HW2

February 13, 2022

Python/MATLAB for Physics

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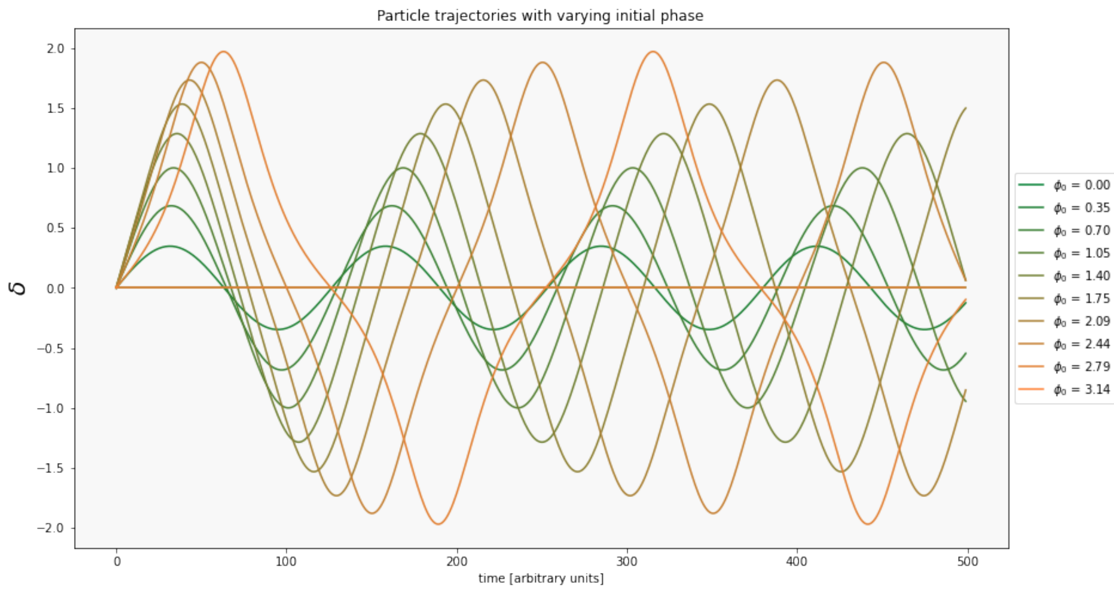
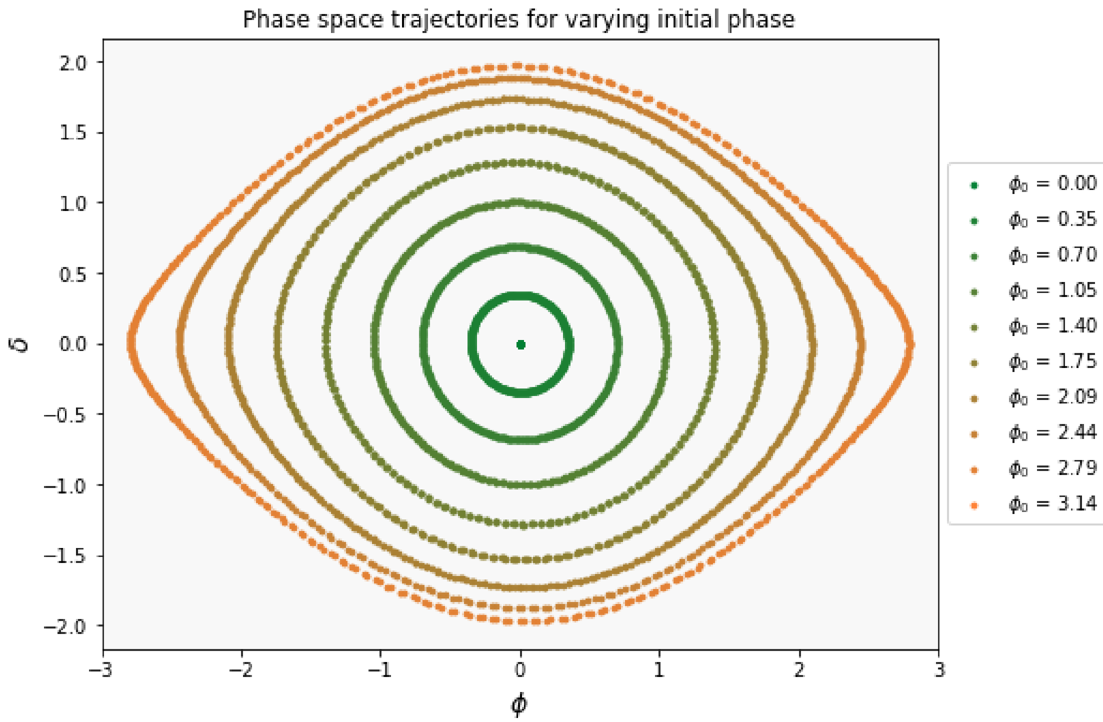
[ ]:
```
%reset -f
import numpy as np
import matplotlib.pyplot as plt
from sympy import *
```

