Chapter 1

Solutions to selected exercises

Many of the exercises may be answered correctly in different ways. Where that is the case, the solution here represents one possible correct answer.

Chapter ?? Ex ??:

1. I should take my sunglasses.
2. It must have been sunny.
3. You took the cookie from the cookie-jar.
4. Colonel Mustard did it in the kitchen with the lead pipe.

Chapter ?? Ex ??:

1. Yes, valid arguments can have a mix of true and false premises.
   e.g., You are a lizard (false). If you are a lizard, you are a reptile (true). Therefore, you are a reptile.
2. Yes, a valid argument can have only false premises.
   e.g., You are a lizard (false). If you are a lizard, you are a mammal (false). Therefore, you are a mammal.
3. Yes, a valid argument can have false premises and conclusion.
   e.g., You are a lizard (false). If you are a lizard, you are a fish (false). Therefore, you are a fish.
4. Yes, an invalid argument can be made valid by adding a new premise.
   e.g., You are alive. Therefore you breathe. This is invalid. But adding 'Everything that is alive breathes' makes the argument valid.
5. No, you can't make a valid argument invalid by adding a new premise. Adding new premises just makes it harder to make all the premises true. Recall that a valid argument has a true conclusion whenever its premises are true, so adding premises can only make it easier to be valid.

Chapter ?? Ex ??: Are these logical truths, falsehoods, or contingent?

1. contingent
2. contingent
3. logical truth
4. logical falsehood
5. contingent

Chapter ?? Ex ??: Which pairs of statements are logically equivalent?

1. Not equivalent
2. Not equivalent
3. Not equivalent
4. Equivalent
5. Equivalent

Chapter ?? Ex ??:

1. Consistent.
2. Consistent.
3. Inconsistent.
4. Consistent.
5. Inconsistent.
6. Inconsistent.
7. Inconsistent.

Chapter ?? Ex ??:

1. Possible. 20=30. Therefore 2 = 3.
2. Impossible. Invalid arguments have counter-examples. Counter-examples make the conclusion false. Logical truths can never be false.
3. Impossible. Valid arguments have a true conclusion if the premises are true, and they are true always, so the conclusion is true always.
4. Impossible. Contingent statements are those that are neither logical truths nor falsehoods.
5. Possible. Any two logical truths are equivalent.
6. Impossible. The contingent statement will be false in a situation where the logical truth is true, so they are not equivalent.
7. Possible. Any two logical falsehoods are equivalent, and inconsistent.
8. Impossible. A falsehood is never true, so all the statements can't be true.
9. Possible. As long as the other statements are mutually inconsistent, adding a logical truth won't make the other statements all true.

Chapter ?? Ex ??:

1. $(R \wedge W)$
2. $(R \wedge \neg S$
3. $\neg(R \wedge S)$
4. $\neg(R \vee S)$
5. $(R \rightarrow (W \wedge G))$
6. $(G \wedge \neg W)$
7. $(R \rightarrow (W \wedge \neg S))$
8. $((R \rightarrow W) \wedge \neg S)$

Chapter ?? Ex ??:
The letters in your symbolisation keys may differ from ours.

1. $(\neg Z \rightarrow \neg C), (Z \rightarrow T), (T \rightarrow \neg C) \therefore \neg C.$
2. $(R \rightarrow S), (R \rightarrow E) \therefore (\neg E \rightarrow (\neg R \wedge \neg S)).$
3. $(R \rightarrow \neg D), (P \rightarrow M), ((P \rightarrow M) \rightarrow (\neg D \rightarrow \neg N)) \therefore (N \rightarrow \neg R).$

Chapter ?? Ex ??:

1. $\neg M$
2. $(M \vee \neg M)$
3. $(G \vee C)$
4. $\neg(C \vee G)$
5. $C \rightarrow \neg(G \vee M)$
6. $\neg M \rightarrow (C \vee G)$

Chapter ?? Ex ??:

1. $(A \wedge B)$
2. $(C \rightarrow E)$
3. $(C \vee A)$
4. $(B \wedge \neg F)$
5. $(\neg A \wedge \neg B)$
6. $(A \wedge B \wedge \neg(E \vee F))$
7. $(F \rightarrow D)$
8. $((\neg A \rightarrow \neg B) \wedge (A \rightarrow B))$
9. $(E \leftrightarrow \neg F)$
10. $((B \wedge D) \rightarrow F)$
11. $\neg(B \wedge D)$

Chapter ?? Ex ??:

1. $(A \wedge B)$
2. $((A \vee B) \rightarrow C)$
3. $(\neg(A \vee B) \rightarrow \neg C)$
4. $(E \vee C)$
5. $((C \vee \neg C) \wedge E)$
6. $((A \vee B) \wedge \neg(A \wedge B))$

Chapter ?? Ex ??:
The letters in your symbolisation keys may differ from ours.

1. $(A \rightarrow E), (\neg D \rightarrow A) \therefore (\neg E \rightarrow D)$.
2. $(R \vee S), (R \rightarrow D), (S \rightarrow C) \therefore (D \vee C)$.
3. $(R \rightarrow (C \wedge \neg N)), (\neg R \rightarrow (N \wedge \neg C)) \therefore ((N \vee C) \wedge \neg(N \wedge C))$.
4. $(\neg M \rightarrow D), (\neg D \rightarrow M), \neg(M \wedge D) \therefore ((M \vee D) \wedge \neg(M \wedge D))$.

Chapter ?? Ex ??:

1. $\neg(S \leftrightarrow (P \rightarrow S))$

P	S	$\neg$	$(S$	$\leftrightarrow$	$(P$	$\rightarrow$	$S))$
1	1	0	1	1	1	1	1
1	0	0	0	1	1	0	0
0	1	0	1	1	0	1	1
0	0	1	0	0	0	1	0

2. $\neg((X \wedge Y) \vee (X \vee Y))$

X	Y	$\neg$	$((X$	$\wedge$	$Y)$	$\vee$	$(X$	$\vee$	$Y))$
1	1	0	1	1	1	1	1	1	1
1	0	0	1	0	0	1	1	1	0
0	1	0	0	0	1	1	0	1	1
0	0	1	0	0	0	0	0	0	0

3. $((\neg P \vee \neg M) \leftrightarrow M)$

P	M	$((\neg$	P	$\vee$	$\neg$	$M)$	$\leftrightarrow$	$M)$
1	1	0	1	0	0	1	0	1
1	0	0	1	1	1	0	0	0
0	1	1	0	1	0	1	1	1
0	0	1	0	1	1	0	0	0

