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Project 2 – April 11th, 2019

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Abstract

The purpose of this project is to study Conditional Value-at-Risk (CVaR) optimization and Robust CVaR with scenario-based optimization using Monte-Carlo simulations. Two simulations will be used: Gaussian Monte-Carlo and Non-normal Monte-Carlo with higher moments. The simulations were used to create factor scenarios for each of the Fama-French factor returns. The Fama-French model was then used to create 5000 asset scenarios which were used to create portfolios which minimize losses using the Conditional Value-at-Risk (CVaR) using a confidence level of 95%. A Robust CVaR portfolio was also created with a robustness level of 90%. It was observed that the portfolios had very similar volatility expectations and realizations, but those created using the Non-normal Monte-Carlo simulations had higher returns and better Sharpe ratios. Accuracy of results can be improved with the inclusion of more assets, observation of the assets for a longer time period, more simulations and inclusion of investor perspectives in the formulation of the problem.

Introduction

The purpose of this project is to study Conditional Value-at-Risk (CVaR) optimization and Robust CVaR with scenario-based optimization using Monte-Carlo simulations. Two simulations will be used: Gaussian Monte-Carlo and Non-normal Monte-Carlo with higher moments.

The project uses weekly adjusted closing prices of the stocks listed in Table 1, from 30-Dec-2011 to 31-Dec-2015 to compute observed asset weekly returns. In addition, historical factor returns for the Fama-French three-factor model corresponding to the period 06-Jan-2012 to 31-Dec-2015 are used. This includes the weekly risk-free rate.

Table 1: Company tickers of the assets considered

F	CAT	DIS	MCD	KO	PEP	WMT	C	WFC	JPM
AAPL	IBM	PFE	JNJ	XOM	MRO	ED	T	VZ	NEM

The portfolios are built using the estimated parameters with the French-Fama three-factor model and CVaR and Robust CVaR investment strategies. The portfolios with will random variables generated from Monte-Carlo and Non-normal Monte Carlo with higher moments. In total, four portfolios will be built.

Stochastic Process & Monte-Carlo Simulations

Arithmetic Random walks

Arithmetic random walks are a form of stochastic (random) process in which randomness is governed by independent normal distributions. In an arithmetic random walk process, the value of a variable, S_t at time t is calculated as follows:

$$S_{t+1} = S_t + \mu + \omega_t$$

where S_{t-1} is the value of the variable at time t-1, μ is the mean of the distribution of the variable S_t and ω_{t-1} is a random variable drawn from a normal distribution with mean of 0 and a standard deviation σ.

Arithmetic random walks are applicable for short time frames (e.g. model intraday price changes). Using arithmetic random walks to simulate longer time frames decreases accuracy due to the increased impact of two drawbacks. One drawback of arithmetic random walks is that they can take on negative values, especially for low initial values, S_0. Negative values may violate constraints in some scenarios. (e.g. Stock prices of assets). Another drawback of arithmetic random walks is that the variable being simulated may be non-stationary. As such, its mean and standard deviation are not constant in time (e.g. academia and industry have shown that asset prices are non-stationary). [1]

Geometric random walks

Geometric random walks are also a form of stochastic (random) process in which randomness is governed by independent normal distributions. Unlike arithmetic random walks, geometric random walk values will always remain positive if the initial variable, S_0, is positive since the order of magnitude change of the future projected random variable changes based on the value of the random variable. Thus, the variables being forecasted do not need to be stationary. Hence, geometric random walks work well for financial assets since the assets cannot be negative in value and are not stationary.
In a geometric random walk process, the value of a variable, S_t at time t is calculated as follows:

$$S_t = S_{t-1} \cdot (1 + r_t)$$

where r_t is a random variable of the percentage change of the variable S from time t-1 to time t. For a sufficiently small-time step such that Δt ≈ dt where dt is an infinitesimally small-time step, the forecasted variable value at iteration t+1 is:

$$S_{t+1} = S_t e^{\left(\mu - \frac{1}{2}\sigma^2\right) \cdot dt + \sigma\sqrt{dt}\varepsilon_t}$$

where σ is the standard deviation of the distribution of the variable being forecasted. [1]

Monte-Carlo Simulation

Monte Carlo simulations are used when there is a significant amount of uncertainty in the process of making a forecast or estimation. [2] Monte Carlo simulations allow for a wide variety of scenarios that are not limited by historical data. [3]Monte Carlo simulations are used to study the impact of uncertainty and risk in forecasting and predicting models. [2] A variable modeled by a Monte Carlo simulation has two components: drift and random input. [2] The drift of the variable is the constant directional movement of the variable, while the random input is used to determine how volatility could affect the variable being forecasted. [2]

One limitation of Monte Carlo simulations is that they ignore other influences and trends that are not built into the model. For example, a Monte Carlo simulation of an asset price over time may assume perfectly efficient markets, ignore macro trends and cyclical factors which could affect the price of the asset. [2]Thus Monte Carlo simulations are limited by the accuracy of the assumptions that are inputted into the model. [3] Another limitation of Monte Carlo simulations is that they do not forecast extreme values for variables at the frequency to which they are observed in reality. [3] Monte Carlo simulations tend not to include the "fat tailed" portion of the distribution from which the forecasted variable is drawn from. [3]

CVaR and robust CVaR optimization

Robust Optimization

Robust optimization is an optimization theory that originates from the establishment of modern decision theory and based on the use of worst case analysis and Wald's maximin model. It deals and seeks the solutions that are optimal for worst-case realization and satisfy all the constraints for a set of uncertain inputs. Robust optimization is widely used in various fields such as statistics, operation research, control theory, finance and portfolio management. [4] The worst case is taking into consideration which means the solution derived from Robust optimization is not flexible but safer. Robust optimization can increase the qualitative robustness, stability and quantitative robustness. [5] In portfolio management, robust model helps to provide consistently accurate output even if there are unforeseen circumstances which cause drastic change in some of the input variables. [6]

CVaR

Conditional Value at Risk (CVaR), also known as the Expected Shortfall (ES), is a risk assessment strategy which aiming to eliminate the drawbacks of VaR (Value at Risk). VaR of a portfolio for some certain time horizon and confidence level $\boldsymbol{\alpha}$ quantifies the loss that the portfolio will exceed with the likelihood of 1-$\boldsymbol{\alpha}$ within the given time horizon. CVaR measures the expected loss that beyond VaR. Compared to VaR, CVaR is a more conservative approach with regard to risk exposure [7]. It is not always clear which method to choose, but CVaR has some apparent advantages over VaR since it is sub-additive and convex which means the overall losses can be reduced by the dispersion of asset portfolios [8]. Overall, CVaR presents a better prediction of potential risk value.

