DATA130004: Homework 8

Due in class on December 21, 2017

1. Exercises 7.1, 7.3.

2. Consider a p-dimensional normal distribution $X = (Y, Z)^\top$ with two partitions $Y \in \mathbb{R}^q, Z \in \mathbb{R}^{p-q}, 0 < q < p$. Correspondingly, the mean of X is $\mu = (\mu_Y, \mu_Z)^\top$ and the covariance of X is
$$\Sigma = \begin{pmatrix} \Sigma_{YY} & \Sigma_{YZ} \\ \Sigma_{ZY} & \Sigma_{ZZ} \end{pmatrix}.$$

   (a) Now derive the conditional distribution of Z given Y.
   Hint: make a non-singular transformation AX where
$$A = \begin{pmatrix} I_q & 0 \\ -\Sigma_{ZY}\Sigma_{YY}^{-1} & I_{p-q} \end{pmatrix}.$$

   (b) Restate the result when assuming $q = 1$, i.e., $Z|Y$ is a random variable conditioning with a $p-1$ dimensional random vector. This result is useful in next lecture.