4. $((A \rightarrow B) \leftrightarrow (\neg B \leftrightarrow \neg A))$

A	B	$((A$	$\rightarrow$	$B)$	$\leftrightarrow$	$(\neg$	B	$\leftrightarrow$	$\neg$	$A))$
1	1	1	1	1	1	0	1	1	0	1
1	0	1	0	0	1	1	0	0	0	1
0	1	0	1	1	0	0	1	0	1	0
0	0	0	1	0	1	1	0	1	1	0

5. $\neg\neg(\neg A \wedge \neg B)$

A	B	$\neg$	$\neg$	$(\neg$	A	$\wedge$	$\neg$	$B)$
1	1	0	1	0	1	0	0	1
1	0	0	1	0	1	0	1	0
0	1	0	1	1	0	0	0	1
0	0	1	0	1	0	1	1	0

6. $(((D \wedge R) \to I) \to \neg(D \vee R))$

D	I	R	$((D$	$\wedge$	$R)$	$\to$	$I)$	$\to$	$\neg$	$(D$	$\vee$	$R))$
1	1	1	1	1	1	1	1	0	0	1	1	1
1	1	0	1	0	0	1	1	0	0	1	1	0
1	0	1	1	1	1	0	0	1	0	1	1	1
1	0	0	1	0	0	1	0	0	0	1	1	0
0	1	1	0	0	1	1	1	0	0	0	1	1
0	1	0	0	0	0	1	1	1	1	0	0	0
0	0	1	0	0	1	1	0	0	0	0	1	1
0	0	0	0	0	0	1	0	1	1	0	0	0

7. $((C \leftrightarrow (D \vee E)) \wedge \neg C)$

C	D	E	$((C$	$\leftrightarrow$	$(D$	$\vee$	$E))$	$\wedge$	$\neg$	$C)$
1	1	1	1	1	1	1	1	0	0	1
1	1	0	1	1	1	1	0	0	0	1
1	0	1	1	1	0	1	1	0	0	1
1	0	0	1	0	0	0	0	0	0	1
0	1	1	0	0	1	1	1	0	1	0
0	1	0	0	0	1	1	0	0	1	0
0	0	1	0	0	0	1	1	0	1	0
0	0	0	0	1	0	0	0	1	1	0

8. $(\neg(G \wedge (B \wedge H)) \leftrightarrow (G \vee \neg(B \vee H)))$

B	G	H	$(\neg$	$(G$	$\wedge$	$(B$	$\wedge$	$H))$	$\leftrightarrow$	$(G$	$\vee$	$\neg$	$(B$	$\vee$	$H))$
1	1	1	0	1	1	1	1	1	0	1	1	0	1	1	1
1	1	0	1	1	0	1	0	0	1	1	1	0	1	1	0
1	0	1	1	0	0	1	1	1	0	0	0	0	1	1	1
1	0	0	1	0	0	1	1	0	0	0	0	0	1	1	0
0	1	1	1	1	0	0	0	1	1	1	1	0	0	1	1
0	1	0	1	1	0	0	0	0	1	1	1	1	0	0	0
0	0	1	1	0	0	0	0	1	0	0	0	0	0	1	1
0	0	0	1	0	0	0	0	0	1	0	1	1	0	0	0

Chapter ?? Ex ??:

1. $A \to A$, $\neg A \to \neg A$, $A \wedge A$, $A \vee A$ are mutually consistent.

A	$(A$	$\to$	$A)$	$(\neg$	A	$\to$	$\neg$	$A)$	$(A$	$\wedge$	$A)$	$(A$	$\vee$	$A)$
1	1	1	1	0	1	1	0	1	1	1	1	1	1	1
0	0	1	0	1	0	1	1	0	0	0	0	0	0	0

2. $A \vee B$, $A \to C$, $B \to C$ are mutually consistent.

A	B	C	$(A$	$\vee$	$B)$	$(A$	$\to$	$C)$	$(B$	$\to$	$C)$
1	1	1	1	1	1	1	1	1	1	1	1
1	1	0	1	1	1	1	0	0	1	0	0
1	0	1	1	1	0	1	1	1	0	1	1
1	0	0	1	1	0	1	0	0	0	1	0
0	1	1	0	1	1	0	1	1	1	1	1
0	1	0	0	1	1	0	1	0	1	0	0
0	0	1	0	0	0	0	1	1	0	1	1
0	0	0	0	0	0	0	1	0	0	1	0

3. $B \wedge (C \vee A)$, $A \to B$, $\neg(B \vee C)$ are mutually *in*consistent.

A	B	C	$(B$	$\wedge$	$(C$	$\vee$	$A))$	$(A$	$\to$	$B)$	$(\neg$	$(B$	$\vee$	$C))$
1	1	1	1	1	1	1	1	1	1	1	0	1	1	1
1	1	0	1	1	0	1	1	1	1	1	0	1	1	0
1	0	1	0	0	1	1	1	1	0	0	0	0	1	1
1	0	0	0	0	0	1	1	1	0	0	1	0	0	0
0	1	1	1	1	1	1	0	0	1	1	0	1	1	1
0	1	0	1	0	0	0	0	0	1	1	0	1	1	0
0	0	1	0	0	1	1	0	0	1	0	0	0	1	1
0	0	0	0	0	0	0	0	0	1	0	1	1	0	0

4. $A \leftrightarrow (B \vee C)$, $C \to \neg A$, $A \to \neg B$ are mutually consistent.

A	B	C	$(A$	$\leftrightarrow$	$(B$	$\vee$	$C))$	$(C$	$\to$	$\neg$	$A)$	$(A$	$\to$	$\neg$	$B)$
1	1	1	1	1	1	1	1	1	0	0	1	1	0	0	1
1	1	0	1	1	1	1	0	0	1	0	1	1	0	0	1
1	0	1	1	1	0	1	1	1	0	0	1	1	1	1	0
1	0	0	1	0	0	0	0	0	1	0	1	1	1	1	0
0	1	1	0	0	1	1	1	1	1	1	0	0	1	0	1
0	1	0	0	0	1	1	0	0	1	1	0	0	1	0	1
0	0	1	0	0	0	1	1	1	1	1	0	0	1	1	0
0	0	0	0	1	0	0	0	0	1	1	0	0	1	1	0

Chapter ?? Ex ??:

1. $A \to A \therefore A$ is invalid.

