

# solvingEquation

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## 2.3.1 Classical statistics for classical data

Proof that the mean of the Poisson distribution maximises the log-likelihood:

From before we know that the likelihood (written here as  $L$ ) is a multiplication of all the individual probabilities:

$$L(\lambda, x = (k_1, k_2, k_3 \dots)) = \prod_{i=1}^{100} f(k_i)$$

$f(k)$  is simply the Poisson density function:

$$f(k) = \frac{e^{-\lambda} \lambda^k}{k!}$$

So if we put those together and take the log of both sides, we get:

$$\log(L(\lambda, x)) = \log\left(\prod_{i=1}^{100} \frac{e^{-\lambda} \lambda^k}{k!}\right)$$

We know that the product log of a product ( $\prod$ ) is the same as the sum ( $\sum$ ) of a log, we can rewrite it as:

$$\log L = \sum_{i=1}^{100} \log\left(\frac{e^{-\lambda} \lambda^k}{k!}\right)$$

We can also break up the fraction, again using the log rules of  $\log(a * b) = \log(a) + \log(b)$  and  $\log(\frac{a}{b}) = \log(a) - \log(b)$ :

$$\log L = \sum_{i=1}^{100} (\log(e^{-\lambda}) + \log(\lambda^k) - \log(k!))$$

Now we can get rid of the powers using  $\log(a^b) = b \log(a)$ . Also  $\log(e) = 1$  because this is the natural log.

$$\log L = \sum_{i=1}^{100} (-\lambda + k \log(\lambda) - \log(k!))$$

Now we want to break apart the sum by extracting terms that do not depend on  $k$ . The final term does not depend on  $\lambda$ , so it is just a constant:

$$\log L = -100\lambda + \log \lambda \left( \sum_{i=1}^{100} k_i \right) + \text{const.}$$

To get the maximum of a function we want the derivative of the function to be equal to 0:

$$\frac{d}{d\lambda} \log L = \frac{d}{d\lambda} (-100\lambda + \log \lambda \left( \sum_{i=1}^{100} k_i \right) + \text{const.}) = 0$$

Using the derivative rules of  $\frac{d}{dx} ax = a$  and  $\frac{d}{dx} \log(x) = \frac{1}{x}$ , and derivative of a constant is 0, we get:

$$-100 + \frac{1}{\lambda} \sum_{i=1}^{100} k_i = 0$$

$$100 = \frac{1}{\lambda} \sum_{i=1}^{100} k_i$$

Multiply by  $\frac{\lambda}{100}$ :

$$\lambda = \frac{1}{100} \sum_{i=1}^{100} k_i = \bar{k}$$

So the  $\lambda$  parameter is the same as the mean  $(\bar{k})$ .