[ ]:
```
tnum = 500

p0_array = np.linspace(0, np.pi, 10)
plt.figure(figsize=(14,8))
plt.title('Particle trajectories with varying initial phase')
color_array = np.linspace(0,1,len(p0_array))

for n in range(len(p0_array)):
    d0 = 0.0
    p0 = p0_array[n]
    d = np.zeros(tnum)*np.nan
    p = np.zeros(tnum)*np.nan
    d[0] = d0
    p[0] = p0

    for i in range(1,tnum):
        d[i] = d[i-1] + 0.05*np.sin(p[i-1])
        p[i] = p[i-1] -0.05*d[i]
    plt.plot(d, color=(color_array[n], 0.5, 0.2), label=r'$\phi_0$ = %.
 ↪2f'%(p0_array[n]))
plt.legend(loc='center left', bbox_to_anchor=(1, 0.5))
plt.xlabel('time [arbitrary units]')
plt.ylabel('$\delta$', fontsize=20)
plt.gca().set_facecolor('#F8F8F8')
```

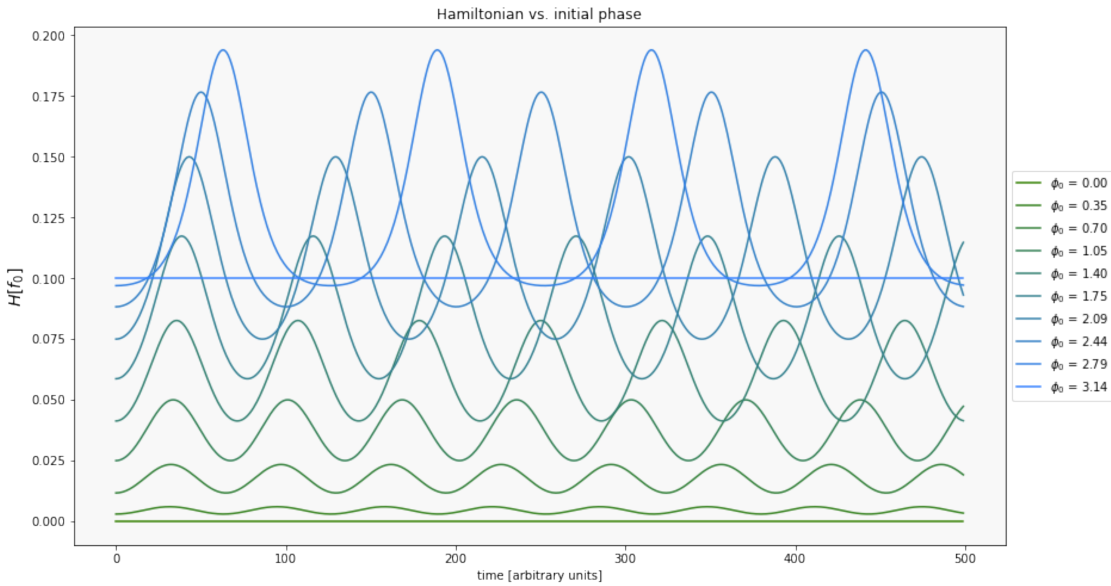


[ ]:
```
# Plot phase space for fun
plt.figure(figsize=(8,6))
plt.title('Phase space trajectories for varying initial phase')

for n in range(len(p0_array)):
    d0 = 0.0
    p0 = p0_array[n]
    d = np.zeros(tnum)*np.nan
    p = np.zeros(tnum)*np.nan
    d[0] = d0
    p[0] = p0

    for i in range(1,tnum):
        d[i] = d[i-1] + 0.05*np.sin(p[i-1])
        p[i] = p[i-1] -0.05*d[i]
    plt.scatter(p, d, s=8.0, color=(1.0*color_array[n], 0.5, 0.2),␣
 ↪label=r'$\phi_0$ = %.2f'%(p0_array[n]))
plt.legend(loc='center left', bbox_to_anchor=(1, 0.5))
plt.xlim(-3,3)
plt.xlabel('$\phi$', fontsize=14)
plt.ylabel('$\delta$', fontsize=14)
plt.gca().set_facecolor('#F8F8F8')
```



[ ]:
```
# Plot the Hamiltonian for a non-zero trajectory
plt.figure(figsize=(14,8))
plt.title('Hamiltonian vs. initial phase')

for n in range(len(p0_array)):
    d0 = 0.0
    p0 = p0_array[n]