A	$(A$	$\to$	$A)$	A
1	1	1	1	1
0	0	1	0	0

2. $A \to (A \wedge \neg A) \therefore \neg A$ is valid.

A	$(A$	$\to$	$(A$	$\wedge$	$\neg$	$A)$	$\neg$	A
1	1	0	1	0	0	1	0	1
0	0	1	0	0	1	0	1	0

3. $A \vee (B \to A) \therefore \neg A \to \neg B$ is valid.

A	B	$(A$	$\vee$	$(B$	$\to$	$A)$	$(\neg$	A	$\to$	$\neg$	$B)$
1	1	1	1	1	1	1	0	1	1	0	1
1	0	1	1	0	1	1	0	1	1	1	0
0	1	0	0	1	0	0	1	0	0	0	1
0	0	0	1	0	1	0	1	0	1	1	0

4. $A \vee B, B \vee C, \neg A \therefore B \wedge C$ is invalid.

A	B	C	$(A$	$\vee$	$B)$	$(B$	$\vee$	$C)$	$\neg$	A	$(B$	$\wedge$	$C)$
1	1	1	1	1	1	1	1	1	0	1	1	1	1
1	1	0	1	1	1	1	1	0	0	1	1	0	0
1	0	1	1	1	0	0	1	1	0	1	0	0	1
1	0	0	1	1	0	0	0	0	0	1	0	0	0
0	1	1	0	1	1	1	1	1	1	0	1	1	1
0	1	0	0	1	1	1	1	0	1	0	1	0	0
0	0	1	0	0	0	0	1	1	1	0	0	0	1
0	0	0	0	0	0	0	0	0	1	0	0	0	0

5. $(B \wedge A) \to C, (C \wedge A) \to B \therefore (C \wedge B) \to A$ is invalid.

A	B	C	$((B \wedge A)$	$\to$	$C)$	$((C \wedge A)$	$\to$	$B)$	$((C \wedge B)$	$\to$	$A)$
1	1	1	1 1 1	1	1	1 1 1	1	1	1 1 1	1	1
1	1	0	1 1 1	0	0	0 0 1	1	1	0 0 1	1	1
1	0	1	0 0 1	1	1	1 1 1	0	0	1 0 0	1	1
1	0	0	0 0 1	1	0	0 0 1	1	0	0 0 0	1	1
0	1	1	1 0 0	1	1	1 0 0	1	1	1 1 1	0	0
0	1	0	1 0 0	1	0	0 0 0	1	1	0 0 1	1	0
0	0	1	0 0 0	1	1	1 0 0	1	0	1 0 0	1	0
0	0	0	0 0 0	1	0	0 0 0	1	0	0 0 0	1	0

Chapter ?? Ex ??:

1. $\mathcal{A}$ and $\mathcal{B}$ have the same truth value on every line of a complete truth table, so $\mathcal{A} \leftrightarrow \mathcal{B}$ is true on every line. It is a logical truth.
2. $(\mathcal{A} \wedge \mathcal{B}) \to \mathcal{C}$ is false on some line of a complete truth table. On that line, $\mathcal{A}$ and $\mathcal{B}$ are true and $\mathcal{C}$ is false. So the argument is invalid.
3. Since $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$ are mutually inconsistent, there is no line on which all three are true, so their conjunction will be a logical falsehood.
4. Since $\mathcal{A}$ is false on every line of a complete truth table, there is no line on which $\mathcal{A}$ and $\mathcal{B}$ are true and $\mathcal{C}$ is false. So the argument is valid.
5. Since $\mathcal{C}$ is true on every line of a complete truth table, there is no line on which $\mathcal{A}$ and $\mathcal{B}$ are true and $\mathcal{C}$ is false. So the argument is valid.
6. $(\mathcal{A} \vee \mathcal{B})$ is equivalent to $\mathcal{A}$. So $(\mathcal{A} \vee \mathcal{B})$ is a logical truth iff $\mathcal{A}$ is; a contradiction iff $\mathcal{A}$ is; and contingent iff $\mathcal{A}$ is.
7. $\mathcal{A}$ and $\mathcal{B}$ have different truth values on at least one line of a complete truth table, and $(\mathcal{A} \vee \mathcal{B})$ will be true on that line. On other lines, it might be true or false. So $(\mathcal{A} \vee \mathcal{B})$ is either a logical truth or it is contingent; it is *not* a contradiction.

Chapter ?? Ex ??:

1. A, $\neg A$ are not equivalent.

A	A	$\neg$	A
1	1	0	1

2. A, $A \vee A$ are equivalent.
3. $A \to A$, $A \leftrightarrow A$ are equivalent.
4. $A \vee \neg B$, $A \to B$ are not equivalent.

A	B	A	$\vee$	$\neg$	B	A	$\to$	B
1	0	1	1	1	0	1	0	0

5. $A \wedge \neg A$, $\neg B \leftrightarrow B$ are equivalent.
6. $\neg(A \wedge B)$, $\neg A \vee \neg B$ are equivalent.
7. $\neg(A \to B)$, $\neg A \to \neg B$ are not equivalent.

A	B	$\neg$	$(A$	$\to$	$B)$	$\neg$	A	$\to$	$\neg$	B
0	0	0	0	1	0	1	0	1	1	0
1	1	0	1	1	1	0	1	1	0	1

8. $(A \to B)$, $(\neg B \to \neg A)$ are equivalent.

Chapter ?? Ex ??:

1. $A \vee (A \to (A \leftrightarrow A)) \therefore A$ is invalid.

A	$(A$	$\vee$	$(A$	$\to$	$(A$	$\leftrightarrow$	$A))$	A
0	0	1	0	1	0	1	0	0

2. $A \leftrightarrow \neg(B \leftrightarrow A) \therefore A$ is invalid.

A	B	$(A$	$\leftrightarrow$	$\neg$	$(B$	$\leftrightarrow$	$A))$	A
0	0	0	1	0	0	1	0	0

3. $A \to B, B \therefore A$ is invalid.

A	B	$(A$	$\to$	$B)$	B	A
0	1	0	1	1	1	0

4. $A \vee B, B \vee C, \neg B \therefore A \wedge C$ is valid.
5. $A \leftrightarrow B, B \leftrightarrow C \therefore A \leftrightarrow C$ is valid.

