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Electrical modelling of homogeneous dielectric barrier discharges under an arbitrary excitation voltage

Shuhai Liu and Manfred Neiger

Lighting Technology Institute (LTI), University of Karlsruhe, Kaiserstrasse 12, D-76128 Karlsruhe, Germany

E-mail: liu_shuhai@hotmail.com and manfred.neiger@etec.uni-karlsruhe.de

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Abstract
In order to quantitatively describe the electrical working principles of dielectric barrier discharges (DBDs), a dynamic electrical model for homogeneous DBDs has been put forward that is composed of a new equivalent circuit for homogeneous DBDs and the equations derived from it. This model is a global and self-consistent model, valid for an arbitrary external excitation voltage. This model reveals instantaneous relations of internal electrical quantities in the gap (gap voltage, internal discharge current and internal power consumption process) to external electrical quantities (external voltage and external total current) and provides the theoretical fundamentals to calculate the temporal development processes of all internal electrical quantities in the discharge gap from the measured external voltage and external total current. The knowledge obtained of dynamic processes of DBDs in the discharge gap explains quantitatively the mechanisms that result in ignition, development and extinction of DBDs and provide physical interpretation of the measured external total current and other phenomena such as memory effect and multiple current pulses in one half period. In this model, several current terms (external total current, external displacement current, external discharge current, internal discharge current and internal displacement current) are introduced to distinguish the different currents involved in DBDs. Moreover, the equations for charge and energy deposition by one discharge and in one half period are derived. Applications of this model to studying a bipolar sine wave excited DBD and a unipolar pulse excited DBD are also included. This model has been proved to be a useful tool to understand DBDs better.

1. Introduction

Dielectric barrier discharges (DBDs) occur usually in filamentary form, i.e. DBDs are created as a number of individual breakdown channels (microdischarges) with very short time duration from several nanoseconds to several microseconds. Accumulation of charges on the dielectric surface is considered to reduce the electric field in the gap and finally to quench the discharge [1]. Filamentary DBDs exhibit a collective behaviour, i.e. individual microdischarges occur discretely in space and time. Increasing the excitation voltage amplitude creates additional microdischarges at new locations. In order to understand the mechanisms that lead to ignition and quenching of DBDs, as well as the other observed phenomena such as the memory effect, self-erasing effect, stationary discharge patterns and double discharges in unipolar DBDs, a knowledge about the dynamic processes of voltage and current in the gap is necessary. However, access to the electrical quantities in the gap has been prevented by the filamentary discharge form and the complicated collective phenomenon of microdischarges.

Under special conditions DBDs can also take place in homogeneous form, depending not only on the gas type and gas pressure, but also on the surface characteristics of dielectric materials. Homogeneous DBDs at atmospheric pressure have been observed in nitrogen, helium and neon [2–9, 11]. In such simplified situations, it can be assumed that free charges are deposited uniformly on the dielectric and thus the gap voltage becomes independent of location. Under this assumption the otherwise inaccessible dynamic process in the gap becomes accessible. A formula for calculation of the temporal development of the gap voltage was given in [3], but this formula is valid only for a symmetric bipolar ac excitation voltage. Moreover, no equation for calculation of other internal electric quantities (internal discharge current and internal power consumption) was provided. The electrical model for DBDs, which allows us to determine simultaneously all the electrical quantities in the gap for an arbitrary external excitation voltage, was put forward by the author [11] and has been partly published in [10, 16]. In this paper, the complete model that was introduced in [11] will be presented and discussed in detail.

2. Equivalent circuit for homogeneous DBDs

For a homogeneous DBD, strictly speaking, homogeneous vertical to the electrical field direction, the following equivalent circuit is introduced. The DBD is assumed to be of parallel plate type, with one electrode covered by a dielectric (see figure 1(*a*)). The equivalent circuit of this configuration is shown in figure 1(*b*), with C_g and C_d representing the equivalent capacitances of gap and dielectric layer, respectively. In the case of double barrier layers on both electrodes, C_d is the total series capacitance of both dielectric layers. The discharge plasma is not represented by a temporally variable resistance as, e.g. in [12–14], but instead by a current source, $I_{p,g}(t)$, representing the conduction discharge current in the gas gap. $I_{p,g}(t)$ is a voltage-controlled current source, i.e. $I_{p,g}(t)$ is a function of $U_g(t)$. By using $I_{p,g}(t)$ instead of $R(t)$, the unknown discharge-dependent non-linear variable $R(t)$ is avoided. The basic idea is that no matter how the internal discharge current $I_{p,g}(t)$ changes there must be a corresponding current change in the external current $I_{t,a}(t)$, which can be measured directly. It is mathematically much simpler to set up a relationship between $I_{p,g}(t)$ and $I_{p,a}(t)$ than between $I_{p,a}(t)$ and $R(t)$. The introduction of $I_{p,g}(t)$ instead of $R(t)$ avoids also the condition that the gap voltage, $U_g(t)$, must be zero after the discharge according to figure 1(*b*). Contrary to the usual treatment that the capacitance C_g disappears or changes with time due to gas breakdown, C_g is assumed here to be constant. The external excitation voltage, $U_a(t)$, can be of arbitrary time-dependent waveform. In this model, loss processes of memory charges are obviously not taken into consideration.