    d = np.zeros(tnum)*np.nan
    p = np.zeros(tnum)*np.nan
    d[0] = d0
    p[0] = p0

    for i in range(1,tnum):
        d[i] = d[i-1] + 0.05*np.sin(p[i-1])
        p[i] = p[i-1] -0.05*d[i]
    #plt.scatter(p, d, s=8.0, color=(1.0*color_array[n], 0.5, 0.2),␣
 ↪label=r'$\phi_0$ = %.2f'%(p0_array[n]))
    H = 0.5*(0.05)*d**2 + 0.05*(1-np.cos(p0))
    plt.plot(H, color=(0.2, 0.5, color_array[n]), label=r'$\phi_0$ = %.
 ↪2f'%(p0_array[n]))
plt.ylabel('$H [f_0]$', fontsize=14)
plt.legend(loc='center left', bbox_to_anchor=(1, 0.5))
plt.xlabel('time [arbitrary units]')
```

```python
plt.gca().set_facecolor('#F8F8F8')
```



[ ]:
```python
L, f1, f2 = symbols(['L','f_1','f_2'])
```

[ ]:
```python
# Horizontal FODO cell matrix
M_QF_H = Matrix([[1,0],[-1/(2*f1),1]])
M_L_H = Matrix([[1,L],[0,1]])
M_QD_H = Matrix([[1,0],[1/(f2),1]])

M_FODO_H = simplify(M_QF_H*M_L_H*M_QD_H*M_L_H*M_QF_H); M_FODO_H
```

[ ]:

$$\begin{bmatrix} -\frac{L^2}{2f_1f_2} + \frac{L}{f_2} - \frac{L}{f_1} + 1 & \frac{L(L+2f_2)}{f_2} \\ \frac{\frac{L^2}{4} - Lf_1 + \frac{Lf_2}{2} + f_1^2 - f_1f_2}{f_1^2f_2} & -\frac{L^2}{2f_1f_2} + \frac{L}{f_2} - \frac{L}{f_1} + 1 \end{bmatrix}$$

[ ]:
```python
# Vertical FODO cell matrix
M_QF_V = Matrix([[1,0],[1/(2*f1),1]])
M_L_V = Matrix([[1,L],[0,1]])
M_QD_V = Matrix([[1,0],[-1/(f2),1]])

M_FODO_V = simplify(M_QF_V*M_L_V*M_QD_V*M_L_V*M_QF_V); M_FODO_V
```

[ ]:

$$\begin{bmatrix} -\frac{L^2}{2f_1f_2} - \frac{L}{f_2} + \frac{L}{f_1} + 1 & \frac{L(-L+2f_2)}{f_2} \\ \frac{-\frac{L^2}{4} - Lf_1 + \frac{Lf_2}{2} - f_1^2 + f_1f_2}{f_1^2f_2} & -\frac{L^2}{2f_1f_2} - \frac{L}{f_2} + \frac{L}{f_1} + 1 \end{bmatrix}$$

