Letter to the Governor Office

Dear Governor of the state of Washington,

We have heard your concern about the traffic congestion in the Greater Seattle area. Self-driving, cooperating vehicles are taken into consideration as an effective solution to alleviate pressure of highways. We construct several models to verify that by introducing a certain proportion of self-driving cars, the whole traffic system of a local region would be improved substantially. In the following parts, we demonstrate a brief analysis on how and to what extent do this state-of-the-art technique helps to do so. Moreover, we address some formulation of policies in terms of flow density, aiming to tackle the real-life traffic problem.

We conduct experiments and observe significant advantages of self-driving vehicles. Our model provides the baseline that allocation strategies of self-driving vehicles vary from peak hours to normal hours. Succinctly put, since the overall speed during peak time is relatively lower, having 80% of all vehicles be self-driving can apparently prevent congestions. On the other hand, during free time, the figure falls to 50% so as to maximize average speed of a section of a highway. With regard to the number of lanes, when car density is in a medium level, one specifical lane for self-driving vehicles is the most ideal. As the number increases, the allocation should decrease (50% for 3 lanes, 40% for 4 lanes and 30% for 5 lanes).

We study the map of highways around Thurston, Pierce, King, and Snohomish counties and look up some background information. We find that among Interstates 5, 90, 405 and Route 520, N-S highways like I-5 and I-405 tend to load heavy traffic. In contrast, roads that link the former two, i.e. E-W roads I-90 and R-520 suffer fewer traffic jams. According to our computation, the car densities of these four in peak hours are approximately equal, about 800 vehicles per mile. In normal hours, traffic on two E-W roads are much lighter. We implement our model upon the traffic data in year 2015 and formulate following policies for normal hours:

- For I-5 and I-405, one lane should be reserved for self-driving vehicles.
- For the intersection of I-5 and R-520, allocate 50% of self-driving vehicles to the north and 40% to the south.
- For the intersection of I-405 and I-90, allocate 50% of self-driving vehicles to the north and 70% to the south. Special pass is not recommended in the above two sections of roads.
- For I-90 and RS-520, Special pass is not recommended. Allocation proportion should be 50%.

The policy for peak hour is relatively simple:

- 80% of the total are SC cars. No special pass for SC, they are mixed with manual cars.

We believe that these policies recommendations represent the most efficient way to alleviate heavy traffic pressure. We hope that they will prove useful in practice to make the Greater Seattle Area a more desired place to travel by.

Yours sincerely,
Team #63205

Contents

Analysis of Traffic Flow With Mixed Manual and Self-driving Vehicles: in the Greater Seattle Area

January 24, 2017

1 Introduction

Self-driving cars, equipped with various sensors indicating the presence of obstacles and the road condition, have been put into use for some time. They can autonomously switch to the ideal mode of driving so that there is less randomness in traffic. Many cities in United States such as New York, Chicago have been trapped in traffic jams due to the limited lanes of roads and too large a number of vehicles on the road. A self-driving, cooperating system may alleviate traffic problems in those big cities. In this paper we analyze the specific difference and cooperation between self-driving and manual cars in the traffic flow and develop a cooperating system for several roads in the Greater Seattle area.

1.1 Restatement of the Problem

We are asked to help the Governor of the state of Washington to analyze the effects of self-driving, cooperating cars on Interstates 5, 90 and 405 and State Route 520, the roads of interest in Seattle area. We should build a model to satisfy the following requirements:

- Reveal the cooperation between self-driving cars and the interaction between self-driving and non-self-driving cars.
- Study the effect brought by self-driving cars under different conditions including number of lanes, density of vehicles and percentage of vehicles using self-driving systems.
- Apply the results of the model to the road of interest around Seattle, connecting the model to reality in the map.

1.2 Literature Review

We searched for works mainly on autonomous cars behavior and the simulation method. Gipps (1981) defined the safety distance based on Newton's second law and the reaction gap for drivers. It is useful in simulating a car-following model. Bose, A., & Ioannou, P. (2001) develop a model to analyze traffic flow with mixed manual and semi-automated vehicles from a micro level to a complicated high level. This is very inspiring for us to design our own model.

Many studies on traffic flow simulation used cellular automaton known as 'N-S' model, proposed by Nagel and Schreckenberg (1992). They described a one-lane road with cells looked as vehicles or gaps. Assigning velocity and acceleration to cells, they simulated car motion successfully.

1.3 Global Assumption

Assumptions are needed to limit and simplify our model. Some global assumptions are listed here. More specific ones are included in the concrete step.

