MIE377

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Purpose

The purpose of this project is to build and compare portfolios using the CAPM, Fama-French and PCA factor models, and Mean Variance Optimization (MVO) and Cardinality-constrained MVO investment strategies. In total, six portfolios are compared.

Methodology

The project uses weekly adjusted closing prices of the stocks listed in Table 1, from 30-Dec-2011 to 31-Dec-2015 to compute observed asset weekly returns. In addition, historical factor returns for the Fama-French three-factor model French corresponding to the period 06-Jan-2012 to 31-Dec-2015 are used. This includes the weekly risk-free rate.

The prices are adjusted closing prices. Adjusting closing prices are beneficial for back-testing as it provides a more accurate representation of the firm's equity value beyond the simple market price. Adjustments can account for dividend distributions, stock splits, and other corporate actions.

Table 1: Company tickers of the assets considered

F	CAT	DIS	MCD	KO	PEP	WMT	C	WFC	JPM
AAPL	IBM	PFE	JNJ	XOM	MRO	ED	T	VZ	NEM

The CAPM, Fama-French and PCA factor models are used to estimate the parameters required for portfolio optimization. The portfolios are built using the estimated parameters from each factor model and the Mean Variance Optimization (MVO) and Cardinality-constrained MVO investment strategies. In total, six portfolios will be built.

Factor Models

The following factor models were implemented in MATLAB to determine the expected returns and covariance matrix of the assets in Table 1 given the weekly prices of each asset from 30-Dec-2011 to 31-Dec-2015.

Capital Asset Pricing Model (CAPM)

The Capital Asset Pricing Model (CAPM) a is single-factor model that can be used to calculate the excess return of an asset above the risk-free rate. The CAPM of the ith asset is given as:

$$r_i - r_f = \alpha_i + \beta_i(f_m - r_f) + \varepsilon_i$$

The variables are defined as follows: r_i : return of asset i, r_f : the risk-free rate, α_i: the intercept from the regression of asset i, β_i: the market factor loading, ($f_m - r_f$): the excess market return factor and ε_i: residual of asset i.
The ideal environment assumption was used for CAPM calculations and as a result, the following assumptions were made:

- $cov(f_m, \varepsilon_i)=0$
- $cov(\varepsilon_i, \varepsilon_j) = 0$ for $i \neq j$

The regression coefficients (α_i and β_i) were solved in closed form from the following equation:

$$\begin{bmatrix} \alpha^T \\ \beta^T \end{bmatrix} = (X^T X)^{-1} X^T Y$$

The variables are defined as follows: Y is a matrix of n asset returns in which each column is the N time observations for a single asset, X = [1 ($f_m - r_f$)] where X is a matrix of factor predictors , 1 is column matrix of ones, ($f_m - r_f$) is the timeseries of the factor market excess return, α is a vector of the intercepts for all n assets and β is a vector of the excess market return factor loading for each asset i.

The expected assets return, μ, was calculated using:

$$\mu = \alpha + \beta \bar{f}$$

The variables are defined as follows: α is a vector of the intercepts for all n assets, β is a vector of the excess market return factor loading for each asset i and $\bar{f}$ is a vector of the geometric means of the excess market return factors for each asset i over the N time steps.

The covariance matrix of the assets, Q, was calculated using:

$$Q = \beta F \beta^T + D$$

The variables are defined as follows: β is a vector of the excess market return factor loading for each asset i, F is the variance of the excess market factor return over N time steps and D is the covariance matrix of the residuals of the n assets.