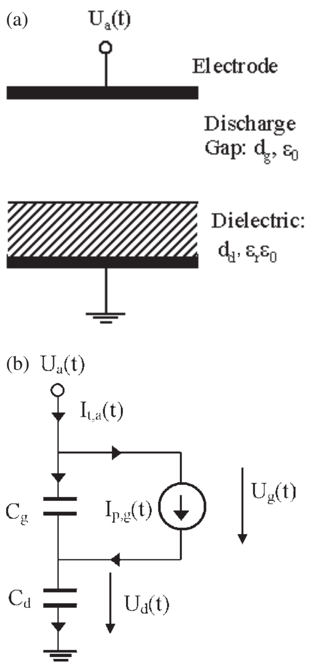

Figure 1. (*a*) The DBD electrode configuration and (*b*) the corresponding equivalent circuit.

3. Derivation of equations for internal electrical quantities in the discharge gap

According to Kirchhoff's theorem, we obtain from figure 1(*b*):

$$\frac{\mathrm{d}U_d(t)}{\mathrm{d}t} = \frac{I_{t,a}(t)}{C_d}, \tag{1}$$

$$\frac{\mathrm{d}U_g(t)}{\mathrm{d}t} = \frac{1}{C_g} I_{v,g}(t) = \frac{1}{C_g}(I_{t,a}(t) - I_{p,g}(t)), \tag{2}$$

$$U_g(t) + U_d(t) = U_a(t), \tag{3}$$

where $U_a(t)$ is the external excitation voltage applied to the DBD cell; $U_d(t)$ the voltage across the dielectric; $U_g(t)$ the voltage across the discharge gap; $I_{t,a}(t)$ the total external current through the DBD cell; $I_{p,g}(t)$ the discharge current in the gap and $I_{v,g}(t)$ the displacement current in the gap. C_d and C_g are equivalent capacitances of the dielectric barrier and the discharge gap, respectively. They are known for a certain electrode configuration and can be calculated by the following equations:

$$C_d = \varepsilon_0 \varepsilon_r \frac{A}{d_d},$$

$$C_g = \varepsilon_0 \frac{A}{d_g},$$

where d_d is the thickness of the dielectric barrier; d_g the thickness of the discharge gap; A the overall discharge area; ε_0 the permittivity of the gas gap and ε_r the dielectric constant of the used dielectric barrier material.

Differentiating equation (3) with respect to time and substituting equations (1) and (2) into equation (3), we obtain:

$$\frac{1}{C_d} I_{t,a}(t) + \frac{1}{C_g}(I_{t,a}(t) - I_{p,g}(t)) = \frac{\mathrm{d}U_a(t)}{\mathrm{d}t}.$$

Rearranging, we obtain an expression for the external current, $I_{t,a}(t)$, which can be measured easily:

$$\begin{aligned} I_{t,a}(t) &= C_g \frac{\mathrm{d}U_a(t)/\mathrm{d}t}{1 + C_g/C_d} + \frac{I_{p,g}(t)}{1 + C_g/C_d} \\ &= \frac{C_{DBD}\,\mathrm{d}U_a(t)}{\mathrm{d}t} + \frac{I_{p,g}(t)}{1 + C_g/C_d} = I_{v,a}(t) + I_{p,a}(t), \end{aligned} \tag{4}$$

where

$$I_{v,a}(t) = \frac{C_{DBD}\,\mathrm{d}U_a(t)}{\mathrm{d}t}, \tag{5}$$

$$I_{p,a}(t) = \frac{I_{p,g}(t)}{1 + C_g/C_d} \tag{6}$$

and $C_{\mathrm{DBD}} = C_{\mathrm{g}}/(1 + C_{\mathrm{g}}/C_{\mathrm{d}})$, i.e. the total capacitance of the DBD configuration. From equation (4) the measured total external current, $I_{\mathrm{t,a}}(t)$, is composed of two terms that have different physical mechanisms. The first term, $I_{\mathrm{v,a}}(t)$, is the external displacement current, i.e. the charging and discharging current of the total capacitance of the DBD configuration due to temporal changing of $U_{\mathrm{a}}(t)$. The second term, $I_{\mathrm{p,a}}(t)$, is the response current in the external circuit if a conduction discharge current, $I_{\mathrm{p,g}}(t)$, flows in the gap. The measured total external current, $I_{\mathrm{t,a}}(t)$, is the sum of the external displacement current, $I_{\mathrm{v,a}}(t)$, and the external discharge current $I_{\mathrm{p,a}}(t)$. For this reason it is not correct to interpret the measured external total current as the discharge current. Because $I_{\mathrm{v,a}}(t)$ is known at a given external voltage waveform, $I_{\mathrm{p,a}}(t)$ is easily calculated from the measured external current, $I_{\mathrm{t,a}}(t)$, using equation (4).

