UNIVERSITY OF COPENHAGEN

XMP: Assignment 2
- Theory

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To find an equivalent definition of NET we first eliminate the general choice in $STORE_x$, which can be done using the definition of general choice.

$$\begin{aligned} STORE_x &= (\mathsf{add}?y \to STORE_{x+y}) \,\square\, (\mathsf{fetch}!x \to STORE_{x-1}) \\ &= (\mathsf{add}?y \to STORE_{x+y} \mid \mathsf{fetch}!x \to STORE_{x-1}) \end{aligned}$$

We then define and calculate the following auxillary process

$$\begin{aligned} SS_x &= STORE_x \parallel STOP_A \\ &= (\mathsf{add}?y \to STORE_{x+y} \mid \mathsf{fetch}!x \to STORE_{x-1}) \parallel STOP_A \end{aligned}$$

we use [2.3.1 L7] where $A = \{|\mathsf{fetch}|, |\mathsf{add}|\}, B = \{|\mathsf{add}|\}, C = \{|\mathsf{fetch}|\}$

$$\begin{aligned} &= \mathsf{fetch}!x \to (STORE_{x-1} \parallel STOP_A) \\ &= \mathsf{fetch}!x \to SS_{x-1} \end{aligned}$$

Finally we define the process NET_x as solving this for $x = 10$ is equivalent to NET

$$NET_x = (STORE_x \parallel (\mathsf{add}!3 \to STOP_A)) \setminus A$$

we first use [4.3 L1] and [2.3.1 L7] with $A = \{|\mathsf{fetch}|, |\mathsf{add}|\}, B = \{\mathsf{add}.3\}, C = \{|\mathsf{fetch}|, \mathsf{add}.3\}$

$$\begin{aligned} &= (\mathsf{add}!3 \to (STORE_{x+3} \parallel STOP_A) \mid \mathsf{fetch}!x \to (STORE_{x-1} \parallel (\mathsf{add}!3 \to STOP_A))) \setminus A \\ &= (\mathsf{add}!3 \to SS_{x+3} \mid \mathsf{fetch}!x \to NET_{x-1}) \setminus A \end{aligned}$$

We then apply the special case of [3.5.1 L10] given as an example just above the law

$$\begin{aligned} &= SS_{x+3} \sqcap (SS_{x+3} \,\square\, (\mathsf{fetch}!x \to NET_{x-1})) \\ &= SS_{x+3} \sqcap (\mathsf{fetch}!(x+3) \to SS_{x+2} \mid \mathsf{fetch}!x \to NET_{x-1}) \end{aligned}$$

We now have the equivalent definition

$$\begin{aligned} NET &= NET_{10} \\ &= SS_{13} \sqcap (\mathsf{fetch}!13 \to SS_{12} \mid \mathsf{fetch}!10 \to NET_9) \end{aligned}$$

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(a) To show that $s \in traces(NET)$ we first use [3.2.3 L1]

$$traces(NET) = traces(SS_{13}) \cup \underline{traces(\text{fetch!13} \rightarrow SS_{12} \mid \text{fetch!10} \rightarrow NET_9)}$$

We then focus of the second expression and use [1.8.1 L3]

$$\begin{aligned} traces(\text{fetch!13} \rightarrow SS_{12} \mid \text{fetch!10} \rightarrow NET_9) = \{t \mid t = \langle\rangle \vee (t_0 = \text{fetch!13} \wedge t' \in traces(SS_{12})) \\ \vee \underline{(t_0 = \text{fetch!10} \wedge t' \in traces(NET_9))}\} \end{aligned}$$

Again we focus on the second expression and use [3.2.3 L1]

$$traces(NET_9) = \underline{traces(SS_{12})} \cup traces(\text{fetch!12} \rightarrow SS_{11} \mid \text{fetch!9} \rightarrow NET_8)$$

We can now choose either of the two expressions (as they both contain fetch!12) but for simplicity we choose the first, and use [1.8.1 L2] thrice

$$\begin{aligned} traces(SS_{12}) &= \{t \mid t = \langle\rangle \vee \underline{(t_0 = \text{fetch!12} \wedge t' \in traces(SS_{11}))}\} \\ traces(SS_{11}) &= \{t \mid t = \langle\rangle \vee \underline{(t_0 = \text{fetch!11} \wedge t' \in traces(SS_{10}))}\} \\ traces(SS_{10}) &= \{t \mid \underline{t = \langle\rangle} \vee (t_0 = \text{fetch!10} \wedge t' \in traces(SS_9))\} \end{aligned}$$

