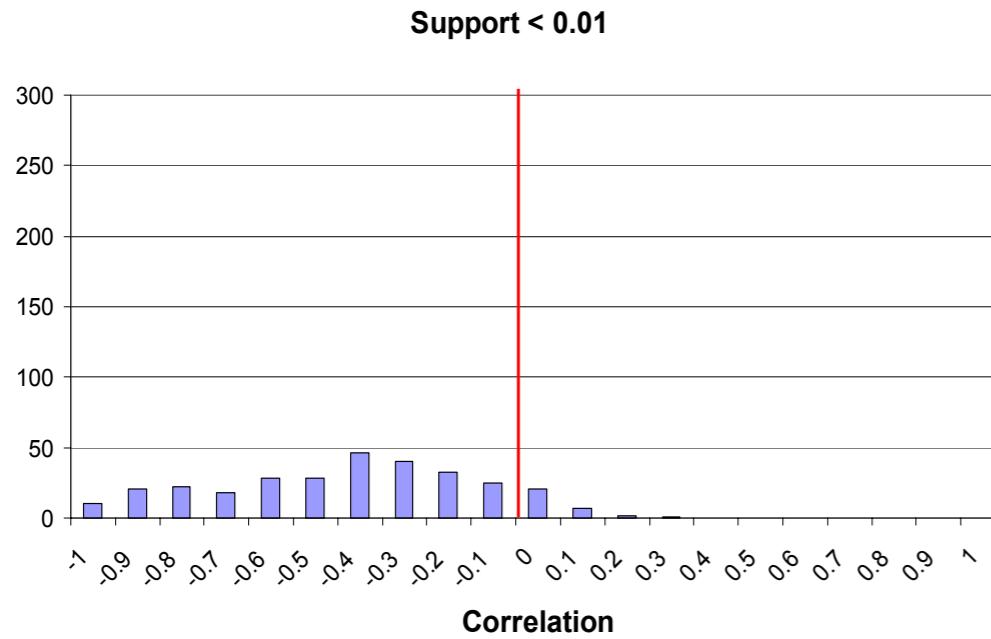
Support-based Pruning

- Most of the association rule mining algorithms use support measure to prune rules and itemsets
- Study effect of support pruning on correlation of itemsets
 - Generate 10000 random contingency tables
 - Compute support and pairwise correlation for each table
 - Apply support-based pruning and examine the tables that are removed