On the other hand, we know from figure 1(*b*):

$$I_{\mathrm{t,a}}(t) = I_{\mathrm{v,g}}(t) + I_{\mathrm{p,g}}(t). \tag{7}$$

Substituting equation (4) into equation (7), we obtain

$$I_{\mathrm{t,a}}(t) = I_{\mathrm{v,g}}(t) + I_{\mathrm{p,g}}(t) = I_{\mathrm{v,a}}(t) + I_{\mathrm{p,a}}(t); \tag{8}$$

the external total current is also equal to the sum of the internal displacement current, $I_{\mathrm{v,g}}(t)$, and the internal conduction discharge current, $I_{\mathrm{p,g}}(t)$. The external response currents of $I_{\mathrm{v,g}}(t)$ and $I_{\mathrm{p,g}}(t)$ are $I_{\mathrm{v,a}}(t)$ and $I_{\mathrm{p,a}}(t)$, respectively. $I_{\mathrm{v,a}}(t)$ and $I_{\mathrm{v,g}}(t)$ are capacitive displacement currents, whereas $I_{\mathrm{p,a}}(t)$ and $I_{\mathrm{p,g}}(t)$ are physically conduction currents due to discharges.

From equation (4) we can calculate the true discharge current, $I_{\mathrm{p,g}}(t)$, in the discharge gap from the measured external current, $I_{\mathrm{t,a}}(t)$:

$$I_{\mathrm{p,g}}(t) = \left(1 + \frac{C_{\mathrm{g}}}{C_{\mathrm{d}}}\right) I_{\mathrm{t,a}}(t) - C_{\mathrm{g}}\frac{\mathrm{d}U_{\mathrm{a}}(t)}{\mathrm{d}t}. \tag{9}$$

It is obviously a mistake to interpret the measured current, $I_{\mathrm{t,a}}(t)$, as the discharge current even if the displacement current is small enough to be ignored. The conduction discharge current, $I_{\mathrm{p,g}}(t)$, is always greater than the external discharge current, $I_{\mathrm{p,a}}(t)$. By introducing different current terms (external total current, $I_{\mathrm{t,a}}(t)$, external displacement current, $I_{\mathrm{v,a}}(t)$ and external discharge current, $I_{\mathrm{p,a}}(t)$, as well as internal displacement current, $I_{\mathrm{v,g}}(t)$ and internal discharge current, $I_{\mathrm{p,g}}(t)$), a clear description of different currents and their different physical mechanisms is achieved.

From equation (1), we obtain

$$U_{\mathrm{d}}(t) = \frac{1}{C_{\mathrm{d}}}\int_0^t I_{\mathrm{t,a}}(\tau)\,\mathrm{d}\tau + U_{\mathrm{d}}(0), \tag{10}$$

where $U_{\mathrm{d}}(0) = U_{\mathrm{d}}(t = 0)$, the initial voltage on C_{d}. Substituting equation (10) into equation (3), we obtain

$$U_{\mathrm{g}}(t) = U_{\mathrm{a}}(t) - U_{\mathrm{d}}(t) = U_{\mathrm{a}}(t) - \frac{1}{C_{\mathrm{d}}}\int_0^t I_{\mathrm{t,a}}(\tau)\,\mathrm{d}\tau - U_{\mathrm{d}}(0). \tag{11}$$

In the following we will determine the values of $U_{\mathrm{d}}(0)$ for different excitation methods of DBDs.

3.1. In the case of bipolar ac voltage excitation

We assume that the applied external ac voltage, $U_{\mathrm{a}}(t)$, e.g. sine wave voltage, is strictly symmetric. For the first breakdown, $U_{\mathrm{d}}(0) = 0$. For stationary DBDs $U_{\mathrm{d}}(0) \neq 0$ due to memory charges deposited by the preceding discharge. If a symmetric ac voltage is used for excitation of DBDs, DBDs take place in a symmetric manner, i.e. all voltage and current quantities including the voltage across the barrier $U_{\mathrm{d}}(t)$ reverse after a half period:

$$\begin{aligned} U_{\mathrm{d}}\left(t + \frac{T}{2}\right) &= \frac{1}{C_{\mathrm{d}}}\int_0^{t+T/2} I_{\mathrm{t,a}}(\tau)\,\mathrm{d}\tau + U_{\mathrm{d}}(0) \\ &= \frac{1}{C_{\mathrm{d}}}\int_0^{T/2} I_{\mathrm{t,a}}(\tau)\,\mathrm{d}\tau + \frac{1}{C_{\mathrm{d}}}\int_{T/2}^{t+T/2} I_{\mathrm{t,a}}(\tau)\,\mathrm{d}\tau + U_{\mathrm{d}}(0) \\ &= \frac{1}{C_{\mathrm{d}}}\int_0^{T/2} I_{\mathrm{t,a}}(\tau)\,\mathrm{d}\tau - \frac{1}{C_{\mathrm{d}}}\int_0^{t} I_{\mathrm{t,a}}(\tau)\,\mathrm{d}\tau + U_{\mathrm{d}}(0) \\ &= -U_{\mathrm{d}}(t). \end{aligned} \tag{12}$$

