Part I

Key Notions of Logic

Chapter 1

What is Logic?

Logic studies which claims can go together without causing problems, and which claims have to be true whenever other ones are. It also studies the methods we use for checking for problems and truth, and their limitations.

That's vague. Let's try again:

- Formal Logic, which is the topic of this textbook, uses formal, symbolic methods for manipulating representations of claims to demonstrate relationships between them. The relationships that we are interested are solely those related to the truth or falsity of the claims, and not what might be known, believed, implied, inferred, or contextually relevant.

- Logic does not study how humans use language to communicate claims, or the cultural or social significance of exactly how a claim is phrased. It is not dependent on which language is used, or the social or moral standing of the logician, or the knowledge, power, or skill of the logician. Logic is person-independent. Or so we would like to think.

- The logics we will introduce in this book are formal, algebraic, and central to the Western philosophical tradition of the ancient Greeks, Indians, and Romans, the Medieval Islamic and Christian scholars, and the modern mathematicians, computer scientists, and linguists. They are the two core, primary logics that underpin formal reasoning today.

- Logic is not a study of how we think or reason, or even a very good one story about how we should reason. It is not psychology, although it is used in psychology.

One of the key attributes of Logic is that it is precise, and that everything we talk about can be defined. So we will attempt to describe our terms more carefully, in preparation for providing full and precise definitions for our logical notions.

1.1 Statements

Logic is a study of relationships between claims. These claims are something like thoughts, and something like sentences. But all we are interested in is the truth or falsity of the claims, not the rest of their properties. Thoughts are hard to examine, so we will just consider sentences. The type of sentence that is used to directly make truth claims is a STATEMENT.

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> If we were being careful, we would talk solely of *propositions*. A proposition is whatever is in common between different statements that 'mean the same thing', whether written or spoken, whether in English, Chinese or Serbian, and even if only thought, or expressed in language. This is because we can apply logic to our thoughts, as well as to sentences, and logic is constant regardless of which language (if any) we use. However, we will continue to talk of statements.

You should not confuse the idea of a statement with the difference between fact and opinion. Often, statements in logic will express things that would count as facts – such as 'Kierkegaard was a hunchback' or 'Kierkegaard liked almonds.' They can also express things that you might think of as matters of opinion – such as, 'Almonds are tasty.' In other words, a statement is not disqualified because we don't know if it is true or false, or because its truth or falsity is a matter of opinion. We only need it to be either true or false, and not anything else.

There are many types of sentences that are not statements, and we will usually ignore them, or re-write them as statements. It's worth considering some of the major classes of non-statements, just so we don't get tripped up.

Questions 'Are you sleepy yet?' is a question. Although your answer might be a statement, the question itself is neither true nor false. e.g., 'What is Logic?' is not a statement, but 'Logic is mysterious' is a statement.

Commands Sentences like 'Wake up!', 'Sit up straight', and so on are commands. Although it might be good for you to wake up, the command is neither true nor false.

Exclamations 'Ouch!' is an exclamation; it is neither true nor false. We will treat 'Ouch, I hurt my toe!' as meaning the same thing as 'I hurt my toe.' The 'ouch' does not explicitly add any information to the claim.

1.2 Statement Relationships

Most statements can be true if the world is one way, and false if the world is another way. This seems obvious, and even uninteresting. But if we reflect on the possible relationships between statements, we might end up asking ourselves questions like these:

1. Can this collection of statements all be true together?
2. Does this statement have to be true if these other statements are true?
3. Do these two statements always have the same truth value as each other?
4. Do these two statements always have different truth values from each other?
5. Is this statement always true, or always false, or in-between?

Let's consider examples of each of these relationships.

#1 'I am a dog' and 'I am a cat' can't both be true; neither can 'Zebras are a type of fish' and 'Zebras are not a type of fish'. What about 'Zach is happy' and 'Zach is sad' – can you be happy and sad at the same time?