Chapter ?? Ex ??:

1. The root of a truth tree is the list of formulas at the beginning of the tree. Trees test whether the root is consistent.
2. You should tick (✓) a formula when you've decomposed it on every open branch below that formula. This is simply to remind yourself that it has been decomposed, and so avoid decomposing it again.
3. A truth tree branch represents a potential valuation that may make the root formulas all true.
4. You should create branches when the formula being decomposed has more than one valuation that would make it true.
5. You should close a branch when it contains any formula and its negation, as we know that branch represents an impossible valuation.
6. An open branch is complete when all the formulas above it either have a check mark, or are atomic. Once you have found a complete open branch, you can stop growing the tree.
7. The collection of atomic formulas in an open branch describe a valuation under consideration. A *complete* open branch represents a valuation where all the formulas in the root are true.

Chapter ?? Ex ??: To be updated shortly.
Chapter ?? Ex ??:

1. (a) $\{P, P \to Q, Q \to \neg P\}$; (b) false.
2. (a) $\neg((P \to Q) \leftrightarrow (Q \to P))$; (b) true.
3. (a) $\{P \wedge Q, \neg R \to \neg Q, \neg(P \wedge R)\}$; (b) true.
4. (a) $\{A \vee B, B \to C, A \leftrightarrow C, \neg C\}$; (b) true.

5. (a) $A \leftrightarrow \neg A$; (b) true.
6. (a) $\{P, P \to Q, \neg Q, \neg A\}$; (b) true.
7. (a) $\{P \to Q, \neg P \vee \neg Q, Q \to P\}$; (b) false.

Chapter ?? Ex ??:

1. $P, P \to Q, Q \to \neg P$ are NOT mutually consistent

1.		P		Root
2.		$P \to Q$ ✓		Root
3.		$Q \to \neg P$ ✓		Root
4.	$\neg P$		Q	$2 \to$
	$\times$ $1, 4$			
5.		$\neg Q$	$\neg P$	$3 \to$
		$\times$ $4, 5$	$\times$ $1, 5$	

2. $(P \to Q) \leftrightarrow (Q \to P)$ is NOT a tautology.

1.	$\neg((P \to Q) \leftrightarrow (Q \to P))$ ✓		Root
2.	$P \to Q$ ✓	$Q \to P$	$1 \leftrightarrow$
3.	$\neg(Q \to P)$ ✓	$\neg(P \to Q)$	$1 \leftrightarrow$
4.	Q	$\vdots$	$3 \neg\!\to$
5.	$\neg P$		$3 \neg\!\to$
6.	$\neg P$ Q		$2 \to$
	$\uparrow$		

P	Q	$(P$	$\to$	$Q)$	$\leftrightarrow$	$(Q$	$\to$	$P)$
0	1	0	1	1	0	1	0	0

3. $P \wedge Q, \neg R \to \neg Q \;\therefore\; P \wedge R$ is valid.

1.	$P \wedge Q$ ✓			Premise
2.	$\neg R \to \neg Q$ ✓			Premise
3.	$\neg(P \wedge R)$ ✓			Neg Conc
4.	P			$1 \wedge$
5.	Q			$1 \wedge$
6.	$\neg\neg R$		$\neg Q$	$2 \to$
			$\times$ $5, 6$	
7.	$\neg P$	$\neg R$		$3 \neg\wedge$
	$\times$ $4, 7$	$\times$ $6, 7$		

4. Every valuation making P, $P \to Q$, and $\neg Q$ true also makes A true.

1.	P		Premise
2.	$P \to Q$		Premise
3.	$\neg Q$		Premise
4.	$\neg A$		Neg Conc
5.	$\neg P$	Q	$2 \to$
	$\times$ $1, 5$	$\times$ $3, 5$	

5. $A \vee B$, $B \to C$, and $A \leftrightarrow C$ cannot all be true but C false.

1.		$A \vee B$ ✓			Root
2.		$B \to C$ ✓			Root
3.		$A \leftrightarrow C$ ✓			Root
4.		$\neg C$			Root
5.		$\neg B$		C $\times$ $4,5$	$2 \to$
6.	A		B $\times$ $5,6$		$1 \vee$; $5 \vee$
7.	A	$\neg A$			$3 \leftrightarrow$
8.	C $\times$ $4,8$	$\neg C$ $\times$ $6,7$			$3 \leftrightarrow$

6. $A \leftrightarrow \neg A$ is a contradiction.

1.	$A \leftrightarrow \neg A$ ✓		Root
2.	A	$\neg A$	$1 \leftrightarrow$
3.	$\neg A$ $\times$ $2,3$	$\neg\neg A$ $\times$ $2,3$	$1 \leftrightarrow$

7. $P \to Q$, $\neg P \vee \neg Q$, and $Q \to P$ are mutually consistent.

1.			$P \to Q$		Root
2.			$\neg P \vee \neg Q$		Root
3.			$Q \to P$		Root
4.			$\neg P$	Q	$1 \to$
5.		$\neg P$	$\neg Q$		$2 \vee$
6.	$\neg Q$ $\uparrow$	P $\times$ $4,6$			$3 \to$

P	Q	$(P$	$\to$	$Q)$	$(\neg$	P	$\vee$	$\neg$	$Q)$	$(Q$	$\to$	$P)$
0	0	0	1	0	1	0	1	1	0	0	1	0

Chapter ?? Ex ??:

1.	$A \leftrightarrow B$ ✓	Premise
2.	$\neg B \rightarrow (C \vee D)$ ✓	Premise
3.	$E \rightarrow \neg C$ ✓	Premise
4.	$(\neg D \wedge F) \vee G$ ✓	Premise
5.	$\neg A \wedge E$ ✓	Premise
6.	$\neg(H \vee G)$ ✓	Neg Conc
7.	$\neg A$	5 $\wedge$
8.	E	5 $\wedge$
9.	$\neg H$	6 $\neg\vee$
10.	$\neg G$	6 $\neg\vee$
11.	$\neg E$ $\times$ (8, 11) ⎮ $\neg C$	3 $\rightarrow$
12.	(under $\neg C$:) $\neg D \wedge F$ ✓ ⎮ G	4 $\vee$
13.	$\neg D$ ⎮ $\times$ (10, 12)	12 $\wedge$
14.	F	12 $\wedge$
15.	$\neg\neg B$ ✓ ⎮ $C \vee D$ ✓	2 $\rightarrow$
16.	(under $C \vee D$:) C ⎮ D	15 $\vee$
17.	B ⎮ $\times$ (11, 16) ⎮ $\times$ (13, 16)	15 $\neg\neg$
18.	(under B:) A ⎮ $\neg A$	1 $\leftrightarrow$
19.	B $\times$ (7, 18) ⎮ $\neg B$ $\times$ (17, 19)	1 $\leftrightarrow$

This argument is valid.

