GEOPHYSICS, VOL. XXXI, NO. 2 (APRIL, 1966), PP. 362–371 4 FIGS., 1 TABLE

THE GRAVITATIONAL ATTRACTION OF A RIGHT RECTANGULAR PRISM†

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The derivation of a closed expression is presented to calculate the vertical component of the gravitational attraction of a right rectangular prism, with sides parallel to the coordinate axis. As any configuration can be expressed as the sum of prisms of various sizes and densities, the computation of the total gravitational effect of bodies of arbitrary shapes at any point outside of or on the boundary of the bodies is straightforward. To calculate the gravitational effect of the "unit" building element a subroutine called Prism has been developed, tested, and incorporated, in one program to calculate terrain corrections, and in another program for three-dimensional analysis of a gravity field.

I. INTRODUCTION

A number of papers have been published on methods of computing the gravitational attraction of simple forms such as the sphere, cylinder, ellipsoid, and prism. For most of these cases, only approximate expressions have been obtained, such that there are restrictions limiting the validity of the expressions near the computation point. In this paper a closed expression is developed for the gravitational attraction of a prism which is valid for any point outside of or on the boundary of the prism. It is possible to describe any arbitrary configuration in terms of building blocks composed of prisms of various dimensions and densities and, hence, to compute the vertical component of the gravitational attraction of any given mass distribution at arbitrarily selected points.

II. THE ATTRACTION OF A PRISM

The magnitude of the attraction of an elementary mass on a unit mass at distance r is given by:

$$\Delta F = G\rho \frac{\Delta v}{r^2}, \tag{1}$$

where G is the gravitational constant, ρ the density and Δv the volume element.

If the angle enclosed by r and the vertical axis is denoted by γ, then the vertical component of the attraction of a body can be obtained by integrating $\Delta F \cos\gamma$ over the volume, i.e.,

$$F_z = G\rho \int_V \frac{dv}{r^2}\cos\gamma = G\rho \int_V \frac{zdz}{r^3} \cdot \tag{2}$$

The problem is simply to carry out this integration for a prism.

Using the cartesian coordinate system shown in Figure 1, (2) becomes:

$$F_z = G\rho \int_{x_1}^{x_2} dx \int_{y_1}^{y_2} dy \cdot \int_{z_1}^{z_2} \frac{zdz}{\sqrt{(x^2+y^2+z^2)^3}} \cdot \tag{3}$$

Carrying out the integration with respect to z and without substituting the limits, one finds:

$$I_1 = \int \frac{zdz}{\sqrt{(x^2+y^2+z^2)^3}} = -\frac{1}{\sqrt{x^2+y^2+z^2}} \cdot \tag{4}$$

Integrating (4) with respect to y gives:

$$I_2 = \int I_1 dy = \int \frac{dy}{\sqrt{x^2+y^2+z^2}} = \ln\left(y + \sqrt{x^2+y^2+z^2}\right). \tag{5}$$

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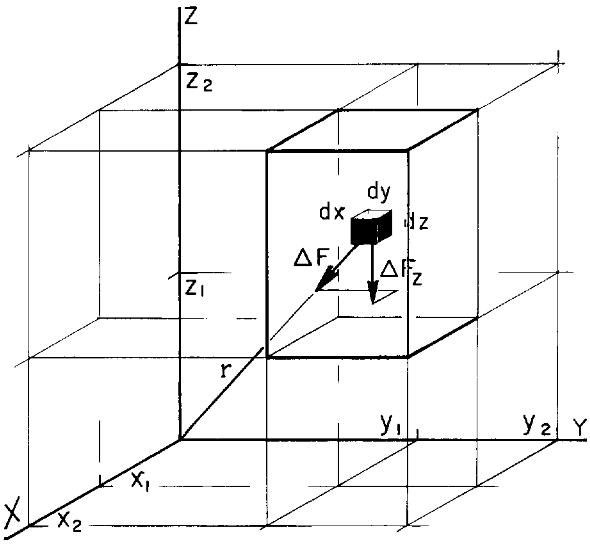

FIG. 1. A right rectangular prism with the volume element and its relation to the Cartesian coordinate system.