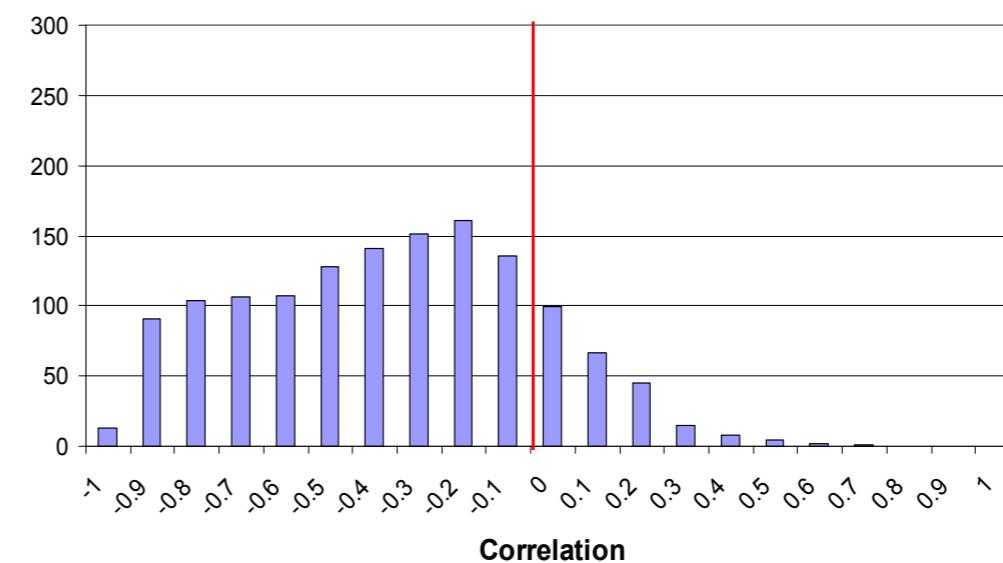
Effect of Support Based Pruning



Effect of Support Based Pruning



Support-based pruning
eliminates mostly
negatively correlated
itemsets



Recall: Mining Association Rules

- Two-step approach:
 1. Frequent Itemset Generation
 - Generate all itemsets whose support $\geq \text{minsup}$
 2. Rule Generation
 - Generate high confidence rules from each frequent itemset, where each rule is a binary partitioning of a frequent itemset

Rule Generation

- Given a frequent itemset L , find all non-empty subsets $f \subset L$ such that $f \rightarrow L - f$ satisfies the minimum confidence requirement
 - If $\{A, B, C, D\}$ is a frequent itemset, candidate rules:

$ABC \rightarrow D,$	$ABD \rightarrow C,$	$ACD \rightarrow B,$	$BCD \rightarrow A,$
$A \rightarrow BCD,$	$B \rightarrow ACD,$	$C \rightarrow ABD,$	$D \rightarrow ABC$
$AB \rightarrow CD,$	$AC \rightarrow BD,$	$AD \rightarrow BC,$	$BC \rightarrow AD,$
$BD \rightarrow AC,$	$CD \rightarrow AB,$		
- If $|L| = k$, then there are $2^k - 2$ candidate association rules (ignoring $L \rightarrow \emptyset$ and $\emptyset \rightarrow L$)

Rule Generation

- How to efficiently generate rules from frequent itemsets?

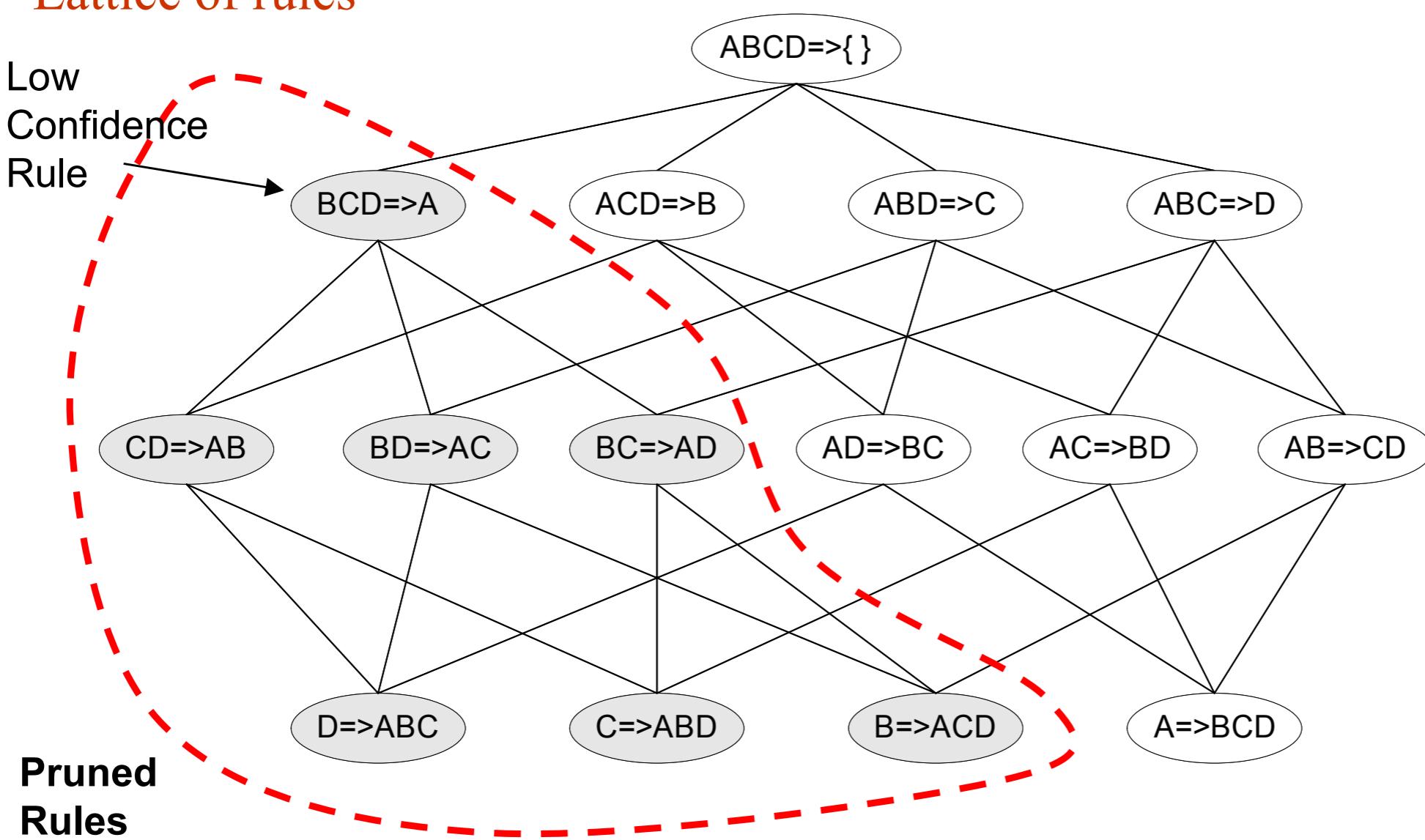
- In general, confidence does not have an anti-monotone property
 $c(ABC \rightarrow D)$ can be larger or smaller than $c(AB \rightarrow D)$
 - But confidence of rules generated from the same itemset has an anti-monotone property
 - e.g., $L = \{A, B, C, D\}$:

$$c(ABC \rightarrow D) \geq c(AB \rightarrow CD) \geq c(A \rightarrow BCD)$$

- ◆ Confidence is anti-monotone w.r.t. number of items on the RHS of the rule

Rule Generation for Apriori Algorithm

Lattice of rules



Evaluating Generated Rules

- Association rule algorithms tend to produce too many rules
 - many of them are uninteresting or redundant
 - Redundant if $\{A,B,C\} \rightarrow \{D\}$ and $\{A,B\} \rightarrow \{D\}$ have same support & confidence
- Interestingness measures can be used to prune/rank the derived patterns
- In the original formulation of association rules, support & confidence are the only measures used

Computing Interestingness Measure

- Given a rule $X \rightarrow Y$, information needed to compute rule interestingness can be obtained from a contingency table

Contingency table for $X \rightarrow Y$

	Y	\bar{Y}	
X	f_{11}	f_{10}	f_{1+}
\bar{X}	f_{01}	f_{00}	f_{0+}
	f_{+1}	f_{+0}	$ T $

f_{11} : support of X and Y

f_{10} : support of X and \bar{Y}

f_{01} : support of \bar{X} and Y

f_{00} : support of \bar{X} and \bar{Y}

Used to define various measures

- support, confidence, lift, Gini, J-measure, etc.

Drawback of Confidence

	Coffee	<u>Coffee</u>	
Tea	15	5	20
<u>Tea</u>	75	5	80
	90	10	100

Association Rule: Tea → Coffee

Confidence= $P(\text{Coffee}|\text{Tea}) =$

Drawback of Confidence

	Coffee	$\bar{\text{Coffee}}$	
Tea	15	5	20
$\bar{\text{Tea}}$	75	5	80
	90	10	100

Association Rule: Tea \rightarrow Coffee

Confidence = $P(\text{Coffee}|\text{Tea}) = 0.75$

but $P(\text{Coffee}) = 0.9$

\Rightarrow Although confidence is high, rule is misleading

$\Rightarrow P(\text{Coffee}|\bar{\text{Tea}}) = 0.9375$

Statistical Measures

- Measures that take into account statistical dependence

$$Lift = \frac{P(Y | X)}{P(Y)} = \frac{\text{conf.}(X \rightarrow Y)}{\text{supp.}(Y)}$$