#2 If 'I am a dog' is true, then 'I am a mammal' is also true. If '2+2=4' is true, then 'Some maths equations are true' is true. If 'Orange is the new black' is true, then 'Either I like strawberry icecream or I don't' is true, even though it has nothing to do with orange.

#3 The pair 'Kim is taller than Sally' and Sally is shorter than Kim' are both true, or both false. So are the pair 'John married Pat' and 'Pat married John'. But the pair 'John loved Pat' and 'Pat loved John' need not be.

#4 Exactly one of the pair 'All giraffes are ugly' and 'Not all giraffes are ugly' is true. But this may not be the case with the pair 'All giraffes are ugly' and 'All giraffes are not ugly' – perhaps only some giraffes are ugly. Nor with 'The Duke of Auckland is bald', and 'The Duke of Auckland is not bald' – there is a Baron of Auckland, but no Duke.

#5: 'I am here now' might always be true whenever it's spoken, but perhaps not when it recorded and played back. What about 'Everyone dies, eventually'? Here's a more complex example: 'If Auckland is south of Beijing and Beijing is south of Oslo, then Auckland is south of Oslo'. And there are statements that seem to always be false such as 'Every dog is a cat', or 'Tomorrow is yesterday', or 'Red is not red'.

Exploring techniques for investigating these five relationships, particularly #2 and #3, will occupy us for the rest of this book. We'll shortly be defining these relationships much more precisely.

1.3 What is Truth?

The only property of statements that logic needs is their truth value. So, what values of truth are there? To avoid a lot of controversial philosophy, we will make some assumptions:

First, that in every possibility, every statement is either true or not true. So any scenario that leaves the truth-value of a statement undetermined will not be considered. For instance, a scenario where it is neither true nor false that I like chocolate is not a possibility we would consider. When dealing with predictions, we might say instead that every statement will eventually either be true or not true.

Second, that if a statement isn't true in a possibility, it is FALSE; that is, every statement is either true or false in a possibility.

Third, that it is impossible that a statement is both true and false.

We will use the symbol '1' for the truth value TRUE and the symbol '0' for the truth value FALSE. We assume that these are the only two truth values, and that every statement has exactly one of these truth values.

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Even if these assumptions seem common-sense to you, they are controversial among philosophers of logic. First of all, there are logicians who want to study statements that are neither true nor false, but have some kind of intermediate level of truth or probability. More controversially, some logicians think we should allow for the possibility of statements being both true and false at the same time.

It's important not to be confused between a statement being true or false, and us knowing whether a statement is true or false. We will often not know the truth value of a statement. But there will be an actual truth of the matter; we merely don't happen to know it. If this occurs, we'll use '?' to represent that we haven't got the information to determine a truth value. This is NOT a third truth value. We will be using just two truth values – true and false – in this book.

If you think that's sorted out all our problems with truth, consider what's known as 'the Liar sentence':

This statement is false.

If it's true, it's false. And if it's false, it's true. So it's obviously both. Or neither. Is it even a statement? We'll just pretend it doesn't exist.

Practice Exercises

At the end of most chapters there are exercises that review and explore the material covered in the chapter. Actually working through some problems is really important, because learning logic is more about developing a way of thinking than it is about memorising facts. Learning it is a surprising practical, hands-on activity.

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Exercise A: Which of the following sentences are statements?

1. The trains are always late.
2. Welcome to the University of Auckland.
3. Tailgating is a major cause of car accidents.
4. How can I stop tailgating?
5. The reason that I like bananas is that they have no bones.
6. When the car ahead passes an object, make sure you can count to four crocodiles before you pass the same object.
7. Just leave them in the bathroom and I'll deal with them on Wednesday afternoon.
8. Logic is the study of deductive arguments.
9. Never engage in light treason.
10. All great things are the cause of their own self-destruction.
11. Only 5% of North Americans can locate New Zealand on a world map.
12. Do at least a few of the exercises from the textbook every day.
13. An apple a day keeps the doctor away.
14. Ask me if I'm a policeman.
15. Are you a policeman?
16. The Mayor said bus passengers should be belted.