Methodology

The project uses weekly adjusted closing prices of the stocks listed in Table 1, from 30-Dec-2011 to 31-Dec-2015 to compute observed asset weekly returns. In addition, historical factor returns for the Fama-French three-factor model corresponding to the period 06-Jan-2012 to 31-Dec-2015 are used. This includes the weekly risk-free rate. With this data, two simulations were created: Gaussian Monte-Carlo and Non-normal Monte-Carlo with higher moments. The simulations were used to create factor scenarios for each of the Fama-French factor returns. The Fama-French model was then used to create 5000 asset scenarios which were used to create portfolios which minimize losses using the Conditional Value-at-Risk (CVaR) using a confidence level of 95%. A Robust CVaR portfolio was also created with a robustness level of 90%. The portfolios were rebalanced every six months. Additionally, analysis conducted on the portfolios is elaborated upon in the Results and Analysis section.

Monte Carlo Simulations and the Fama-French 3 Factor Model

The Monte Carlo simulations were used to generate scenarios of the factor loadings present in the Fama-French 3 Factor Model. The Fama-French three-factor model uses the following regression model to predict the excess returns of an asset:

$$r_i - r_f = \alpha_i + \beta_{im}\left(f_m - r_f\right) + \beta_{is}SMB + \beta_{iv}HML + \varepsilon_i$$

Gaussian process

A Gaussian process is a stationary stochastic process in which every step in time is influenced by a normally (Gaussian) distributed random variable. A variable modelled by a Gaussian process can be represented as follows:

$$f_s = \bar{f} + \omega_s$$

The variables in the above equation are defined as follows: f_s is a realization of the factor that correspond to the scenarios s = 1,…,S, is the factor expected return that was calculated from the geometric mean of the historic factor returns and is the error term which is a normally distributed random variable with mean 0 and variance from the matrix *F*, the factor covariance matrix. The random numbers f_s were generated in the MATLAB code using the MATLAB function *mvnrnd* where f_s is normally distributed with mean and variance from the matrix *F*, the factor covariance matrix for s = 1….,5000. The residual error term ε_s was generated using the MATLAB function *mvnrnd* where ε_s is normally distributed with mean of 0 and variance from the matrix *D* where the matrix *D* is a diagonal matrix of residual variances.

Non-normal Stochastic Process with Higher Moments

The non-normal Monte Carlo variant incorporates additional moments of distribution (skewness and kurtosis) as well as the mean and variance moments of distribution used in the Gaussian Monte Carlo Simulation. It was hypothesized that the return of the portfolio would increase by including higher moments in the process of selecting random variables. The inclusion of higher moments would yield a more realistic set of solutions that follow the historic data of the factor returns which would subsequently lead to better CVaR optimizations. Skewness is the degree of distortion from the normal distribution (symmetrical bell curve) in a set of data. [4] Kurtosis measures the extreme values at either tail.

The random variables in the non-normal Monte Carlo Simulation of the factor loadings was drawn from a distribution that had the same skewness of the historical data of factor returns from the previous year. The skewness was calculated using the MATLAB function skewness [5]. The random variables in the non-normal Monte Carlo Simulation of the factor loadings was drawn from a distribution that had the same kurtosis of the historical data of factor returns from the previous year. The MATLAB kurtosis was used to calculate the kurtosis of the historical data factor returns [6]. The random numbers for the Non-Normal Monte Carlo Simulation was generated using the MATLAB function *pearsrnd*. As each factor had a different distribution of historic factor returns, random numbers were drawn for each factor. Thus each factor was simulated individually.

Copula and rank correlation was used to synthetically correlate the random variables generated by the *pearsnd* function [7]. First, the MATLAB function *copularnd* was used to generate sets of 5000 random numbers that have correlations calculated from the correlation matrix of the historic factor returns [8]. Subsequently, Spearman's rank correlation was used to transform the three independent Pearson samples of the three factors into correlated data. Spearman Rank Correlation Coefficient is a distribution-free rank statistic that measures the strength of association between two variables. The randomly generated numbers from *copularnd* was sorted and the indices of the original random variables in the sorted list (from smallest to largest) was used to sort the numbers generated from the *pearsrnd* function. The random numbers, f_s, were generated in the MATLAB code using the MATLAB function *mvnrnd* where f_s is normally distributed with mean and variance from the matrix *F*, the factor covariance matrix for s = 1, ….,5000.

Conditional Value at Risk(CVaR)

CVaR is defined as:

$$CVaR_\alpha(x) = \frac{1}{(1-\alpha)} \int_{\mathrm{f(x,r)} \geq VaR_\alpha(x)} \mathrm{f(x,r)p(r)\,dr}$$

where x is the portfolio, r is the random vector of returns, p(r) is the density of our random asset returns, f(x,r) is the loss of portfolio for a realization of our random asset return r.