- **Accesses to a highway are limited.** That is, there are only one entrance and one exit except for intersections within highways. Pedestrians are not allowed to go on a highway.
- **Highways are straight and flat, without traffic lights.**
- **Drivers all follows the traffic rule.** Nobody has a chance to exceed the speed limitation of 60 miles per hour. Drivers all drive within his or her own lane, so do the SCs.
- **Drivers only look ahead.** Drivers may make some judgement on road like stepping on the gas or the brake. They are assumed to neglect situations behind them.
- **No difference on the configuration of cars.** To simplify the model, we assume that the vehicles are all the same from the outlook to the acceleration system.

1.4 Model Overview

For convenience, self-driving, cooperating cars will be written as SC, and manual-driving cars will be written as MC in the following sections.

We first study the microscopic case, considering the interaction between two cars, which includes SC+MC, SC+SC and MC+MC cases. Then we construct a computer simulation that all vehicles are assumed to travel on a long straight road with only one lane. Through this simplified model we find out the basic relationship between several variables and the effect of SC. Next we extend our model to several lanes case to find whether there is a optimal policy for SC system. After assessing stability and sensitivity of our model, we apply it to the real-world problem, that is to put it into use with road information data.

2 Construction of the Model

2.1 List of Variables

Symbol	Definition
N	The total number of cars on a straight road
n	Counting number of one car
l_n	Length of the n^{th} car
a_n	Acceleration for the n^{th} car
d_n	Deceleration for the n^{th} car
d'_n	Randomly deceleration for the n^{th} car
q	Probability of a driver to slow down his or her speed
t	Counting time
Δt	Counting time interval
$v_n(t)$	Velocity for the n^{th} car at counting time t
v_{max}	Limited velocity on highway
$x_n(t)$	Shifted distance for the n^{th} car at counting time t
G_n^m	Safe gap for a MC, kept between itself and the fronting car
G_n^s	Safe gap for a SC, kept between itself and the fronting car
r^m	Response time for a MC in an emergency case
r^s	Response time for a SC in an emergency case
C	Index of the crowded degree
p	proportion of SC of the total batch of cars
ρ	Density of cars on a length-finite road
v_a	velocity of one trial
m	numbers of vehicles with velocity under a certain low speed

2.2 The Two-car Case

The model starts from single-lane driving mode, considering drivers' response time, acceleration, safety distance and other facts to find the similarity and difference between SC and MC. Safety distance plays an important role in highway driving, on both sides of drivers and traffic flow. A driver must keep an appropriate distance with the front car to prevent accident. On the other hand, distance of vehicles can reveal the traffic situation to some extent. Safety distance is helpful in our model to detect the effect of self-driving cars.

The definition of safety distance is the distance that keep a car from rear-ending when the front car brakes suddenly. It is related to the present velocity of both cars($v_n(t)$ and $v_{n+1}(t)$), the response time of driver(r_n) and the max deceleration(d_n). Let G_n^m and G_n^s denote the safety distance for SC and MC separately. Figure 1 is a demo as to compute them.

Through Newton's law of motion, we derive the formula for G_n as:

$$\begin{aligned} G_n &= x_{n+1}(t) - x_n(t) - l_{n+1} \\ &= v_n(t) \cdot r + \frac{v_n^2(t)}{2d_n} - \frac{v_{n+1}^2(t)}{2d_n} \end{aligned}$$

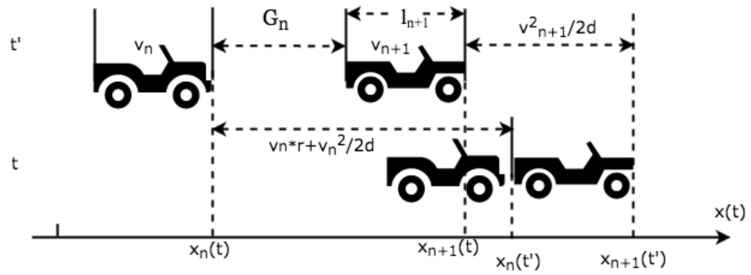


Figure 1: The diagram of safety distance

From the above derivation we know that the safety distance has positive correlation with velocity and response time. A significant characteristic of SC is that its response time is extremely little, so that the safety distance an SC needs can be shorter.

Another fact is that human-beings may have something on mind when they drive, which makes them slow down at a unpredictable pace. Artificial Intelligence(AI) is stable at this point of view. In the simulation step, we may add a randomly-slow-down fact to manual drivers. These are the two main differences between SC and MC when constructing our model, thus the two models are mostly the same except for several parameters and a random deceleration item.

