UNIVERSITY OF COPENHAGEN

XMP: Exam
- Theory

Jens Fredskov (chw752)

January 16, 2014

Variation 1

a)

We first consider our given processes to eliminate the general choice in $PROD$.

$$PROD = (\mathsf{in}?x \rightarrow \mathsf{chk}!(x-4) \rightarrow PROD) \,\square\, (\mathsf{rej}?y \rightarrow \mathsf{chk}!(y-2) \rightarrow PROD)$$
$$= (\mathsf{in}?x \rightarrow \mathsf{chk}!(x-4) \rightarrow PROD) \mid (\mathsf{rej}?y \rightarrow \mathsf{chk}!(y-2) \rightarrow PROD) \qquad [3.3.1\ \text{L5}]$$

$$QUAL = \mathsf{chk}?z \rightarrow ((\mathsf{out}!z \rightarrow QUAL) \lhd ok(z) \rhd (\mathsf{rej}!z \rightarrow QUAL))$$

We now define an auxiliary process $TMP0_x$

$$TMP0_x = (\mathsf{chk}!x \rightarrow PROD) \parallel QUAL$$
$$= \mathsf{chk}.x \rightarrow (PROD \parallel ((c!z \rightarrow QUAL) \lhd ok(z) \rhd (\mathsf{rej}!z \rightarrow QUAL))) \qquad [2.3.1\ \text{L4A}]$$
$$= \mathsf{chk}.x \rightarrow ((PROD \parallel (c!z \rightarrow QUAL)) \lhd ok(z) \rhd (PROD \parallel (\mathsf{rej}!z \rightarrow QUAL))) \qquad [\text{LCD}]$$

With the sets $A = \{|\mathsf{in}|, |\mathsf{rej}|\}, B = \{\mathsf{out}.x\}, C = \{|\mathsf{in}|, \mathsf{out}.x\}$, the left hand side of the *if-then*-clause becomes

$$LHS = PROD \parallel (\mathsf{out}!x \rightarrow QUAL)$$
$$= (\mathsf{in}?z \rightarrow ((\mathsf{chk}!(z-4) \rightarrow PROD) \parallel (\mathsf{out}!x \rightarrow QUAL))$$
$$\mid \mathsf{out}!x \rightarrow (PROD \parallel QUAL)) \qquad [2.3.1\ \text{L7}]$$
$$= (\mathsf{in}?z \rightarrow \mathsf{out}!x \rightarrow ((\mathsf{chk}!(z-4) \rightarrow PROD) \parallel QUAL)$$
$$\mid \mathsf{out}!x \rightarrow \mathsf{in}?z \rightarrow ((\mathsf{chk}!(z-4) \rightarrow PROD) \parallel QUAL)) \qquad [2.3.1\ \text{L5A/B}]$$
$$= (\mathsf{in}?z \rightarrow \mathsf{out}!x \rightarrow TMP0_{z-4} \mid \mathsf{out}!x \rightarrow \mathsf{in}?z \rightarrow TMP0_{z-4})$$

With the sets $A = \{|\mathsf{in}|, |\mathsf{rej}|\}, B = \{\mathsf{rej}.x\}, C = \{|\mathsf{in}|, \mathsf{rej}.x\}$, the right hand side becomes

$$RHS = PROD \parallel (\mathsf{rej}!x \rightarrow QUAL)$$
$$= (\mathsf{in}?z \rightarrow ((\mathsf{chk}!(z-4)PROD) \parallel (\mathsf{rej}!x \rightarrow QUAL))$$
$$\mid \mathsf{rej}.x \rightarrow ((\mathsf{chk}!(x-2) \rightarrow PROD) \parallel QUAL)) \qquad [2.3.1\ \text{L7}]$$
$$= (\mathsf{in}?z \rightarrow STOP \mid \mathsf{rej}.x \rightarrow TMP0_{x-2})$$

We then put the side together again

$$TMP0_x = \mathsf{chk}.x \rightarrow ((\mathsf{in}?z \rightarrow \mathsf{out}!x \rightarrow TMP0_{z-4} \mid \mathsf{out}!x \rightarrow \mathsf{in}?z \rightarrow TMP0_{z-4})$$
$$\lhd ok(x) \rhd (\mathsf{in}?z \rightarrow STOP \mid \mathsf{rej}.x \rightarrow TMP0_{x-2}))$$

We now define a new process which is the same as before, but with CR concealed.

$$TMP1_x = TMP0_x \setminus CR$$
$$= (\mathsf{chk}.x \rightarrow ((\mathsf{in}?z \rightarrow \mathsf{out}!x \rightarrow TMP0_{z-4} \mid \mathsf{out}!x \rightarrow \mathsf{in}?z \rightarrow TMP0_{z-4})$$
$$\lhd ok(x) \rhd (\mathsf{in}?z \rightarrow STOP \mid \mathsf{rej}.x \rightarrow TMP0_{x-2}))) \setminus CR$$
$$= ((\mathsf{in}?z \rightarrow \mathsf{out}!x \rightarrow TMP0_{z-4} \mid \mathsf{out}!x \rightarrow \mathsf{in}?z \rightarrow TMP0_{z-4})$$
$$\lhd ok(x) \rhd (\mathsf{in}?z \rightarrow STOP \mid \mathsf{rej}.x \rightarrow TMP0_{x-2})) \setminus CR \qquad [3.5.1\ \text{L5}]$$
$$= (((\mathsf{in}?z \rightarrow \mathsf{out}!x \rightarrow TMP0_{z-4} \mid \mathsf{out}!x \rightarrow \mathsf{in}?z \rightarrow TMP0_{z-4})$$
$$\setminus CR) \lhd ok(x) \rhd ((\mathsf{in}?z \rightarrow STOP \mid \mathsf{rej}.x \rightarrow TMP0_{x-2}) \setminus CR)) \qquad [\text{LCD}]$$

Then twice with sets $B = \{|\mathsf{in}|, \mathsf{out}.x\}, C = \{|\mathsf{chk}|, |\mathsf{rej}|\}$

$$\begin{aligned} &= ((\mathsf{in}?z \to \mathsf{out}!x \to \mathit{TMP1}_{z-4} \mid \mathsf{out}!x \to \mathsf{in}?z \to \mathit{TMP1}_{z-4}) \\ &\quad \lhd ok(x) \rhd ((\mathsf{in}?z \to \mathit{STOP} \mid \mathsf{rej}.x \to \mathit{TMP0}_{x-2}) \setminus \mathit{CR})) && [3.5.1\ L8] \end{aligned}$$

Finally with sets $B = \{|\mathsf{in}|, \mathsf{rej}.x\}, C = \{|\mathsf{chk}|, |\mathsf{rej}|\}$

$$\begin{aligned} &= ((\mathsf{in}?z \to \mathsf{out}!x \to \mathit{TMP1}_{z-4} \mid \mathsf{out}!x \to \mathsf{in}?z \to \mathit{TMP1}_{z-4}) \\ &\quad \lhd ok(x) \rhd (\mathit{TMP1}_{x-2} \sqcap (\mathit{TMP1}_{x-2} \,\Box\, (\mathsf{in}?z \to \mathit{STOP})))) && [3.5.1\ L10] \end{aligned}$$

To eliminate the general choice after hiding CR we define a new process

$$\begin{aligned} \mathit{TMP2}_x &= \mathit{TMP1}_x \,\Box\, (\mathsf{in}?z \to \mathit{STOP}) \\ &= ((\mathsf{in}?z \to \mathsf{out}!x \to \mathit{TMP1}_{z-4} \mid \mathsf{out}!x \to \mathsf{in}?z \to \mathit{TMP1}_{z-4}) \\ &\quad \lhd ok(x) \rhd (\mathit{TMP1}_{x-2} \sqcap (\mathit{TMP1}_{x-2} \,\Box\, (\mathsf{in}?z \to \mathit{STOP})))) \,\Box\, (\mathsf{in}?z \to \mathit{STOP}) \\ &= (((\mathsf{in}?z \to \mathsf{out}!x \to \mathit{TMP1}_{z-4} \mid \mathsf{out}!x \to \mathsf{in}?z \to \mathit{TMP1}_{z-4}) \,\Box\, (\mathsf{in}?z \to \mathit{STOP})) \\ &\quad \lhd ok(x) \rhd ((\mathit{TMP1}_{x-2} \sqcap (\mathit{TMP1}_{x-2} \,\Box\, (\mathsf{in}?z \to \mathit{STOP}))) \,\Box\, (\mathsf{in}?z \to \mathit{STOP}))) && [LCD] \end{aligned}$$

The left hand side of the *if-then*-clause then becomes

$$\begin{aligned} \mathit{LHS} &= (\mathsf{in}?z \to \mathsf{out}!x \to \mathit{TMP1}_{z-4} \mid \mathsf{out}!x \to \mathsf{in}?z \to \mathit{TMP1}_{z-4}) \,\Box\, (\mathsf{in}?z \to \mathit{STOP}) \\ &= (\mathsf{in}?z \to ((\mathsf{out}!x \to \mathit{TMP1}_{z-4}) \sqcap \mathit{STOP}) \mid \mathsf{out}!x \to \mathsf{in}?z \to \mathit{TMP1}_{z-4}) && [3.3.1\ L5] \end{aligned}$$

And the right hand side becomes

$$\begin{aligned} \mathit{RHS} &= (\mathit{TMP1}_{x-2} \sqcap (\mathit{TMP1}_{x-2} \,\Box\, (\mathsf{in}?z \to \mathit{STOP}))) \,\Box\, (\mathsf{in}?z \to \mathit{STOP}) \\ &= (\mathit{TMP1}_{x-2} \,\Box\, (\mathsf{in}?z \to \mathit{STOP})) \sqcap ((\mathit{TMP1}_{x-2} \,\Box\, (\mathsf{in}?z \to \mathit{STOP})) \,\Box\, (\mathsf{in}?z \to \mathit{STOP})) && [3.3.1\ L6] \\ &= \mathit{TMP2}_{x-2} \sqcap \mathit{TMP2}_{x-2} && [3.3.1\ L1\text{-}L3] \\ &= \mathit{TMP2}_{x-2} && [3.2.1\ L1] \end{aligned}$$

We then put the side together again

$$\begin{aligned} \mathit{TMP2}_x &= ((\mathsf{in}?z \to ((\mathsf{out}!x \to \mathit{TMP1}_{z-4}) \sqcap \mathit{STOP}) \mid \mathsf{out}!x \to \mathsf{in}?z \to \mathit{TMP1}_{z-4}) \\ &\quad \lhd ok(x) \rhd \mathit{TMP2}_{x-2}) \end{aligned}$$

Finally we can use this in the definition of the earlier process

$$\begin{aligned} \mathit{TMP1}_x &= ((\mathsf{in}?z \to \mathsf{out}!x \to \mathit{TMP1}_{z-4} \mid \mathsf{out}!x \to \mathsf{in}?z \to \mathit{TMP1}_{z-4}) \\ &\quad \lhd ok(x) \rhd (\mathit{TMP1}_{x-2} \sqcap \mathit{TMP2}_{x-2})) \end{aligned}$$

We are now ready to find an equivalent definition for *MILL* using the subprocesses we have created.

$$\begin{aligned} \mathit{MILL} &= (\mathit{PROD} \parallel \mathit{QUAL}) \setminus \mathit{CR} \\ &= (\mathsf{in}?x \to ((\mathsf{chk}!(x-4) \to \mathit{PROD}) \parallel \mathit{QUAL})) \setminus \mathit{CR} && [2.3.1\ L5A] \\ &= \mathsf{in}?x \to (\mathit{TMP0}_{x-4} \setminus \mathit{CR}) && [3.5.1\ L5] \\ &= \mathsf{in}?x \to \mathit{TMP1}_{x-4} \end{aligned}$$

b)

One trace which could make $MILL$ deadlock is $s = \langle \mathsf{in}.106, \mathsf{in}.106 \rangle$. We first show that $s \in traces(MILL)$.

$$\begin{aligned}
traces(MILL) &= traces(\mathsf{in}?106 \to TMP1_{102}) \\
&= \{t \mid t = \langle\rangle \lor \underline{(t_0 = \mathsf{in}.x \land t' \in traces(TMP1_{102}))}\} && [1.8.1\ \mathrm{L2}] \\
traces(TMP1_{102}) &= traces(TMP1_{100} \sqcap TMP2_{100}) \\
&= traces(TMP1_{100}) \cup \underline{traces(TMP2_{100})} && [3.2.3\ \mathrm{L1}] \\
traces(TMP2_{100}) &= traces(\mathsf{in}?106 \to ((\mathsf{out}!100 \to TMP1_{102}) \sqcap STOP) \\
&\qquad\qquad \mid \mathsf{out}!100 \to \mathsf{in}?106 \to TMP1_{102})) \\
&= \{t \mid t = \langle\rangle \lor \underline{(t_0 = \mathsf{in}.106 \land t' = traces((\mathsf{out}!100 \to TMP1_{102}) \sqcap STOP))} \\
&\qquad\qquad \lor (t_0 = \mathsf{out}.100 \land t' = traces(\mathsf{in}?106 \to TMP1_{102}))\} && [1.8.1\ \mathrm{L3}] \\
traces((\mathsf{out}!100 \to TMP1_{102}) \sqcap STOP) &= traces((\mathsf{out}!100 \to TMP1_{102})) \cup \underline{traces(STOP)} && [3.2.3\ \mathrm{L1}] \\
traces(STOP) &= \{\underline{\langle\rangle}\} && [1.8.1\ \mathrm{L1}]
\end{aligned}$$

Notice that we here have substituted the variables with actual values, as this means we easily can choose the correct side of the *if-else*-clause. We now show that $MILL \;/\; s = STOP$.

$$\begin{aligned}
MILL \;/\; s &= (\mathsf{in}?x \to TMP1_{x-4}) \;/\; \langle \mathsf{in}.106, \mathsf{in}.106 \rangle \\
&= TMP1_{102} \;/\; \langle \mathsf{in}.106 \rangle && [1.8.3\ \mathrm{L3A}] \\
&= (TMP1_{100} \;/\; \langle \mathsf{in}.106 \rangle) \sqcap (TMP2_{100} \;/\; \langle \mathsf{in}.106 \rangle) && [3.2.3\ \mathrm{L2}] \\
&= ((\mathsf{out}!100 \to TMP1_{102}) \;/\; \langle\rangle) \sqcap (TMP2_{100} \;/\; \langle \mathsf{in}.106 \rangle) && [1.8.3\ \mathrm{L3}] \\
&= (\mathsf{out}!100 \to TMP1_{102}) \sqcap ((\mathsf{out}!100 \to TMP1_{102}) \sqcap STOP) \;/\; \langle\rangle) && [1.8.3\ \mathrm{L3A}] \\
&= (\mathsf{out}!100 \to TMP1_{102}) \sqcap STOP && [3.2.1\ \mathrm{L1}]
\end{aligned}$$

Now depending on which side of the internal choice $MILL$ chooses we get that $MILL \;/\; s = STOP$.

To see that we indeed have a deadlock we consider the state of $PROD$ and $QUAL$. After the first action $QUAL$ will be trying to send rej as the log was too long. However as only $PROD$ has the in-channel it is allowed to choose both of its branches, and thus it might choose the first as this does not require participation from the environment ($QUAL$). In this situtation $PROD$ will be waiting on chk which $QUAL$ is unwilling to do at the moment, and $QUAL$ will be waiting on rej which $PROD$ is unwilling to do. Thus we clearly have a deadlock.

c)

If we consider the trace from the previous question we could simply add one more action such that $t = \langle \mathsf{in}.106, \mathsf{in}.106, \mathsf{out}.100 \rangle$. We have already calculated $MILL \;/\; s$. Thus we can just calculate $refusals(MILL \;/\; s)$. We first use [3.4.1 L4]. Then since $STOP$ refuses the entire alphabet, per [3.4.1 L1], we have that $\{\mathsf{out}.100\} \in refusals(MILL \;/\; s)$. We however also have that $\langle \mathsf{out}.100 \rangle \in traces(MILL \;/\; s)$, as per [3.2.3 L2] and [1.8.3 L3A]. This proves that that $MILL \;/\; s$, and therefore also $MILL$, may behave nondeterministically.

Variation 2

a)

b)

c)

Variation 3

a)

b)

We consider a trace where we read the a value which is rejected first time around thrice (the third does not have to reject). This could e.g. be

$$t = \langle \mathsf{in}.108, \mathsf{in}.108, \mathsf{in}.108 \rangle$$

To show that this can create a deadlock we consider the state of $QUAL$ and $PROD''$ after the trace. For $PROD''$ the trace we use will have rej and chk injected and the trace then becomes

$$t_1 = \langle \mathsf{in}.108, \mathsf{chk}.104, \mathsf{rej}.104, \mathsf{in}.108, \mathsf{chk}.104, \mathsf{in}.108 \rangle$$

For $QUAL$ we remove in and inject rej and chk, giving us the following trace

$$t_2 = \langle \mathsf{chk}.104, \mathsf{rej}.104, \mathsf{chk}.104 \rangle$$

We now run the two processes in parallel to show that we reach a point after t where $MILL''$ will have be in a state of deadlock.

$$\begin{aligned} PROD' \ / \ t_1 &= ((\mathsf{in}?x \rightarrow \mathsf{chk}!(x-4) \rightarrow PRODI) \\ &\quad ||| \ (\mathsf{rej}?y \rightarrow \mathsf{chk}!(y-2) \rightarrow PRODR)) \ / \ t_1 \\ QUAL \ / \ t_2 &= (\mathsf{chk}?z \rightarrow ((\mathsf{out}!z \rightarrow QUAL) \\ &\quad \lhd \ ok(z) \rhd (\mathsf{rej}!z \rightarrow QUAL))) \ / \ t_2 \end{aligned}$$

$PROD'$ first read from in

$$\begin{aligned} PROD' \ / \ t_1 &= ((\mathsf{chk}!(104) \rightarrow PRODI) \\ &\quad ||| \ (\mathsf{rej}?y \rightarrow \mathsf{chk}!(y-2) \rightarrow PRODR)) \ / \ t_1' && [3.6.2 \text{ L3}] + [1.8.3 \text{ L3A}] \\ QUAL \ / \ t_2 &= (\mathsf{chk}?z \rightarrow ((\mathsf{out}!z \rightarrow QUAL) \\ &\quad \lhd \ ok(z) \rhd (\mathsf{rej}!z \rightarrow QUAL))) \ / \ t_2 \end{aligned}$$

Both are now willing to do chk

$$\begin{aligned} PROD' \ / \ t_1 &= (PRODI \\ &\quad ||| \ (\mathsf{rej}?y \rightarrow \mathsf{chk}!(y-2) \rightarrow PRODR)) \ / \ t_1'' && [3.6.2 \text{ L3}] + [1.8.3 \text{ L3A}] \\ QUAL \ / \ t_2 &= ((\mathsf{out}!104 \rightarrow QUAL) \\ &\quad \lhd \ ok(104) \rhd (\mathsf{rej}!104 \rightarrow QUAL)) \ / \ t_2' && [1.8.3 \text{ L3A}] \end{aligned}$$

$ok(104)$ is false and, both are now willing to do rej

$$\begin{aligned}
PROD' \;/\; t_1 &= ((\mathsf{in}?x \to \mathsf{chk}!(x-4) \to PRODI) \\
&\quad ||| \; (\mathsf{chk}!(102) \to PRODR)) \;/\; \langle \mathsf{in}.108, \mathsf{chk}.104, \mathsf{in}.108 \rangle && [3.6.2\ \text{L3}] + [1.8.3\ \text{L3A}] \\
QUAL \;/\; t_2 &= (QUAL) \;/\; \langle \mathsf{chk}.104 \rangle && [5.5.1\ \text{L8}] + [1.8.3\ \text{L3A}]
\end{aligned}$$

$PROD'$ now does another read from in

$$\begin{aligned}
PROD' \;/\; t_1 &= ((\mathsf{chk}!(104) \to PRODI) \\
&\quad ||| \; (\mathsf{chk}!(102) \to PRODR)) \;/\; \langle \mathsf{chk}.104, \mathsf{in}.108 \rangle && [3.6.2\ \text{L3}] + [1.8.3\ \text{L3A}] \\
QUAL \;/\; t_2 &= (\mathsf{chk}?z \to ((\mathsf{out}!z \to QUAL) \\
&\quad \lhd\; ok(z) \;\rhd\; (\mathsf{rej}!z \to QUAL))) \;/\; \langle \mathsf{chk}.104 \rangle
\end{aligned}$$

This is the turning point. Instead of handling the rejected log first we let $PRODI$ do chk together with $QUAL$

$$\begin{aligned}
PROD' \;/\; t_1 &= (PRODI \\
&\quad ||| \; (\mathsf{chk}!(102) \to PRODR)) \;/\; \langle \mathsf{in}.108 \rangle && [3.6.2\ \text{L3}] + [1.8.3\ \text{L3A}] \\
QUAL \;/\; t_2 &= ((\mathsf{out}!104 \to QUAL) \\
&\quad \lhd\; ok(104) \;\rhd\; (\mathsf{rej}!104 \to QUAL)) && [1.8.3\ \text{L3A}]
\end{aligned}$$

Finally $PROD'$ reads another value from in and the system then looks as following

$$\begin{aligned}
PROD' \;/\; t_1 &= (\mathsf{chk}!(104) \to PRODI) \; ||| \; (\mathsf{chk}!(102) \to PRODR) && [3.6.2\ \text{L3}] + [1.8.3\ \text{L3A}] \\
QUAL \;/\; t_2 &= \mathsf{rej}!104 \to QUAL && [5.5.1\ \text{L8}]
\end{aligned}$$

Clearly we have now reached a deadlock as both of $PRODI$ and $PRODR$ wants to send on chk, which $QUAL$ is unwilling to receive, and $QUAL$ wants to send on rej which both $PRODI$ and $PRODR$ is unwilling to receive.

c)