Exercise B: For each statement in Exercise A, write down one statement that is true whenever the original statement is true, one which is true exactly when it is false, and one that is never true when it is true, but can be false sometimes when it is false.

Chapter 2
Logic and Arguments

2.1 Arguments

One important use for Logic is evaluating arguments; sorting the good from the bad. People sometimes use the word 'argument' to talk about belligerent shouting matches. Logic is not concerned with such teeth-gnashing and hair-pulling. For logicians, an argument is not a disagreement, it is a reasoned series of statements culminating in a conclusion, such as in maths.

To be perfectly general, we can define an ARGUMENT as a series of statements. All the statements are premises, except for the final sentence, which is the conclusion. An argument whose conclusion follows from its premises will be called a VALID argument.

An argument, as we will understand it, is something more like this:

Either the butler or the gardener did it.
The butler didn't do it.
$\therefore$ The gardener did it.

We have here a series of sentences. The three dots on the final line of the argument are read as 'therefore'. They indicate that this sentence is the *conclusion* of the argument. The two sentences before that are the *premises* of the argument.

We will discuss some further concepts that apply to arguments in a natural language like English, so that we begin with a clear understanding of what arguments are and what it means for an argument to be valid. Later we will 'translate' arguments from English into a formal language, explore some of the limitations of this process, then move to working predominantly with arguments in the formal language. We will approach logic in this order as we want validity, as defined in the formal language, to have at least some of the important features of natural-language validity.

2.2 Introducing Validity

Some arguments seem to be 'logical', even if we don't have much background as to what's going on. Recall this argument:

Either the butler or the gardener did it.
The butler didn't do it.
∴ The gardener did it.

We don't have any context for what the statements refer to. Perhaps you suspect that 'did it' here means they committed some unspecified crime. You might imagine that the argument occurs in a mystery novel or TV show, perhaps spoken by a detective working through the evidence.

But even without having any of this information, you hopefully agree that the argument is a good one in the sense that the conclusions seems to follow from the premises. That is, if both the premises are true (whatever they refer to), the conclusion must be true as well. There just seems to be no possibility that the premises could be true and the conclusion false. So we say it is VALID.

But not all arguments are valid. For example:

If the driver did it, the maid didn't do it.
The maid didn't do it.
∴ The driver did it.

This argument is different from the previous one in an important respect. If the premises are true, the conclusion need not also be true. It is possible that neither the driver nor the maid did it; in this possibility both the premises are true, but the conclusion is false. Here, the conclusion does not follow from the premises alone. So we say it is INVALID.

So arguments where the conclusion follows from the premises are called valid, and those where the conclusion does not follow from the premises are invalid. Although we still hasn't said exactly what 'follows from' means, it's a solid start:

> An argument is VALID if and only if the conclusion follows from the premises.

> An argument is INVALID if and only if it is not valid.

Now we just need to pin down 'follows from', which seems to involve whether there is a possibility that the argument can go wrong. So we'll next explore what a possibility is, and what it means for an argument to go wrong.

2.3 Counter-examples

For a conclusion to follow from its premises, the truth of the premises must *guarantee* the truth of the conclusion. So if there is a possibility that the premises are true and the conclusion is not, the conclusion cannot follow from the premises, and we say the argument is INVALID.

This possibility that the argument can go wrong is a COUNTEREXAMPLE to the argument. For example, in the second argument above, we pointed to a possibility that neither the driver nor maid did it, but some other person did. This was a counter-example to the argument, showing the argument was invalid.

There can be more than one counterexample to an argument. For instance, in the above argument, perhaps neither the maid nor the driver 'did it' because no one 'did it' at all. Perhaps there was a misunderstanding or deception. This serves equally well as a counterexample. But you only *need* one. One failure is enough to show than an argument is invalid; after that, we are just being mean.

