The Relation between Pen Force and Pen-Point Kinematics in Handwriting

Article *in* Biological Cybernetics · August 1990

DOI: 10.1007/BF00203451

2 authors:

Lambert Schomaker
University of Groningen

Réjean Plamondon
Polytechnique Montréal

Biol. Cybern. 63, 277–289 (1990)

Biological Cybernetics

The Relation between Pen Force and Pen-Point Kinematics in Handwriting

L. R. B. Schomaker [1] **and R. Plamondon** [2]

[1] Nijmegen Institute for Cognition Research and Information Technology (NICI), University of Nijmegen, P.O. Box 9104, 6500 HE Nijmegen, The Netherlands
[2] Laboratoire Scribens, Département de Génie Électrique, Ecole Polytechnique, CP 6079 succ "A", Montréal H3C 3A7, Canada

Received February 20, 1989/Accepted in revised form March 14, 1990

Abstract. This study investigates the spectral coherence and time-domain correlation between pen pressure (axial pen force, APF) and several kinematic variables in drawing simple patterns and in writing cursive script. Two types of theories are prevalent: "biomechanical" and "central" explanations for the force variations during writing. Findings show that overall coherence is low (< 0.5) and decreases with pattern complexity, attaining its lowest value in cursive script. Looking at subjects separately, it is found that only in a small minority of writers "biomechanical coupling" between force and displacement takes place in cursive handwriting, as indicated by moderate to high negative overall correlations. The majority of subjects display low coherence and correlation between kinematics and APF. However, APF patterns in cursive script reveal a moderate to high replicatability, giving support to the notion of a "centrally" controlled pen pressure. The sign of the weak residual average correlation between APF and finger displacement, and between APF and wrist displacement is negative. This indictes that small biomechanical effects may be present, a relatively higher APF corresponding to finger flexion and wrist radial abduction. On the whole, however, variance in APF cannot be explained by kinematic variables. A motor task demanding mechanical impedance control, such as handwriting, apparently introduces a complexity that is not easily explained in terms of a passive mass-spring model of skeleto-muscular movement.

1 Introduction

Generally, researchers of handwriting movements in the fields of signature verification, forensic studies, and in biophysical or psychomotor studies have recognized the importance of the *pen pressure*[1] on the writing surface as an important dependent variable. For instance, in signature verification, the force exerted by the pen on the paper during handwriting appears to be a discriminating parameter between individual writers (Hale and Paganini, 1980; Crane and Ostrem 1983; Deinet et al. 1987). Also, writer identification on the basis of normal handwriting samples is greatly improved if the pen-force signal is known (Maarse et al. 1986a). Thus, in the writer identification or signature verification problem, the pen-force signal is an important source of information (Plamondon and Lorette 1989). On the other hand, the number of studies exploring pen force is rather limited and little is known about the underlying control process.

Methods to measure pen force differ greatly. Sometimes, the pen force is measured directly with some kind of transducer during writing so that its time function is known. In the case where the transducer is mounted in the pen, and measures force along the longitudinal axis of the pen, we will speak about Axial Pen Force (APF). In the case where the transducer is located under the writing surface, normal pen force (NPF) is measured. In the latter case, the wrist is typically located on a separate supporting surface. At other times, as in forensic handwriting analysis, pen force is inferred from the static properties of the handwriting, i.e., trace thickness and depth (Baier et al. 1987) and the paper characteristics (Deinet et al. 1987), but the pen-force time function is not known. Another measure that is sometimes used is the pen-grip force (Kobayashi 1981). In what follows, however, we will only be concerned with time-varying APF or NPF. Axial Pen Force and Normal Pen Force are related by:

$$F_N(t) = -F_A(t)\sin(\varphi(t)) \qquad (1)$$

where φ is the angle between the longitudinal pen axis and the writing plane.

A central question to be solved is the relationship between the pen-tip kinematics and pen force. Essentially two viewpoints are relevant: the biomechanical hypothesis and the central control hypothesis.

[1] Since the actual area of the pen point is rarely included in the measurements, pen pressure will be referred to as *pen force* in this article

Pen-Force Variation as a Passive, Biomechanical Process

In this view, the pen-force changes during writing are seen as a consequence of biomechanical factors related to the kinematics of the movements. Dooijes (1983) relates APF variations to the pen-tip displacement in the vertical direction, supposedly brought about by the forefinger in many subjects which is "... pushing the pen into the paper surface during down strokes" (a stroke is generally defined as the trajectory segment between two consecutive minima in the tangential pen-tip velocity).

In this paper we would like to propose an approach that describes the pen force problem in terms of a mechanical impedance (Hogan 1985) or compliance control problem (Asada and Slotine 1986; Mason 1982). A mechanical impedance is a system which accepts motion input and yields force output (Hogan 1985). Suppose we wanted to let a robot system produce cursive script on some writing surface. We could define the motor task in terms of a pen-tip trajectory formation problem. In this situation the moving system has to control a lot of intra-corporal degrees of freedom (body df, bdf) in joint and actuator space. In the extra-corporal *spatial* domain, however, the movement in the air towards the writing surface demands the control of six (3 translational, 3 rotational) extra-corporal degrees of freedom (task df, tdf), while controlling zero degrees of freedom in the extra-corporal *force* domain since there is as yet no contact. However, at the moment of contact, making point-to-plane contact with the pen held in the end effector, the control problem is transformed into a five spatial tdf and one force tdf problem. The force is applied to the paper surface and compensated by a component, normal to the writing plane (NPF) and a frictional component along the writing plane. No torques are required by a point-to-plane contact. Clearly, the requirements of force control should be part of the motor task description. A possible description in handwriting is: "apply force in such a way that friction is overcome and a clear, legible trace is left behind". Thus, apart from the trajectory formation, mechanical impedance control is required. Since the writing system has to overcome surface (say, Coulomb) friction, additional force components have to be present along the X- and Y-axes, that are linearly related to NPF (Deinet et al. 1987). These additional force components complicate the control problem. There are task constraints, however, to make things easier for our robot. No rotation around the longitudinal pen axis is required for normal cursive script, so we can neglect this tdf. Furthermore, pen orientation does not have to be controlled explicitly (can be held approximately constant) since it is not part of the specific task requirements. In the human writer, the average orientation angle of the pen depends on hand anatomy and on personal preference, and variations seldom exceed a maximum amplitude of ten degrees in the normal cursive handwriting size, which is about 2.5 mm for an ⟨a⟩, on average. The movement system can concentrate on the pen tip's trajectory formation and on mechanical impedance, i.e., on regulating the normal pen force to produce a continuous trace of sufficient thickness and on overcoming friction in the XY plane by exerting an appropriate force along the X and Y axes.

According to a "pure" biomechanical hypothesis, variations in pen force are directly related to the peculiarities of the multi-degree-of-freedom, non-orthogonal effector configuration that a human hand in fact is. In this view, movements intended to take place in the XY plane are accompanied by inadvertent force variations along the Z-axis because the system is not exhibiting ideal active or passive compliance or both. If the system is geared to high stiffness, force variations will be of high amplitude; if the system is highly compliant, force variations will be reduced. However, in any case, the result will be a strong coupling between pen-tip kinematics and pen-force variations. As Fig. 1 shows, the writing hand is a polyarticular system consisting of a closed kinematic chain. It is polyarticular in the sense that each tendon spans a considerable number of joints, going from its muscular attachment in the forearm to the distal finger tip. A grossly simplified biomechanical model describes APF as the consequence of compressing a viscoelastic system by moving the surface contact point in the direction of the normal at a fixed hinge. It also shows that in such a system, pen angle is directly related to pen-tip position. Empirical evidence (1 subject) for the latter point is found in Deinet et al. (1987).

For example, in the pen-grip style with the palmar part of the wrist resting virtually flat on the writing surface, the finger flexion and extension will lead to

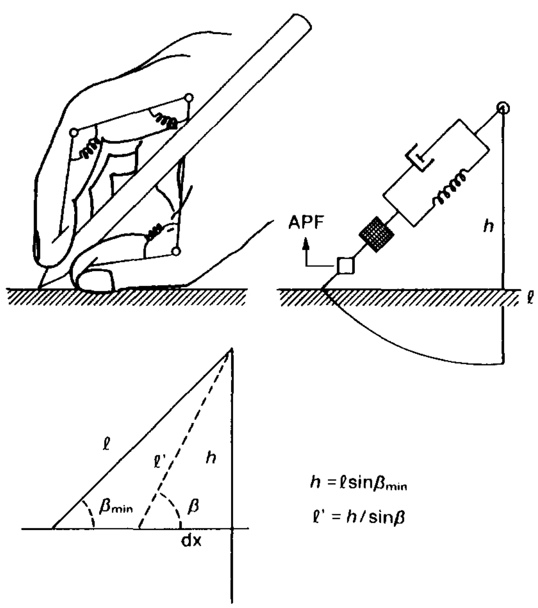

Fig. 1. A simplified biomechanical model relating planar movement to axial pen pressure (h is fixed height of hinge, l is distance from hinge to pen tip, l' is compressed length, β is current pen angle, β_{min} is minimum angle for surface contact, dX is current distance between pen tip and the normal)

larger variations in pen angle than wrist adduction and abduction, as an observation of the rear end of the pen during simple linear writing movements will reveal. The biomechanical hypothesis is attractive from the point of view of control efficiency. An appealing theory on skeleto-muscular motor control states that movements are brought about by the planning of muscle length ratios at target positions (Bizzi et al. 1976; Morasso and Mussa Ivaldi 1987; Hogan 1985). In this view, movement is an equilibrium trajectory of minimum potential energy caused by the elastic energy that is stored by muscular (co-) contraction. This type of control obliterates a temporally fine-grained trajectory planning between intermediate target positions. Similarly, the application of force to external objects is the direct result of the difference between the stored elastic energy state and the state the motor system is forced to maintain after obstruction by an external object. In handwriting, the obstruction is presented by the pen, yielding pen-grip force, and by the writing surface, yielding NPF and friction. In equilibrium theory, the planned virtual trajectory would be located spatially beneath the writing surface. If we assume that the elastic energy potential function E_p of the end effector is smooth (a valley), that the movement direction coincides with the major or minor stiffness axis (Hogan 1985), and that the movement does not cross the equilibrium point, there is a linear relation between small displacements and force. Under the same assumptions, force will generally covary strongly with displacement in complex movements patterns, too, since E_p is monotonically increasing with distance from the equilibrium point. The exception is the special case of the isotonic trajectories in which the shape of the movement pattern is fully determined by a constant force constraint.

Pen-Force Variation as an Actively Controlled Process

However, it can also be hypothesized that variations in pen force are actively regulated by a central nervous system (CNS) process, independent of the trajectory control. For example, Kao et al. (1983) found an increase in normal pen force (NPF) as the patterns to be copied became increasingly complex. Another finding of this study was that pen force increased during the production of a single pattern. Furthermore, there are many older (German) studies, relating pen force to high-level constructs such as personality or mental state (Kraepelin 1899; Kretschmer 1934; Steinwachs 1969). A problem with these latter theories is that they do not attempt to describe important physical aspects of the pen-force control problem.

Leaving aside hypotheses that attach weight to high-level constructs, such as, e.g., mental stress, as causing the pen-force variations (Steinwachs 1969), it can be hypothesized that in the process of learning to write the letter shapes (allographs), the writer adopts his own strategy or style of controlling pen force during trajectory formation. According to this viewpoint, the main intention of the movements is to produce spatial shapes within a certain amount of time. The shape of the pen-force time function would be only indirectly of importance: its average level should be just high enough to produce a trace of sufficient thickness. If force variations are indeed purely a matter of personal writing style, the result would be a complex, subject-dependent relation between pen-tip kinematics and pen force.

The question of whether pen force is a natural, physical consequence of finger movement or an independently controlled variable is especially important in models of handwriting. Plamondon and Maarse (1989) give an overview of 14 models of handwriting from the point of view of systems theory. These models are two-dimensional and do not incorporate pen-force or mechanical impedance control. Ideally, to be included in these models, the pen-force signal should be independent of the movement control signals. Also, before developing a coupled oscillator model (Beek and Beek, 1988) of pen-force control, one must know if there is any coupling at all.

Although the separation of passive from active aspects in the handwriting process is a very complicated problem, and probably only partly solvable because the nervous system makes efficient use of the biomechanical and physiological characteristics of the effector and sensor systems in an integrated fashion, it seems worthwhile to test to what extent pen force is related to movement.

In a pilot study on the handwriting and drawing movements of two subjects, two methods of analysis were performed to test the relation between movement and pen force. First, it was argued that a simple first-order correlation would not suffice because of phase or time differences between the movement (displacement, velocity, acceleration and angular velocity) and the force signal. Therefore, a cross-correlation analysis was performed. Results revealed that the cross-correlation never displayed a consistent and reproducible clear peak value above 0.8 at a fixed delay, and correlation values were lowest if the movements involved scribbles or cursive handwriting. Subsequently, a second type of analysis was performed, that was based on the assumption that the combined linear contribution of planar displacement, velcoity and acceleration yielded, by biomechanical coupling, an axial component of pen force. The latter analysis (linear multiple regression) did not yield consistent results in terms of signal significance or the proportion of explained variance. The conclusions of the pilot experiment were threefold. First, it appeared that it was of essential importance to control the pen-grip style of the subjects in order to allow for a comparison of finger and wrist contributions to the movement. For example, short straight lines of length 1 cm at an angle of 45 deg can be produced by the wrist, the fingers, or a combination of both, depending on the forearm attitude. Second, it was evident that, in order to rule out either the "biomechanical" or the "central" explanation for pen-force variations, a larger number of subjects and recording was necessary. Third, it can be argued that the lack of consistent findings is caused by the fact that the relation between movement and force is only significant within a limited frequency band, e.g., the 5 Hz periodicity in handwriting (Teulings and Maarse et al. 1984; Maarse 1986b), and that a lumped correlation measure hides such a dependency.

It is hypothesized that if pen force is the direct consequence of biomechanical loading and unloading of the wrist and finter muscles, it should covary with the movement produced by the stroke production process, regardless of the complexity of the drawing pattern as a whole, e.g., pen force invariably going up in downward strokes. In one study the fingers are mentioned as having a larger effect on pen-force variation than wrist movement (Dooijes 1983).

According to several handwriting models, writing movements are generated by a system that produces bell-shaped tangential velocity profiles ("strokes") of the effector (Morasso and Mussa Ivaldi 1982), along with the production of bell-shaped angular velocity profiles (Plamondon 1987, 1989). A possible coupling (synergism) between this (CNS) stroke production mechanism and the pen pressure should be revealed by high coherence between tangential velocity and/or angular velocity on the one hand, and APF on the other hand. In this case, a hypothesis that can be put forward is that pen force will be increased at stroke transitions, where the tangential velocity is low and the angular velocity and curvature are high. We use the term tangential velocity instead of the more general term curvilinear velocity because we are dealing with planar movement.

In order to determine the existence and strength of linear relationships between movement and axial pen force, we will calculate the coherence spectrum for several types of handwriting patterns. The Cartesian displacement coordinates will be transformed into an estimate of the oblique system that represents the directions of wrist and finger movement, respectively. This provides the opportunity to separate the wrist and finger contributions to the axial pen force. Also, the coherence between APF and tangential velocity as well as angular velocity will be determined. A set of drawing patterns will be used, varying in complexity from straight lines to scribbles and cursive script.

2 Methods

2.1 Data Acquisition

Subjects. Sixteen right-handed students, five male and eleven female, with an average age of 23.3 years, participated in the experiment. Subjects were not informed of the purpose of the experiment (i.e. that "pen pressure" was being measured).

Materials. The movements of the tip of the writing stylus was recorded by means of a large-size writing tablet (Calcomp 9000). The sampling frequency was 105.2 Hz, samples having a resolution of 0.025 mm and an accuracy of 0.25 mm in both X and Y directions. The tablet was connected to a PDP 11/45 computer via a 9600 baud serial line. The laboratory-made writing stylus was equipped with a strain-gauge force tranducer, measuring axial pen force in the 0–10 N range. The stylus contained a normal ball-point refill in tight contact with the force transducer. The analog signal from the pen-force transducer was low-pass filtered (second-order Butterworth, −3 dB at 17.5 Hz) and A/D converted with a resolution of 10 bits. Data were stored on magnetic tape and copied to a VAXstation 2000 computer where the actual analyses were done. Software was written in Fortran-77.

Procedure. The subject's task was to write predefined patterns or cursive words on a DIN A4 paper sheet on the writing tablet. The tablet was placed in such a way that the subject was sitting in a convenient position, writing at a preferred angle, just as in a normal writing situation. Patterns had to be written at a pace corresponding to normal writing speed. The recording of a single drawing pattern lasted 12 sec. The duration of the writing of a single word is writer-dependent, but the maximum duration was set at 12 s. Before the actual recording took place, subjects had the opportunity to accustom themselves to the experimental set-up and to the writing patters that were to be used. The writing patterns were practised three times each. To eliminate arm movements, the forearm was placed and fixed in an adjustable special-purpose cuff attached to the digitizer (Maarse et al. 1986b). The forearm was fixed in such a way that its inner side was parallel to the vertical axis (Y) of the digitizer. In order to allow free movement of the hand, the ulnar side of the processus styloideus ulnae was just above the top edge of the cuff (Fig. 2).

Writing patterns were indicated by simple icons on the response sheet (Fig. 3), on which six patterns were randomly distributed, and amounted to ten trials per pattern. The following writing patterns were used. In conditon "F" (fingers), the subject had to make an oscillating writing movement at a preferred frequency, producing a short (maximally 6 mm) straight line by moving the fingers only, holding the wrist still, in a relaxed attitude. In condition "W" (wrist), the subject was asked to perform similar writing movements, in this case producing a straight line by using the wrist

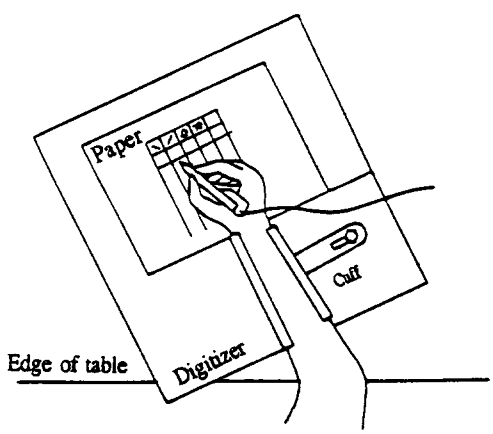

Fig. 2. The recording set-up with digitizer tablet and forearm cuff

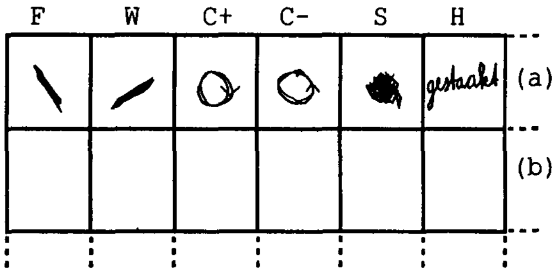

Fig. 3. The stimuli (**a**) and the writing area (**b**) on a DIN A4 response sheet. Patterns: straight finger movements (F), straight wrist movements (W), clockwise circling (C+), counter-clockwise circling (C−), scribbling (S), and cursive handwriting (H)

only, and holding the fingers still in a relaxed attitude. In a third condition, "C−", the subject had to draw counter-clockwise circles, about 5 to 6 mm in diameter. In a similar fourth condition, "C+", the circles were drawn in a clockwise fashion. In the fifth condition, "S", the subject had to draw scribbles, aiming at a spatial range of 6 by 6 mm, maximally. In the sixth condition, "H" (handwriting), the subject had to write the Dutch word "gestaakt" ("struck") in cursive style, without pen-lifts. This word was selected because it contains body-sized letters as well as ascenders and descenders, and is not too long. Care was taken to optimize the dynamic range of the wrist and finger movements, since the forearm was fixed. The subject was instructed to hold the writing hand relaxed in its preferred position. Finally, the response sheet was positioned with the left hand until the pen tip pointed to the center of the white response area below the stimulus pattern. No pen lifting was allowed during the trials.

2.2 Data Processing

Per subject, a data set of 10 trials × 6 patterns × 1280 samples × 3 coordinates (X, Y, APF) was collected (460.8 kb). From each trial in the drawing pattern conditions, the middle 1024 samples (9.733 s) were used in the analyses, thereby removing possible artefacts appearing during the initial and final periods of 128 samples (0.122 s) at the beginning and at the end of a trial. From each trial in the text condition, the middle 256 samples (2.433 s) of the written word were used (average word duration was 4.9 s). The signals, horizontal displacement (S_x), and vertical displacement (S_y), were obliquely transformed (Dooijes 1983), using:

$$
\begin{aligned}
\phi &= \mu - \lambda \\
c_1 &= \cos(\lambda) + \sin(\lambda)/\tan(\phi) \\
c_2 &= \sin(\mu) + \cos(\mu)/\tan(\phi) \\
c_3 &= -\tan(\lambda) \cdot c_2 \\
c_4 &= -c_1/\tan(\mu) \\
S_w &= c_1 \cdot S_x + c_4 \cdot S_y \\
S_f &= c_3 \cdot S_x + c_2 \cdot S_y
\end{aligned}
\tag{2}
$$

where λ is the estimated angle for the axis of the wrist system, with respect to the Cartesian x-axis, and μ is the estimated angle of the axis of the finger system with respect to the Cartesian x-axis. The angle ϕ represents the angle between the wrist and finger axes. The wrist angle is obtained by estimating the angle of the written line from the (S_x, S_y) coordinates in the "W" trials of a subject by linear regression. The finger axis angle is obtained by estimating the angle of the written line from the (S_x, S_y) coordinates in the "F" trials of a subject by linear regression. The application of Eq. (2) transforms the data to the estimated "internal" effector coordinate system, with wrist activity indicated by S_w, and finger activity indicated by S_f.

The displacement signals S_x, S_y, wrist activity (S_w), finger activity (S_f), and axial pen force (APF), were differentiated, using a five-point convolution window with Lagrange weights (1/12, −8/12, 0, 9/12 and −1/12, Abramowitz and Stegun 1970). The frequency domain transfer function of this differentiator is linear up to about 13 Hz in our case. Thus, the signals V_x, V_y, wrist velocity (V_w), finger velocity (V_f) and differentiated APF (i.e., dAPF) were obtained. From V_x and V_y, the tangential velocity (V_a) and angular velocity (V_θ) were calculated. The reason for the time-domain differentiation is twofold: (*a*) it removes low-frequency variations that would lead to large bias errors in the low-frequency range of the Fourier transform to be performed later, and (*b*) it keeps spectral components in the frequency range of interest (3–13 Hz) intact. Differentiation has virtually no effect on the coherence function estimate (see Appendix). Of each signal, the Fast Fourier Transform (FFT) was calculated per trial per condition per subject, after tapering with a 10% cosine window (Bendat and Piersol 1971; van Boxtel and Schomaker 1983). Bandwidth resolution (B_r) before smoothing was 0.103 Hz except in the case of the handwriting condition where B_r was equal to 0.411 Hz. The Fourier spectrum was transformed to a power spectral density function (PSDF). Also, cross power spectral density functions (CSDF) were calculated for the following comparisons: dAPF vs wrist velocity V_w, dAPF vs finger velocity V_f, dAPF vs tangential velocity V_a, and, finally, dAPF vs angular velocity (V_θ), a signal closely related to curvature.

The PSDF and CSDF were then smoothed with a rectangular window ($l-5$) in order to increase the reliability of the individual spectral estimates and to make it possible to calculate the spectral coherence function (Bendat and Piersol 1971). Then, per subject, per condition, the PSDF and CSDF spectra were averaged over the ten trials in a condition to obtain ensemble averages. This yields $2 \times 5 \times 10 = 100$ statistical degrees of freedom for the average smoothed PSDF and CSDF per subject per condition. To obtain a general estimate of the PSDFs and coherence functions per condition, however, the ensemble average spectra were again averaged over the sixteen subjects. The PSDFs were normalized to unit area before averaging, and the obtained condition average was rescaled to physical units again. A condition average PSDF has

16 × 100 = 1600 statistical degrees of freedom. The coherence functions underwent Fisher's Z transform before averaging, the average being converted to the coherence domain again. The squared coherence (also called Magnitude Squared Coherence or MSC) is given by:

$$\gamma^2(f) = \frac{|G_{uv}(f)|^2}{(G_u(f) \cdot G_v(f))} \tag{3}$$

where

$\gamma^2(f)$ = squared spectral coherence function

$G_{uv}(f)$ = cross spectral density function (CSDF)

$G_u(f)$ = power spectral density function (PSDF), signal u

$G_v(f)$ = PSDF, signal v

Note the similarity to the formula for the Pearson correlation. In fact, a single spectral coherence estimator is a coefficient of determination for the relationship between two variables in a specific frequency band. Other measures that will be used are the average APF level and its standard deviation and Pearson correlation between APF and displacement.

In order to test for non-stationarity, run tests were performed on all signals of each trial. The runs were determined by dividing each sample record into 10 segments of equal duration and calculating the 10 mean square values and their median value. This procedure captures non-stationarities in the mean and the variance of the signal. It is assumed that data are (weakly) stationary if maximally 5% of the trials exhibit a number of runs that has a probability of less than 0.05 of originating from a random process.

3 Results

The run tests revealed the following percentages (N = 960) of sample records yielding a number of runs with $p < 0.05$; dAPF 3.23%, V_w 2.08%, V_f 3.02%, V_θ 3.33%, all below 5%. There was no systematic relation between number of runs and condition. Table 1 shows the results for the preferred angles of the lines drawn in the conditions W and F. From the mean difference value, it can be inferred that the wrist and finger systems have approximately orthogonal movement axes, given the forearm attitude used. Figure 4 shows a superposition of the patterns produced by wrist movement and by finger movement in a trial of the W and F conditions, respectively. The widths of the patterns indicate that the wrist movements are more accurate than the finger movements, a finding consistent with earlier studies (Maarse et al. 1986b). Figure 5 shows the dAPF and V_f signals of a single trial in the clockwise circling (C+) condition, a time segment of 1 s within this trial and the shape of both the total circling pattern and the selected 1 s segment.

Figures 6 and 7 show the results of the comparisons of dAPF with V_w and V_f (wrist and finger domain velocities). The figures are scaled in physical units to enable comparison. The smoothness of the handwriting spectra (panel H) as compared to the simple drawing spectra (panels C+, C−, F, W, S) is due to the shorter sample record duration in the former, resulting in a lower spectral resolution.

Table 1. Average preferred angles in degrees for the linear wrist and finger movements with respect to the X-axis of the digitizer, and their difference. Note that the forearm is aligned with the Y-axis of the digitizer

Subject	Wrist	Fingers	F-W
01	36	136	100
02	26	132	106
03	44	127	83
04	35	129	95
05	36	127	91
06	36	141	105
07	52	136	85
08	35	134	98
09	48	128	79
10	31	146	115
11	22	132	110
12	39	130	91
13	39	127	88
14	42	123	81
15	53	159	106
16	48	134	86
Mean	39	134	95

The Axial Pen-Force PSDFs

From the figures it is clear that all PSDFs (dAPF and kinematics) have a peak in the area of 2 and 5 Hz. The peak in the dAPF spectrum occurs at about the same frequency as the peak in the kinematics spectra, small deviations being due to the estimation error. The main difference between the dAPF and kinematics spectra is the relatively larger amount of dAPF power in the range above 8 Hz, for all conditions. The most probable explanation is the contribution of friction with its hysteresis effects, and the paper surface irregularities in the APF signal. In fact, hysteresis could be inferred clearly in the single-subject dAPF PSDFs of six of the

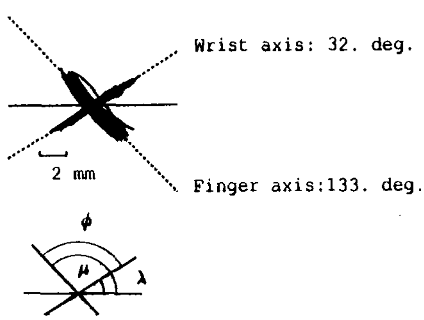

Fig. 4. Superimposed wrist and finger patterns of one subject, from trials in conditions W and F, respectively, and the estimated main axes of movement

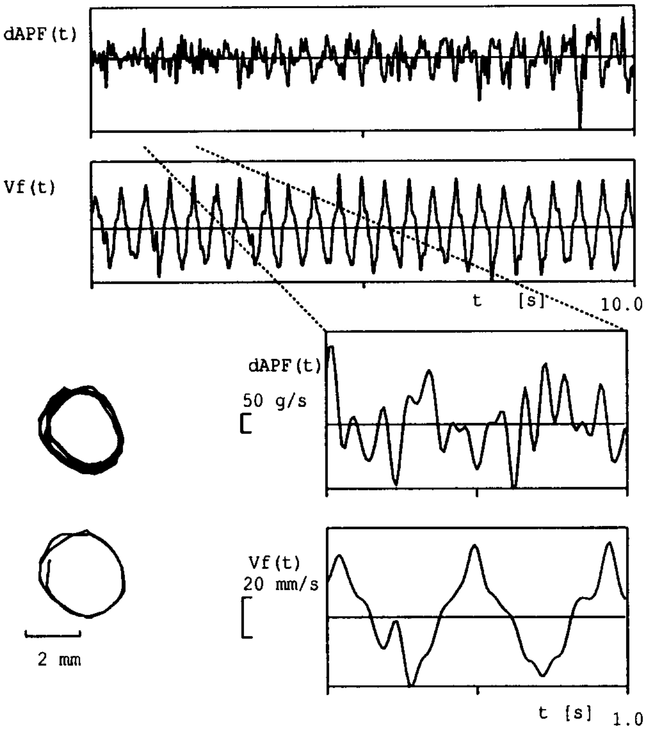

Fig. 5. An example dAPF and V_f time functions and pattern shape of a single trial and a segment of 1 s duration of this trial in the clockwise circling condition C+(subject 15)

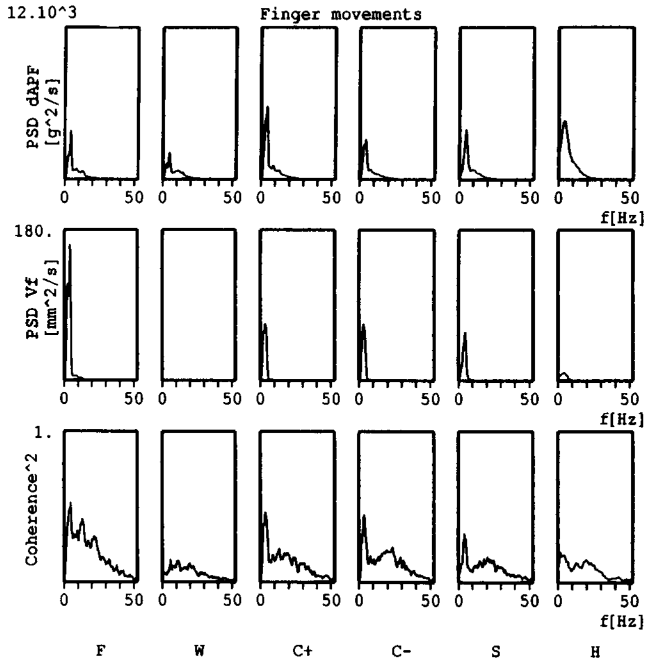

Fig. 7. Power spectral density functions and squared coherence spectra for dAPF and finger velocity V_f. Conditions are: straight finger movements (F), straight wrist movements (W), clockwise circling (C+), counter-clockwise circling (C−), scribbling (S) and cursive handwriting (H)

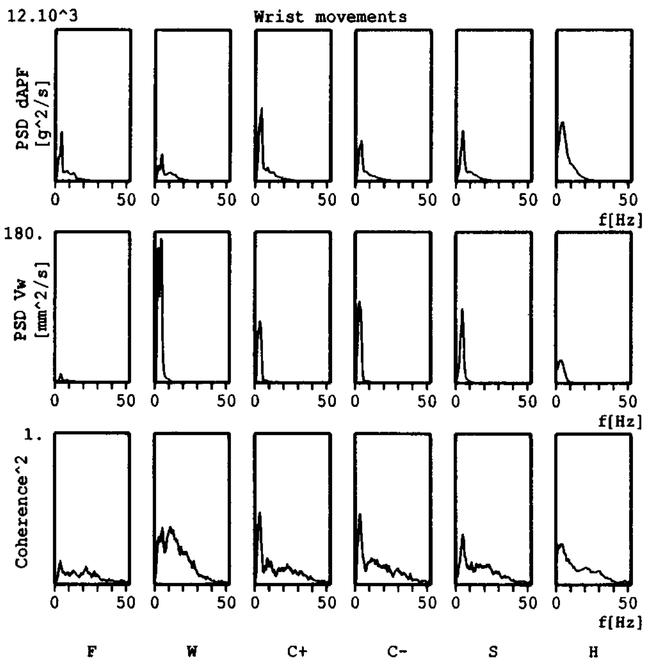

Fig. 6. Power spectral density functions and squared coherence spectra for dAPF and wrist velocity V_w. Conditions are: straight finger movements (F), straight wrist movements (W), clockwise circling (C+), counter-clockwise circling (C−), scribbling (S) and cursive handwriting (H)

sixteen subjects, showing peaks up to the second harmonic. The reduced remainders of these higher harmonics can be seen in the average dAPF PSDF of the clockwise circling (panel C+) and the finger movement (panel F) conditions. Overall dAPF power is greatest in handwriting (panel H), intermediate in scribbles (panel S) and circling (panels C+, C−), and small in straight finger and wrist movements (panels F and W). The average, variance and time trend of the primitive APF signal are shown in Table 2. The average APF is not related to variance in this series of conditions. In cursive script, there is a positive time trend, in the other conditions, APF decreases slowly during a trial.

The Kinematics PSDFs

The kinematics PSDFs in circling and scribbling are roughly comparable in shape and area (Figs. 6 and 7, panels C+, C− and S). The differences in peak power values between circling and scribbling spectra reflect differences in spatial movement amplitude, rather than differences in periodicity, as can be inferred from peak width. When subjects are asked to produce scribbles, the temporal behavior is thus not as irregular as the spatial result would lead one to suspect. Scribbling is performed faster (average peak frequency 4.5 Hz) than circling (3.5 Hz).

Furthermore, as could be expected, the power of movements along the finger axis is greatly reduced when wrist movement were requested (Fig. 7, panel W). This suppression takes place to a lesser extent with respect to the power of movements along the wrist axis in case of finger movements (Fig. 6, panel F).

Coherence Between dAPF and V_w

In the comparison between dAPF and the wrist velocity V_w, it appears that a maximum peak coherence (0.42 to 0.44) is reached in circling movements (Fig. 6, panels C+ and C−). The difference in peak coherence between clockwise and counter-clockwise circling is small. This means that maximally 40% of the power in dAPF at the fundamental movement frequency can be explained by wrist movements, if the movements are circular. In straight wrist movements (W), the peak coherence is somewhat lower (0.35) but the more striking feature is that coherence is smeared out over a broad band from 5 to 20 Hz, (Fig. 6, panel W). In straight finger movements, the wrist contribution to dAPF is negligible. The shape of the coherence spectrum for the scribbling movements is comparable to that for circling, but the peak coherence is lower (0.3). In producing cursive handwriting (panel H), peak coherence is still lower (0.25) and the shape of the coherence spectrum is broad banded.

Coherence Between dAPF and V_f

In the comparison between dAPF and the finger velocity, peak coherence values and coherence spectrum shape are similar to those in the $d\text{APF} - V_w$ comparison, with the exception of straight finger movement condition. The finger velocity in straight finger movements (Fig. 7, panel F) explains 0.49 of the dAPF power at the fundamental oscillation frequency, which is the maximum peak coherence value obtained in this study. The coherence spectrum shape is of the broadband type with peaks at the fundamental frequency 4.5 Hz, at 13.0 Hz and at 21.7 Hz. The coherence between the wrist velocity and dAPF is very low in straight finger movements (0.14).

Coherence Between dAPF and V_a or V_θ

The coherence between dAPF and tangential velocity or between dAPF and angular velocity V_θ never reached a value above 0.3 in any condition.

First-Order Correlation Between APF and S_f or S_w

An analysis of variance on the Z-transformed first-order (Pearson) correlation between APF and S_w and between APF and S_f revealed the significant main effects Condition ($p < 0.0001$, 5 df), Effector (i.e., Wrist vs Fingers) ($p < 0.0001$, 1 df), Subject ($p < 0.0001$, 15 df). The interactions were: a trivial Effector*Condition ($p < 0.0001$, 5 df) due to the F and W conditions, Subject*Effector ($p < 0.0001$, 15 df), Subject*Condition ($p < 0.0001$, 75 df). The finger movements were slightly more strongly correlated to APF (mean $r = -0.23$) than the wrist movements (mean $r = -0.14$). Note that in this analysis the sign of the correlation biases the average, unlike the case of mean coherence values. There is only one positive correlation (+0.09), between APF and finger displacement in the W condition. The correlation figures should be squared for comparison with coherence values. The largest mean correlation found was −0.39 for the fingers in the clockwise circling condition (C+). Clockwise circling (C+) yielded higher correlation values than counter-clockwise circling (C−) (Table 3).

To exclude the possible influence of pen angle variation, and consequent variation in the pen force, a test was performed with a modified tablet controller from which pen angle could be derived with an accuracy of 3 deg (Maarse et al. 1988). The controller device was only available after collection of the main data set. Recordings ($T = 9$ s) never revealed correlations below 0.96 between axial and normal pen force in any of the

Table 2. Average APF masures for all conditions, in [g] unless otherwise stated. Note the maximum variance and positive time trend in the cursive script condition (H). APF(0) and APF(n) denote linearly estimated initial and final force level, be is the gain of the time trend, r is correlation of APF vs time

	mean	var[g^2]	APF(0)	APF(n)	b[g/s]	r[ ]
S	83.6	284.5	91.3	75.9	−2	−0.27
W	87.5	250.6	95.0	80.1	−1	−0.29
C$^+$	108.9	295.9	116.0	101.8	−1	−0.25
H	109.7	806.4	91.6	127.7	+8	+0.39
F	113.5	259.2	121.2	105.8	−1	−0.26
C$^-$	115.3	307.7	124.0	106.6	−2	−0.29

Table 3. Average first-order (Pearson) correlations over subjects ($N = 16$) between APF and finger displacements S_f and between APF and wrist displacement S_w for all conditions

Effector:	Condition:	F	W	C+	C−	S	H	Mean
R(APF, Sf)	(Fingers)	−0.36	−0.09	−0.39	−0.24	−0.20	−0.22	−0.23
R(APF, Sw)	(Wrist)	−0.02	−0.15	−0.33	−0.11	−0.14	−0.10	−0.14
Mean		−0.20	−0.03	−0.36	−0.	−0.17	−0.17	−0.16

conditions (N = 4 subjects). The reason for this strong relationship is the small amplitude of the pen angle variations (2.5 deg) with respect to the average value (50 deg).

Results thus far seem to indicate that, in cursive script, the relation between pen force and kinematics is rather weak, and that only in simple movement patterns a coherence of intermediate value can be observed. However, from a visual analysis of the single-subject time records, the impression was that the correlation between APF and kinematics is actually waxing and waning in time. We will now proceed to analyze this behavior for the cursive script condition (H). In order to determine the development of the relation between APF and vertical displacement over time, an instantaneous (running) Pearson r ($r^{51}_{\mathrm{APF},\,Y}(t)$) (see Appendix) was calculated, using a window width of 51 samples, corresponding to about half a second, or at least a number of five strokes. This window-width value is not critical, as long as it is large enough to contain several strokes, and small enough to fluctuate within the sample record (the written word "gestaakt"). For simplicity, the raw vertical displacement $Y(t)$ was chosen. This signal contains a large proportion of finger movement (Table 1). It appears that roughly three types of subjects were present.

Type *a* subjects (10 in 16) show a high number of sign reversals (> 4) of the correlation $r^{51}_{\mathrm{APF},\,Y}(t)$ (Fig. 8a). Locally, however, absolute correlation values of 0.8 in $r^{51}_{\mathrm{APF},\,Y}(t)$ were not uncommon. Overall correlation (and coherence) was low. Especially interesting in type *a* subjects is the fact that the shape of APF(t), $Y(t)$ and the correlation time function $r^{51}_{\mathrm{APF},\,Y}(t)$ were well replicated over trials in the cursive script conditions.

Tye *b* subjects (2 in 16) show a much more smooth pattern of $r^{51}_{\mathrm{APF},\,Y}(t)$, with a limited number of brief sign reversals, and a relatively high but negative correlation value (Fig. 8b). There is a medium inter-trial consistency.

Type *c* subjects (4 in 16) show noisy displacement and APF patterns and a consequently low correlation with kinematics (Fig. 8c).

In order to track down the origin of the fluctuations in $r^{51}_{\mathrm{APF},\,Y}(t)$, a measure of replicatability of both APF(t) and $Y(t)$ was needed. We chose to calculate the average correlations, via Fisher's Z transform, between replications of a word for APF, yielding $R_{\mathrm{APF},\,\mathrm{APF}}$, and for the vertical displacement, yielding $R_{Y,\,Y}$. These measures will indicate the degree to which the writer was able to replicate the APE or S_y patterns over different realizations of the written word, given the beginning of the first down stroke in ⟨g⟩ as the time synchronization reference. For comparison, the within-trial average correlation between APF and vertical displacement, $R_{\mathrm{APF},\,Y}$ was also calculated. The next measure calculated was the number of runs or phases, N_{phases}, in $r^{51}_{\mathrm{APF},\,Y}(t)$, in order to provide a measure of the complexity of the relation between APF(t) and $Y(t)$.

Figure 9 shows the distribution of subjects in a two-dimensional space of correlation complexity (N_{phases}) versus the average correlation between vertical

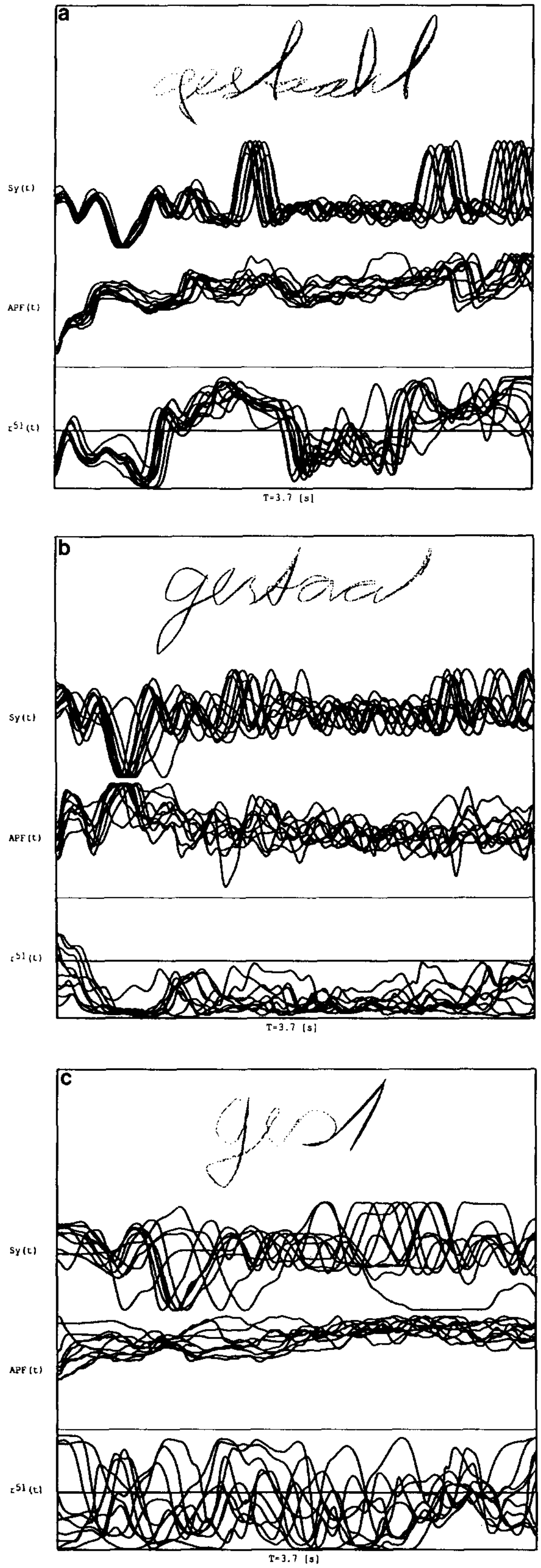

Fig. 8 a–c. Instantaneous Pearson r^{51} between APF and vertical displacement S_y during the production of a cursive word in three types of subjects. The relative APF level is coded in grey scale (high force in black). **a** Type *a* subject, showing APF patterning. **b** Type *b* subject, showing biomechanical coupling. **c** Type *c* subject, showing noisy APF patterns

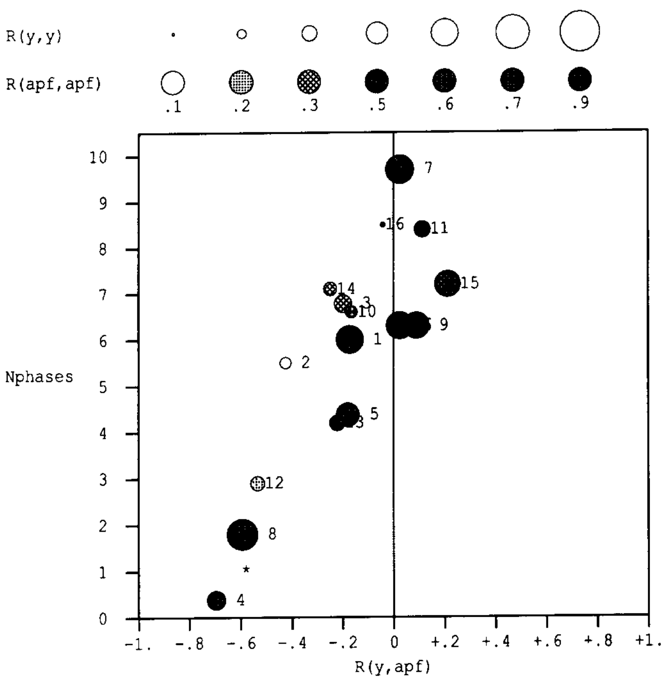

Fig. 9. Average number of phases in the instantaneous Pearson r, versus average Pearson r between vertical displacement and APF ($R_{(y,\,\mathrm{APF})}$), for sixteen subjects and a calligrapher (*). The average inter-pattern correlation ("replicatability") for vertical displacement ($R_{(y,\,y)}$) is represented by circle radius, and for APF($R_{(\mathrm{APF},\,\mathrm{APF})}$) by grey scale. Subjects are numbered 1 to 16

displacement and APF ($R_{\mathrm{APF},\,Y}$). The replicatability of the APF and vertical displacement pattern S_y, as reflected in average inter-pattern correlations also are shown ($R_{\mathrm{APF},\,\mathrm{APF}}$ and $R_{Y,\,Y}$). Typical type *a* subjects are numbers seven and fifteen, typical type *b* subjects are numbers four and eight. Subjects two, ten, twelve and sixteen are typical of the type *c* category: note the small radius and light shading that are indicative of low

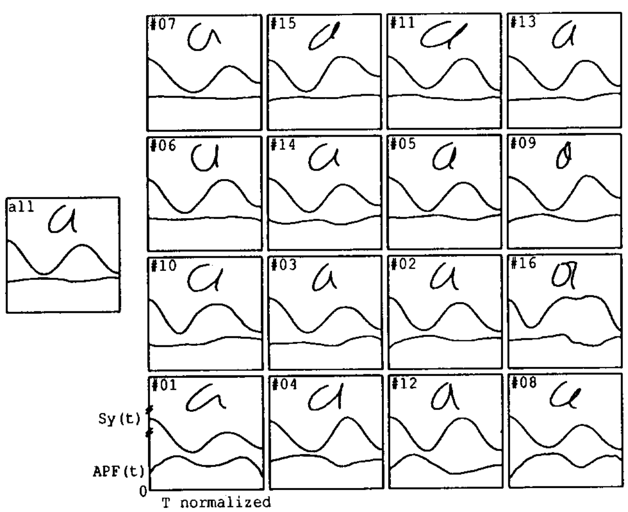

Fig. 10. Average time-normalized functions of APF and S_y and the shapes of the first letter ⟨a⟩ in the word "gestaakt" for each subject (n = 10 replications), and the overall average pattern. Scaling of APF and S_y is in arbitrary units for each panel, APF is plotted with zero origin, S_y is plotted with floating origin

inter-trial replicatability for both vertical displacement and APF for these subjects. Interestingly, from recordings of a calligrapher it was found that this person can be classified as a type *b* subject, as indicated by an asterisk (Fig. 9). Table 4 contains the within and between-pattern correlations for all subjects, for the whole word "gestaakt" (Table 4a) and for the time segment that corresponds to the first letter ⟨a⟩ (Table 4b). This letter was selected because it did not display allographic variation over the subjects. All subjects wrote it much in the same way, such that a clean ensemble average over 160 replications could be obtained (Schomaker and Thomassen 1986). As could be expected from the $r^{51}_{\mathrm{APF},\,Y}(t)$ fluctuations, locally, within the ⟨a⟩, a somewhat stronger relation between APF and S_y can be found: another two subjects display a value of $R_{\mathrm{APF},\,Y}$ that is less than -0.6. Subjects six, ten and sixteen display elevated writing times. Trend removal from APF and S_y yielded very similar figures. Figure 10 shows the time-normalized ensemble averages (n = 10) of ⟨a⟩ per subject and the overall average (n = 160). Apart from the time normalization, no DC or amplitude normalization was performed in the calculation of the averages, but because of between-subject differences in the average APF level, Figure 10 is scaled to fit optimally, assuming an APF origin of 0. The panels are sorted in an order of increased relative APF variance, with the result that subjects with a low absolute correlation between APF and S_y are predominant in the top row, with an approximately flat APF profile, and subjects with a higher (negative) correlation between APF and S_y are predominant in the bottom row. Note the different APF patterns for each subject.

4 Discussion

The moderate coherence values obtained indicate that, at least for the majority of writers, a simple biomechanical coupling between APF and kinematics is unlikely. The residual non-ideal or non-linear relation that exists, attains its greatest strength in simple linear movements, with low average pressure levels, deteriorating as movement complexity increases. Since the periodicity of the axial pen-force variations is the same as the periodicity of the movements, it must be the phase relation between the two that is time-variant or noisy. This latter explanation is supported by the finding that the first-order correlation between pen force and displacement fluctuates over time. The pen angle can be discarded as a cause of this phase jitter because it is coupled to the pen-tip kinematics. With regard to pure spatial factors, like points of high curvature, the data reveal that there is no coherence between APF and tangential velocity or angular velocity. Since points of low velocity and high angular velocity correspond to the high-curvature points, high-order inter-relationships of this kind can be excluded. On the whole, pen force appears to be a separate control variable.

The mean first-order correlation over the subjects, between APF and wrist displacement, and between

Table 4. Average correlations (N = 10 replications) between APF and S_y, between the S_y functions of different replications and between APF functions of different replications of the word "gestaakt" (4a) and its first letter ⟨a⟩ (4b), for all subjects. Also shown are the average APF level and its standard deviation and the average writing time

Table 4a. The word "gestaakt"

Subject	*R*(APF, *Y*)	*R*(*Y*, *Y*)	*R*(APF, APF)	*M* [g]	*SD* [g]	*T* [s]
01	−0.17	0.60	0.43	57	18	3.419
02	−0.43	0.24	0.11	60	12	2.974
03	−0.20	0.38	0.32	128	16	4.917
04	−0.70	0.39	0.47	85	18	4.312
05	−0.18	0.50	0.73	212	25	3.592
06	0.02	0.57	0.55	65	14	10.133
07	0.02	0.61	0.43	95	11	3.319
08	−0.59	0.65	0.45	100	25	2.596
09	0.09	0.57	0.65	114	25	3.361
10	−0.17	0.25	0.40	175	24	8.663
11	0.11	0.35	0.55	66	23	3.761
12	−0.54	0.31	0.14	83	19	5.990
13	−0.22	0.34	0.47	58	11	3.564
14	−0.25	0.29	0.36	97	15	5.471
15	0.21	0.56	0.77	140	19	3.736
16	−0.04	0.11	0.41	114	17	8.107

Table 4b. First ⟨a⟩ in "gestaakt"

Subject	*R*(APF, *Y*)	*R*(*Y*, *Y*)	*R*(APF, APF)	*M* [g]	*SD* [g]	*T* [s]
01	−0.19	0.52	0.43	154	7	0.278
02	−0.04	0.64	0.54	146	7	0.236
03	−0.21	0.76	0.60	88	8	0.373
04	−0.72	0.79	0.57	83	6	0.309
05	−0.60	0.53	0.36	67	20	0.287
06	0.28	0.79	−0.01	105	9	0.796
07	−0.12	0.76	0.46	226	28	0.247
08	−0.61	0.84	0.81	125	11	0.215
09	−0.41	0.67	0.48	72	17	0.328
10	−0.10	0.78	0.75	100	5	0.462
11	−0.25	0.71	0.31	74	5	0.279
12	−0.46	0.33	0.46	210	13	0.542
13	−0.60	0.64	0.62	89	11	0.326
14	0.01	0.83	0.66	130	16	0.419
15	0.19	0.77	0.67	57	6	0.309
16	0.00	0.49	0.01	93	19	0.694

APF and finger displacement, shows comparable results to the coherence analysis but the values are lower, supporting the hypothesis that residual biomechanical coupling takes place predominantly at the modal movement frequency. There are indications that the residual correlations are due to biomechanical effects. The sign of these correlations is mostly negative, relatively higher APF corresponding to finger flexion and wrist radial abduction. On the whole however, variance in APF cannot be explained by kinematic variables. The fact that APF is somewhat stronger coupled (Table 3) with movement in the clockwise circling condition than it is in counter-clockwise circling could be explained by the existence of a curl term in the stiffness matrix (Hogan 1985) that has to be overruled in clockwise movement. More specifically, this finding points to a larger stiffness of the thumb subsystem as compared to the opposing finger system.

For the cursive script condition, a more detailed analysis revealed three categories of subjects. Type *a* subjects ("APF patterns") displayed a complex but replicatable relationship between APF and displacement. The replicatability of the pen-force pattern and the instantaneous force-displacement correlation pattern both support the notion of independent, feedforward control of the force by the CNS in many subjects. It is well known that in handwriting at least one, but most probably several, strokes are planned in advance (Hulstijn and van Galen 1983; Stelmach and Teulings 1983). Transmission delays exclude the possibility of a continuously monitored pen-tip *displacement* in a neural feedback loop. The average observed writing speed is eight to twelve strokes per second in the adult cursive writer. In type *a* subjects, it is quite likely that CNS advance planning or feedforward control is also the case with the pen *force* aspects (average force level and impedance) of the writing movement as it is with the trajectory formation (Rack 1981).

In a small minority of subjects (type *b*, "biomechanics"), there can be a strong coupling between axial pen force and movement kinematics. The sign of the correlation between vertical displacement and APF is negative, which means that APF does indeed increase in down strokes (Dooijes 1983), at least in this group of writers.

A third group of subjects is characterized by low replicatability of both kinematics as well as APF (type *c*, "shakers"). It is as if these writers do not have a stable internal representation for cursive script movements.

In the current experiment, most subjects fall into category *a*. Here, the correlation between the force and displacement is time-variant, biphasic, and subject-dependent. The writer's strategy might be, at some time or in some specific writing context, to actively pre-program mechanical impedance during movement, thereby minimizing pen-force variations. This can be achieved by an anticipatory lowering of the amount of agonist/antagonist co-contraction. If the writer overcompensates, the sign of the resulting force-displacement correlation will be the inverse of the sign of the correlation in the case of uncompensated biomechanical force variations. If the writer fails to compensate or even increase stiffness, e.g., if the trajectory formation control temporarily requires more resources, the force-displacement correlation will be determined by the amount of noise in the neuro-muscular force control and by the biomechanics. In human handwriting, it is unlikely that the stiffness regulation mechanism is Cartesian, plane-oriented such as is used in the robotical seam welding of curved surfaces. If a planar target trajectory requires a high degree of hand stiffness along the X and Y axes for positional accuracy, this generally will have a strong effect on the stiffness along the Z axis, too. Considering the effects of pen-to-paper friction, current findings show that in using the relatively low-friction ball point stylus, the friction influence on APF is nearly constant, as witnessed by the high correlation between APF and NPF. Dooijes (1984) estimates the friction to be about 4% of the NPF value in the 0–2 N range. However, more study is needed on this topic.

Conclusions

The levels of the coherence and first-order correlation between pen force and pentip kinematics in drawing simple patterns and in cursive script are rather low for a majority of writers. However, the replicatability of the pen-force pattern for a given word and the replicatability of the instantaneous correlation pattern between vertical displacement and pen force shows that the lack of overall coherence cannot be explained by an external source of random noise. A possible explanation is the presence of a separate control component that regulates pen force in an idiosyncratic fashion for each writer. One may speculate that this is possible since pen force is an extraneous, invisible variable, the time function of which is not explicitly addressed in the course of learning cursive script. The current findings are consistent with the known high discriminatory value of pen pressure in writer identification. A major implication for handwriting modeling (Hollerbach 1981; Edelman and Flash 1987; Schomaker et al. 1989) is that trajectory control can be separated from pen-force control. The availability of a pen angle signal will allow for more detailed analyses, e.g., a decomposition of the axial pen force into 3-dimensional components. As a preliminary result however, it appears that pen angle variations are too small to explain a large proportion of the pen-force variance. The control of pen force during handwriting could be a paradigmatic example of how a biological manipulator handles mechanical impedance. Further studies will be needed on questions regarding the flexibility of the centrally controlled portion of pen force control in adapting to the various requirements of the motor task. This can be done by trying to teach the writer a given strategy of mechanical impedance control by means of artificial feedback about pen force. Such an experiment would reveal the learning ability of the human movement control system as compared to the teaching of the inverse dynamics solution to an artificial neural network (Kawato et al. 1987).

5 Appendix

1. The differentiation of two signals does not influence their coherence spectrum. Assume the signals $u(t)$ and $v(t)$, that are transformed by a linear operator $h(t)$, then

$$U'(f) = H(f) \cdot U(f)$$
$$V'(f) = H(f) \cdot V(f)$$

The cross-spectral density will then be given by

$$\begin{aligned} Gu'v'(f) &= U'_{\mathrm{conj}}(f) \cdot V'(f) \\ &= |H(f)|^2 \cdot U_{\mathrm{conj}}(f) \cdot V(f) \end{aligned}$$

and the squared coherence function will be

$$\begin{aligned} \gamma^2(f) &= |G_{u'v'}(f)|^2/(|U'(f)|^2 \cdot |V'(f)|^2) \\ &= G_{uv}(f)|^2/(|U(f)|^2 \cdot |V(f)|^2) \end{aligned}$$

because of a $|H(f)|^4$ component in both numerator and denominator. In the numerical estimation, however, the coherence at $f = 0$ (DC) will be reduced because of discretization, if $h(t)$ is a differentiator. This is not of importance for the present purpose.

2. The instantaneous or running Pearson correlation is given by:

$$r^w_{xy}(j) = \frac{\sigma^{w2}_{xy}(j)}{\sigma^w_{xx}(j)\sigma^w_{yy}(j)}$$

where j is the discrete time index, w is a time window width, σ^2_{xy} is the covariance and σ_x and σ_y are standard deviations. Time window calculation of σ's is done by:

$$\sigma^{w}_{uv}(j) = \left(1/w \sum_{i=j-w/2}^{j+w/2} (u_i - \mu_u)(v_i - \mu_v \right)^{1/2}$$

where $u \neq v$ for the square root of the covariance and $u = v$ for the standard deviation. The actual value used for w is 51.

Acknowledgements. This study was supported by grants to Lambert Schomaker from the Dutch science organization NWO, project 560-259-020, and from the European Esprit Programme, project 419. This study was also supported by grants to Réjean Plamondon from NSERC O915 and from The Netherlands Institute for Advanced Study, 1989–1990. Calligrapher's data courtesy of R. G. J. Meulenbroek.

References

Abramowitz M, Stegun LA (1970) Handbook of mathematical functions. Dover, New York, p 914

Asada H, Slotine J-JE (1986) Robot analysis and control. Wiley, New York

Baier PE, Teder W, Hussong J (1987) Future trends in automatic document analysis. Proceedings of the Third International Symposium on Handwriting and Computer Applications, Montreal. Ecole Polytechnique Montreal, pp. 146–151

Beek PJ, Beek WJ (1988) Tools for constructing dynamical models of rhythmic movement. Hum Mov Sci 7:301–342

Bendat JS, Piersol AG (1971) Random data: analysis and measurement procedures. Wiley, London

Bizzi E, Polit A, Morasso P (1976) Mechanisms underlying achievement of final head position. J Neurophysiol 39:435–444

Boxtel A van, Schomaker LRB (1983) Motor unit firing rate during static contraction indicated by the surface EMG power spectrum. IEEE Trans Biomed Engin 30:601–609

Crane HD, Ostrem JS (1983) Automatic signature verification using a three-axis force-sensitive pen. IEEE Trans Syst Man Cybern 13:329–337

Deinet W, Linke M, Rieger B (1987) Analyse der Schreibdynamik. Technical Report, Bundeskriminalamt (BKA-Technische Forschung), Wiesbaden, February 1987

Dooijes EH (1983) Analysis of handwriting movements. PhD Dissertation. University of Amsterdam, Amsterdam

Edelman S, Flash T (1987) A model of handwriting. Biol Cybern 57:25–36

Hale WJ, Paganini BJ (1980) An automatic personal verification system based on signature writing habits, Proceedings of the 1980 Carnahan Conference on Crime Countermeasures. University of Kentucky Lexington, pp 121–125

Hogan N (1985) The mechanics of multi-joint posture and movement control. Biol Cybern 52:315–331

Hollerbach JM (1981) An oscillation theory of handwriting. Biol Cybern 39:139–156

Hulstijn W, van Galen GP (1983) Programming in handwriting: reaction time and movement time as a function of sequence length. Acta Psychol 54:23–49

Kao HSR, Shek DTL, Lee ESP (1983) Control modes and task complexity in tracing and handwriting performance. Acta Psychol 54:69–77

Kawato M, Furukawa K, Suzuki R (1987) A hierarchical neural network model for control and learning of voluntary movement. Biol Cybern 57:169–185

Kobayashi T (1981) Some experimental studies on writing behavior. Hiroshima Forum Psychol 8:27–38

Kraepelin E (1899) Allgemeine Psychiatrie Bd 2. Thieme Leipzig, pp 362–370

Kretschmer E (1934) A text-book of medical psychology (translated). Milford, Oxford University Press, London, pp 215–220

Maarse FJ, Schomaker LRB, Teulings HL (1986) Kenmerkende verschillen in individueel schrijfgedrag: Automatische identificatie van schrijvers. Ned Tijdschr Psychol 41:41–47

Maarse FJ, Schomaker LRB, Thomasen AJWM (1986) The influence of changes in the effector coordinate systems on handwriting movements. In: Kao HSR, van Galen GP, Hoosain R (eds) Graphonomics: contemporary research in handwriting, North-Holland, Amsterdam, pp 33–46

Maarse FJ, Janssen HJJ, Dexel F (1988) A special pen for an XY tablet. In Maarse FJ, Mulder LJM, Sjouw WPB, Akkerman AE (eds) Computers in psychology: methods, instrumentation, and psychodiagnostics. Swets and Zeitlinger, Amsterdam, pp 133–139

Mason MT (1982) Compliance and force control for computer controlled manipulators. In: Brady M, Hollerbach JM, Johnson TL, Lozano-Pérez T, Mason MT (eds) Robot motion: planning and control. MIT, Cambridge, pp 373–404

Morasso P, Mussa Ivaldi FA (1982) Trajectory formation and handwriting: a computational model. Biol Cybern 45:131–142

Morasso P, Mussa Ivaldi FA (1987) Computational models of handwriting. Proceedings of the Third International Symposium on Handwriting and Computer Applications, Montreal. Ecole Polytechnique Montreal pp 8–9

Plamondon R (1987) What does differential geometry tell us about handwriting generation? In: Plamondon R, Suen CY, Deschênes JG, Poulin G (eds) Proceedings of the Third International Symposium on Handwriting and Computer Applications, Montreal. Ecole Polytechnique Montreal, pp 25–27

Plamondon R (1989) Handwriting control: a functional model. In: Cotterill RMJ (ed) Models of brain function. Cambridge University Press, Cambridge, pp 563–574

Plamondon R, Lorette G (1989) Automatic signature verification and writer identification: the state of the art. Pattern Recognition 22:107–131

Plamondon R, Maarse FJ (1989) An evaluation of motor models of handwriting: IEEE Trans Syst Man Cybern 19:1060–1072

Rack PMH (1981) Limitations of somatosensory feedback in control of posture and movement. In: Brooks VB (ed) Handbook of physiology, Sect. 1: The nervous system, vol 2: motor control, part 1. Americal Physiological Society Bethesda, pp 225–256

Schomaker LRB, Thomassen AJWM (1986) On the use and limitations of averaging handwriting signals. In: Kao HSR, GP van Galen, Hoosain R (eds) Graphonomics: contemporary research in handwriting. North-Holland, Amsterdam, pp 225–238

Schomaker LRB, Thomassen AJWM, Teulings HL (1989) A computational model of cursive handwriting. In: Plamondon R, Suen CY, Simner ML (eds) Computer recognition and human production of handwriting. World Scientific, Singapore, pp 153–177

Steinwachs F (1969) Mikromotorische Tonusregistrierungen und ihre diagnostische Moeglichkeiten. In: Jahrbuch, Landesamt fuer Forschung des Landes Nordrhein-Westfalen. Westdeutscher Verlag, Opladen

Stelmach GE, Teulings H-L (1983) Response characteristics of prepared and restructured handwriting. Acta Psychol 54:51–67

Teulings HL, Maarse FJ (1984) Digital recording and processing of handwriting movements. Hum Mov Sci 3:193–217

Dr. L.R.B. Schomaker
Nijimegen Institute for Cognition Research
and Information Technology (NICI)
University of Nijmegen
P.O. Box 9104
6500 HE Nijmegen
The Netherlands