Fama-French Three-Factor Model (FF)

The Fama-French three-factor model (FF model) is a factor model with three factors that attempts to explain the excess return of an asset based on its exposure to the market, size and value (Book-to-Market ratio). The excess returns of an asset using the FF model is as follows:

$$r_i - r_f = \alpha_i + \beta_{im}(f_m - r_f) + \beta_{is} SMB + \beta_{iv} HML + \varepsilon_i$$

The variables are defined as follows: r_i is the asset return, r_f is the risk-free rate, α_i is the intercept of the i^{th} asset from regression, β_{im} is is the market factor loading of asset i, ($f_m - r_f$) is the excess market return factor, β_{is} is the size factor of asset i, SMB is the size factor computed as the average return of a portfolio composed of small cap stocks minus the average return of large cap stocks, β_{iv} is the value factor loading of asset i

and HML is the value factor calculated as the average return of a portfolio comprised of stocks with the highest Book-to-Market ratio minus the stocks with the lowest Book-to-Market ratio and ε_i is the residual of asset i from regression.
The ideal environment was not used as it cannot be assumed that the three factors (market exposure, size and value) are not correlated. However, the following assumptions were made:

- $cov(f_m, \varepsilon_i)=0$
- $cov(\varepsilon_i, \varepsilon_j) = 0$ for $i \neq j$

The regression coefficients (α_i and β_i) were solved in closed form from the following equation:

$$\begin{bmatrix} \alpha^T \\ \beta^T \end{bmatrix} = (X^T X)^{-1} X^T Y$$

The variables are defined as follows: Y is a matrix of n asset returns in which each column is the N time observations for a single asset, $X = [\ 1\ (f_m - r_f)\ SMB\ HML\ ]$ where X is a matrix of factor predictors, $(f_m - r_f)$ is a column vector of the timeseries of the factor excess market return, SMB is a column vector of the timeseries of the size factor, HML is a column vector of the timeseries of the value factor,, α is a vector of the intercepts for all n assets and β is a vector of the excess market return factor loading for each asset i.

The expected assets return, μ, was calculated using:

$$\mu = \alpha + \beta \bar{f}$$

The variables are defined as follows: α is a vector of the intercepts for all n assets, β is a vector of the factor loading for each asset i and $\bar{f}$ is a vector of the geometric means of the factors for each asset i over the N time steps.

The covariance matrix of the assets, Q, was calculated using:

$$Q = \beta F \beta^T + D$$

The variables are defined as follows: β is a vector of the factor loading for each asset i, F is the covariance matrix the factors over N time steps and D is the covariance matrix of the residuals of the n assets.

Principal Components Analysis (PCA)

The Principal Components Analysis reduces dimensionality of the data to create a factor model using the given returns.

Given a matrix of returns, *R*, for *n* assets over a time period *T*, a *n* x *n* sample covariance matrix, *C*, of the returns is created where C_{ij} is the the covariance between asset *i* and *j*. The sample covariance matrix is created using the following procedure:

1. Calculate the arithmetic mean of the returns for each asset: $\bar{x} = \frac{1}{T} X 1_T$
2. Calculate the de-meaned matrix: $X^* = X - \bar{x} 1'_T$
3. Calculate the sample covariance matrix: $C = \frac{1}{T} X^* (X^*)'$

The covariance matrix is then decomposed using the matrix of eigenvectors and the diagonalized matrix of the eigenvalues: $C = VDV^{-1}$where D is the diagonal matrix of the eigenvalues $D = diag(\lambda_1, \lambda_2 ... \lambda_n)$ and $\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_n$, V is the matrix of eigenvectors where each vector p_i in $V = [p_1, p_2 ... p_n]$ is associated with the eigenvalue λ_i, and $VV^{-1} = I$.

The factor for asset *i* at time *t* is then calculated using the following:

$$f_{it} = p_{i1}R_{1t} + p_{i2}R_{2t} + \ldots + p_{in}R_{nt}$$

The factors are then used to calculate the mean and variance as indicated in the French-Fama Model.

The number of factors can be limited to K factors, where $1 \leq K \leq n$. This project uses $K = 3$ to directly compare the PCA portfolio with the French-Fama portfolio since both use three factors to determine the expected mean and asset covariance.

The principal components are orthogonal and thus uncorrelated. Additionally, it is assumed that the residuals from the factor model are uncorrelated.

Investment Strategies

The following investment strategies were implemented in MATLAB to create portfolios of the assets provided in Table 1 using the returns and covariance matrices calculated using the three factor models discussed above.