Now that we know what it means for a conclusion to follow from its premises, we can define this:

> A COUNTER-EXAMPLE to the argument $B_1, \ldots, B_n \therefore A$ is the possibility that $B_1, \ldots, B_n$ are all true and A is not true.

> A statement A FOLLOWS FROM $B_1, \ldots, B_n$ if and only if there is no counter-example to the argument $B_1, \ldots, B_n \therefore A$.

For these definitions to be useful, we'll still need to define what a possibility is, and what it means to be true in a possibility. But we are getting closer.

Logicians are in the business of making the concept of 'possibility' more precise, and investigating which arguments are valid under various types of possibility. If we take 'possibility' to mean 'hypothetical scenario' like the counterexample to the second argument, then the first argument counts as valid. If we imagine a scenario in which either the butler or the gardener did it, and also the butler didn't do it, we are automatically imagining a scenario in which the gardener did it. So any hypothetical scenario in which the premises of our first argument are true automatically makes the conclusion of our first argument true. This makes the first argument valid.

However, there are several ways of unpacking the concept of 'hypothetical'. Can we change past history, or natural laws, or the meaning of words? How hypothetical can we get?

2.4 Possible Possibilities

Making ‘possibility’ more specific by interpreting it as ‘hypothetical scenario’ is not enough the end of the story. Which hypotheses can we use? If we limit ourselves by not varying from our existing beliefs, then we can’t use reason to consider what might have been, or even that we might be wrong! Perhaps we should limit hypotheses so they don’t conflict with the laws of nature? Or with the laws of meaning? Or with the laws of morality? Which hypotheses we are prepared to consider determines which arguments we count as valid.

Normally, arguments in natural language arise in a context, and we (mostly) know what is fixed and what is not. A research scientist might consider alternative laws of nature in their arguments, but an engineer probably shouldn’t. A poet might allow words to change their meaning partway through a composition, but a technical writer shouldn’t. And so forth.

However, as logicians, when we are just looking at the logic on an argument, we don’t have any of these constraints. Instead, our constraints are driven by our notion of Truth. Every statement much be true or false; and no statement can be true and not true (false). So as long as we aren’t forced to concede that a statement is both true and false (or without a truth value), we should be prepared to make any hypothesis, so as to allow the widest range of possibilities, and the best chance of finding a counter-example.

Once we come to apply our counter-example to an argument, we might well decide that the counter-example is not applicable. We’ll discuss what happens then a little later in the book.

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We might have allowed possibilities with incomplete information, so some statements are neither true nor false. Or even possibilities where we have inconsistent evidence, and some statements are both true and false. Either of these choices lead to different types of logic.

Each logic will also fix the meaning of a few words, and our hypotheses cannot alter their meanings. These LOGICAL TERMS help to define the logical possibilities for the logic we use. Adding or changing the logical terms will change what is possible to hypothesise, and thus what can be a counter-example, and which arguments are valid.

In the logic that we look at in the first parts of this book, we will hold fixed the meaning of ‘not’, ‘and’, ‘or’, and ‘if’. In the second half of the book we will add ‘exists’, ‘all’, and eventually ‘identical’.

2.5 Validity and Truth

If an argument is valid, and all its premises are true, then its conclusion *must* be true. That's why we value validity so highly. But validity *alone* doesn't require the premises (or conclusion) to be true. Consider this example:

  Oranges are either fruit or musical instruments.
  Oranges are not fruit.
$\therefore$ Oranges are musical instruments.

The conclusion of this argument is ridiculous. Nevertheless, it follows from the premises. *If* both premises are true, *then* the conclusion just has to be true. So the argument is valid. Don't let the meanings of the words distract you, nor whether the premises are true, or whether they even make sense. None of that is part of Logic.

Conversely, having true premises and a true conclusion is not enough to make an argument valid. Consider this example:

  London is in England.
  Beijing is in China.
$\therefore$ Paris is in France.

