

DATA130004: Homework 9

Due in class on December 28, 2017

1. Exercises 9.1, 9.2, 9.3, 9.7.
2. In Bayesian inference, if the posterior distributions $p(\theta|x)$ are in the same family as the prior probability distribution $\pi(\theta)$, the prior and posterior are then called conjugate distributions, and the prior is called a conjugate prior for the likelihood function. Let $x = (x_1, \dots, x_n)^\top$ consist of n i.i.d. random samples. Prove the following well-known conjugate results by specifying the posterior distribution type and parameters.
 - (a) When the variance parameter is known, the Gaussian family is conjugate to itself (or self-conjugate) with respect to a Gaussian likelihood function. That is, if the likelihood function is Gaussian, choosing a Gaussian prior over the mean will ensure that the posterior distribution is also Gaussian.
 - (b) Gamma distribution is the conjugate prior for the rate parameter of Poisson distribution. That is, given data are generated according to $\text{Poisson}(\lambda)$, specify a $\text{Gamma}(\alpha, \beta)$ prior for λ , then the posterior distribution $p(\lambda|x)$ is still Gamma.