

University of California, Los Angeles

Practice for Midterm 1

Math 32A

Date: Oct 17, 2025

Instructor: Jack Sempliner

Name: \_\_\_\_\_

Discussion Section: \_\_\_\_\_

UID: \_\_\_\_\_

**Please read the following instructions carefully.**

- You have 50 minutes to complete this exam. This question booklet contains 3 questions, 4 pages (including the cover) for a total of 40 points/marks.
- Check to see if any pages are missing. Please use a separate sheet for rough work. Carefully cross out marks on the page from false starts or scratch work, leaving only the calculations relevant to your answer.
- All the questions are compulsory and all the notations have their usual meaning.
- No outside resources are allowed, beyond your cheat sheet. No electronic devices may be used.

| Question | Points | Score |
|----------|--------|-------|
| 1        | 10     |       |
| 2        | 10     |       |
| 3        | 20     |       |
| Total:   | 40     |       |

1. Consider the plane  $\text{Pl}$  defined by the following equation

$$2x - 3y + z = 5$$

and the line  $\text{L}$  defined by the following parameterization

$$\vec{l}(t) = (1, 2, -4) + t \cdot (3, 1, 2).$$

- (a) (5 points) Find the intersection of the line  $\text{L}$  with the plane  $\text{Pl}$ .

- (b) (5 points) Compute the projection of the vector  $(3, 1, 2)$  onto the plane  $\text{Pl}$ .

2. (10 points) Find the area of the triangle with vertices at the points

$$P = (1, 0, 0)$$

$$Q = (0, 1, 0)$$

$$R = (0, 0, 1).$$

3. Let  $\mathbb{E}$  be the vector  $\hat{j} = (0, 1, 0)$ , and let  $\mathbb{B}$  be the vector  $\hat{k} = (0, 0, 1)$ . Suppose a particle traces out a path  $\vec{r}(t)$  such that the acceleration vector  $\vec{a}(t) = \vec{r}''(t)$  satisfies the vector equation

$$\vec{a}(t) = (\mathbb{E} + \vec{v} \times \mathbb{B})$$

where here  $\vec{v}(t) = \vec{r}'(t)$  is the velocity function along the path.

- (a) (10 points) Suppose the particle has initial velocity  $\vec{v}(0) = (v_0, 0, 0)$ . Find explicit equations expressing the components of  $\vec{a}(t)$  in terms of the components of  $\vec{v}(t)$ . Partially solve to express  $\hat{k} \cdot \vec{v}(t)$  as a function of time.

- (b) (5 points) Let  $\vec{w}(t) = \vec{v}(t) - (1, 0, 0)$ . Show using part (a) that the norm  $\|\vec{w}\|$  is constant as a function of  $t$ .

- (c) (5 points) It can be deduced from part (b) that the velocity function

$$\vec{v}(t) = (1 + (v_0 - 1) \cos(t), -(v_0 - 1) \sin(t), 0).$$

Taking this formula for  $\vec{v}(t)$  as a given, and given the initial position  $\vec{r}(0) = (0, 0, 0)$ , solve for the position of the particle  $\vec{r}(t)$  as a function of time.