Chapter ?? Ex ??:

1. The scope of $\exists x$ is Fx and the scope of $\exists y$ is Gy.
2. The scope of $\exists x$ is Fx and the scope of $\exists y$ is Fy.
3. The scope of $\exists x$ is $(Fx \wedge \exists y Gy)$ and the scope of $\exists y$ is Gy.
4. The scope of $\exists x$ is $(Fx \rightarrow \exists y((Gy \vee Hy) \wedge \neg\forall z(Fx \vee Gz)))$, the scope of $\exists y$ is $((Gy \vee Hy) \wedge \neg\forall z(Fx \vee Gz))$ and the scope of $\forall z$ is $(Fx \vee Gz)$.
5. The scope of $\exists x$ is $(Ga \wedge Hx))$.

Chapter ?? Ex ??:

1. $Za \wedge Zb \wedge Zc$
2. $Rb \wedge \neg Ab$
3. $Lcb \to Mb$
4. $(Ab \wedge Ac) \to (Lab \wedge Lac)$
5. $\exists x(Rx \wedge Zx)$
6. $\forall x(Ax \to Rx)$
7. $\forall x\big[Zx \to (Mx \vee Ax)\big]$
8. $\exists x(Rx \wedge \neg Ax)$
9. $\exists x(Rx \wedge Lcx)$
10. $\forall x\big[(Mx \wedge Zx) \to Lbx\big]$
11. $\forall x\big[(Mx \wedge Lax) \to Lxa\big]$
12. $\exists x Rx \to Ra$
13. $\forall x(Ax \to Rx)$
14. $\forall x\big[(Mx \wedge Lcx) \to Lax\big]$
15. $\exists x(Mx \wedge Lxb \wedge \neg Lbx)$

Chapter ?? Ex ??:

1. $\forall x[Cxp \to Dx]$
2. $Cjp \wedge Fj$
3. $\exists x[Cxp \wedge Fx]$
4. $\neg\exists x[Sxj]$
5. $\forall x\big[(Cxp \wedge Mx) \to Dx\big]$
6. $\neg\exists x[Cxp \wedge Mx]$
7. $\exists x[Cjx \wedge Sxe \wedge Fj]$
8. $Spe \wedge Mp$
9. $\forall x\big[(Sxp \wedge Mx) \to \neg\exists y Cyx\big]$
10. $\exists x[Sxj \wedge \exists y[Cyx] \wedge Fj]$
11. $\forall x\big[Dx \to \exists y(Sxy \wedge My \wedge Dy)\big]$
12. $\forall x\big[(Fx \wedge Dx) \to \exists y[Cxy \wedge Dy]\big]$

Chapter ?? Ex ??:

1. $\forall x[Cx \to (Bx \wedge Sx)]$.
2. $\neg\exists x[Wx \wedge Sx]$
3. $\exists x\exists y[Cx \wedge Cy \wedge x \neq y]$
4. $\exists x\exists y[Jx \wedge Ox \wedge Jy \wedge Oy \wedge x \neq y]$
5. $\forall x\forall y\forall z\big[(Jx \wedge Ox \wedge Jy \wedge Oy \wedge Jz \wedge Oz) \to (x = y \vee x = z \vee y = z)\big]$
6. $\exists x\exists y\big[Jx \wedge Bx \wedge Jy \wedge By \wedge x \neq y \wedge \forall z[(Jz \wedge Bz) \to (x = z \vee y = z)]\big]$
7. $\exists x_1\exists x_2\exists x_3\exists x_4\big[Dx_1 \wedge Dx_2 \wedge Dx_3 \wedge Dx_4 \wedge x_1 \neq x_2 \wedge x_1 \neq x_3 \wedge x_1 \neq x_4 \wedge x_2 \neq x_3 \wedge x_2 \neq x_4 \wedge x_3 \neq x_4 \wedge \neg\exists y[Dy \wedge y \neq x_1 \wedge y \neq x_2 \wedge y \neq x_3 \wedge y \neq x_4]\big]$

8. $\exists x\big[Dx \wedge Cx \wedge \forall y[(Dy \wedge Cy) \to x = y] \wedge (Bx \wedge Sx)\big]$
9. $\forall x\big[(Ox \wedge Jx) \to Wx\big] \wedge \exists x\big[Mx \wedge \forall y[My \to x = y] \wedge Wx\big]$
10. $\exists x\big[Dx \wedge Cx \wedge \forall y[(Dy \wedge Cy) \to x = y] \wedge (Wx \wedge Sx)\big] \to$ $\exists x \forall y[(Wx \wedge Sx) \leftrightarrow x = y]$
11. wide scope: $\neg\exists x\big[Mx \wedge \forall y(My \to x = y) \wedge Jx\big]$
    narrow scope: $\exists x\big[Mx \wedge \forall y(My \to x = y) \wedge \neg Jx\big]$
12. wide scope: $\neg\exists x \exists z\big[Dx \wedge Cx \wedge Mz \wedge \forall y[(Dy \wedge Cy) \to x = y] \wedge$ $\forall y[(My \to z = y) \wedge x = z]\big]$
    narrow scope: $\exists x \exists z\big[Dx \wedge Cx \wedge Mz \wedge \forall y[(Dy \wedge Cy) \to x = y] \wedge$ $\forall y[(My \to z = y) \wedge x \neq z]\big]$

Chapter ?? Ex ??:

1. True
2. False
3. True
4. False
5. False
6. False
7. True
8. True
9. False
10. True
11. True
12. False
13. True
14. False
15. True

Chapter ?? Ex ??:

1. $\forall x \forall y(Gxy \to \exists z Gxz)$

1.	$\neg\forall x\,\forall y\,Gxy \to \exists z\,[Gxz]$ ✓	Root
2.	$\exists x\,\neg\forall y\,Gxy \to \exists z\,[Gxz]$ ✓a	1 $\neg\forall$
3.	$\neg\forall y\,Gay \to \exists z\,[Gaz]$ ✓	2 $\exists$ a
4.	$\exists y\,\neg(Gay \to \exists z\,[Gaz])$ ✓b	3 $\neg\forall$
5.	$\neg(Gab \to \exists z\,[Gaz])$	4 $\exists$ b
6.	Gab	5 $\neg\to$
7.	$\neg\exists z\,[Gaz]$ ✓	5 $\neg\to$
8.	$\forall z\,[\neg Gaz]\ \backslash b$	7 $\neg\exists$
9.	$\neg Gab$	8 $\forall$ b
	$\times$	

2. $\forall x Fx \vee \forall x(Fx \to Gx)$

1.	$\neg(\forall x\,[Fx] \vee \forall x\,[Fx \to Gx])$ ✓	Root
2.	$\neg\forall x\,[Fx]$ ✓	1 ¬∨
3.	$\neg\forall x\,[Fx \to Gx]$ ✓	1 ¬∨
4.	$\exists x\,[\neg Fx]$ ✓a	2 ¬∀
5.	$\neg Fa$ ✓	4 ∃ a
6.	$\exists x\,[\neg(Fx \to Gx)]$ ✓b	3 ¬∀
7.	$\neg(Fb \to Gb)$ ✓	3 ∃ b
8.	Fb	7 ¬ →
9.	$\neg Gb$	7 ¬ →
	↑	

	F	G
a	0	?
b	1	0

3. $\forall x(Fx \to (\neg Fx \to \forall y Gy))$

1.	$\neg\forall x\,[Fx \to (\neg Fx \to \forall y\,[Gy])]$ ✓	Root
2.	$\exists x\,[\neg(Fx \to (\neg Fx \to \forall y\,[Gy]))]$ ✓	1 ¬∀
3.	$\neg(Fa \to (\neg Fa \to \forall y\,[Gy]))$ ✓	2 ∃ a
4.	Fa	3 ¬ →
5.	$\neg(\neg Fa \to \forall y\,[Gy])$	3 ¬ →
6.	$\neg Fa$	5 ¬ →
7.	$\neg\forall y\,[Gy]$	5 ¬ →
	×	

4. $\exists x Jx \leftrightarrow \neg\forall x \neg Jx$

1.	$\neg(\exists x\,[Jx] \leftrightarrow \neg\forall x\,[\neg Jx])$ ✓		Root
2.	$\exists x\,[Jx]$ ✓a	$\neg\exists x\,[Jx]$	1 ¬ ↔
3.	$\neg\neg\forall x\,[\neg Jx]$ ✓	$\neg\forall x\,[\neg Jx]$	1 ¬ ↔
4.	$\forall x\,[\neg Jx]$ ✓		3 ¬¬
5.	Ja		2 ∃ a
6.	$\neg Ja$		4 ∀ a
7.	×	$\forall x\,[\neg Jx]\ \backslash b$	2 ¬∃
8.		$\exists x\,[\neg\neg Jx]$ ✓b	3 ¬∀
9.		$\neg\neg Jb$	8 ∃ b
10.		$\neg Jb$	7 ∀ b
		×	

5. $\exists x(Fx \vee \neg Fx)$

1.	$\neg\exists x\,[Fx \vee \neg Fx]$ ✓	Root
2.	$\forall x\,[\neg(Fx \vee \neg Fx)]\ \backslash a$	1 $\neg\exists$
3.	$\neg(Fa \vee \neg Fa)$ ✓	2 $\forall$ a
4.	$\neg Fa$	3 $\neg\vee$
5.	$\neg\neg Fa$	3 $\neg\vee$
	$\times$	

6. $\forall x(Fx \vee Gx) \rightarrow (\forall yFy \vee \exists xGx)$

1.	$\neg(\forall x\,[Fx \vee Gx] \rightarrow (\forall y\,[Fy] \vee \exists x\,[Gx]))$ ✓	Root
2.	$\forall x\,[Fx \vee Gx]\ \backslash a$	1 $\neg\rightarrow$
3.	$\neg(\forall y\,[Fy] \vee \exists x\,[Gx])$ ✓	1 $\neg\rightarrow$
4.	$\neg\forall y\,[Fy]$ ✓	3 $\neg\vee$
5.	$\neg\exists x\,[Gx]$ ✓	3 $\neg\vee$
6.	$\exists y\,[\neg Fy]$ ✓a	4 $\neg\exists$
7.	$\neg Fa$	6 $\neg\exists$
8.	$\forall x\,[\neg Gx]\ \backslash a$	5 $\neg\exists$
9.	$Fa \vee Ga$	2 $\forall$ a
10.	Fa Ga	9 $\vee$
11.	$\times$ $\neg Ga$	8 $\forall$ a
	$\times$	

Chapter ?? Ex ??:

1. Fa, Ga $\therefore$ $\forall x(Fx \rightarrow Gx)$

1.	Fa	Premise
2.	Ga	Premise
3.	$\neg\forall x\,[Fx \rightarrow Gx]$ ✓	Neg Conc
4.	$\exists x\,[\neg(Fx \rightarrow Gx)]$ ✓b	3 $\neg\forall$
5.	$\neg(Fb \rightarrow Gb)$	4 $\exists$ b
6.	Fb	5 $\neg\rightarrow$
7.	$\neg Gb$	5 $\neg\rightarrow$
	$\uparrow$	

	F	G
a	1	1
b	1	0

2. Fa, Ga $\therefore$ $\exists x(Fx \wedge Gx)$

1.	Fa	Premise
2.	Ga	Premise
3.	$\neg\exists x\,[Fx \wedge Gx]$ ✓	Neg Conc
4.	$\forall x\,[\neg(Fx \wedge Gx)]\ \backslash a$	3 $\neg\exists$
5.	$\neg(Fa \wedge Ga)$ ✓	4 $\forall$ a
6.	$\neg Fa \qquad \neg Ga$	5 $\neg\wedge$
	$\times \qquad \times$	

3. $\forall x \exists y Lxy$ $\therefore$ $\exists x \forall y Lxy$

1.	$\forall x\,\exists y\,[Lxy]\ \backslash a$	Premise
2.	$\neg\exists x\,\forall y\,[Lxy]$ ✓	Neg Conc
3.	$\forall x\,\neg\forall y\,[Lxy]\ \backslash a$	2 $\neg\exists$
4.	$\exists y\,[Lay]$ ✓b	1 $\forall$ a
5.	Lab	4 $\exists$ b
6.	$\neg\forall y\,[Lay]$ ✓	3 $\forall$ a
7.	$\exists y\,[\neg Lay]$ ✓c	6 $\neg\forall$
8.	$\neg Lac$	7 $\exists$ c
	$\uparrow$	

L	a	b	c
a	?	1	0
b	1	?	?
c	1	?	?