(b) We calculate $NET \,/\, s$ as

$$\begin{aligned} NET \,/\, s &= (SS_{13} \sqcap (\text{fetch!13} \rightarrow SS_{12} \mid \text{fetch!10} \rightarrow NET_9)) \,/\, s \\ &= (\text{fetch!13} \rightarrow SS_{12} \mid \text{fetch!10} \rightarrow NET_9) \,/\, s && [3.2.3\ L2] \\ &= NET_9 \,/\, \langle \text{fetch.12}, \text{fetch.11} \rangle && [1.8.3\ L2\ \&\ L3] \\ &= (SS_{12} \,/\, \langle \text{fetch.12}, \text{fetch.11} \rangle) \sqcap \\ &\quad ((\text{fetch!12} \rightarrow SS_{11} \mid \text{fetch!9} \rightarrow NET_8) \,/\, \langle \text{fetch.12}, \text{fetch.11} \rangle) && [3.2.3\ L2] \\ &= SS_{10} \sqcap ((\text{fetch!12} \rightarrow SS_{11} \mid \text{fetch!9} \rightarrow NET_8) \,/\, \langle \text{fetch.12}, \text{fetch.11} \rangle) && [2 \times 1.8.3\ L2\ \&\ L3A] \\ &= SS_{10} \sqcap (SS_{11} \,/\, \langle \text{fetch.11} \rangle) && [1.8.3\ L2\ \&\ L3] \\ &= SS10 \sqcap SS_{10} && [1.8.3\ L2\ \&\ L3A] \\ &= SS10 && [3.2.1\ L1] \end{aligned}$$

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(a) To show that $\{\mathsf{fetch.10}, \mathsf{fetch.42}\} \in \mathit{refusals}(NET)$ we first use [3.4.1 L4]

$$\mathit{refusals}(NET) = \mathit{refusals}(SS_{13}) \cup \mathit{refusals}(\mathsf{fetch!13} \rightarrow SS_{12} \mid \mathsf{fetch!10} \rightarrow NET_9)$$

We now only need to look at the first set of refusals as this process only wants to do $\mathsf{fetch!13}$ we have, using [3.4.1 L2], that

$$\begin{aligned} \mathit{refusals}(SS_{13}) &= \{x \mid x \subseteq (\alpha SS_x - \mathsf{fetch!13})\} \\ &\Rightarrow \{\mathsf{fetch.10}, \mathsf{fetch.42}\} \in \mathit{refusals}(SS_{13}) \\ &\Rightarrow \{\mathsf{fetch.10}, \mathsf{fetch.42}\} \in \mathit{refusals}(NET) \end{aligned}$$

At the same time we can also see that $\{\mathsf{fetch.13}\} \notin \mathit{refusals}(SS_{13})$ thus for it to not be in $\mathit{refusals}(NET)$ we need now only check that it is also not in the second set. We then use [3.4.1 L3]

$$\begin{aligned} \mathit{refusals}(\mathsf{fetch!13} \rightarrow SS_{12} \mid \mathsf{fetch!10} \rightarrow NET_9) &= \{x \mid x \subseteq (\alpha SS_x - \{\mathsf{fetch!13}, \mathsf{fetch!10}\})\} \\ &\Rightarrow \{\mathsf{fetch.13}\} \notin \mathit{refusals}(\mathsf{fetch!13} \rightarrow SS_{12} \mid \mathsf{fetch!10} \rightarrow NET_9) \end{aligned}$$

Thus we have that $\{\mathsf{fetch.13}\} \notin \mathit{refusals}(NET)$

(b) To show that $\{\mathsf{fetch.13}, \mathsf{fetch.42}\} \in \mathit{refusals}(NET \mathbin{/} s)$ and that $\{\mathsf{fetch.10}\} \notin \mathit{refusals}(NET \mathbin{/} s)$ we use [3.4.1 L2]

$$\begin{aligned} \mathit{refusals}(NET \mathbin{/} s) &= \mathit{refusals}(SS_{10}) \\ &= \{x \mid x \subseteq (\alpha SS_x - \mathsf{fetch!10})\} \\ &\Rightarrow \{\mathsf{fetch.13}, \mathsf{fetch.42}\} \in \mathit{refusals}(NET \mathbin{/} s) \wedge \{\mathsf{fetch.10}\} \notin \mathit{refusals}(NET \mathbin{/} s) \end{aligned}$$