The premises and conclusion of this argument are all true, but the argument is invalid. If Paris were to declare independence from France, then the conclusion would be false, while the premises would remain true. This is a counter-example to the argument, so the argument is invalid.

The important thing to remember is that validity is not about the truth or falsity of the statements in the argument. It is about whether it is hypothetically *possible* for all the premises to be true and the conclusion to be not true at the same time. What is actually true does not affects whether an argument is valid or not. Logic doesn't value actual facts above any other possibility.

There are also good arguments that are invalid. Consider this one:

  Every winter so far, it has rained in Auckland.
$\therefore$ It will rain in Auckland this coming winter.

This seems like good reasoning, but the argument is invalid. Even if it has rained in Auckland every winter thus far, it remains possible that Auckland will stay dry all through the coming winter. There are many good, reliable arguments that don't guarantee their conclusions. But determining what makes them good or reliable is again a matter of actual facts. So we don't care about weaker notions than validity for the same reason that we don't care about stronger notions than validity. Reality – ugh!

Practice Exercises

Exercise A: Which of the following arguments are logically valid? Which are invalid? Why?

Socrates is a man.
All men are carrots.
∴ Socrates is a carrot.

Jacinda Ardern was either born in Berlin or she was once a nurse.
Jacinda Ardern was never a nurse.
∴ Jacinda Ardern was born in Berlin.

If I light the match, the paper will burn.
I do not light the match.
∴ The paper will not burn.

Confucius was either from France or from Luxembourg.
Confucius was not from Luxembourg.
∴ Confucius was from France.

If the world ends today, then I will not need to get up tomorrow morning.
I will need to get up tomorrow morning.
∴ The world will not end today.

Joe is now 19 years old.
Joe is now 87 years old.
∴ Bob is now 20 years old.

⋆ **Exercise B:** Could there be:

1. A valid argument that has one false premise and one true premise?
2. A valid argument that has only false premises?
3. A valid argument with only false premises and a false conclusion?
4. An invalid argument that can be made valid by adding a new premise?
5. A valid argument that can be made invalid by adding a new premise?

For each question: if so, give an example argument; if not, explain why not.

Chapter 3
Other logical notions

In §1.2, we asked five key questions about statements. Our definition of validity in Chapter 2 gave us a precise definition of the second question. And our discussion of possibility allows us to define the remainder. Our questions were:

1. Can this collection of statements all be true together?
2. Does this statement have to be true if these other statements are true?
3. Do these two statements always have the same truth value as each other?
4. Do these two statements always have different truth values from each other?
5. Is this statement always true, or always false, or in-between?

We can now re-frame them in terms of possibilities:

1. Is there a possibility where this collection of statements are all true?
2. Is there a possibility where this statement is false and these other statements are all true?
3. Is there a possibility where these two statements have different truth values?
4. Is there a possibility where these two statements have the same truth value?
5. Is there a possibility where this statement is false? One where it is true?

Question #1 is about consistency of a set. Question #2, as we've seen, is about validity of an argument. Question #3 is about a contradictory pair of statements. Question #4 is about equivalence of a pair of statements. And Question #5 is about logical truth, logical falsehood, and contingency.

3.1 Consistency

Consistency is one of the two central logical notions that we'll be using – along with validity. We like our beliefs to be consistent with each other, and tend to reject some of a set of statements if they can't all be true together.

Consider these two statements:

B1. Jane's only brother is shorter than her.
B2. Jane's only brother is taller than her.

Logic alone cannot tell us which, if either, of these statements is true. Yet we can say that *if* the first statement (B1) is true, *then* the second statement (B2) must be false. And if B2 is true, then B1 must be false. They might both be false, if Jane has 2 brothers, or none. But there is no possibility that both statements are true together. This motivates the following definition:

> A set of statements are MUTUALLY CONSISTENT if and only if there is a possibility that they are all true together.

> A set of statements are MUTUALLY INCONSISTENT if and only if there is no possibility that they are all true together.