[ ]:
```python
# Horizontal stability condition
H_stable = (M_FODO_H[0,0] + M_FODO_H[1,1])/2 # half of the trace
H_stable
```

[ ]:
$$-\frac{L^2}{2f_1f_2} + \frac{L}{f_2} - \frac{L}{f_1} + 1$$

[ ]:
```python
V_stable = (M_FODO_V[0,0] + M_FODO_V[1,1])/2 # half of the trace
V_stable
```

[ ]:
$$-\frac{L^2}{2f_1f_2} - \frac{L}{f_2} + \frac{L}{f_1} + 1$$

[ ]:
```python
# substitute single variables for L/f1, L/f2
a, b = symbols(['a', 'b'])
H_stable = H_stable.subs({L/f1:a, L/f2:b})
H_stable
```

[ ]:
$$-\frac{ab}{2} - a + b + 1$$

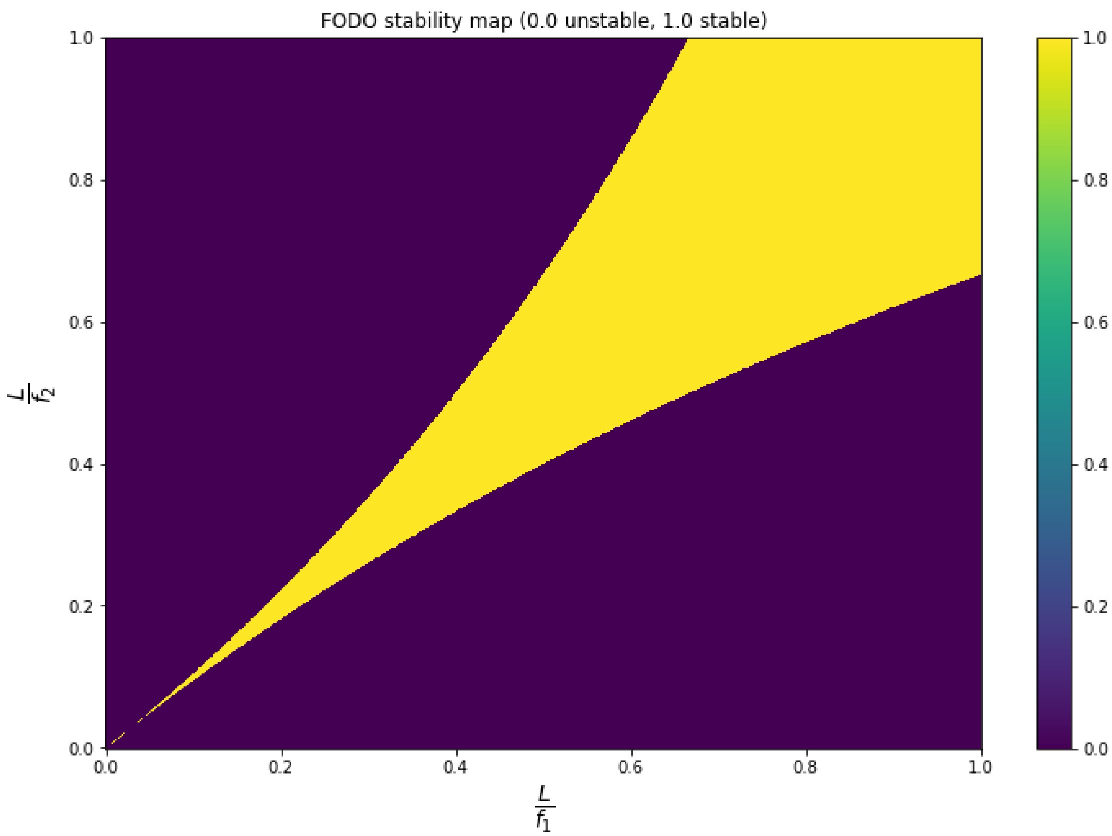
[ ]:
```python
V_stable = V_stable.subs({L/f1:a, L/f2:b})
V_stable
```