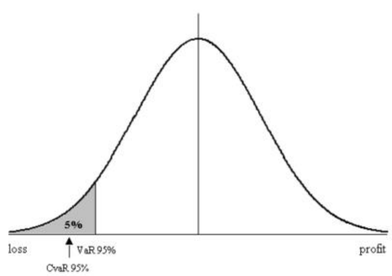

Figure 1: Graphical interpretation of CVaR

VaR can be calculated by the following equation:

$$VaR_\alpha(x) = \min\{\gamma \in R | \Psi(x,\gamma) \geq \alpha\}$$

where $\Psi(x,y)$is the cumulative function (CDF) of loss associated with x:

$$\Psi(x,y) = \int_{\mathrm{f(x,r)<y}} \mathrm{p(r)\,dr}$$

Compared to minimizing portfolio variance, one could only try to minimize the downside risk under CVaR optimization. It is formulated as

$$\begin{gathered}\min_{x}\ CVaR_{\alpha}(x)\\ s.t.\ \ x \in X\end{gathered}$$

where χ is the set of admissible portfolios. However, this is not a convex optimization.

In optimizing CVaR$_{\alpha}$(x), we introduce an auxiliary variable γ as a placeholder for VaR during the optimization process. Then consider the function $F_{\alpha}(x,\gamma)$defined with respect to γ:

$$\begin{gathered}F_{\alpha}(x,\gamma) = \gamma + \frac{1}{(1-\alpha)S}\int (f(x,r)-\gamma)^{+}p(r)dr\\ \text{where } (f(x,r)-\gamma)^{+} = \max\{f(x,r)-\gamma, 0\}\end{gathered}$$

CVaR$_{\alpha}$(x) equals the minimum value over the function F$_{\alpha}$(x, VaR$_{\alpha}$(x)). Thus, the CVaR$_{\alpha}$(x) optimization problem becomes

$$\begin{gathered}\min_{x,z,\gamma}\ \gamma + \frac{1}{(1-\alpha)S}\sum_{s=1}^{S} z_s\\ s.t.\ \ z_s \geq 0 \qquad s = 1, \dots, S\\ z_s \geq f(x,\hat{r}_s) - \gamma \quad s = 1, \dots, S\\ x \in X\end{gathered}$$

In this project, we are asked to implement an optimization model that attempts to minimize CVaR while maximizing the portfolio expected return. The optimization model is derived as

$$\begin{gathered}\min_{x,z,\gamma}\ \gamma + \frac{1}{(1-\alpha)S}\sum_{s=1}^{S} z_s - \lambda\mu^{T}x\\ s.t.\ \ z_s \geq 0 \qquad s = 1, \dots, S\\ z_s \geq -\hat{r}_s^{T}x - \gamma \quad s = 1, \dots, S\end{gathered}$$

Short selling allowed and we use λ=0.1 confidence level is 0.95, number of assets is 20 and there will be 5000 scenarios. We are using two simulation methods to get ${\hat{r}_s}^{T}$. Finally, 'linprog' is used to find the optimal portfolio and retrieve the optimal portfolio weights.

Robust CVaR

To robustify the CVaR model, we can take into consideration, the uncertainty of constructing an uncertainty set around the estimated parameters. For a box uncertainty set, we have:

$$\mu_{true} = U(\mu_{true}) \in R^n : |\mu_{true} - \mu|^T \leqq \delta^2$$

where μ_{true} is the vector of 'true' expected returns and $\delta^2$is the maximum distance assumed to be existed between the estimate and the 'true' value of the expected returns.

The distance δ_i can be set proportionally to the standard errors:

$$\delta_i = \varepsilon_i(\Theta^{\mathbf{1/2}})_{ii}$$

where $\varepsilon_1$is a sizing parameter, providing probabilistic guarantee. In this project, we would assume the error follows a normal distribution, then we can determine $\varepsilon_1$by using the inverse of the cumulative distribution function. For this project, we have a 90% confidence level. We apply the function norminv() in MATLAB to get the corresponding ε_1. Then, we change $\mu^T x$ into $\mu^T x - \delta^T |x|$.

In order to deal with the absolute value, we can introduce an extra variable y to transform the constraints of the robust optimization into

$$y_i \geq x_i$$
$$y_i \geq -x_i$$
$$\mu^T x - \delta^T y \geq R$$

Therefore, our robust CVaR model is as follows:

$$\min_{x,z,\gamma} \gamma + \frac{1}{(1-\alpha)S} \sum_{s=1}^{S} z_s - \lambda(\mu^T x - \delta y)$$
$$s.t. z_s \geq 0 \qquad s = 1, \dots, S$$
$$z_s \geq -\hat{r}_s^T x - \gamma \quad s = 1, \dots, S$$

Difference between CVaR & Robust CVaR

The nominal CVaR model for this project is

$$\min_{x,z,\gamma} \gamma + \frac{1}{(1-\alpha)S} \sum_{s=1}^{S} z_s - \lambda \mu^T x$$
$$s.t. \qquad z_s \geq 0 \qquad s = 1, \dots, S$$
$$z_s \geq -\hat{r}_s{}^T x - \gamma \; s = 1, \dots, S$$
$$1^T x = 1$$

And the Robust CVaR model is

$$\min_{x,z,\gamma} \gamma + \frac{1}{(1-\alpha)S} \sum_{s=1}^{S} z_s - \lambda(\mu^T x - \delta y)$$
$$s.t. z_s \geq 0 \qquad s = 1, \dots, S$$
$$z_s \geq -\hat{r}_s^T x - \gamma \quad s = 1, \dots, S$$
$$y_i \geq x_i$$
$$y_i \geq -x_i$$
$$1^T x = 1$$

Compared with the nominal CVaR model, $\lambda\delta y$ is added on the objective function in Robust CVaR model. The equations: $y_i \geq x_i$and $y_i \geq -x_i$are also added into the constraints in Robust CVaR model to make sure that y denotes the absolute value of x.

Robust optimization has higher computational cost since it has more constraints that the program needs to be braced against. Additionally, it is expected that the robust portfolio returns will be lower since it is braced against a larger set of constraints.