Note that this tree is infinite, as both #1/#4 and #3/#6 will generate new atomic letters every time they are decomposed. The greyed values are not read from the tree, but instead 'guessed' to complete the counter-example.

4. $\exists x(Fx \wedge Gx)$, $Fb \leftrightarrow Fa$, $Fc \rightarrow Fa \therefore Fa$

1.	$\exists x\,[Fx \wedge Gx]$ ✓			Premise
2.	$Fb \leftrightarrow Fa$			Premise
3.	$Fc \rightarrow Fa$			Premise
4.	$\neg Fa$			Neg Conc
5.	$Fd \wedge Gd$			1 $\exists$ d
6.	$\neg Fc$		Fa	3 $\rightarrow$
			$\times$	
7.	Fb	$\neg Fb$		2 $\leftrightarrow$
8.	Fa	$\neg Fa$		2 $\leftrightarrow$
9.	$\times$	Fd		5 $\wedge$
10.		Gd		5 $\wedge$
		$\uparrow$		

	F	G
a	0	?
b	0	?
c	0	?
d	1	1

5. $\forall x \exists y Gyx \therefore \forall x \exists y(Gxy \vee Gyx)$

1.	$\forall x\,[\exists y\, Gyx]\ \backslash a$	Premise
2.	$\neg\forall x\,[\exists y\, Gxy \vee Gyx]$ ✓	Neg Conc
3.	$\exists x\,[\neg\exists y\, Gxy \vee Gyx]$ ✓a	2 $\neg\forall$
4.	$\neg\exists y\,[Gay \vee Gya]$ ✓	3 $\exists$ a
5.	$\exists y\, Gya$ ✓b	1 $\forall$ a
6.	Gba	5 $\exists$ b
7.	$\forall y\,[\neg(Gay \vee Gya)]\ \backslash b$	4 $\neg\exists$
8.	$\neg(Gab \vee Gab)$ ✓	7 $\forall$ b
9.	$\neg Gab$	8 $\neg\vee$
10.	$\neg Gba$	8 $\neg\vee$
	$\times$	

Chapter ?? Ex ??:

1. $b = b \to (Rca \lor Pb), a = b \to (Rcb \lor Pb), b = b \to (Rcb \lor Pb)$
2. $a = a \to (Rca \lor Pb), a = b \to (Rca \lor Pa), a = a \to (Rca \lor Pa)$.
3. $c = b \to (Rca \lor Pb), a = b \to (Rcc \lor Pb), c = b \to (Rcc \lor Pb)$.
4. $a = b \to (Raa \lor Pb)$.
5. $a = c \to (Rca \lor Pb), a = b \to (Rca \lor Pc), a = c \to (Rca \lor Pc)$.
6. $a = b \to (Rba \lor Pb)$.

Chapter ?? Ex ??:

1. $\forall x(x = a \lor x = b)$, $\neg(Fa \land Ga)$, $Gb \to Hb \therefore \forall x((Fx \land \neg Hx) \to \neg Gx)$

1.	$\forall x\,[x = a \lor x = b] \backslash c$				Premise
2.	$\neg(Fa \land Ga)$ ✓				Premise
3.	$Gb \to Hb$ ✓				Premise
4.	$\neg\forall x\,[(Fx \land \neg Hx) \to \neg Gx]$ ✓				Neg Conc
5.	$\exists x\,[\neg((Fx \land \neg Hx) \to \neg Gx)]$ ✓ a				4 $\neg\forall$
6.	$\neg((Fc \land \neg Hc) \to \neg Gc)$ ✓				5 $\exists$ c
7.	$Fc \land \neg Hc$ ✓				6 $\neg \to$
8.	$\neg\neg Gc$ ✓				6 $\neg \to$
9.	Gc				8 $\neg\neg$
10.	Fc				7 $\land$
11.	$\neg Hc$				7 $\land$
12.	$c = a \lor c = b$ ✓				1 $\forall$ c
13.	$c = a$		$c = b$		12 $\lor$
14.	$\neg(Fc \land Gc)$ ✓				2, 13 $[a/c]$
15.	$\neg Fc$	$\neg Gc$			14 $\neg\land$
16.	$\times$	$\times$	$Gc \to Hc$ ✓		3, 13 $[b/c]$
17.			$\neg Gc$	Hc	16 $\to$
			$\times$	$\times$	

2. $\exists xFx, \forall x\forall y((Fx \wedge Fy) \to x = y) \therefore \exists x\forall y(Fy \leftrightarrow (x = y))$

1.	$\forall x\,[Fx]$ ✓a	Premise
2.	$\forall x\,\forall y\,[(Fx \wedge Fy) \to x = y]\ \backslash a$	Premise
3.	$\neg\exists x\,\forall y\,[Fy \leftrightarrow (x = y)]$ ✓	Neg Conc
4.	$\forall x\,[\neg(\forall y\,[Fy \leftrightarrow (x = y)])]\ \backslash a$	3 ¬∃
5.	Fa	1 ∃ a
6.	$\neg(\forall y\,[Fy \leftrightarrow (a = y)])$ ✓	4 ∀ a
7.	$\exists y\,[\neg(Fy \leftrightarrow (a = y))]$ ✓b	6 ¬∀
8.	$\neg(Fb \leftrightarrow (a = b))$ ✓	7 ∃ b
9.	$Fb \qquad \neg Fb$	8 ¬ ↔
10.	$\neg(a = b) \qquad a = b$	8 ¬ ↔
11.	$\forall y\,[(Fa \wedge Fy) \to a = y]\ \backslash b$	2 ∀ a
12.	$(Fa \wedge Fb) \to a$ ✓	11 ∀ b
13.	$\neg Fa$ (right branch)	9, 10 $[b/a]$
14.	$\neg(Fa \wedge Fb)$ ✓ $\qquad a = b$ × $\qquad$ × (right branch)	12 →
15.	$\neg Fa$ × $\qquad \neg Fb$ ×	14 ¬∧