Mean-Variance Optimization (MVO)

Mean-Variance Optimization is an investment technique used to create a portfolio of assets in which the aim is to minimize the risk of the portfolio such that the portfolio return matches or exceeds investor expectations for that given portfolio. In this case, the desired return is the arithmetic mean of the expected returns for each asset. Additionally, the sum of the elements of x must add up to 1 and short selling is not allowed, hence the budget is not exceeded and each element of x must be non-negative.

Mathematically, the following must be satisfied:

$$minimize\ \frac{1}{2}x^TQx \ \ such\ that\ x^T\mu \ \geq R, 1^Tx = 1, 0 \leq x_i$$

Where Q is the covariance matrix of assets, x is a vector of weights where the x_i indicates the portion of the budget that will be invested in asset i, μ is the vector of expected returns for each asset, R is the target return, and 1 is a column of ones.

The MATLAB function quadprog was used to compute x.

Cardinality-Constrained MVO

Cardinality-Constrained MVO is an investment technique used to create a portfolio of assets in which the aim is to minimize the risk of the portfolio such that the portfolio returns match or exceed investor expectations for that given portfolio and only a certain number of assets can be in the portfolio at a given time. It has the same objective as MVO but the number of assets that can be in the constructed portfolio is limited to a pre-defined constrained. That is:

$$minimize\ v^T Q y\ \ such\ that\ x^T\mu\ \geq R,\ 1^T x\ =\ 1, 1^T y\ =\ K, 0\ \leq x_i\ \leq y_i$$

Where y is 1 if the asset is in the portfolio and 0 otherwise, K is the number of assets in the portfolio, Q is the covariance matrix of assets, x is a vector of weights where the x_i indicates the portion of the budget that will be invested in asset i, μ is the vector of expected returns for each asset, R is the target return, and 1 is a column of ones.

The function gurobi was used to compute x.

Summary of Computational Results

The tables below compare each portfolio's standard deviation, average return, and transaction costs. Table 2 and Table 3 display the average realized portfolio standard deviation and annualized returns respectively. Table 4 and Table 5 display the average expected portfolio standard deviation and annualized returns respectively. The transaction costs, displayed in Table 6, are not included in calculating the returns. Table 7 displays the adjusted R^2 values for each factor model and Table 8 displays the significance of each factor model – determined by the F-test.

Additionally, Figure 1 depicts the value of each portfolio throughout the three-year term. Figures 2-7 display each portfolio's assets throughout the three year term.

Table 2: Realized portfolio standard deviation for portfolios created using CAPM, FF and PCA models and MVO and Cardinality Constrained MVO investment strategies

MVO (CAPM)	MVO Card (CAPM)	MVO (FF)	MVO Card (FF)	MVO (PCA)	MVO Card (PCA)
0. 0115	0. 0114	0. 0109	0. 0110	0. 0107	0. 0107

Table 3: Realized geometric mean of annualized weekly portfolio returns using CAPM, FF and PCA models and MVO and Cardinality Constrained MVO investment strategies

MVO (CAPM)	MVO Card (CAPM)	MVO (FF)	MVO Card (FF)	MVO (PCA)	MVO Card (PCA)
0. 1171	0. 1172	0. 1263	0. 1267	0. 1258	0. 1258

Table 4: Expected portfolio standard deviation for portfolios created using CAPM, FF and PCA models and MVO and Cardinality Constrained MVO investment strategies

MVO (CAPM)	MVO Card (CAPM)	MVO (FF)	MVO Card (FF)	MVO (PCA)	MVO Card (PCA)
0.0108	0.0108	0.0114	0.0114	0.0118	0.0118

Table 5: Expected geometric mean of annualized weekly portfolio returns using CAPM, FF and PCA models and MVO and Cardinality Constrained MVO investment strategies

MVO (CAPM)	MVO Card (CAPM)	MVO (FF)	MVO Card (FF)	MVO (PCA)	MVO Card (PCA)
0.1259	0.1258	0.1265	0.1265	0.1464	0.1464

Table 6: Period transaction costs for portfolios created using CAPM, FF and PCA models and MVO and Cardinality Constrained MVO investment strategies