Results

The figure below displays the four different portfolio values over time from January 1st, 2013 to January 1st, 2016. The % loss of each portfolio over time is indicated in **Appendix I**. The composition of each portfolio at each period is indicated by figures 3-6. The portfolios were rebalanced after 6-month period. The rebalancing periods are evident in figures 3-6 but not so evident in figure 2.

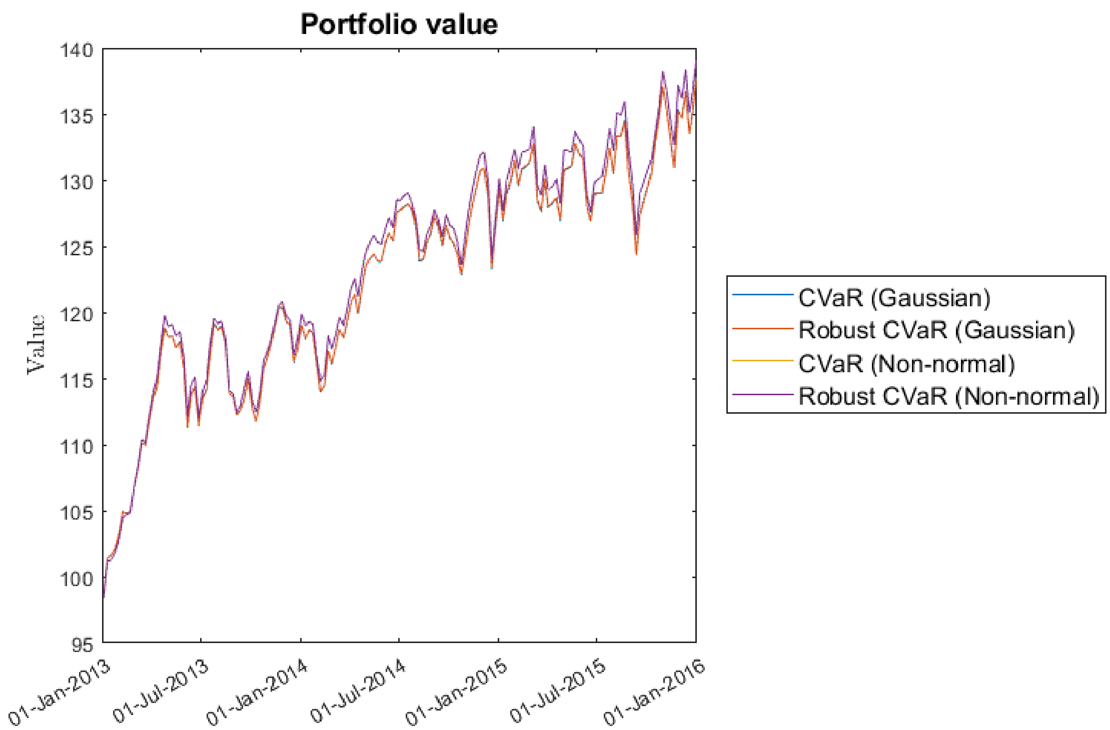

Figure 2: Portfolio values over time

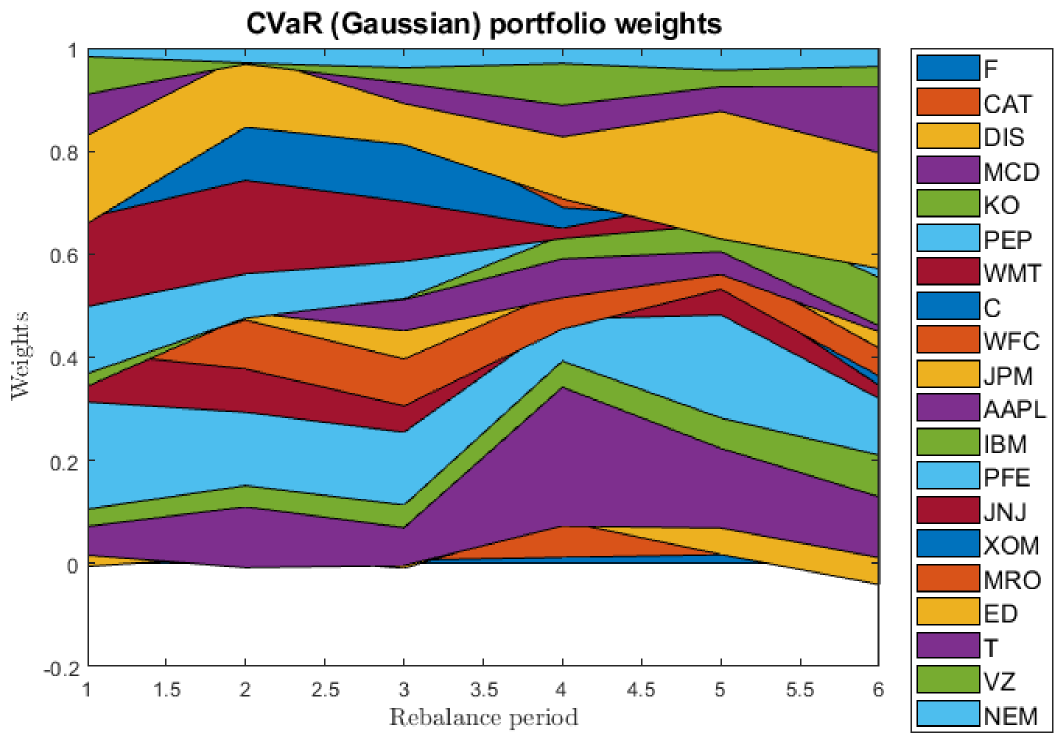

Figure 3: Portfolio composition of the portfolio created using a Gaussian Monte-Carlo simulation and CVaR.

