# Quiz Solutions

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## 1 Tuesday

Problem 1.a: $\mathbf{v}, \mathbf{w}$ make an angle of $5\pi/6$, so

$$\begin{aligned} \mathbf{v} \cdot \mathbf{w} &= ||\mathbf{v}|| \cdot ||\mathbf{w}|| \cdot \cos(5\pi/6) \\ &= 2 \cdot \sqrt{3} \cdot (-\cos(\pi/6)) = -3. \end{aligned}$$

Problem 1.b: Using the formula from 1.a,

$$(\mathbf{v} + 2 \cdot \mathbf{w}) \cdot (\mathbf{v} + 2 \cdot \mathbf{w}) = 8 + 6 - 12 = 2$$

thus $||\mathbf{v} + 2 \cdot \mathbf{w}|| = \sqrt{2}$.

Problem 1.c:

$$\begin{aligned} ||2 \cdot \mathbf{v} + 3 \cdot \mathbf{w}||^2 &= (2 \cdot \mathbf{v} + 3 \cdot \mathbf{w}) \cdot (2 \cdot \mathbf{v} + 3 \cdot \mathbf{w}) \\ &= 4||\mathbf{v}||^2 + 9||\mathbf{w}||^2 + 12 \cdot (-3) \\ &= 6 \end{aligned}$$

so we get $\sqrt{6}$.

Problem 2: to find the equation of the plane we just need a point and a normal vector, so we take $R = (1, 1, 1)$ as our point. We know that the vector $\vec{RP} \times \vec{RQ}$ is perpendicular to $\vec{RP}, \vec{RQ}$ both of which lie parallel to the plane, so as long as it is nonzero we win. Given that

$$\vec{RP} = (1, -4, -2)$$

$$\vec{RQ} = (3, 6, 0)$$

we calculate the cross product as the "determinant" of the matrix

$$\begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -4 & -2 \\ 3 & 6 & 0 \end{pmatrix}.$$

Solving, we get the vector $6 \cdot (2, 1, 3)$. For finding the equation of the plane, the scalar factor of 6 will not matter. Thus an equation of the plane is

$$2 \cdot (x - 1) - (y - 1) + 3 \cdot (z - 1) = 0.$$

## 2 Thursday

Problem 1: We have the lines via parameterizations

$$\vec{\ell}_1(t) = (1,2,3) + t \cdot (2,-1,1)$$

$$\vec{\ell}_2(t) = (3,1,2) + t \cdot (1,1,-1).$$

To find their intersection we set

$$\vec{\ell}_1(t) = \vec{\ell}_2(s)$$

and we get, looking at components:

$$1 + 2t = 3 + s$$

$$2 - t = 1 + s$$

$$3 + t = 2 - s.$$

Using the first equation, we see that $s = 2t - 2$, plugging this into the other two equations we get

$$3t = 3$$

$$3t = 1$$

which are contradictory, thus there is no point of intersection.

Problem 2.a: We calculate the cross product of $\vec{v}, \vec{w}$ and save the answer for part (b) (however for this question we only need its magnitude). We calculate the cross product as the "determinant" of the matrix

$$\begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 2 & -1 & 3 \end{pmatrix}.$$

and we get the vector $(5,-5,-5)$ which has length $5\sqrt{3}$, which is the desired area.

Problem 2.b: using the cross product from before, we see that an equation of the plane is

$$x - y - z = 0.$$