# solvingEquation

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### 2.3.1 Classical statistics for classical data

Proof that the mean of the Poisson distribution maximises the log-likelihood:

From before we know that the liklihood (written here as L) is a multiplication of all the inidividual probabilities:

$$L(\lambda, x = (k_1, k_2, k_3...)) = \prod_{i=1}^{100} f(k_i)$$

f(k) is simply the Poisson density function:

$$f(k) = \frac{e^{-\lambda}\lambda^k}{k!}$$

So if we put those together and take the log of both sides, we get:

$$\log(L(\lambda, x)) = \log(\prod_{i=1}^{100} \frac{e^{-\lambda}\lambda^k}{k!})$$

We know that the product log of a product ($\prod$) is the same as the sum ($\sum$) of a log, we can rewrite it as:

$$\log L = \sum_{i=1}^{100} \log(\frac{e^{-\lambda}\lambda^k}{k!})$$

We can also break up the fraction, again using the log rules of $\log(a * b) = log(a) + log(b)$ and $\log(\frac{a}{b}) = \log(a) - log(b)$:

$$\log L = \sum_{i=1}^{100} (\log(e^{-\lambda}) + \log(\lambda^k) - \log(k!))$$

Now we can get rid of the powers using $\log(a^b) = b\log(a)$. Also $\log(e) = 1$ because this is the natural log.

$$\log L = \sum_{i=1}^{100} (-\lambda + k\log(\lambda) - \log(k!))$$

Now we want to break apart the sum by extracting terms that do not depend on k.The final term does not depend on lambda, so it is just a constant:

$$logL = -100\lambda + \log\lambda(\sum_{i=1}^{100} k_i) + const.$$

To get the maximum of a function we want the derivative of the function to be equal to 0:

$$\frac{d}{d\lambda}\log L = \frac{d}{d\lambda}(-100\lambda + \log\lambda(\sum_{i=1}^{100} k_i) + const.) = 0$$

Using the derivative rules of $\frac{d}{dx}ax = a$ and $\frac{d}{dx}\log(x) = \frac{1}{x}$, and derivative of a constant is 0, we get:

$$-100 + \frac{1}{\lambda}\sum_{i=1}^{100} k_i = 0$$

$$100 = \frac{1}{\lambda}\sum_{i=1}^{100} k_i$$

Multiply by $\frac{\lambda}{100}$:

$$\lambda = \frac{1}{100}\sum_{i=1}^{100} k_i = \overline{k}$$

So the $\lambda$ paramter is the same as the mean ($\overline{k}$).

## Likelihood for the binomial distribution

$$f(\theta|n, y) = f(y|n, \theta) = \binom{n}{y}\theta^y(1-\theta)^{(n-y)}$$

To avoid large-number multiplications we take the log of both sides (log likelihood):

$$\log f(\theta|n, y) = \log(\binom{n}{y}\theta^y(1-\theta)^{(n-y)})$$

We break up the product using the log rule $\log(ab) = \log(a) + \log(b)$:

$$\log f(\theta|n, y) = \log\binom{n}{y} + \log\theta^y + \log(1-\theta)^{(n-y)})$$

We bring down the exponents using the $\log(a^b) = b\log(a)$ rule:

$$\log f(\theta|n, y) = \log\binom{n}{y} + y\log\theta + (n-y)\log(1-\theta)$$

This is the formula used in the text.