B1 and B2 are *mutually inconsistent*, while the following two statements are mutually consistent:

B1. Jane's only brother is shorter than her.
B2. Jane's only brother is younger than her.

We can ask about the mutual consistency of any number of statements. For example, consider the following four statements:

G1. There are at least four giraffes at the wild animal park.
G2. There are exactly seven gorillas at the wild animal park.
G3. There are not more than two martians at the wild animal park.
G4. Every giraffe at the wild animal park is a martian.

G1 and G4 together tell us there are at least four martian giraffes at the park. This conflicts with G3, which implies that there are no more than two martian giraffes there. So the statements G1–G4 are mutually inconsistent. They cannot all be true together.

3.2 Pairs of Statements

We can also consider the logical relations between exactly two statements. For example:

> John was awarded a VC medal after he died.
> John was awarded a VC medal before he died.

These two statements are both contingent, since John might not have been awarded a medal or died at all. But if either of the statements is true, then they both are; and if either of the statements is false, then they both are. When two statements have the same truth value in every possibility, we say that they are LOGICALLY EQUIVALENT.

> Two statements are LOGICALLY EQUIVALENT if and only if they have the same truth value in every possibility.

Let's change that example slightly:

> John was awarded a VC medal after he died.
> John died after he was awarded a VC medal.

Now, if either of those statements is true, the other is false (assuming that John was awarded only one Victoria's Cross, and that he died). And if either of them is false, the other is true! When two statements have different truth values in every possibility, we say that they are LOGICALLY CONTRADICTORY.

> Two statements are LOGICALLY CONTRADICTORY if and only if they have different truth values in every possibility.

If two statements are logically equivalent, then they can be swapped (or 'substituted' for each other) in an argument without affecting the validity of the argument. This can allow us to simplify an argument by replacing long, complex statements with shorter or clearer, equivalent ones. This ability becomes even more useful once we start reasoning using formulas.

Logically contradictory statements are also useful, for a different reason. If we are trying to construct a counter-example, and produce contradictory statements, we know that our attempt at a counter-example has failed. This observation will be the key insight behind the techniques introduced in parts **??** and **??**.

3.3 Single Statements

Finally, we can also consider the logical properties of single statements. For example:

1. It is raining.
2. It is both raining here and not raining here.
3. Either it is raining here, or it is not.

In order to know if statement 1 is true, we would need to look outside or check the weather channel. It might be true; it might be false. A statement which might be true and might be false is called CONTINGENT.

But we do not need to check the weather to determine whether or not statement 2 is true. It must be false; it's not consistent with itself. It is a LOGICAL FALSEHOOD.

Statement 3 is different again; it's always true. Regardless of what the weather is like, it is either raining or it is not. Statement 3 is a LOGICAL TRUTH. This is also sometimes known as a tautology. Tautologies are important in logic, although primarily for historical reasons.

> A statement is a LOGICAL TRUTH if and only if there is no possibility that is it false.

> A statement is a LOGICAL FALSEHOOD if and only if there is no possibility that is it true.

> A statement is CONTINGENT if and only if there is a possibility that is it true and a possibility that it is false.

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Something might *always* be true and still be contingent. For instance, if there never were a time when the universe contained fewer than two things, then the statement 'At least two things exist' would always be true. Yet the statement is contingent: we can imagine that the universe only contains a single photon, and then the statement would have been false.

Here are a couple of odd facts: Every argument that has a logical truth as a conclusion is valid, and every argument that has a logical falsehood as a premise is valid. Check the definition of validity, and those of logical truth and falsehood, to see why this must be true.

3.4 Summary of Logical Terms

- An argument is VALID if there is no possibility that the premises are all true and the conclusion is false; it is INVALID otherwise.
- A collection of statements is MUTUALLY CONSISTENT if there is a possibility that they are all true; otherwise it is MUTUALLY INCONSISTENT.
- Two statements are LOGICALLY EQUIVALENT if, for each possibility, they are both true or both false.
- Two statements are LOGICALLY CONTRADICTORY if, for each possibility, one is true and the other false.
- A LOGICAL TRUTH is a statement that has no possibility of being false.
- A LOGICAL FALSEHOOD is a statement that has no possibility of being true.
- A CONTINGENT statement is neither a logical truth nor a logical falsehood; it is possibly true and possibly false.