$$Interest = \frac{P(X, Y)}{P(X)P(Y)}$$

$$PS = P(X, Y) - P(X)P(Y)$$

$$\phi\text{-coefficient} = \frac{P(X, Y) - P(X)P(Y)}{\sqrt{P(X)[1 - P(X)]P(Y)[1 - P(Y)]}}$$

Example: Lift/Interest

	Coffee	$\overline{\text{Coffee}}$	
Tea	15	5	20
$\overline{\text{Tea}}$	75	5	80
	90	10	100

Association Rule: Tea \rightarrow Coffee

Confidence= $P(\text{Coffee}|\text{Tea}) = 0.75$

but $P(\text{Coffee}) = 0.9$

$\Rightarrow \text{Lift} = 0.75/0.9 = 0.8333$

Drawback of Lift/Interest

	Y	\bar{Y}	
X	10	0	10
\bar{X}	0	90	90
	10	90	100

	Y	\bar{Y}	
X	90	0	90
\bar{X}	0	10	10
	90	10	100

$$Lift = \frac{0.1}{(0.1)(0.1)} = 10$$

$$Lift = \frac{0.9}{(0.9)(0.9)} = 1.11$$

Statistical independence:

If $P(X,Y) = P(X)P(Y)$ \Rightarrow Lift = 1

Comparing Different Measures

10 examples of contingency tables:

Rankings of contingency tables using various measures:

Example	f_{11}	f_{10}	f_{01}	f_{00}
E1	8123	83	424	1370
E2	8330	2	622	1046
E3	9481	94	127	298
E4	3954	3080	5	2961
E5	2886	1363	1320	4431
E6	1500	2000	500	6000
E7	4000	2000	1000	3000
E8	4000	2000	2000	2000
E9	1720	7121	5	1154
E10	61	2483	4	7452

#	ϕ	λ	α	Q	Y	κ	M	J	G	s	c	L	V	I	IS	PS	F	AV	S	ζ	K
E1	1	1	3	3	3	1	2	2	1	3	5	5	4	6	2	2	4	6	1	2	5
E2	2	2	1	1	1	2	1	3	2	2	1	1	1	8	3	5	1	8	2	3	6
E3	3	3	4	4	4	3	3	8	7	1	4	4	6	10	1	8	6	10	3	1	10
E4	4	7	2	2	2	5	4	1	3	6	2	2	2	4	4	1	2	3	4	5	1
E5	5	4	8	8	8	4	7	5	4	7	9	9	9	3	6	3	9	4	5	6	3
E6	6	6	7	7	7	7	6	4	6	9	8	8	7	2	8	6	7	2	7	8	2
E7	7	5	9	9	9	6	8	6	5	4	7	7	8	5	5	4	8	5	6	4	4
E8	8	9	10	10	10	8	10	10	8	4	10	10	10	9	7	7	10	9	8	7	9
E9	9	9	5	5	5	9	9	7	9	8	3	3	3	7	9	9	3	7	9	9	8
E10	10	8	6	6	6	10	5	9	10	10	6	6	5	1	10	10	5	1	10	10	7

Subjective Interestingness Measure

- Objective measure:

- Rank patterns based on statistics computed from data
- e.g., 21 measures of association (support, confidence, Laplace, Gini, mutual information, Jaccard, etc).

- Subjective measure:

- Rank patterns according to user's interpretation
 - ◆ A pattern is subjectively interesting if it contradicts the expectation of a user (Silberschatz & Tuzhilin)
 - ◆ A pattern is subjectively interesting if it is actionable (Silberschatz & Tuzhilin)