The integration of (5) with respect to x is slightly more complicated. One can proceed as follows:

$$I_3 = \int I_2 dx = \int \ln\left(y + \sqrt{x^2 + y^2 + z^2}\right) dx$$

$$= x \ln\left(y + \sqrt{x^2 + y^2 + z^2}\right)$$

$$- \int \frac{x^2 dx}{\left(y + \sqrt{x^2 + y^2 + z^2}\right)\sqrt{x^2 + y^2 + z^2}} . \tag{6}$$

Letting

$$u = y + \sqrt{x^2 + y^2 + z^2},$$

then

$$x^2 = (u - y)^2 - y^2 - z^2,$$

and

$$dx = \frac{(u - y)du}{\sqrt{(u - y)^2 - y^2 - z^2}};$$

thus the integral in (6) becomes

$$I = \int \frac{x^2 dx}{\left(y + \sqrt{x^2 + y^2 + z^2}\right)\sqrt{x^2 + y^2 + z^2}} = \int \frac{\sqrt{u^2 - 2uy - z^2}}{u} du$$

$$= u^2 - 2uy - z^2 - y \ln\left(u - y + \sqrt{u^2 - 2uy - z^2}\right) - z \arcsin \frac{-uy - z^2}{u\sqrt{z^2 + y^2}} .$$

Transforming back to the original variable and noting that

$$u^2 - 2uy - z^2 = \sqrt{y^2 + x^2 + z^2 + 2y\sqrt{x^2 + y^2 + z^2} - 2y^2 - 2y\sqrt{x^2 + y^2 + z^2} + z^2 - z^2} = x,$$

then

$$I = x - y \ln\left(x + \sqrt{x^2 + y^2 + z^2}\right) - z \arcsin - \frac{z^2 + y^2 + y\sqrt{x^2 + y^2 + z^2}}{\left(y + \sqrt{x^2 + y^2 + z^2}\right)\sqrt{y^2 + z^2}} .$$

When the limits of integration are substituted, the first term in I drops out and the following general expression to calculate the vertical component of the attraction of a prism is obtained:

$$F_z = G\rho \left|\left|\left| x \ln (y + r) + y \ln (x + r) - z \arcsin \frac{z^2 + y^2 + yr}{(y + r)\sqrt{y^2 + z^2}} \right|_{z_1}^{z_2} \right|_{y_1}^{y_2} \right|_{x_1}^{x_2}, \tag{7}$$

where equation (7) is valid only when the limits z_1, z_2; y_1, y_2; and x_1, x_2 are substituted. When either the x, y, or both axes are crossed, the integration must be carried out from the lower limit to the axis, then from the axis to the upper limit, the sum of these integrations giving the required effect. To describe all possible situations in the procedure followed in this paper to evaluate equation (7), explicit expressions for four sets of limits are required. Two of these expressions are given in full below:

(a) The prism is completely contained within any one of the four xy quadrants; then using the absolute values of the limits, F_z takes the form:

$$\begin{aligned}
F_z/G\rho = {} & x_2 \ln (y_2 + \sqrt{x_2^2 + y_2^2 + z_1^2}) - x_1 \ln (y_2 + \sqrt{x_1^2 + y_2^2 + z_1^2}) \\
& + y_2 \ln (x_2 + \sqrt{x_2^2 + y_2^2 + z_1^2}) - y_2 \ln (x_1 + \sqrt{x_1^2 + y_2^2 + z_1^2}) \\
& + z_1 \arcsin \frac{z_1^2 + y_2^2 + y_2^2\sqrt{x_2^2 + y_2^2 + z_1^2}}{(y_2 + \sqrt{x_2^2 + y_2^2 + z_1^2})\sqrt{y_2^2 + z_1^2}} \\
& - z_1 \arcsin \frac{z_1^2 + y_2^2 + y_2\sqrt{x_1^2 + y_2^2 + z_1^2}}{(y_2 + \sqrt{x_1^2 + y_2^2 + z_1^2})\sqrt{y_2^2 + z_1^2}} \\
& - x_2 \ln (y_1 + \sqrt{x_2^2 + y_1^2 + z_1^2}) + x_1 \ln (y_1 + \sqrt{x_1^2 + y_1^2 + z_1^2}) \\
& - y_1 \ln (x_2 + \sqrt{x_2^2 + y_1^2 + z_1^2}) + y_1 \ln (x_1 + \sqrt{x_1^2 + y_1^2 + z_1^2}) \\
& - z_1 \arcsin \frac{z_1^2 + y_1^2 + y_1\sqrt{x_2^2 + y_1^2 + z_1^2}}{(y_1 + \sqrt{x_2^2 + y_1^2 + z_1^2})\sqrt{y_1^2 + z_1^2}} \\
& + z_1 \arcsin \frac{z_1^2 + y_1^2 + y_1\sqrt{x_1^2 + y_1^2 + z_1^2}}{(y_1 + \sqrt{x_1^2 + y_1^2 + z_1^2})\sqrt{y_1^2 + z_1^2}} \\
& - x_2 \ln (y_2 + \sqrt{x_2^2 + y_2^2 + z_2^2}) + x_1 \ln (y_2 + \sqrt{x_1^2 + y_2^2 + z_2^2}) \\
& - y_2 \ln (x_2 + \sqrt{x_2^2 + y_2^2 + z_2^2}) + y_2 \ln (x_1 + \sqrt{x_1^2 + y_2^2 + z_2^2}) \\
& - z_2 \arcsin \frac{z_2^2 + y_2^2 + y_2\sqrt{x_2^2 + y_2^2 + z_2^2}}{(y_2 + \sqrt{x_2^2 + y_2^2 + z_2^2})\sqrt{y_2^2 + z_2^2}} \\
& + z_2 \arcsin \frac{z_2^2 + y_2^2 + y_2\sqrt{x_1^2 + y_2^2 + z_2^2}}{(y_2 + \sqrt{x_1^2 + y_2^2 + z_2^2})\sqrt{y_2^2 + z_2^2}} \\
& + x_2 \ln (y_1 + \sqrt{x_2^2 + y_1^2 + z_2^2}) - x_1 \ln (y_1 + \sqrt{x_1^2 + y_2^2 + z_1^2}) \\
& + y_1 \ln (x_2 + \sqrt{x_2^2 + y_1^2 + z_2^2}) - y_1 \ln (x_1 + \sqrt{x_1^2 + y_1^2 + z_2^2}) \\
& + z_2 \arcsin \frac{z_2^2 + y_1^2 + y_1\sqrt{x_2^2 + y_1^2 + z_2^2}}{(y_1 + \sqrt{x_2^2 + y_1^2 + z_2^2})\sqrt{y_1^2 + z_2^2}} \\
& - z_2 \arcsin \frac{z_2^2 + y_1^2 + y_1\sqrt{x_1^2 + y_1^2 + z_2^2}}{(y_1 + \sqrt{x_1^2 + y_1^2 + z_2^2})\sqrt{y_1^2 + z_2^2}}\,.
\end{aligned} \tag{8}$$

