LOG(M) PROJECT ZERO

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2-D (N=1)

In the case when $n = 1$ we are attempting to count the number of saturated ideals for our Hilbert polynomial for when $\lambda = (1^r)$ and our Hilbert polynomial becomes $H_I(d) = r$.
Our problem of counting the Ideals up to saturation then boils down to counting the number of ways of laying down r rows along either the x-axis or y-axis. To count this we simply consider the weak composition of k things into 2 buckets which gives us that the number of saturated ideals for when $n = 1$ is

$$\binom{r+2-1}{2-1} = \binom{r+1}{1} = r+1$$

3-D (N=2)

Now when we consider the case when $n = 1$, our lambda partition becomes $\lambda = (2^p, 1^r)$ and our Hilbert polynomial is of the form $H_I(d) = kd + c$ where $k, c \in \mathbb{R}$ and must satisfy some conditions which we discussed in the past. Further, to make our count simpler note that if we lay planes in any configuration along the $x-y, y-z, x-z$ planes it does not affect how we lay down rows in the future. Keeping this in mind if we let $n_2(p)$ be the number of ways to lay down p planes and $n_1(r)$ the number of ways to lay down r rows then the total number of ways to lay down these objects becomes

$$n_2(p)n_1(r)$$

First, when considering $n_2(p)$ we just have to count the number of ways to place p planes 3 places, these being the $x-y, y-z, x-z$ planes. So then we have to consider a weak composition of p objects into 3 buckets when finding a closed form for $n_2(p)$. So now we have

$$n_2(p) = \binom{p+3-1}{3-1} = \binom{p+2}{2}$$

Now when finding a closed form for $n_1(r)$ lets first consider the number of ways to place r rows along 3 different axis x, y, z. This boils down to a weak composition of r objects into 3 buckets which is

$$\binom{r+3-1}{3-1} = \binom{r+2}{2}$$

Now this only gives us the number of ways to place r rows into 3 different slots. However, within each slot there are additional ways to arrange these rows. So if we have c rows along a particular axis and we consider the projection of these rows onto the plane of the remaining two axis the number of ways t arrange those is $f_2(c)$ which is the number of young diagrams or integer partitions for the integer c. So now we have that the total number of Ideals up to saturation for $\lambda = (2^p, 1^r)$ is

$$n_2(p)n_1(r) = \binom{p+2}{2} \sum_{i=1}^{\binom{r+2}{2}} [f_2(c_{x_i})f_2(c_{y_i})f_2(c_{z_i})]$$

Where i is an index over all weak compositions of r rows into 3 axes