Rules are concluded from above for an ideal vehicle on a single-lane highway:

- **The safety distance** The distance from a car's forefront to the front car's rear-end should be kept equal or greater than the calculated result.

$$G_n^m = v_n(t) \cdot r^m + \frac{v_n^2(t)}{2d_n} - \frac{v_{n+1}^2(t)}{2d_n}$$

proportion

$$G_n^s = v_n(t) \cdot r^s + \frac{v_n^2(t)}{2d_n} - \frac{v_{n+1}^2(t)}{2d_n}$$

- **Acceleration need** An ideal vehicle tends to reach the highest speed whenever it is possible. When a driver or a sensor on an SC detects that the front distance is larger than safety distance($x_{n+1}(t) - x_n(t) > G_n$), it will accelerate until it reaches speed limit or the safety velocity. That is:

$$v_n(t+1) = min\{v_n(t) + a_n, v_{max}, \frac{x_{n+1}(t) - x_n(t)}{\Delta t}\}$$

- **Deceleration rule** When the actual distance is less than safety distance, it is necessary to decelerate. That is:

$$v_n(t+1) = max\{min[v_n(t), \frac{x_{n+1}(t) - x_n(t)}{\Delta t}], 0\}$$

- **Randomly slow down(only for manual drivers)**Uncertainty is added to each manual driver, that is one may slow done at deceleration d_n' with probability q at any moment.

$$v_n(t+1) = max\{v_n - d_n', 0\}$$

2.3 Single-lane Simulation

2.3.1 Simulation Design

A straight single-lane road of 5km is designed for test. The road is composed of 1,000 cells. Each cell is of 5 meters, which is set to be the length of each car(l_n). The counting time interval(Δt) is 1 second, and the situation is updated each second. Now we can change every variable to the unit of cell. v_{max} is around 60 miles, that is around 19 cells per second, written as 19c for convenience.

We 'stick' the road's entrance and exit so that the model works under periodical condition. Thus we can easily determine the traffic volume by setting the initial cars density and initial velocity of them. Cars density can be calculated by $\rho = N/1000$.

Further assumption More assumptions in detail are necessary for the specific model.

- Manual drivers take the same behavior against emergency. The response time of human-beings is 1.5s and for autonomous driver, 0.1s.
- Except for the randomly slowing down phenomenon, both SC and MC drivers tend to drive at the highest speed within safe range.

Evaluation method There are two ways to evaluate the traffic flow, to be used in studying the effect of SC.

- **Congestion Rate** In such a closed system, cars will pile up due to random deceleration and following style of them, displayed in Figure 2.

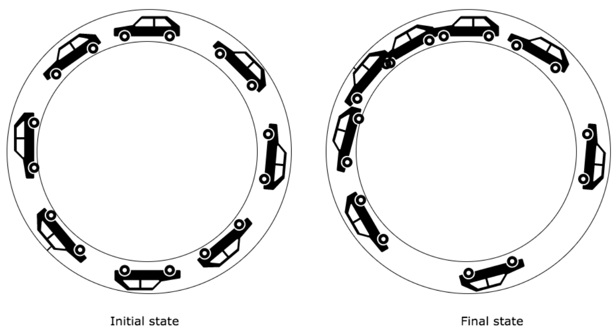
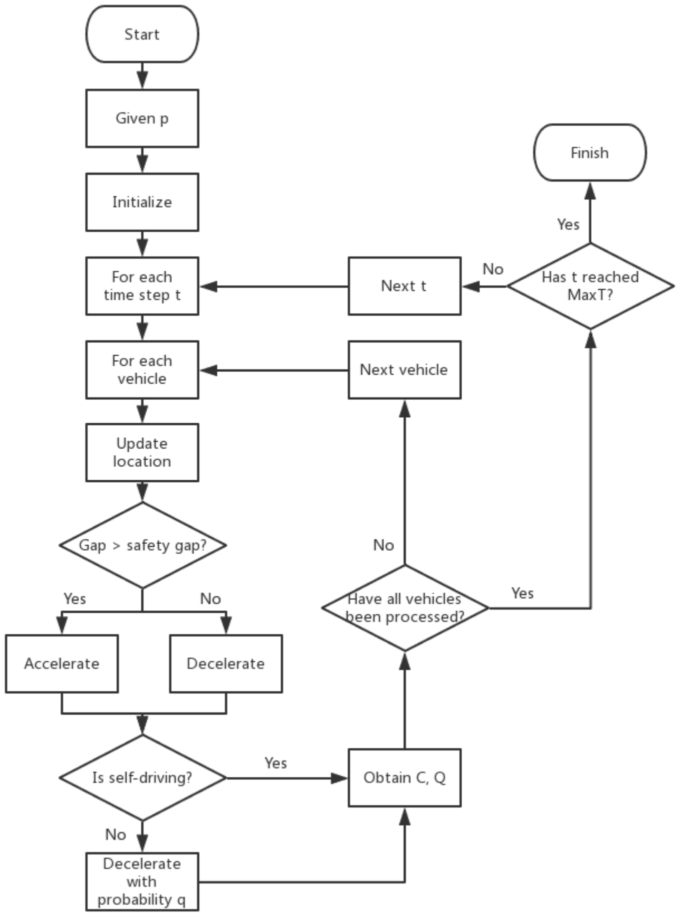