Substituting equation (10) into equation (12), and rearranging, we obtain

$$U_{\mathrm{d}}(0) = -\frac{1}{2C_{\mathrm{d}}}\int_0^{T/2} I_{\mathrm{t,a}}(\tau)\,\mathrm{d}\tau. \tag{13}$$

Substituting equation (13) into equations (10) and (11), we obtain the solutions for both gap voltage and dielectric voltages under the above initial conditions:

$$U_{\mathrm{d}}(t) = \frac{1}{C_{\mathrm{d}}}\int_0^{t} I_{\mathrm{t,a}}(\tau)\,\mathrm{d}\tau - \frac{1}{2C_{\mathrm{d}}}\int_0^{T/2} I_{\mathrm{t,a}}(\tau)\,\mathrm{d}\tau, \tag{14}$$

$$U_{\mathrm{g}}(t) = U_{\mathrm{a}}(t) - \frac{1}{C_{\mathrm{d}}}\int_0^{t} I_{\mathrm{t,a}}(\tau)\,\mathrm{d}\tau + \frac{1}{2C_{\mathrm{d}}}\int_0^{T/2} I_{\mathrm{t,a}}(\tau)\,\mathrm{d}\tau. \tag{15}$$

Here the time zero can be an arbitrary point in time. So the trigger zero on the oscilloscope can be applied directly as time zero in the above equations. For this reason the value of $U_{\mathrm{d}}(0)$ is dependent on the selected time zero. As observed in [11], discharge current pulses have the same polarity in a rising or falling half period. Here the rising half period means the time duration from a negative peak to the following positive peak of the external voltage. Similarly, the falling half period means the time duration from a positive peak to the following negative peak. The peak points of the external voltage are transition points for discharge polarity. For this reason one period is divided into a rising and a falling half period instead of the usual division into a positive and a negative half period according to the value of the external voltage. Both peaks can be regarded as time zero. If we take a negative peak as time zero, so $U_{\mathrm{d}}(0)$ becomes a constant,

$$\begin{aligned} U_{\mathrm{d}}(0) &= -\frac{1}{2C_{\mathrm{d}}}\int_0^{T/2} I_{\mathrm{t,a}}(\tau)\,\mathrm{d}\tau \\ &= -\frac{C_{\mathrm{g}}}{C_{\mathrm{g}} + C_{\mathrm{d}}}U_{\mathrm{ap}} - \frac{\int_0^{T/2} I_{\mathrm{p,g}}(\tau)\,\mathrm{d}\tau}{2(C_{\mathrm{g}} + C_{\mathrm{d}})}, \end{aligned} \tag{16}$$

where U_{ap} is the peak value of the external voltage, and the integral in the second term in equation (16) is memory charges.

It is a constant. Substituting equation (16) into equations (14) and (15) and rearranging, we obtain

$$U_{\rm d}(t) = \frac{C_{\rm g}}{C_{\rm g}+C_{\rm d}}U_{\rm a}(t) + \frac{1}{C_{\rm g}+C_{\rm d}} \times \left[\int_0^t I_{\rm p,g}(\tau)\,{\rm d}\tau - \frac{1}{2}\int_0^{T/2} I_{\rm p,g}(\tau)\,{\rm d}\tau\right], \tag{17}$$

$$U_{\rm g}(t) = \frac{C_{\rm d}}{C_{\rm g}+C_{\rm d}}U_{\rm a}(t) + \frac{1}{C_{\rm g}+C_{\rm d}} \times \left[\frac{1}{2}\int_0^{T/2} I_{\rm p,g}(\tau)\,{\rm d}\tau - \int_0^t I_{\rm p,g}(\tau)\,{\rm d}\tau\right]. \tag{18}$$

Equations (17) and (18) have clear physical meanings. It is seen clearly from equations (17) and (18) that two physical procedures determine together $U_{\rm g}(t)$ and $U_{\rm d}(t)$. The first term in equations (17) and (18) results from the external voltage source, $U_{\rm a}(t)$. $U_{\rm a}(t)$ is divided between $C_{\rm d}$ and $C_{\rm g}$ according to the voltage division law of two series capacitors. The second term results from charge deposition by the internal current source, $I_{\rm p,g}(t)$. The internal conduction current, $I_{\rm p,g}(t)$, deposits free charges on $C_{\rm d}$ and $C_{\rm g}$ in parallel, and therefore causes a voltage change on $C_{\rm d}$ and $C_{\rm g}$:

$$\Delta U = \frac{\Delta Q}{C_{\rm g}+C_{\rm d}} = \frac{1}{C_{\rm g}+C_{\rm d}} \times \left[\frac{1}{2}\int_0^{T/2} I_{\rm p,g}(\tau)\,{\rm d}\tau - \int_0^t I_{\rm p,g}(\tau)\,{\rm d}\tau\right]. \tag{19}$$

Not only free charges deposited in the momentary half period but also memory charges left from the preceding half period (the first term in equation (19)) cause voltage changes across $C_{\rm d}$ and $C_{\rm g}$. The memory charges left from the preceding half period cause the so-called 'memory effect' in DBDs, i.e. memory charges increase the amplitude of $U_{\rm g}(t)$ in the next half period and help to trigger a discharge in the next half period at a lower external voltage than the first discharge (memory effect). Under unipolar pulse excitation [10] memory charges even trigger alone a discharge at zero external voltage (charge-induced discharge).

The external voltage source, $U_{\rm a}(t)$, changes periodically from $-U_{\rm ap}$ to $+U_{\rm ap}$ in the rising half period and vice versa from $+U_{\rm ap}$ to $-U_{\rm ap}$ in the falling half period. The total free charges, $\Delta Q(t)$ (the two integral terms in the bracket in equations (17)–(19)), change from the negative half to the positive half amount of total deposited charges by $I_{\rm p,g}(t)$ in one half period for the rising half period, and vice versa for the falling half period.

As shown in equation (18), charge deposition by the internal conduction discharge current, $I_{\rm p,g}(t)$, reduces the electric field in the gap spacing and finally leads to extinction of the discharge, whereas the voltage across the dielectric increases by the same value. Half the free charges transferred by $I_{\rm p,g}(t)$ in one half period is used to neutralize the memory charges, and the other half is left as new memory charges with opposite polarity for the next half period. This alternating process occurs every half period. The fact that the memory charges are equal to half the total charges deposited by $I_{\rm p,g}(t)$ in one half period is an important theoretical prediction from equation (18). This conclusion is valid under the ideal condition of no loss of memory charges.

3.2. In the case of unipolar pulse excitation

If unidirectional voltage pulses are used for DBD excitation, besides the discharge at the rising front or at the top of the voltage pulses (primary discharge), an opposed discharge (secondary discharge) occurs at the falling flank [10]. This secondary discharge removes the memory charges deposited by the primary discharge. This is referred to as a 'self-erasing effect'. Due to this effect, memory charges left for the next unipolar voltage pulse are negligible. Therefore not only for the first discharge but also for the following stationary discharges there are no memory charges available for primary discharges. If we choose the time zero before a primary discharge, then equations (10) and (11) read as follows:

$$U_{\rm d}(t) = \frac{1}{C_{\rm d}}\int_0^t I_{\rm t,a}(\tau)\,{\rm d}\tau = \frac{C_{\rm g}}{C_{\rm g}+C_{\rm d}}U_{\rm a}(t) + \frac{1}{C_{\rm g}+C_{\rm d}}\int_0^t I_{\rm p,g}(\tau)\,{\rm d}\tau, \tag{20}$$

$$U_{\rm g}(t) = U_{\rm a}(t) - U_{\rm d}(t) = \frac{C_{\rm d}}{C_{\rm g}+C_{\rm d}}U_{\rm a}(t) - \frac{1}{C_{\rm g}+C_{\rm d}}\int_0^t I_{\rm p,g}(\tau)\,{\rm d}\tau. \tag{21}$$

The first term in equations (20) and (21) results from the external voltage source, $U_{\rm a}(t)$. $U_{\rm a}(t)$ is divided between $C_{\rm d}$ and $C_{\rm g}$ according to the voltage division of two series capacitors. The second term results from free charges deposited by the internal current source, $I_{\rm p,g}(t)$. The internal conduction current, $I_{\rm p,g}(t)$, charges $C_{\rm d}$ and $C_{\rm g}$ in parallel. This raises the voltage across $C_{\rm d}$ and at the same time reduces the voltage across $C_{\rm g}$ by the same value (see the second term in equations (20) and (21)). Before the primary discharge the internal voltage, $U_{\rm g}(t)$, rises with the external voltage (e.g. a positive pulse) until the primary discharge occurs. The internal conduction current of this primary discharge, $I_{\rm p,g}(t)$, reduces $U_{\rm g}(t)$ partly or even totally due to charge deposition, and eventually extinguishes the primary discharge. When the external voltage falls, the first term in equation (21) decreases, but the second term remains, so that $U_{\rm g}(t)$ could decrease to zero and further to a negative value that can be high enough to trigger a negative discharge. Unlike the primary discharge, the secondary discharge is not extinguished by charge accumulation but by the available energy stored by memory charges during the primary discharge.