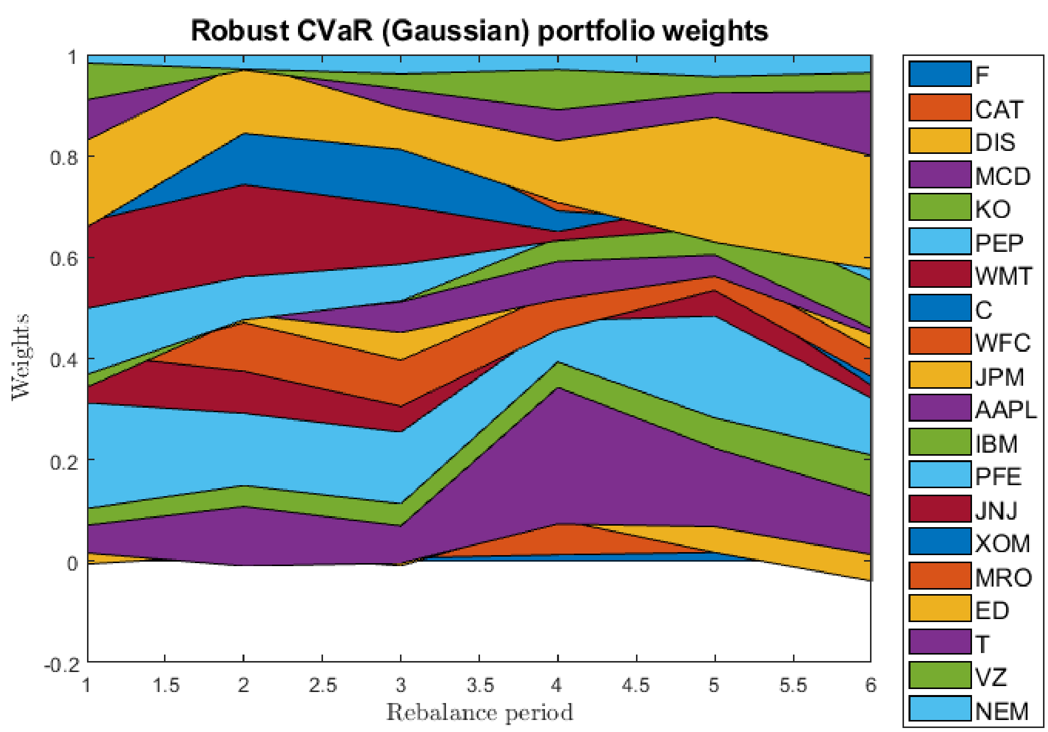

Figure 4: Portfolio composition of the portfolio created using a Gaussian Monte-Carlo simulation and Robust CVaR.

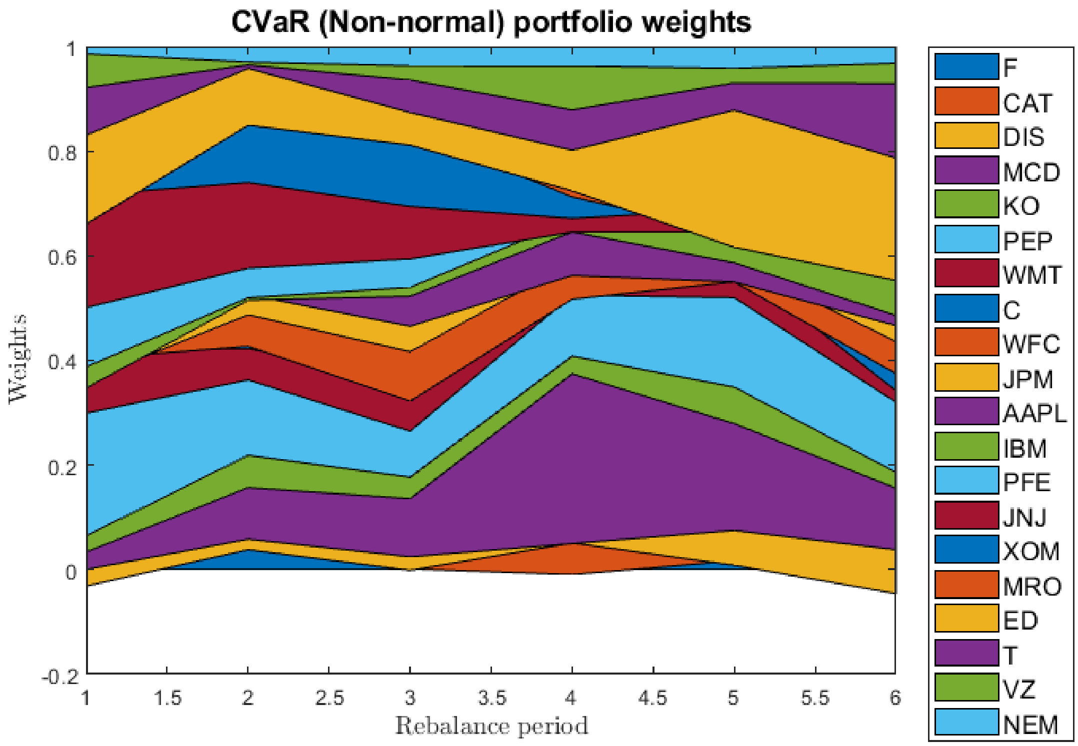

Figure 5: Portfolio composition of the portfolio created using a Non-normal Monte-Carlo simulation with higher moments and CVaR.