Practice Exercises

Exercise A: Are these logical truths, logical falsehoods, or contingent?

1. Rangi Topeora, a Ngāti Toa leader, signed the Treaty of Waitangi.
2. A woman signed the Treaty of Waitangi.
3. No woman has ever signed the Treaty of Waitangi.
4. If Rangi Topeora signed the Treaty of Waitangi, then someone has.
5. If anyone has ever signed the Treaty of Waitangi, it was Rangi Topeora.

⋆ **Exercise B:** Are these logical truths, logical falsehoods, or contingent?

1. Elephants dissolve in water.
2. Wood is a light, durable substance useful for building things.
3. If wood were a good building material, it would be good for building.
4. I live in a three story building that is two stories tall.
5. If gerbils were mammals they would nurse their young.

Exercise C: Which pairs of statements are logically equivalent?

1. Elephants dissolve in water.
   If you put an elephant in water, it will disintegrate.
2. All mammals dissolve in water.
   If you put an elephant in water, it will disintegrate.
3. John Key is the 39th Prime Minister of New Zealand.
   Jacinda Arden is the 40th Prime Minister of New Zealand.
4. Jacinda Arden is the 40th Prime Minister of New Zealand.
   Jacinda Arden is the Prime Minister of New Zealand immediately after the 39th Prime Minister.
5. Elephants dissolve in water.
   All mammals dissolve in water.

⋆ **Exercise D:** Which pairs of statements are logically equivalent?

1. Sarah Murphy is a biathlete.
   Valerie Adams is a shot putter.
2. Sarah Murphy represented New Zealand at the Olympics.
   Valerie Adams represented New Zealand at the Olympics.
3. All biathletes are cross-country skiers.
   Sarah Murphy is a cross-country skier.
4. Sarah Murphy won more medals at the Olympics than Valerie Adams.
   Valerie Adams won fewer medals at the Olympics than Sarah Murphy.
5. Sarah Murphy represented New Zealand at the Olympics, or she didn't.
   Valerie Adam represented New Zealand at the Olympics, or she didn't.

Exercise E: Consider the following statements:

G1 There are at least four giraffes at the wild animal park.
G2 There are exactly seven gorillas at the wild animal park.
G3 There are not more than two Martians at the wild animal park.
G4 Every giraffe at the wild animal park is a Martian.

Which combinations of statements are mutually consistent?

1. Statements G2, G3, and G4
2. Statements G1, G3, and G4
3. Statements G1, G2, and G4
4. Statements G1, G2, and G3

⋆ **Exercise F:** Consider the following statements.

M1 All people are mortal.
M2 Hypatia is a person.
M3 Hypatia will never die.
M4 Hypatia is mortal.

Which combinations of statements are mutually consistent?

1. Statements M2 and M3
2. Statements M1 and M4
3. Statements M1, M2, and M3
4. Statements M1, M2, and M4
5. Statements M1, M3, and M4
6. Statements M2, M3, and M4
7. Statements M1, M2, M3, and M4

⋆ **Exercise G:** Which of the following is possible?
If it is possible, give an example. If it is not possible, explain why.

1. A valid argument, the conclusion of which is a logical falsehood
2. An invalid argument, the conclusion of which is a logical truth
3. A valid argument, whose premises are all logical truths, and whose conclusion is contingent
4. A logical truth that is contingent
5. Two logically equivalent statements, both of which are logical truths
6. Two logically equivalent statements, one of which is a logical truth and one of which is contingent
7. Two logically equivalent statements that are mutually inconsistent
8. A mutually consistent collection that contains a logical falsehood
9. A mutually inconsistent set of statements that contains a logical truth