As a special case of (8), letting $z_1 = 0$ and $z_2 = h$ and rearranging the terms, (8) becomes:

$$
\begin{aligned}
F_z/G\rho = {} & x_2\left\{\ln\frac{y_2+\sqrt{x_2^2+y_2^2}}{y_2+\sqrt{x_2^2+y_2^2+h^2}}-\ln\frac{y_1+\sqrt{x_2^2+y_1^2}}{y_1+\sqrt{x_2^2+y_1^2+h^2}}\right\} \\
& - x_1\left\{\ln\frac{y_2+\sqrt{x_1^2+y_2^2}}{y_2+\sqrt{x_1^2+y_2^2+h^2}}-\ln\frac{y_1+\sqrt{x_1^2+y_1^2}}{y_1+\sqrt{x_1^2+y_1^2+h^2}}\right\} \\
& + y_2\left\{\ln\frac{x_2+\sqrt{x_2^2+y_2^2}}{x_2+\sqrt{x_2^2+y_2^2+h^2}}-\ln\frac{x_1+\sqrt{x_1^2+y_2^2}}{x_1+\sqrt{x_1^2+y_2^2+h^2}}\right\} \\
& - y_1\left\{\ln\frac{x_2+\sqrt{x_2^2+y_1^2}}{x_2+\sqrt{x_2^2+y_1^2+h^2}}-\ln\frac{x_1+\sqrt{x_1^2+y_1^2}}{x_1+\sqrt{x_1^2+y_1^2+h^2}}\right\} \\
& + h\left\{\arcsin\frac{y_2^2+h^2+y_2\sqrt{x_2^2+y_2^2+h^2}}{(y_2+\sqrt{x_2^2+y_2^2+h^2})\sqrt{y^2+h^2}}\right. \\
& - \arcsin\frac{y_2^2+h^2+y_2\sqrt{x_1^2+y_2^2+h^2}}{(y_2+\sqrt{x_1^2+y_2^2+h^2})\sqrt{y_2^2+h^2}} \\
& - \arcsin\frac{y_1^2+h^2+y_1\sqrt{x_2^2+y_1^2+h^2}}{(y_1+\sqrt{x_2^2+y_1^2+h^2})\sqrt{y_1^2+h^2}} \\
& \left. + \arcsin\frac{y_1^2+h^2+y_1\sqrt{x_1^2+y_1^2+h^2}}{(y_1+\sqrt{x_1^2+y_1^2+h^2})\sqrt{y_1^2+h^2}}\right\}. \qquad (9)
\end{aligned}
$$

Designating the terms within the brackets by $T_1, T_2, \cdots T_{12}$ one obtains the simple form:

$$F_z/G\rho = x_2\{T_1-T_2\}-x_1\{T_3-T_4\}+y_2\{T_5-T_6\}-y_1\{T_7-T_8\}+h\{T_9-T_{10}-T_{11}+T_{12}\}. \qquad (10)$$