The instantaneous power input, $P_{\rm t,a}(t)$, from the external circuit into the DBD cell can be calculated as follows:

$$P_{\rm t,a}(t) = U_{\rm a}(t) \times I_{\rm t,a}(t). \tag{22}$$

The instantaneous power consumed by the plasma discharge in the gap can be calculated from equation (23):

$$P_{\rm p,g}(t) = U_{\rm g}(t) \times I_{\rm p,g}(t). \tag{23}$$

The equations derived above show instantaneous relations between internal and external electrical quantities for homogeneous DBDs under arbitrary excitation voltage waveforms and provide a useful tool for an experimental determination of internal electrical quantities in the discharge

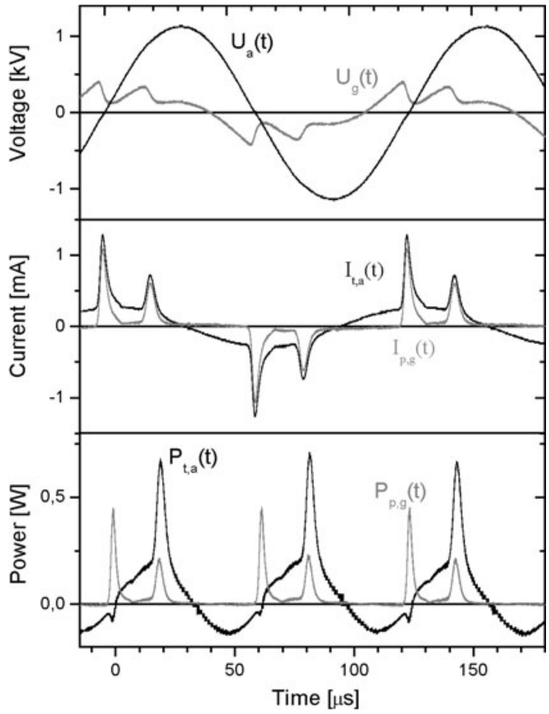

Figure 2. The internal temporal dynamic process in the gap of a two–two mode DBD excited by a bipolar sine wave voltage (neon, 1.5 bar, gap 2 mm, quartz 2 × 1.25 mm, cell diameter 2.9 cm): measured $U_a(t)$ and $I_{t,a}(t)$; calculated $U_g(t)$, $I_{p,g}(t)$, $P_{p,g}(t)$ and $P_{t,a}(t)$.

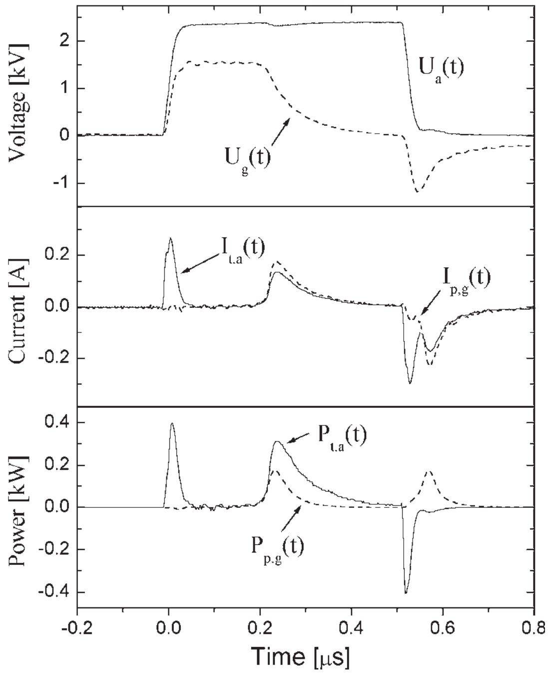

Figure 3. The internal temporal dynamic process in the gap of a unipolar-pulsed DBD (air, 50 mbar, gap 4 mm, quartz 2 × 2 mm, cell diameter 4 cm): measured $U_a(t)$ and $I_{t,a}(t)$; calculated $U_g(t)$, $I_{p,g}(t)$, $P_{p,g}(t)$ and $P_{t,a}(t)$.

gap. By measuring the external voltage, $U_a(t)$, and the external total current, $I_{t,a}(t)$, the temporal evolution of all the internal electrical quantities in the gap can be calculated. The temporal evolution process of internal electrical quantities obtained provides a detailed insight into the dynamic processes of DBDs in the discharge gap and a quantitative explanation of experimental observations. As examples, we show the time evolution process of internal electrical quantities for bipolar sine wave excitation in figure 2 and unipolar pulse excitation in figure 3, respectively.

As shown in figure 2, the true discharge current in the gap is extracted from the measured external total current. There are two discrete discharge current pulses with the same polarity in each rising or falling half period (two–two mode). The occurrence and extinction of DBDs are completely controlled by the gap voltage. If the gap voltage reaches the ignition voltage, a discharge occurs. If the gap voltage is not high enough to sustain the discharge, the discharge ceases. The gap voltage increases with the external voltage in the discharge-free time and decreases during the discharge phase. The memory charges enhance the gap voltage in the next half period. This process is described quantitatively by equation (18). The ignition voltages for the first and second discharges are 400 and 335 V, much lower than the breakdown voltage given by the Paschen curve. The discharge extinguishing voltage is about 130 V for both discharges. By comparing the instantaneous power input from the external circuit with the actual power consumption by discharges in the gap, we see an interesting power storage and releasing process that results from memory charges.

As shown in figure 3, DBDs driven by unipolar pulses show different electrical behaviour. There are no memory charges and no energy stored in the interpulse interval. After each external voltage pulse, all internal and external electric quantities return approximately to zero. The power input proceeds only during the primary discharge. One part of the injected energy from the external circuit directly supports the primary discharge; the rest is stored by memory charges, to be released later to energize the secondary discharge at the pulse falling flank or shortly after the pulse falling flank. The memory charges and the corresponding stored energy are almost completely erased by the secondary discharge. No matter how high the pulse top is, only the one–one mode is observed. The ignition voltage of the primary discharge is much higher than that under sine wave excitation, but the gap voltage is much more strongly reduced after the primary discharge. Whereas the primary discharge is energized directly by the external circuit and extinguished by charge deposition, the secondary discharge is energized by the energy stored by memory charges and ceases after this energy is consumed.

4. Equations for deposited charges and energy

In the above derivation, we obtained instantaneous relationships of internal dynamic quantities ($I_{p,g}(t)$, $U_g(t)$, $P_{p,g}(t)$) with the external dynamical quantities $U_a(t)$ and $I_{t,a}(t)$. In this section, we derive the equations for following time-integration quantities, internal charge transfer and external charge transfer and energy deposition by one discharge or in one half period by introducing two new quantities, internal discharge ignition voltage, $U_{z,g}$ and internal discharge extinguishing voltage, $U_{l,g}$.

We assume that at time point $t = t_1$ the discharge begins to fire and at $t = t_2$ the discharge extinguishes. We define the internal and external charge transfer by one discharge as follows:

$$Q_{\mathrm{p,g}} = \int_{t_1}^{t_2} I_{\mathrm{p,g}}(\tau)\,\mathrm{d}\tau,$$

$$Q_{\mathrm{p,a}} = \int_{t_1}^{t_2} I_{\mathrm{p,a}}(\tau)\,\mathrm{d}\tau.$$

Integrating equation (6) and substituting the above definitions into it, we obtain

$$Q_{\mathrm{p,a}} = \frac{Q_{\mathrm{p,g}}}{1 + C_{\mathrm{g}}/C_{\mathrm{d}}}. \tag{24}$$

From equation (24) we know that the internal charge transfer is not equal to the external charge transfer, but larger by a factor of $(1 + C_{\mathrm{g}}/C_{\mathrm{d}})$. Substituting equation (4) into equation (11), we obtain

$$U_{\mathrm{g}}(t_1) = \frac{U_{\mathrm{a}}(t_1)}{1 + C_{\mathrm{g}}/C_{\mathrm{d}}} + \frac{C_{\mathrm{g}}}{C_{\mathrm{g}} + C_{\mathrm{d}}} U_{\mathrm{a}}(0) - U_{\mathrm{d}}(0) = U_{\mathrm{z,g}}, \tag{25}$$

$$U_{\mathrm{g}}(t_2) = \frac{U_{\mathrm{a}}(t_2)}{1 + C_{\mathrm{g}}/C_{\mathrm{d}}} + \frac{C_{\mathrm{g}}}{C_{\mathrm{g}} + C_{\mathrm{d}}} U_{\mathrm{a}}(0) - \frac{Q_{\mathrm{p,g}}}{C_{\mathrm{g}} + C_{\mathrm{d}}} - U_{\mathrm{d}}(0) = U_{\mathrm{l,g}}. \tag{26}$$

From equations (25) and (26) we obtain the equation for internal charges transferred by one discharge:

$$Q_{\mathrm{p,g}} = (C_{\mathrm{g}} + C_{\mathrm{d}})(U_{\mathrm{z,g}} - U_{\mathrm{l,g}}) + C_{\mathrm{d}}(U_{\mathrm{a}}(t_2) - U_{\mathrm{a}}(t_1)). \tag{27}$$

The energy consumed by one discharge can be calculated as follows:

$$\begin{aligned} E_{\mathrm{p,g}} &= \int_{t_1}^{t_2} I_{\mathrm{p,g}}(\tau) U_{\mathrm{g}}(\tau)\,\mathrm{d}\tau \\ &= \left[\frac{U_{\mathrm{a}}(0)}{1 + C_{\mathrm{d}}/C_{\mathrm{g}}} - U_{\mathrm{d}}(0)\right] Q_{\mathrm{p,g}} \\ &\quad - \frac{Q_{\mathrm{p,g}}^2}{2(C_{\mathrm{g}} + C_{\mathrm{d}})} + \int_{t_1}^{t_2} \frac{I_{\mathrm{p,g}}(\tau) U_{\mathrm{a}}(\tau)}{(1 + C_{\mathrm{g}}/C_{\mathrm{d}})\,\mathrm{d}\tau}. \end{aligned} \tag{28}$$

We assume that the external voltage is constant during the short discharge duration, i.e. $U_{\mathrm{a}}(t_1) \approx U_{\mathrm{a}}(t_2)$. Then equation (27) can be simplified to the following form:

$$Q_{\mathrm{p,g}} = (C_{\mathrm{g}} + C_{\mathrm{d}})(U_{\mathrm{z,g}} - U_{\mathrm{l,g}}). \tag{29}$$

Substituting equation (29) into equation (24), we obtain

$$Q_{\mathrm{p,a}} = C_{\mathrm{d}}(U_{\mathrm{z,g}} - U_{\mathrm{l,g}}). \tag{30}$$

Equations (29) and (30) show that both internal and external charge transfer by one discharge are proportional to the difference between internal discharge firing voltage and discharge extinguishing voltage.

Equation (28) can also be simplified under the condition $U_{\mathrm{a}}(t_1) \approx U_{\mathrm{a}}(t_2)$. Introducing equations (25) and (29) into equation (28), we obtain

$$\begin{aligned} E_{\mathrm{p,g}} &= \left[\frac{U_{\mathrm{a}}(t_1)}{1 + C_{\mathrm{g}}/C_{\mathrm{d}}} + \frac{C_{\mathrm{g}}}{C_{\mathrm{g}} + C_{\mathrm{d}}} U_{\mathrm{a}}(0) - U_{\mathrm{d}}(0)\right] Q_{\mathrm{p,g}} \\ &\quad - \frac{Q_{\mathrm{p,g}}^2}{2(C_{\mathrm{g}} + C_{\mathrm{d}})} = U_{\mathrm{z,g}} Q_{\mathrm{p,g}} - \frac{1}{2}(U_{\mathrm{z,g}} - U_{\mathrm{l,g}}) Q_{\mathrm{p,g}} \\ &= \frac{1}{2}(U_{\mathrm{z,g}} + U_{\mathrm{l,g}}) Q_{\mathrm{p,g}} = \frac{1}{2}(C_{\mathrm{g}} + C_{\mathrm{d}})(U_{\mathrm{z,g}}^2 - U_{\mathrm{l,g}}^2). \end{aligned} \tag{31}$$

Equations (30) and (31) are not new. They have already been obtained from an analytical model of DBD in [15]. Unlike the relation between internal and external charge transfer, the amount of internal energy consumed by discharges must be equal to the injected external energy in every period according to energy conservation, although the instantaneous power input from the external circuit is not equal to the power instantaneously consumed by the discharges. If more than one discharge occurs in each half period, the deposited charge and energy in one half period are the sum of the charges and energy deposited by all single discharges:

$$Q_{\mathrm{p,g}}(\text{half period}) = \sum_{j=1}^{n} (Q_{\mathrm{p,g}})_j, \tag{32}$$

$$E_{\mathrm{p,g}}(\text{half period}) = \sum_{j=1}^{n} (E_{\mathrm{p,g}})_j, \tag{33}$$

where n represents the number of discharges occurring in one half period.

$E_{\mathrm{p,g}}$, $Q_{\mathrm{p,g}}$ and $Q_{\mathrm{p,a}}$ are all expressed from state parameters C_{g}, C_{d}, $U_{\mathrm{z,g}}$ and $U_{\mathrm{l,g}}$. Because C_{g}, C_{d}, $U_{\mathrm{z,g}}$ and $U_{\mathrm{l,g}}$ are constant for given experimental conditions (electrode configuration, discharge gas and gas pressure), by using equations (29)–(33) we can predict charge transfer and energy input for the applied reactor. This is very useful in practice for the design of proper reactors, e.g. generation of a required amount of ozone or UV radiation.

5. Summary and conclusions

A dynamic electrical model for homogeneous DBDs has been put forward. This dynamic model includes a new equivalent circuit for homogeneous DBDs and several equations have been derived. These equations reveal the dynamic relationships between internal instantaneous electrical quantities (internal gap voltage, $U_{\mathrm{g}}(t)$, internal discharge current, $I_{\mathrm{p,g}}(t)$) and external electrical quantities (external voltage, $U_{\mathrm{a}}(t)$, and the external total current, $I_{\mathrm{t,a}}(t)$). These equations allow us to calculate simultaneously the temporal evolution processes of all internal electrical quantities ($U_{\mathrm{g}}(t)$, $I_{\mathrm{p,g}}(t)$ and $P_{\mathrm{p,g}}(t)$) from the measured external voltage, $U_{\mathrm{a}}(t)$, and the external total current, $I_{\mathrm{t,a}}(t)$, for an arbitrary external voltage waveform. The access to the dynamic evolution processes in the gap provides useful information to understand the electrical working principle of DBD devices under different excitation methods and to find new ways to improve DBD technology (e.g. unipolar-pulsed DBD).

Several current terms (external total current, external displacement current, external discharge current, internal discharge current and internal displacement current) have been introduced to distinguish their different physical mechanisms. Their instantaneous relationships are quantified. It was predicted that two mechanisms (time variation of external voltage and discharge in the gap) determine the measured total current in the external circuit, from which the true discharge current in the gap can be extracted. The gap voltage and the voltage across the dielectric are determined by two sources: the external voltage source and the free charges deposited by the internal discharge current source in both the momentary and preceding half periods. Their contribution is quantified. In addition, it was predicted that the number of memory charges is equal to half the deposited charge in each half period for a bipolar sinusoidal voltage excitation.

Furthermore, we have also derived equations for the following electrical quantities: internal charge deposition and external charge transfer by one discharge, and energy deposition by one discharge, as well as their corresponding counterparts in a half period, if multiple discharges occur in a half period.

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