3. $\forall x\forall y(Fx \to x = y), \forall x(Fx \vee Gx) \therefore (\forall xFx \vee \forall xGx)$

1.	$\forall x\,\forall y\,[Fx \to x = y]\ \backslash b$	Premise
2.	$\forall x\,[Fx \vee Gx]\ \backslash b$	Premise
3.	$\neg(\forall x\,[Fx] \vee \forall x\,[Gx])$ ✓	Neg Conc
4.	$\neg\forall x\,[Fx]$ ✓	3 ¬∨
5.	$\neg\forall x\,[Gx]$ ✓	3 ¬∨
6.	$\exists x\,[\neg Fx]$ ✓a	4 ¬∀
7.	$\neg Fa$	6 ∃ a
8.	$\exists x\,[\neg Gx]$ ✓b	5 ¬∀
9.	$\neg Gb$	8 ∃ b
10.	$Fb \vee Gb$ ✓	2 ∀ b
11.	$Fb \qquad Gb$	10 ∨
12.	$\forall y\,[Fb \to b = y]\ \backslash a \qquad$ ×	1 ∀ b
13.	$Fb \to b = a$ ✓	12 ∀ a
14.	$\neg Fb \qquad b = a$	13 →
15.	× $\qquad Fa$ ×	11, 14 $[b/a]$

4. $\exists x \forall y(Fy \leftrightarrow (x = y)) \therefore \exists x Fx$

1.	$\exists x\, [\forall y\, [Fy \leftrightarrow (x = y)]] \checkmark a$		Premise
2.	$\neg \exists x\, [Fx] \checkmark$		Neg Conc
3.	$\forall x\, [\neg Fx] \setminus a$		2 $\neg\exists$
4.	$\forall y\, [Fy \leftrightarrow (a = y)] \setminus a$		1 $\exists$ a
5.	$\neg Fa$		3 $\forall$ a
6.	$Fa \leftrightarrow (a = a)$		4 $\forall$ a
7.	Fa	$\neg Fa$	6 $\leftrightarrow$
8.	$a = a$	$\neg(a = a)$	6 $\leftrightarrow$
9.	$\times$	$a = a$	Identity
		$\times$	

5. $\exists x \forall y(Fy \leftrightarrow (x = y)) \therefore \forall x \forall y((Fx \wedge Fy) \to (x = y))$

1.	$\exists x\, \forall y\, [Fy \leftrightarrow (x = y)] \checkmark c$		Premise
2.	$\neg \forall x\, \forall y\, [(Fx \wedge Fy) \to (x = y)] \checkmark$		Neg Conc
3.	$\exists x\, [\neg \forall y\, [(Fx \wedge Fy) \to (x = y)]] \checkmark a$		2 $\neg\forall$
4.	$\neg \forall y\, [(Fa \wedge Fy) \to (a = y)] \checkmark$		3 $\exists$ a
5.	$\exists y\, [\neg((Fa \wedge Fy) \to (a = y))] \checkmark b$		4 $\neg\forall$
6.	$\neg((Fa \wedge Fb) \to (a = b)) \checkmark$		5 $\exists$ b
7.	$Fa \wedge Fb \checkmark$		6 $\neg \to$
8.	$\neg(a = b) \checkmark$		6 $\neg \to$
9.	Fa		7 $\wedge$
10.	Fb		7 $\wedge$
11.	$\forall y\, [Fy \leftrightarrow (c = y)] \setminus a, b$		1 $\exists$ c
12.	$Fa \leftrightarrow (c = a) \checkmark$		11 $\forall$ a
13.	Fa	$\neg Fa$	12 $\leftrightarrow$
14.	$c = a$	$\neg(c = a)$	12 $\leftrightarrow$
15.	$Fb \leftrightarrow (c = b) \checkmark$	$\times$	11 $\forall$ b
16.	Fb / $\neg Fb$		15 $\leftrightarrow$
17.	$c = b$ / $\neg(c = b)$		15 $\leftrightarrow$
18.	$a = b$ / $\times$		$14, 17\ [c/a]$
	$\times$		

6. $\exists x \forall y (Fy \leftrightarrow (x = y))$, $\forall x (Fx \vee Gx)$ $\therefore$ $\forall x \forall y ((Gx \vee Gy) \vee x = y)$

1.	$\exists x\, [\forall y\, [Fy \leftrightarrow (x = y)]] \checkmark c$		Premise
2.	$\forall x\, [Fx \vee Gx] \setminus a, b$		Premise
3.	$\neg \forall x\, [\forall y\, [(Gx \vee Gy) \vee x = y]] \checkmark$		Neg Conc
4.	$\exists x\, [\neg \forall y\, [(Gx \vee Gy) \vee x = y]] \checkmark a$		3 $\neg\forall$
5.	$\neg \forall y\, [(Ga \vee Gy) \vee a = y] \checkmark$		4 $\exists$ a
6.	$\exists y\, [\neg((Ga \vee Gy) \vee a = y)] \checkmark b$		5 $\neg\forall$
7.	$\neg((Ga \vee Gb) \vee a = b) \checkmark$		6 $\exists$ b
8.	$\neg(Ga \vee Gb) \checkmark$		7 $\neg\vee$
9.	$\neg(a = b) \checkmark$		7 $\neg\vee$
10.	$\neg Ga$		8 $\neg\vee$
11.	$\neg Gb$		8 $\neg\vee$
12.	$\forall y\, [Fy \leftrightarrow (c = y)] \setminus a, b$		1 $\exists$ c
13.	$Fa \vee Ga \checkmark$		2 $\forall$ a
14.	Fa	Ga	13 $\vee$
15.	$Fb \vee Gb \checkmark$	$\times$	2 $\forall$ b
16.	Gb / Fb		15 $\vee$
17.	$\times$ / $Fa \leftrightarrow (c = a) \checkmark$		12 $\forall$ a
18.	Fa / $\neg Fa$		17 $\leftrightarrow$
19.	$c = a$ / $\neg(c = a)$		17 $\leftrightarrow$
20.	$Fb \leftrightarrow (c = b) \checkmark$ / $\times$		12 $\forall$ a
21.	Fb / $\neg Fb$		20 $\leftrightarrow$
22.	$c = b$ / $\neg(c = b)$		20 $\leftrightarrow$
23.	$a = b$ / $\times$		$19, 22\ [c/a]$
	$\times$		