Figure 2: The diagram of a possible procedure in the periodic system

  We define m=numbers of vehicles with velocity under 1c and the crowded degree coefficient to be $C = \frac{m}{N}$, indicating how many cars are trapped in a traffic jam.

- **Quantity of flow** Our model takes 10,000 intervals in one trial and chooses the last 2,000 states because they may represent a relatively stable state. We use the average velocity among all the cars on road and the last 2,000 intervals to compute an average velocity of one trial v_a. The quantity of flow is defined as $Q = v_a \cdot \rho$

An illustration of the model is shown below.



2.3.2 Model Results

We first use the Congestion Rate(C) to indicate the state of the road. Since every moment has a present state, we use the average Congestion Rate of the last 2,000 times to reduce deviation. To see the relationship between parameters like proportion of SC clearly, initial density, turn to Figure 3 and 4.

Figure 3 shows the trend of C with respect to the initial car density ρ at every proportion p. As the initial total number of cars increase, the Congestion Rate goes up monotonously. For a fixed number of cars on road, the crowded is smaller as the proportion of SC increases. For instance, when there are totally 300 cars on a 5km-long road, the Congestion Rate is over 90% without usage of self-driving cars. Half usage of SC can reduce crowded cars to around 63% and 90% usage of SC can reduce the index to less than 50%. This is clearer in Figure 4. In one experiment, initial car density is set to be 300 of 1000 cells, and the final state w.r.t p is like a down-hill road. The result goes down quickly when $p < 0.4$ and $p > 0.8$. Now turn back to Figure 3. There is another fact to notice. It can be seen that the increasing trend is almost linear when $p > 0.5$, but not obvious when $p < 0.3$. This is because when the car density is over 30%, the randomness in the 90% manual drivers can cause a traffic jam that make the crowded rate more than 0.9. However more autonomous vehicles cannot change the fact.

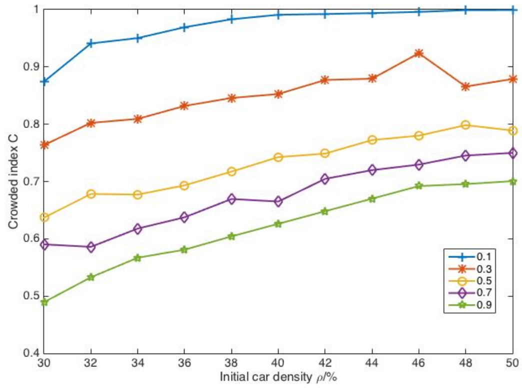


Figure 3: Congestion Rate under different initial density and proportion of SC

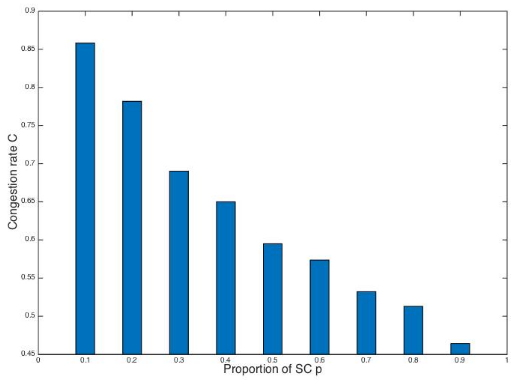


Figure 4: Congestion Rate under same initial density, with respect to the proportion of SC

Now we use the second evaluation method, flow rate of the road (Q), to see if there is other interesting facts. See Figure 5 and Figure 6 for details.

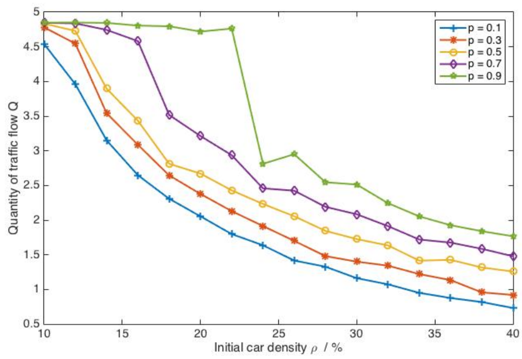


Figure 5: Quantity of traffic flow under different initial density and proportion of SC

The conclusion is similar to the Congestion Rate. That is as the proportion of SC goes up, the average velocity on the road rises, leading to rise in quantity of flow. From Figure 5 we found that 90% SC in use can double 10% SC in use when initial car density is 30%.