[ ]:
$$-\frac{ab}{2} + a - b + 1$$

[ ]:
```python
# Convert to a function
Htrace = lambdify([a, b], H_stable)
Vtrace = lambdify([a, b], V_stable)
```

[ ]:
```python
n = 1000
range = np.linspace(0,1.0,n)
X, Y = np.meshgrid(range, range)


plt.figure(figsize=(12,8))
plt.title('FODO stability map (0.0 unstable, 1.0 stable)')
plt.pcolormesh(X, Y, np.logical_and(Htrace(X,Y)<1, Vtrace(X,Y)<1),␣
 ↪shading='auto')
plt.xlabel(r'$\frac{L}{f_1}$', fontsize=20)
plt.ylabel(r'$\frac{L}{f_2}$', fontsize=20)
plt.colorbar()
```

[ ]: <matplotlib.colorbar.Colorbar at 0x7fe076a82070>



[ ]:
```
x = Function('x')
B, w, f0, w0, t, C1, C2 = symbols(['beta', 'omega', 'f_0', 'omega_0', 't',␣
 ↪'C1', 'C2'], real=True)
eq = Eq(x(t).diff(t,t) - B*x(t).diff(t) + w**2*x(t),f0*sin(w0*t)); eq
```

[ ]:

$$-\beta\frac{d}{dt}x(t) + \omega^2 x(t) + \frac{d^2}{dt^2}x(t) = f_0\sin(\omega_0 t)$$

[ ]:
```
soln = dsolve(eq, x(t)).simplify().rhs; soln
```

[ ]:

$$\frac{\beta f_0\omega_0\cos(\omega_0 t) + f_0\omega^2\sin(\omega_0 t) - f_0\omega_0^2\sin(\omega_0 t) + \left(C_1 e^{\frac{t\left(\beta-\sqrt{\beta^2-4\omega^2}\right)}{2}} + C_2 e^{\frac{t\left(\beta+\sqrt{\beta^2-4\omega^2}\right)}{2}}\right)\left(\beta^2\omega_0^2 + \omega^4 - 2\omega^2\omega_0^2 + \omega_0^4\right.}{\beta^2\omega_0^2 + \omega^4 - 2\omega^2\omega_0^2 + \omega_0^4}$$

[ ]:
```
soln = soln.subs({Symbol('C1'):0.0, Symbol('C2'):0.0}); soln
```

[ ]:

$$\frac{\beta f_0\omega_0\cos(\omega_0 t) + f_0\omega^2\sin(\omega_0 t) - f_0\omega_0^2\sin(\omega_0 t)}{\beta^2\omega_0^2 + \omega^4 - 2\omega^2\omega_0^2 + \omega_0^4}$$

[ ]:
```
soln = soln.subs(t, 0)*(w**2/f0); soln
```

[ ]:

$$\frac{\beta\omega^2\omega_0}{\beta^2\omega_0^2 + \omega^4 - 2\omega^2\omega_0^2 + \omega_0^4}$$

[ ]:
```
soln1 = soln.subs(B, 0.1*w0); soln1
```

[ ]:
$$\frac{0.1\omega^2\omega_0^2}{\omega^4 - 2\omega^2\omega_0^2 + 1.01\omega_0^4}$$

[ ]:
```
soln2 = soln.subs(B, 0.05*w0); soln2
```

[ ]:
$$\frac{0.05\omega^2\omega_0^2}{\omega^4 - 2\omega^2\omega_0^2 + 1.0025\omega_0^4}$$