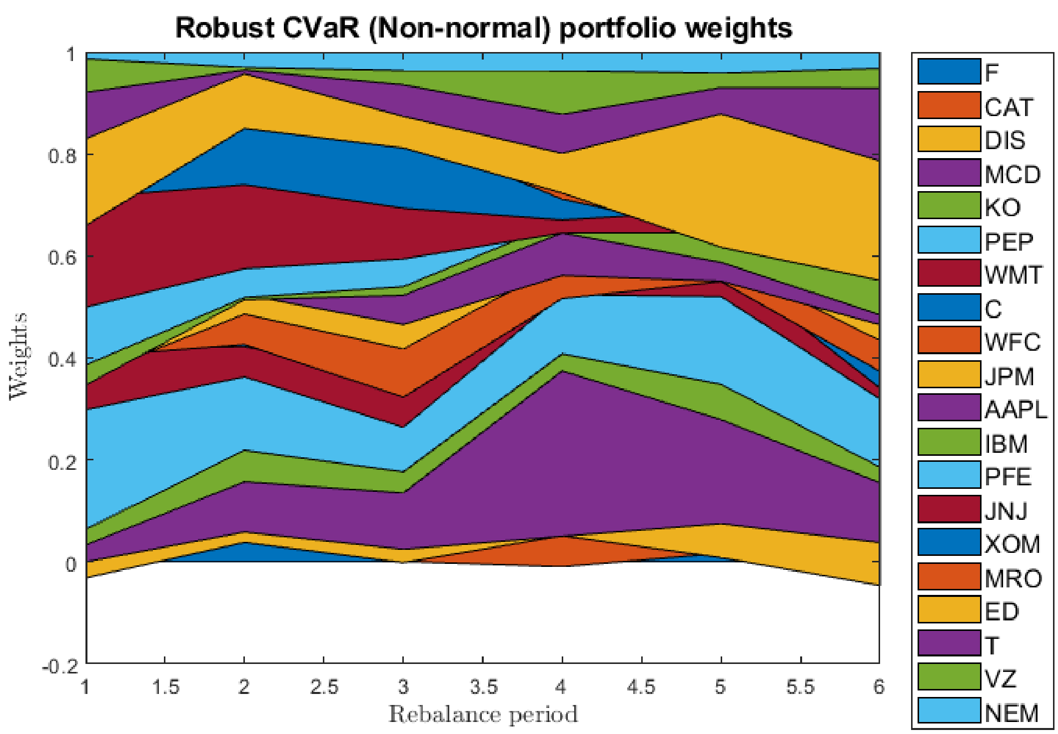

Figure 6: Portfolio composition of the portfolio created using a Non-normal Monte-Carlo simulation with higher moments and Robust CVaR.

Analysis

The generated portfolios can be compared in various ways. Primarily, investors are interested in high returns and low downward risk. The portfolio values at the end of the 3-year period are indicated in Table 2. The table also compares the expected and realized returns of each portfolio, and each portfolio's Sharpe Ratio.

Table 2: Portfolio values at the end of the 3-year term, and their expected & realized returns

Portfolio	End of Term (3 years) Portfolio Values	Sharpe Ratio	Average Period Returns (annualized)	
			Expected	Realized
CVaR (Gaussian)	$137.80	1.7676	0.1410	0.1189
Robust CVaR (Gaussian)	$137.70	1.7672	0.1402	0.1186
CVaR (Non-normal)	$139.10	1.7822	0.1481	0.1223
Robust CVaR (Non-normal)	$139.10	1.7797	0.1481	0.1223

The portfolio created using Gaussian Monte-Carlo simulations and Robust CVaR was expected to perform slightly worse than the non-robust portfolio. Although, both portfolios performed equally well. Furthermore, table 2 indicates robust and non-robust portfolios had equally high returns. Additionally, the portfolios which were created using non-normal Monte-Carlo simulations with higher moments, performed much better as expected during each term as indicated in table 2, but also as expected as explained in the Background section. The latter two portfolios had the highest expected and realized returns indicating that portfolios created using the Non-normal Monte-Carlo simulations perform better than those formed using Gaussian Monte-Carlo simulations.

To further compare the portfolios, the volatility of each portfolio is displayed in figure 7. They are relatively very similar to one another and hence do not provide much insight. Since each portfolio is composed of the same set of stocks, correlated volatility amongst the portfolio's is expected. If the set of assets was larger, volatility correlation should decrease.

Although, it is observed that in 2013, the expected and realized volatilities diverged noticeably more than in other years. This may be due to the unexpected great year for US stocks, as mentioned by forbes [14]. It may also be caused by investor speculation that the US federal reserve may cut back on its purchases of bonds [15]. Impacts of macroeconomic activities are further elaborated upon in the Discussion section.

The Sharpe ratio is a measure of the portfolio's reward to risk [16]. In figure 7, it is observed that the volatility of the portfolios are very similar to each other. That is, in the figure, the dashed lines almost overlap. Additionally, the risk-free rate is the same for all four portfolios. Hence, the reward to risk of the portfolios should bring one to the same conclusion as the reward. That is the case: the Non-normal portfolios have a better ratio than others. Additionally, the Sharpe ratios identify that the Robust CVaR portfolios have lower reward to risk ratios.

