

# DATA130004: Homework 1

Due in class on September 26, 2018

1. Suppose that  $X$  and  $Y$  are continuous random variables with density  $f(\cdot)$  and  $g(\cdot)$  respectively, and  $a$  and  $b$  are constants. Prove the following arguments
  - (a)  $E(aX + b) = aE(X) + b$ .
  - (b)  $E(X + Y) = E(X) + E(Y)$ .
  - (c) If  $X$  and  $Y$  are independent, then  $E(XY) = E(X)E(Y)$ .
  - (d)  $Var(b) = 0$ .
  - (e)  $Var(aX + b) = a^2Var(X)$ .
  - (f)  $Var(X + Y) = Var(X) + Var(Y) + 2Cov(X, Y)$ .
  - (g) If  $X$  and  $Y$  are independent, then  $Var(X + Y) = Var(X) + Var(Y)$ .
2. Show that the correlation coefficient of two random variables  $X$  and  $Y$ , denoted by  $\rho_{XY}$ , is bounded within  $[-1, 1]$ . (Hint: use Cauchy-Schwarz inequality.)
3. Find a linear transformation for  $X \sim \text{Unif}(a, b)$  such that the transformed random variable follows  $\text{Unif}(0, 1)$ .