[ ]:
```
a = Symbol('alpha')
soln1 = soln1.subs(w0, a*w); simplify(soln1)
```

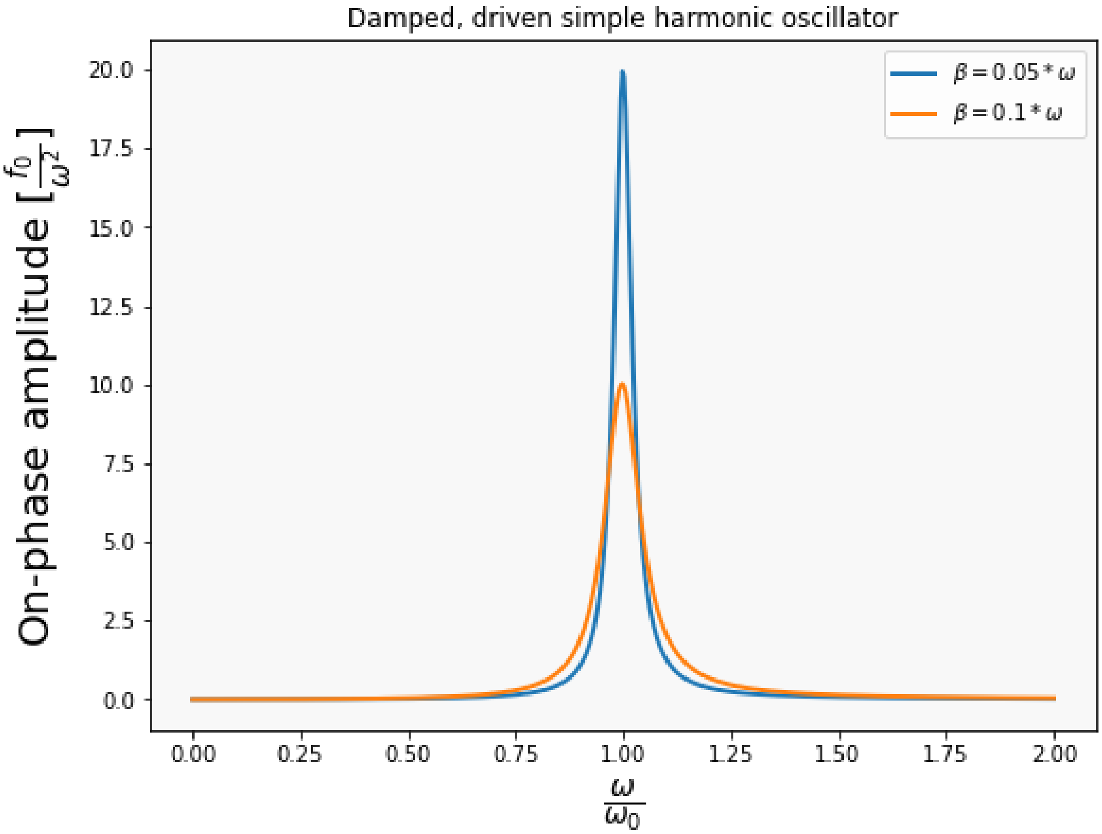
[ ]:
$$\frac{0.1\alpha^2}{1.01\alpha^4 - 2\alpha^2 + 1}$$

[ ]:
```
soln2 = soln2.subs(w0, a*w); simplify(soln2)
```

[ ]:
$$\frac{0.05\alpha^2}{1.0025\alpha^4 - 2\alpha^2 + 1}$$

[ ]:
```
soln1_func = lambdify(a, soln1.subs(w0, a*w))
soln2_func = lambdify(a, soln2.subs(w0, a*w))

a_array = np.linspace(0.0,2.0,500)

plt.figure(figsize=(8,6))
plt.title('Damped, driven simple harmonic oscillator')
plt.plot(a_array, soln2_func(a_array), label=r'$\beta=0.05*\omega$',␣
 ↪linewidth=2.0)
plt.plot(a_array, soln1_func(a_array), label=r'$\beta=0.1*\omega$', linewidth=2.
 ↪0)
plt.xlabel(r'$\frac{\omega}{\omega_0}$', fontsize=20)
plt.ylabel(r'On-phase amplitude [$\frac{f_0}{\omega^2}$]', fontsize=20)
plt.gca().set_facecolor('#F8F8F8')
plt.legend()
```

[ ]: <matplotlib.legend.Legend at 0x7fe0639d4c10>



Damped, driven simple harmonic oscillator
$\beta = 0.05 * \omega$
$\beta = 0.1 * \omega$
On-phase amplitude [$\frac{f_0}{\omega^2}$]
$\frac{\omega}{\omega_0}$

[ ]: