DATA130013: Homework 1

Due in class on March 22, 2018

1. Shumway's book (4th ed.) Problems 1.2, 1.5, 1.8, and 1.25(a).

2. Prove the following properties of $\rho(h)$, the ACF of a stationary time series process,

   (a) $\rho(h) = \rho(-h)$

   (b) $-1 \leq \rho(h) \leq -1$, when do $\rho(h) = 1$ and $\rho(h) = -1$ hold?

3. The correlation coefficient we discussed is called Pearson's product-moment coefficient. Given two random variables X, Y, it is defined as

$$\text{corr}(X, Y) = \frac{\text{cov}(X, Y)}{\sqrt{\text{var}(X)\text{var}(X)}}.$$

It is claimed that such correlation only measures the *linear* dependence between random variables. Verify the following example by calculating $\text{corr}(X, Y)$. Let $X \sim \text{Unif}(-1, 1)$ be a uniform distribution on $(-1, 1)$ and $Z \sim \text{Unif}(0, \frac{1}{10})$. Assume X and Z are independent. Let $Y = X^2 + Z$.

   (a) Use the conditional structure Y given X, write down the joint density function of X, Y, say $f(x, y)$.

   (b) Display the region where $f(x, y) > 0$, which is called the support of $f(x, y)$. Describe the dependence structure between X and Y.

   (c) Calculate $\text{corr}(X, Y)$.