Period t	MVO (CAPM)	MVO Card (CAPM)	MVO (FF)	MVO Card (FF)	MVO (PCA)	MVO Card (PCA)
1	0.1791	0.1791	0.217	0.217	0.2676	0.2676
2	0.2003	0.2009	0.2213	0.2213	0.2698	0.2698
3	0.3413	0.3657	0.3741	0.3741	0.4434	0.4434
4	0.303	0.2974	0.3095	0.3097	0.3217	0.3218
5	0.1027	0.1038	0.1169	0.1145	0.2868	0.2868
Total	1.1264	1.1469	1.2388	1.2366	1.5893	1.5894

Table 7: Average adjusted R^2 value for each factor model at each period.

Period	1	2	3	4	5	6
CAPM	0.3331	0.3937	0.3601	0.3098	0.3313	0.3771
FF	0.3331	0.3937	0.3601	0.3098	0.3313	0.3771
PCA	0.4968	0.5548	0.5465	0.492	0.4821	0.5416

Table 8: Percent of factors which were statistically significant (F-test) for each factor model at each period

Period	1	2	3	4	5	6
CAPM	0.8	1	0.9	0.85	0.85	0.85
FF	0.8	1	0.9	0.85	0.85	0.85
PCA	0.95	1	1	1	0.95	1

Figure 1: Weekly expected returns for portfolios created using CAPM, FF and PCA models and MVO and Cardinality Constrained MVO investment strategies

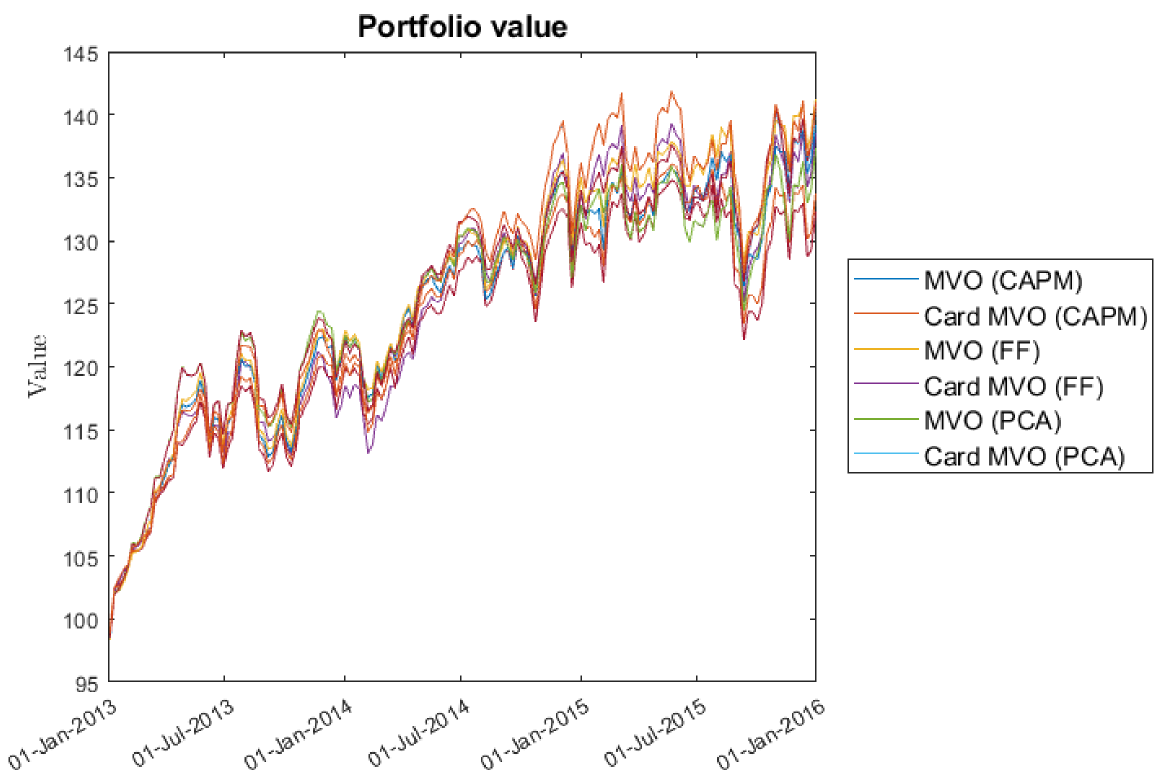


Figure 2: Portfolio asset weights for portfolio created using CAPM model and MVO investment strategy