(*b*) The y axis is crossed, i.e. the signs of x_1 and x_2 are different. Since the vertical component of the attraction of the same mass below and above the y axis is the same, the integral is evaluated from 0 to x_2 and from 0 to x_1 (absolute values):

$$
\begin{aligned}
P_1 = {} & x_2\left\{\ln\frac{y_2+\sqrt{x_2^2+y_2^2}}{y_2+\sqrt{x_2^2+y_2^2+h^2}}-\ln\frac{y_1+\sqrt{x_2^2+y_1^2}}{y_1+\sqrt{x_2^2+y_1^2+h^2}}\right\} - 0 \\
& + y_2\left\{\ln\frac{x_2+\sqrt{x_2^2+y_2^2}}{x_2+\sqrt{x_2^2+y_2^2+h^2}}-\ln\frac{y_2}{\sqrt{y_2^2+h^2}}\right\} \\
& - y_1\left\{\ln\frac{x_2+\sqrt{x_2^2+y_1^2}}{x_2+\sqrt{x_2^2+y_1^2+h^2}}-\ln\frac{y_1}{\sqrt{y_1^2+h^2}}\right\} \\
& + h\left\{\arcsin\frac{y_2^2+h^2+y_2\sqrt{x_1^2+y_2^2+h^2}}{(y_2+\sqrt{x_2^2+y_2^2+h^2})\sqrt{y_2^2+h^2}}\right. \\
& - \arcsin\frac{y_2^2+h^2+y_2\sqrt{y_2^2+h^2}}{(y_2+\sqrt{y_2^2+h^2})\sqrt{y_2^2+h^2}} \\
& - \arcsin\frac{y_1^2+h^2+y_1\sqrt{x_2^2+y_1^2+h^2}}{(y_1+\sqrt{x_2^2+y_1^2+h^2})\sqrt{y_1^2+h^2}} \\
& \left. + \arcsin\frac{y_1^2+h^2+y_1\sqrt{y_1^2+h^2}}{(y_1+\sqrt{y_1^2+h^2})\sqrt{y_1^2+h^2}}\right\},
\end{aligned}
$$

and

$$P_2 = x_1 \left\{ \ln \frac{y_2 + \sqrt{x_1^2 + y_2^2}}{y_2 + \sqrt{x_1^2 + y_2^2 + h^2}} - \ln \frac{y_1 + \sqrt{x_1^2 + y_1^2}}{y_1 + \sqrt{x_1^2 + y_1^2 + h^2}} \right\} - 0$$

$$+ y_2 \left\{ \ln \frac{x_1 + \sqrt{x_1^2 + y_2^2}}{x_1 + \sqrt{x_1^2 + y_2^2 + h^2}} - \ln \frac{y_2}{\sqrt{y_2^2 + h^2}} \right\}$$

$$- y_1 \left\{ \ln \frac{x_1 + \sqrt{x_1^2 + y_1^2}}{x_1 + \sqrt{x_1^2 + y_1^2 + h^2}} - \ln \frac{y_1}{\sqrt{y_1^2 + h^2}} \right\}$$

$$+ h \left\{ \arcsin \frac{y_2^2 + h^2 + y_2\sqrt{x_1^2 + y_2^2 + h^2}}{(y_2 + \sqrt{x_1^2 + y_2^2 + h^2})\sqrt{y_2^2 + h^2}} \right.$$

$$- \arcsin \frac{y_2^2 + h^2 + y_2\sqrt{y_2^2 + h^2}}{(y_2 + \sqrt{y_2^2 + h^2})\sqrt{y_2^2 + h^2}}$$

$$- \arcsin \frac{y_1^2 + h^2 + y_1\sqrt{x_1^2 + y_1^2 + h^2}}{(y_1 + \sqrt{x_1^2 + y_1^2 + h^2})\sqrt{y_1^2 + h^2}}$$

$$\left. + \arcsin \frac{y_1^2 + h^2 + y_1\sqrt{y_1^2 + h^2}}{(y_1 + \sqrt{y_1^2 + h^2})\sqrt{y_1^2 + h^2}} \right\}.$$

Using the terms $T_1, T_2 \cdots T_{12}$, P_1 and P_2 take the following form:

$$P_1 = x_2\{T_1 - T_2\} + 0 + y_2 \left\{ T_5 - \ln \frac{y_2}{\sqrt{y_2^2 + h^2}} \right\} - y_1 \left\{ T_7 - \ln \frac{y_1}{\sqrt{y_1^2 + h^2}} \right\}$$
$$+ h\{T_9 - 1 - T_{11} + 1\},$$

$$P_2 = x_1\{T_3 - T_4\} + 0 + y_2 \left\{ T_6 - \ln \frac{y_2}{\sqrt{y_2^2 + h^2}} \right\} - y_1 \left\{ T_8 - \ln \frac{y_1}{\sqrt{y_1^2 + h^2}} \right\}$$
$$+ h\{T_{10} - 1 - T_{12} + 1\}.$$

