DATA130013: Homework 9

Due in class on June 8, 2017

1. Given a partition of a d-dimensional Gaussian distribution $X = (X_1, X_2)^\top$. Correspondingly, the mean and covariance of X are specified as

$$\mu = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}, \qquad \Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}.$$

   Prove that the conditional distribution of X_1 given $X_2 = x_2$ is still Gaussian. Derive its conditional mean and covariance.

2. Consider a univariate input space, a closed interval $[-5, 5]$. Generate and plot five independent draws (each draw is a function!) from a Gaussian process with zero mean function and covariance function

$$K(s_1, s_2) = \exp\left\{ -\frac{1}{2}(s_1 - s_2)^2 \right\}.$$

   (Hint: The specification of the covariance function implies a distribution over functions. To see this, we can draw samples from the distribution of functions evaluated at any number of points; in detail, we choose a number of input points, and write out the corresponding covariance matrix using $K(\cdot, \cdot)$ elementwise. Then we generate a random Gaussian vector with this covariance matrix.)

   Compare the trajectories of the random draws when we change the covariance function to be

$$K(s_1, s_2) = \exp\left\{ -\frac{1}{2}|s_1 - s_2| \right\}.$$

   Comment on the characteristics of the two covariance functions.