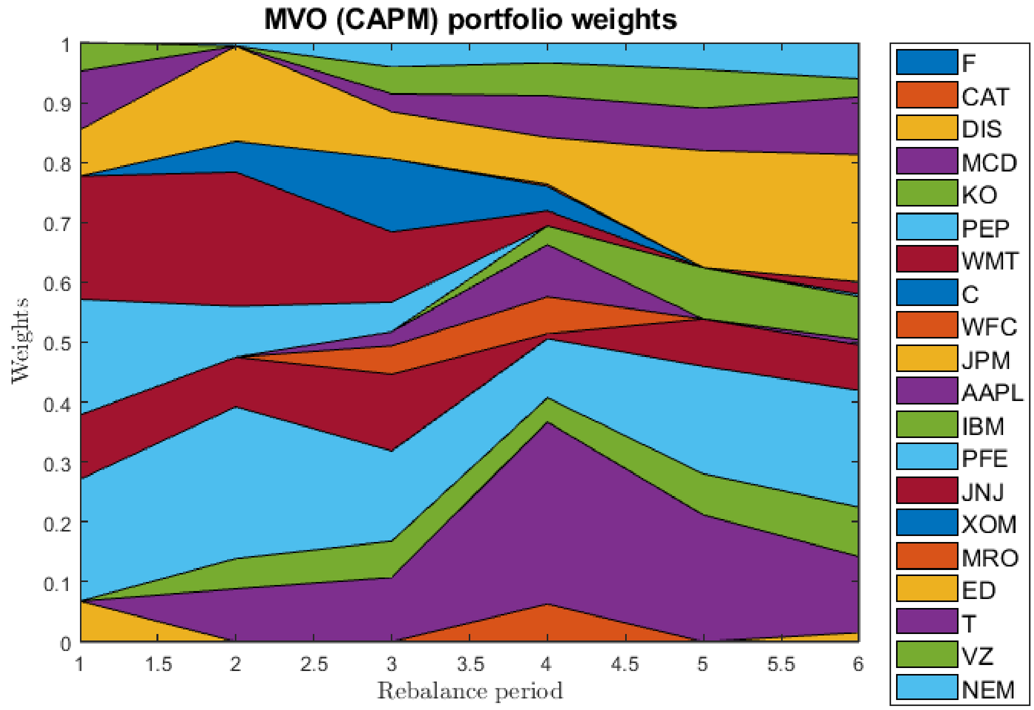


Figure 3: Portfolio asset weights for portfolio created using CAPM model and Cardinality Constrained MVO investment strategy