Then adding P_1 and P_2 one finds:

$$F_z/G\rho \equiv P_1 + P_2 = x_2\{T_1 - T_2\} + x_1\{T_3 - T_4\} + y_2\{T_5 + T_6\} - y_1\{T_7 + T_8\}$$
$$+ h\{T_9 + T_{10} - T_{11} - T_{12}\} - 2\left\{ y_2 \ln \frac{y_2}{\sqrt{y_2^2 + h^2}} - y_1 \ln \frac{y_1}{\sqrt{y_1^2 + h^2}} \right\}. \quad (11)$$

Other special cases, when crossing the x axis (y_1 and y_2 having different signs) or crossing the origin (both y_1, y_2 and x_1, x_2 with different signs), can be obtained similarly.

Subsequent to this development of equation (7), it was found that Sorokin (1951) and Haáz (1953) also published solutions to this problem. The solutions they obtained are given below using their notation. Sorokin carried out the integration differently and obtained (page 370, equation (426)):

$$\Delta g = -f\sigma \left\| \left\| \left\| \xi \ln (\eta + R) + \eta \ln (\xi + R) + \zeta \operatorname{arctg} \frac{\zeta R}{\xi \eta} \right|_{\xi_1}^{\xi_2} \right|_{\eta_1}^{\eta_2} \right|_{\zeta_1}^{\zeta_2}.$$

Table 1. Calculated terrain corrections for twelve gravity stations shown in Figure 2

Station number	UTM Coordinates X_m	UTM Coordinates Y_m	h_{ft}	Δg	Δg_A	ϵ
	2	3	4	5	6	7
8174	6 793 889.7	569 909.1	1795	1.09	1.11	±.11
9810	6 794 529.7	573 172.9	1622	3.31	3.36	±.14
8193	6 792 197.2	571 837.6	1831	1.44		
9815	6 795 525.5	571 852.6	1936	2.04	2.08	±.13
9813	6 795 878.6	571 687.2	2063	3.68	3.69	±.16
8135	6 796 148.7	571 687.4	1908	1.62		
1	6 793 921.4	570 604.2	−380	8.29	8.26	±.10
3	6 794 073.7	571 291.7	−685	13.43	13.43	±.06
5	6 794 229.6	571 229.0	−745	14.45	14.46	±.06
7	6 794 350.0	572 610.0	−780	13.07		
14	6 794 861.6	571 697.5	−805	14.39		
16	6 793 164.5	571 631.4	−440	9.03		

Haáz applied Euler's theorem of homogeneous functions to the second derivative of the potential of a prism, thus obtaining the first derivative and the potential itself without integration. His result for the vertical component of the attraction (page 62) is quoted below:

$$\phi_z = -a \log (b + r) - b \log (a + r) + c \operatorname{arctg} \frac{ab}{cr} \cdot$$

Although these equations, including (7), do not seem to agree, it has been verified that they are identical.

Having obtained the expressions for all cases, a subroutine called Prism has been written in Fortran II. Although the arithmetic is fairly simple, some care must be exercised to obtain the proper special case to be used for a given set of input values. Extensive testing shows that the subroutine provides the correct value for the vertical component of the gravitational attraction of a prism on a unit particle at P, if P is outside or on the boundary of the prism. In the following section two applications of the Prism subroutine are given. See also Nagy (1966).

III. APPLICATIONS

a) *Terrain corrections*

The Prism subroutine has been used in a program to calculate terrain corrections. The principle of the method is described as follows: the local area, whose terrain effect is to be taken into account, is subdivided into prisms by a grid system with intervals dx and dy. The bases of these prisms are at sea level, and the tops are defined by the estimated elevations, $H_{i,j}$. The elevation difference between a compartment and the station, together with the horizontal coordinates of the compartment, are calculated and fed into the Prism subroutine, which computes the exact gravitational effect of that compartment on the gravity station. The sum of the effects of all compartments gives the terrain correction. More detail will be given in a forthcoming paper.

This program has been applied for an area 10×10 km surrounding the New Quebec Crater (Figure 2). A grid interval of 100 m for both x and y has been used producing 10,000 compartments. The elevation of the water level in the crater is 1,620 feet above sea level, with a maximum depth of 810 ft. The surrounding topography varies from 1,530 to 2,156 ft. For the calculations it was assumed that the top of each prism was a plane surface parallel to the xy plane, and that all prisms had the same density. The standard deviations of $H_{i,j}$ for compartments on land was estimated at ±5 ft and for compartments on water at ±25 ft. Terrain corrections were calculated for 130 gravity stations of which twelve are shown in Figure 2. The coordinates, elevation and computed terrain corrections of these stations are listed in Table 1.

