DATA130013: Homework 8

Due in class on June 1, 2017

1. Shumway's book Problems 4.6, 4.8 and 4.9. (Read Example 4.10 first.)

2. Recall that the spectral density of X_t is defined as

$$f(\omega) = \sum_{h=-\infty}^{\infty} \gamma(h) e^{-2\pi i \omega h}, \qquad \omega \in [-\frac{1}{2}, \frac{1}{2}],$$

where the autocovariance function $\gamma(h)$ satisfying absolute summability. Verify the following properties of $f(\omega)$:

(a) $f(\omega) \geq 0$.

(b) $f(\omega)$ is even: $f(\omega) = f(-\omega)$.

(c) $f(\omega) = f(1-\omega)$. (Check its periodicity.)

(d) For all integers h,

$$\gamma(h) = \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{2\pi i \omega h} f(\omega)\, \mathrm{d}\omega.$$