Note that there's a changing point in the plot for $p = 0.7$ and $p = 0.9$, and it is more remarkable

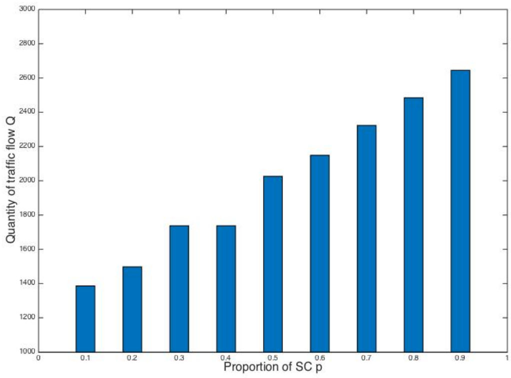


Figure 6: Quantity of traffic flow under same initial density, with respect to the proportion of SC

for $p = 0.9$. Result can be concluded as: for $p = 0.9$, the increase on traffic flow is significantly large when traffic is not so crowded ($\rho < 25\%$), getting plain when ρ is getting greater.

2.4 Multi-lane Model

Next we want to exploit the influence of the number of lanes and make a policy for periods in a day, like drawing a lane dedicated to SC.

2.4.1 Model Design

The aim of the system The two-lane freeway is designed to maximize the quantity of traffic flow. There are two policies for the system. Case 1 is that the two lanes are open for both SC and MC, and in case 2 SC and MC are separated.

Variables There are three variables in this model that determine the effect of SC proportions: number of lanes, initial car density and SC proportion. Another decisive condition is that whether a lane is dedicated to SC or not. We run the model in Matlab to see if there is an equilibrium point in those variables. The following deduction tells why there can exist an equilibria.

Deduction Suppose a two-lane freeway, which we divide into SC Lane and MC Lane. If we assign 10% of the total cars to be self-driving cars, of course the SC Lane will moving fast. However, the MC Lane will be bothered with more serious traffic jams since the car density is 1.8 times than the undivided road.

Modification Lane changing is not considered in the single-lane model. Therefore we must modify the model a little for lane changing. A car needs to change its lane when the front car is too slow and the front gap is too small to accelerate.

Further assumption To simplify the optimization problem, we temporarily construct a multi-lane freeway, without consideration of in/out of ramps.

Evaluation method Since the quantity of traffic flow (Q) is positively correlated with the average velocity(v_a), we use v_a weighted by proportion in every lane to evaluate the fluency of the multi-lane system.

2.4.2 Model Results

We firstly consider the policy that only one road may be dedicated to SC under some circumstances. Though experiments with different variables setting, we get different shape of line charts with respect to car density and the dividing policy.

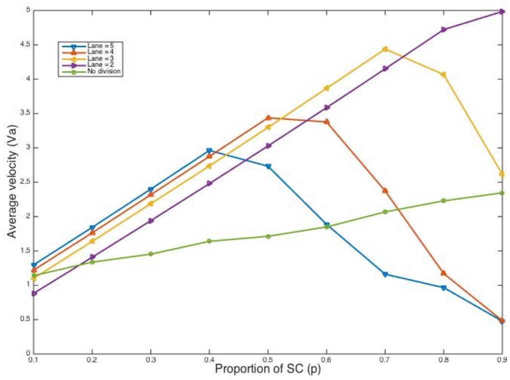


Figure 7: Average velocity varying with proportion p, under different numbers of lanes when the initial density is 30%

A baseline is drawn in green to show the improvement of dividing policy as the two types of cars mix on several lanes. From Figure 7 we can tell that as the proportion of SC goes up, the traffic flow may get slower than mixed lanes. Another important information is the tipping point for $Lane = 3, 4, 5$. This is the best proportion for the specific numbers of lanes. For instance, 50% of the total vehicles being SC, driving on an assigned lane can be the best scheme for a four-lane road when car density is 30%.

We want to find out the best solution for different numbers of lanes and different traffic states. Mini-plots are displayed below to show more results.

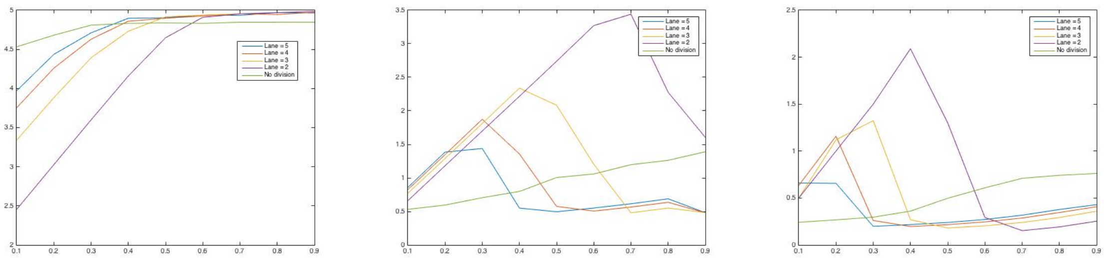


Figure 8: Average velocity varying with proportion p, under different numbers of lanes when the initial density $\rho = 100, 500, 800$

Additionally, we should take into account whether the proportion can work. An impossible case is that the number of cars on a road exceeds 1,000.

The discussion for dividing two lanes to SC is similar to sinlge-lane case.

We need to make a specific policy on various types of road and every hour in a day. We use the results collected above to reach the optimal solution. Here we define four types of traffic state, using car density as a feature, listed in Table 1.

Traffic state	Number of Lanes	Policy (proportion of SC)	Policy (number of lanes of SC)
Light ($0 < \rho < 10\%$)	2,3,4,5	50%	0
Medium ($10\% < \rho < 50\%$)	2	90%	1
	3	70%	1
	4	50%	1
	5	40%	1
Heavy ($50\% < \rho < 80\%$)	2	70%	1
	3	50%	1
	4	60%	2
	5	30%	1
Congestion ($80\% < \rho < 100\%$)	2	40%	1
	3	80%	0
	4	80%	0
	5	80%	0

Table 1: Policy on four traffic states

Comment: Extra policy on division does not make a lot of difference when traffic is very light or any car can barely move. When there's no significant improvement as the proportion of SC increases, we can just choose the smallest proportion number.

3 Model Application in the Greater Seattle Area

3.1 Background and Data

Background knowledge We apply the model to Route 5, 90, 405, 520 around Seattle to help alleviate the heavy traffic in those areas. Interstate 5, length 1,381 miles, is one of the most important interstates in America, connecting Mexico and Canada. We just care about the $100 \sim 218$ milepost of it, within Seattle Area. Interstate 405 is a bypass of Interstate 5 and is regarded as the busiest freeway in America. It starts from El Toro and ends with San Fernando, both intersect with Route 5. Interstate 90 and State Route 520 run across the area surrounded by I-5 and I-405.

Further assumption Situation is more complicated in reality than in the model, so we make some reasonable assumptions for the routes.

- **Peak hours** One of twenty-four hours in a day is defined as peak hour when the road is the most congested. 8% of the daily traffic volume occurs during peak hours.
- **Speed limit** The nominal speed limit for all these roads is 60 miles per hour.
- **Access limit** For both Interstate and State Route, accesses are limited. That is, only the four roads can intersect with each other.
- **Lanes condition** The data set provides us with a Lane width of 12 feet and varying numbers of lanes. Traffic laws allow vehicles on emergency hard shoulder at a low speed when traffic is very busy. Therefore we add a lane to roads during peak hours.

- **Traffic volume** Except for peak hour, traffic volume is the same in every hour. Since there is only a total count of traffic volume, we suppose that the volume of two direction on a road are equal.

Data analysis We have a dataset of the four roads covering number of Lanes for both directions and the average daily traffic volume. The task is to use the volume to estimate the density so that we can fit the actual traffic state into policies Table 1. Simple calculation is done to the original dataset by Excel. The known and unknown terms are listed below.

	Term (for each lane)
Known	The length of each section
	Average daily traffic
	Average peak flow rate
	Average Normal flow rate
Unknown	The density of peak hours
	The density of normal hours

3.2 Computing the Road Density

Greenshields Model In 1935, Greenshields[3] proposed a linear model, using density to estimate the average speed of cars.

$$v = v_f(1 - \frac{\rho}{\rho_c}),$$

where v_f denotes the velocity under free condition and ρ_c denotes the congestion density. We have already deducted the correlation of Q, v and ρ in the form $Q = \rho v$. Then the correlation of Q and ρ is that

$$Q = \rho v_f(1 - \frac{\rho}{\rho_c}).$$

Thus ρ can be solved by a quadratic equation in which coefficients are Q, ρ_c and v_f.

There are two positive roots for the equation. Its physical meaning is that a certain quantity

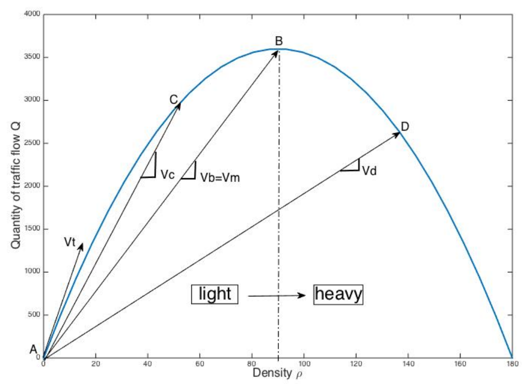


Figure 9: Illustration for quadratic equation of Q and ρ. The physical meaning of tangent line is the present velocity.

of traffic flow may has two possible states. One is for heavy traffic and one is for light traffic. For instance, if Q is very small. There may have been a congestion on road and the cars can hardly move. On the other and, the density can be so small that the period of time is very long between two cars' emergence.

3.3 Specific Policy for SC

After solving the car density, we can fit the roads of interest into the model evaluation and give the specific policy for self-driving cooperating cars.

In peak hour, the density is over 800 vehicles per kilometers ($\rho > 80\%$) for all of the roads. Situation may vary at other time in a day. For the busiest road I-405 and some sections of I-5, the density is over 10%. For less traveled road, the density is around 5%. Detailed computed results is shown in Figure 10.

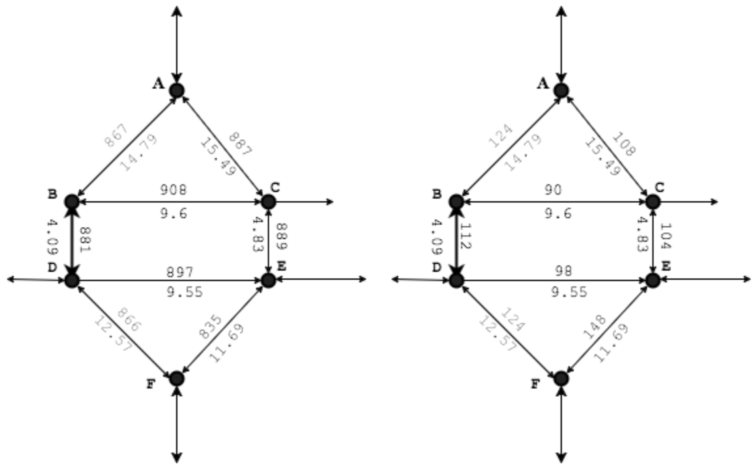


Figure 10: Car density and road length diagram. The 6 points denotes the intersection position of the four roads.The number below the segment stands for the length of the road. The number over the arrow indicates how many cars there are per kilometer. Picture on the left shows the peak hours and picture on the right shows the normal hours.

Correspondingly, we develop a specific policy for those sections of route, listed in Table 2.

Route No.	Segment	Traffic state	Proportion of SC	Number of lanes of SC
5	AB, BD, DF	Peak	80%	0
	AB	Normal	50%	1
	BD, DF	Normal	40%	1
90	DE	Peak	80%	0
	DE	Normal	50%	0
405	AC, CE, EF	Peak	80%	0
	AC, CE	Normal	50%	1
	EF	Normal	70%	1
520	BC	Peak	80%	0
	BC	Normal	50%	0

Table 2: Specific policies on the road of interest

4 Analysis of the Model

4.1 Convergence

We wait for 10,000 time step to let the simulation get to a stable final state. To monitor convergence, we plot curves on $C - t$ and $v_a - t$. The two indexes float within a certain range after 1,000 time steps. So the result we obtain is a converged one.

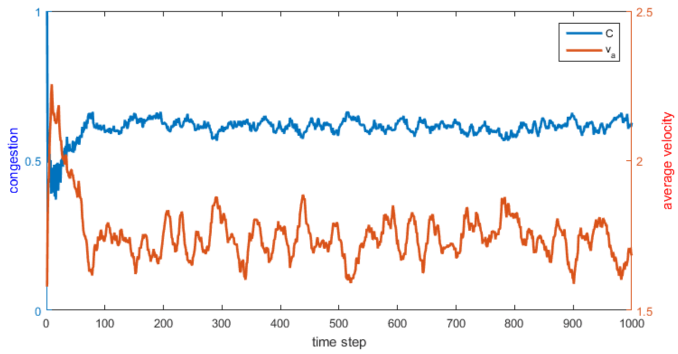


Figure 11: Congestion rate-Time step

4.2 Sensitivity and Stability

Initial Speed When we simulate the traffic on a freeway, the initial speed is set at a medium level. To validate the stability of the simulation, we try initial speed from 0 to 2c to see if the final state is all the same. The fluctuation is within 3.7%, which implies that the model is not changed by the initial speed setting.