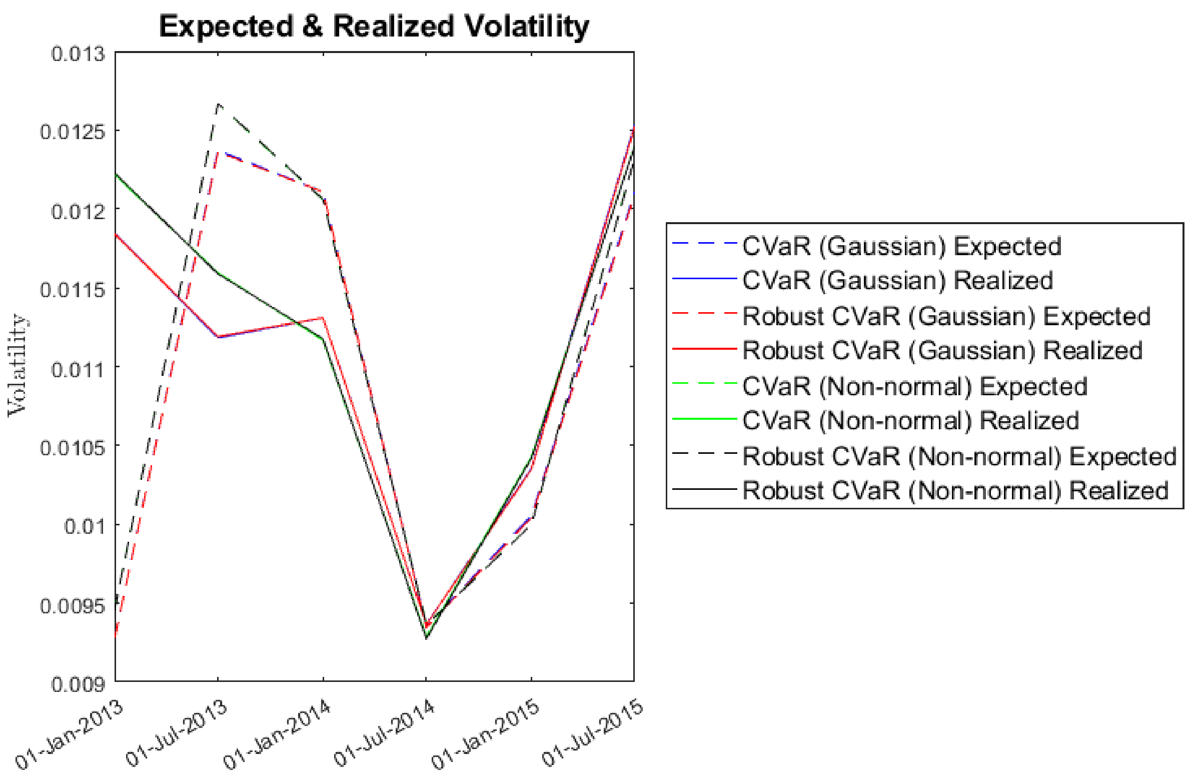

Figure 7: Volatility of each portfolio, expected & realized.

The portfolio losses, as shown in Appendix I, are unimodal. This indicates that the portfolio losses and gains are evenly distributed throughout the three-year term. This is not a favourable outcome since CVaR is used to minimize losses. One would expect the curves should be right-skewed since only losses are minimized but this is not the case.

Discussion

The results indicate that the portfolios created using the non-normal Monte-Carlo simulations fare better than those created using the Gaussian Monte-Carlo simulations. This is expected since it incorporates additional moments of distribution (skewness and kurtosis) as well as the mean and variance moments of distribution used in the Gaussian Monte Carlo Simulation. The inclusion of higher moments of historical data yields a more realistic set of projections of the factor returns which subsequently lead to better CVaR optimizations. As was hypothesized, the returns of the portfolios increased by including higher moments.

Although the results are as hypothesized, there are a few limitations of this project that can be addressed to yield more accurate results: inclusion of more than 20 assets, comparing the portfolios within different time periods, creating more than 5000 simulations and including more investor preferences.

This project only uses 20 assets to create the portfolio. The limited number of assets reduces the scope of the project because when creating the portfolios, the truly best assets available in the market cannot be used. Using the true best assets would increase portfolio returns. Furthermore, with just 20 assets, macroeconomic events are more likely to disrupt the expected outcomes of portfolios. For example, newly introduced tax incentives for technology companies are not taken into account when forming the portfolios, although the implications of these incentives may have a large impact on the assets themselves. Having more assets negates this drawback since a smaller fraction of the assets in our portfolios are impacted by such events.

The portfolios generated were only observed from 2013 to 2016. Since economic conditions are constantly changing, the asset price movement from 2013 to 2016 may not reflect the true nature of these assets. Although observing them for a longer period does not guarantee that their natural trends will be observed, a longer time period increases the probability of this occurring.

Using robustness to stabilize the portfolios was not very beneficial. Instead, portfolio performance may be improved by incorporating investor views. For example, in 2013, major US banks were sued [17]. This would negatively impact the banks - an insight that historical data cannot provide. As such, including investor views and meaningful insights would increase the portfolio performance.

Another limitation is the computational power available to perform Monte-Carlo simulations. In this project, 5000 asset scenarios were simulated. Although random sampling 5000 times should lead to the desired results (Gaussian/ Non-normal distributions of asset paths), the outcome can be improved with more scenarios. With more scenarios, the portfolio expected mean and expected variance may yield more accurate and reliable results. With just 5000 scenarios, the results seem to vary with different trials. Hence, the results are not conclusive.

Conclusion

Two Monte Carlo simulations were conducted on the factor returns of the Fama-French 3 Factor Model. One using random variables distributed on a normal distribution and another using random variables that were distributed on a non-normal distribution which included the kurtosis and skewness of the historical factor returns.

CVaR measures the expected loss beyond VaR for given confidence level α. Robust optimization takes the uncertainty of portfolio expected return into consideration.

Portfolios created with non-normal Monte-Carlo simulations outperformed the portfolios formed using normally distributed Monte-Carlo simulations when comparing realized returns and Sharpe ratios. Additionally, robustness does not have a significant impact on the results and has lower Sharpe ratios. Hence, Robust CVaR should not be used since it adds to the computational costs.

Although, the results are not conclusive since they seem to vary significantly every trial. Accuracy of results can be improved with the inclusion of more assets, observation of the assets for a longer time period, more simulations and inclusion of investor perspectives in the formulation of the problem.

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Appendix I: Portfolio Losses

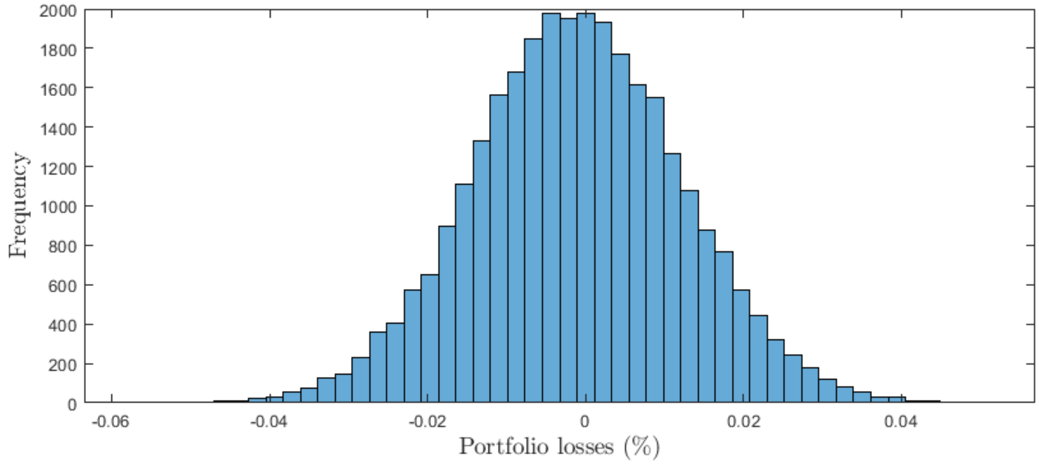

Figure 8: Historical loss distribution of the portfolio created using a Gaussian Monte-Carlo simulation and CVaR.

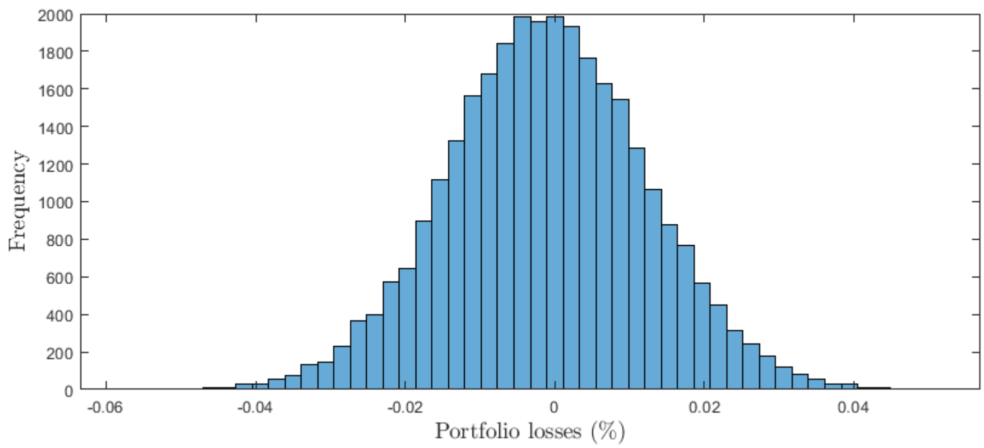

Figure 9: Historical loss distribution of the portfolio created using a Gaussian Monte-Carlo simulation and Robust CVaR.

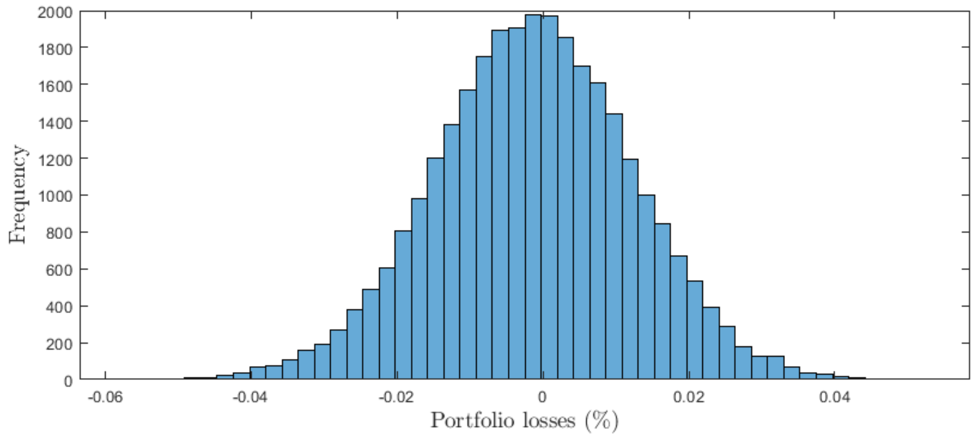

Figure 10: Historical loss distribution of the portfolio created using a Non-normal Monte-Carlo simulation with higher moments and CVaR.

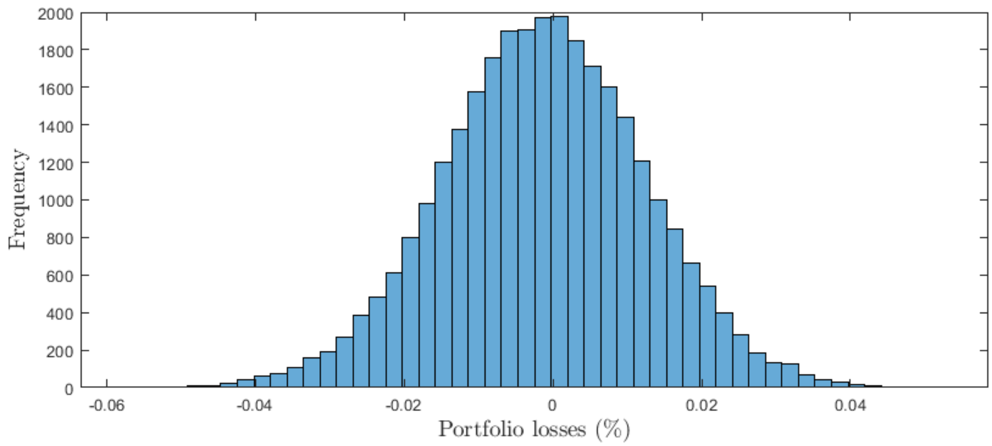

Figure 11: Historical loss distribution of the portfolio created using a Non-normal Monte-Carlo simulation with higher moments and Robust CVaR.