To assess the effect of the errors in the elevations, $H_{i,j}$, on the computed terrain corrections, provision was made for error analysis by using the Monte Carlo technique. Pseudo-random numbers of magnitude proportional to the standard deviation of $H_{i,j}$, were superimposed on each

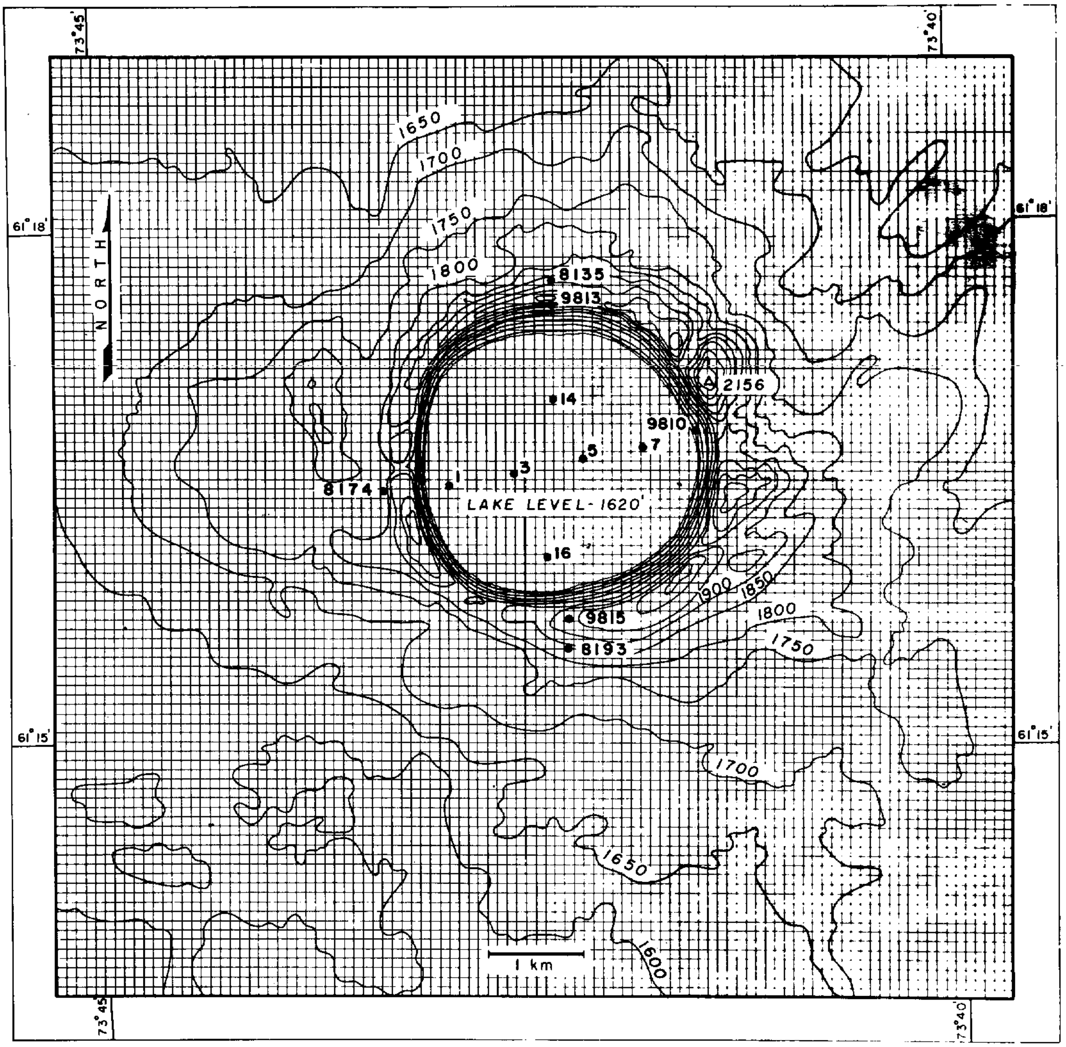

FIG. 2. The New Quebec Crater with surrounding topography. Contour interval 50 ft.