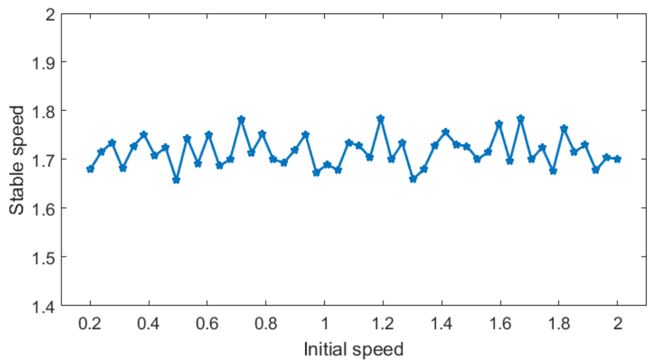


Figure 12: Final speed-Initial speed

Deceleration of randomly slowing down We add 10% disturbance to the deceleration and the congestion. Either the average velocity or the congestion rate changes a little. The deviation is less than 5%.

Randomly slowing down rate Again, 10% deviation is added to the randomly slowing down rate. The deviation of two evaluation indexes reports little dependence on the slowing down rate.

5 Evaluation of the Model

Strengths

- The simplified model is based on the safety distance and makes sure that safety goes first, which is most important in traffic system.
- It efficiently simulates different road situations through Matlab programming, and gets a regulation with respect to the parameters of interest.
- It gives a reasonable answer to the congestion problem in the Greater Seattle Area. Road data are carefully used and well-embedded in the model so the validation step is naturally taken.
- Sensitivity analysis shows that the model is robust with disturbance and does not depend largely on a certain one of the parameters.

Weakness

- Maybe we can come up with a new evaluation method besides congestion rate and quantity of traffic flow. The current evaluation index is simple and maybe not so adequate.
- The data process step can be improved. The distribution of density (ρ) is too dense and that reduce individuality in roads. More meticulous work can be done to the dataset and make it more reliable, with models other than Greenshields'.

Further Study

- Consider more actual conditions: types of cars and drivers' behavior may differ from person to person. A more detailed discussion on over-taking can be done to the model.
- Add the network of self-driving, cooperating car to consideration. Driving mode for a car does not depend on the front car, but all of the cars in the big area. A self-driving car can get the road information by obtaining the other SC's position and speed. Thus, it can make a optimal route selection.
- Dynamic plan can be made. The policy regulates the proportion of SC and MC, neglecting the flow of cars. Further study can learn the pattern of a manual driver or an autonomous driver when they face such a policy. How would they choose and what would be the consequence of such selection remains to be discussed.

6 Conclusion

Review of the model

- We first discuss a two-car situation, which is a car following model based on safety distance.
- Next we used the thought of cellular automaton in simulating a one-dimensional traffic system to study the traffic flow and proportion of SC.

- The single-lane model is then extended to a multi-lane model. We obtain equilibrium point and tipping point to decide which is the best solution to congestion.
- We used the given data and map to calculate related variables. They were fit in the model and used for a specific policy.
- Disturbance and switching initialization are done to test the robustness of our model. We found the convergence step, stability and sensitivity, and the most appropriate parameters setting.

Conclusion of results

- From the single-lane model we can see an increase trend of traffic capacity as proportion of SC goes up.
- Studying the tipping point in multi-lane model, we made a set of policies for different time period and different number of lanes. The policies are listed in Table 1.
- Apply the model on the data, we decide a specific policy for roads of interest in the Greater Seattle Area, listed in Table 2.
- After the sensitivity analysis and the application in real problem, we find that our model is a robust and adaptive model.

References

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Appendices

Appendix A Algorithm

The algorithm for simulation, single-lane and multi-lane.

Algorithm 1 Stimulating procedure

```
Initialize the mode. Set initial velocity, car density and other parameters.
for t = 1 to 10000 do
  Update location: x_n = x_n + v_n
  if new gap is greater than safety distance then
    Update v_n = v_n + a_n
  else
    Update v_n = v_n - d_n
  end if
  if Bernoulli(q) event happens for MC then
    Update v_n = v_n - d'_n
  end if
  Obtain the Congestion Rate (C) and calculate the quantity of flow (Q).
end for
```

Algorithm 2 Double lane situation

```
for every time step t do
  if mod (t, 2) == 1 then
    for every vehicle in the right lane do
      if lane changing condition satisfied then
        Switch the vehicle to the left lane
        Update gaps, speeds in both lanes
      end if
    end for
  else
    for every vehicle in the left lane do
      if lane changing condition satisfied then
        Switch the vehicle to the right lane
        Update gaps, speeds in both lanes
      end if
    end for
  end if
end for
```