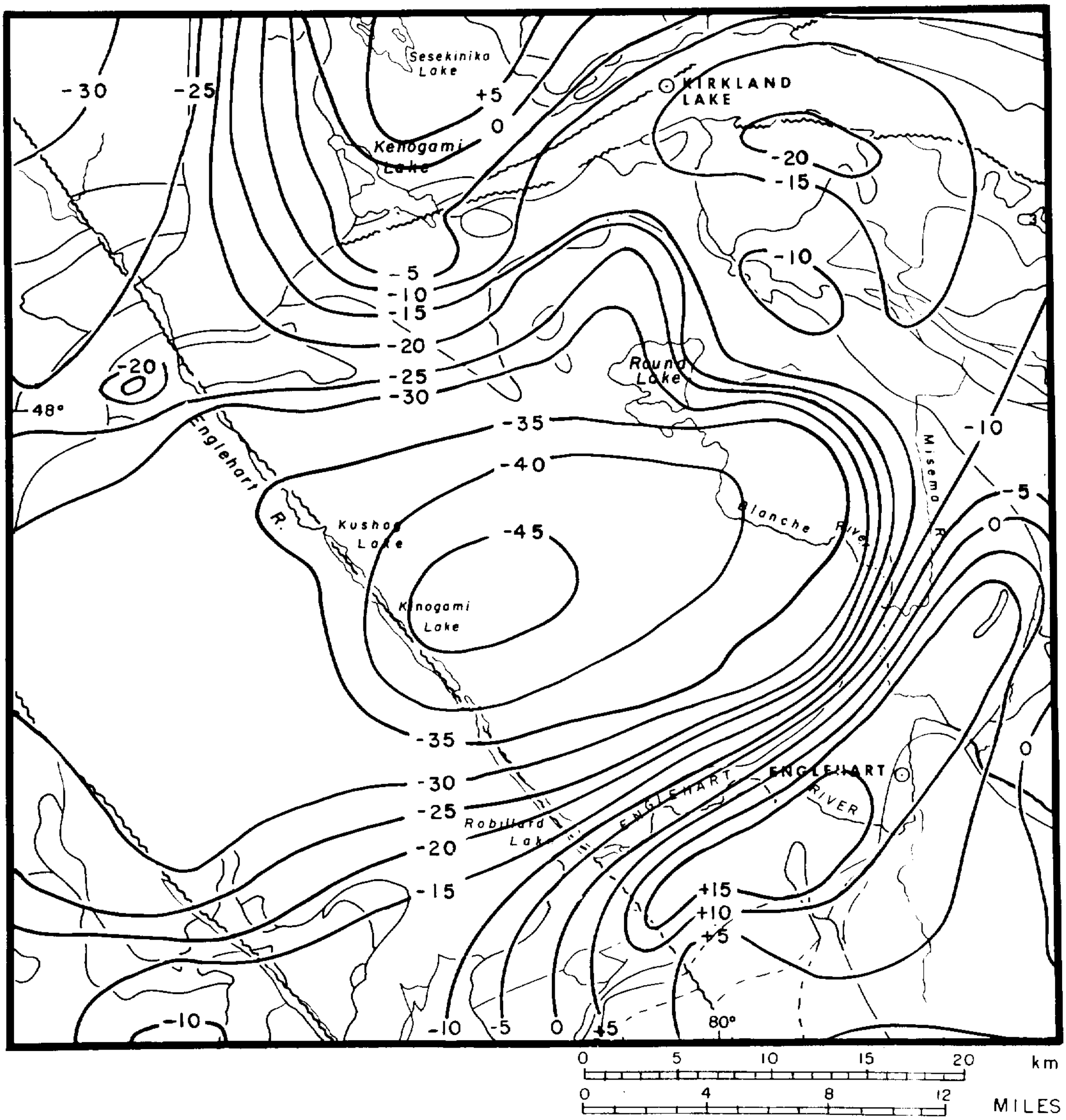

FIG. 3. Residual Bouguer anomaly map for the vicinity of Kirkland Lake, Ontario. Contour interval 5 mgal.

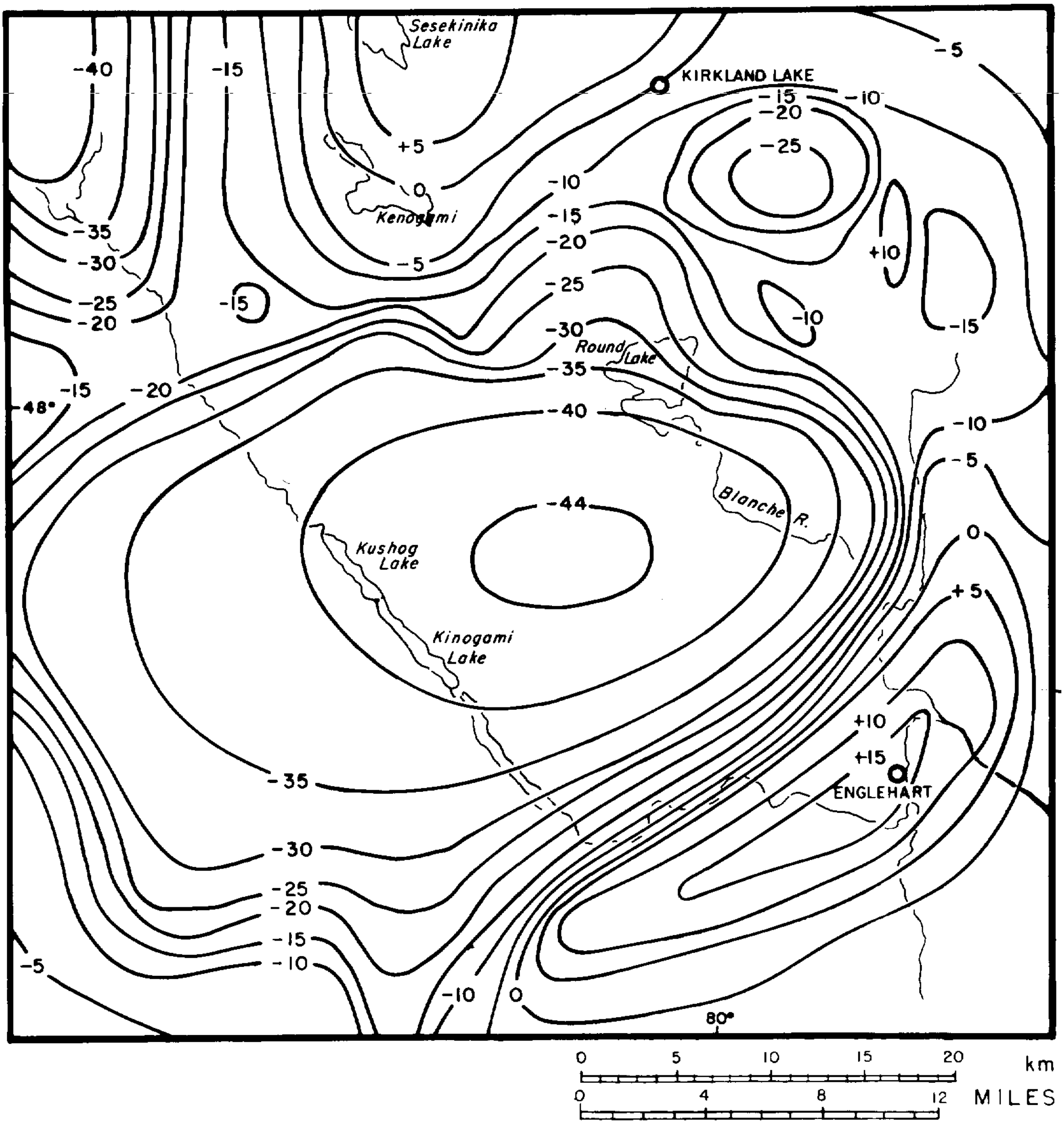

FIG. 4. Synthetic gravity contour map resulting from three-dimensional analysis.

estimated elevation. Then the terrain correction was calculated with this new set of input data. This computation has been repeated 36 times for each station using new sets of random numbers each time. Table 1 gives the average, Δg_A, of the 36 calculated terrain corrections with their respective standard deviations, ϵ, for seven stations. As can be seen from Table 1 the terrain correction Δg calculated from the original input (column 5) differs from Δg_A by less than $\pm\epsilon$.

b) *Three-dimensional analysis*

On the suggestion of J. van Boeckel of the Dominion Observatory, another program, for three-dimensional analysis, has been developed around the Prism subroutine. The principle involved is simple: the sum of the gravitational effects of prisms of given dimensions and densities is calculated at specific points. These initial dimensions and densities are determined from existing information, usually from geological maps of the area. It is noted here that there is no restriction on the dimensions of prisms or their distance from the computation point (other than that the point may not be inside a prism). The calculated values then are compared to the given anomalies and the differences are successively eliminated by modifying the block arrangement, number of blocks and/or densities. This program has been used by van Boeckel to explain the residual gravity anomaly field shown in Figure 3. After four modifications the gravitational attraction of 80 blocks was evaluated at 625 grid points. The values were plotted and the resulting contoured map is shown in Figure 4.

The two examples above illustrate the power and flexibility of the application of the Prism method to problems associated with gravity fields. The application to other problems is limited only by the availability of input data and by memory space in the computers. For example, in terrain corrections one could easily include variations in density.

It is now possible by using the Prism subroutine to test some of the assumptions about the geology of an area; to analyze different sources of errors and calculate their effect on the output; to derive different degrees of approximations for the gravitational effect of the prism for practical computations and estimate their accuracies; to obtain "regional" gravity anomalies; to carry out geological corrections to obtain "residuals"; and to compare different isostatic hypotheses.

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