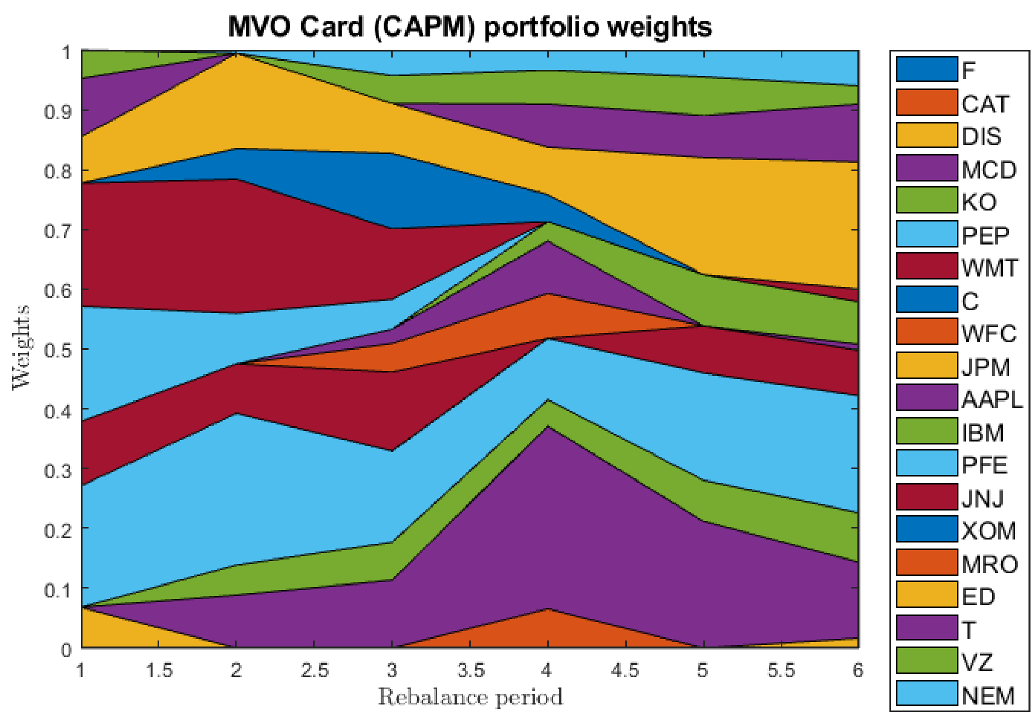


Figure 4: Portfolio asset weights for portfolio created using FF model and MVO investment strategy

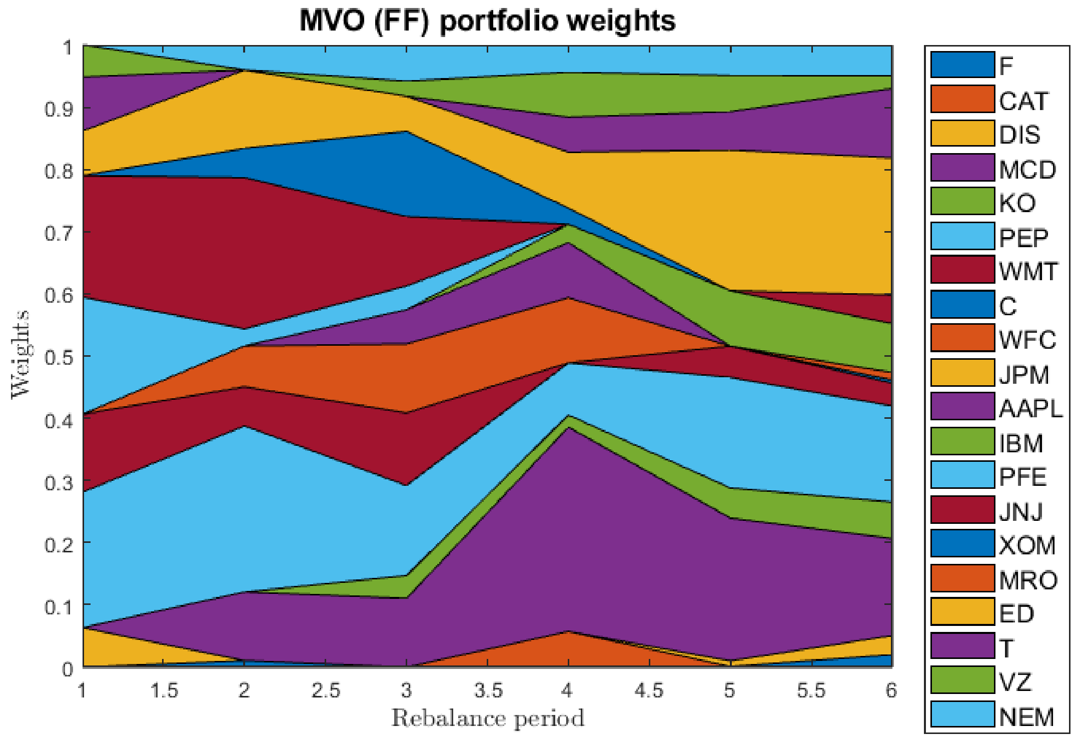


Figure 5: Portfolio asset weights for portfolio created using FF model and Cardinality Constrained MVO investment strategy

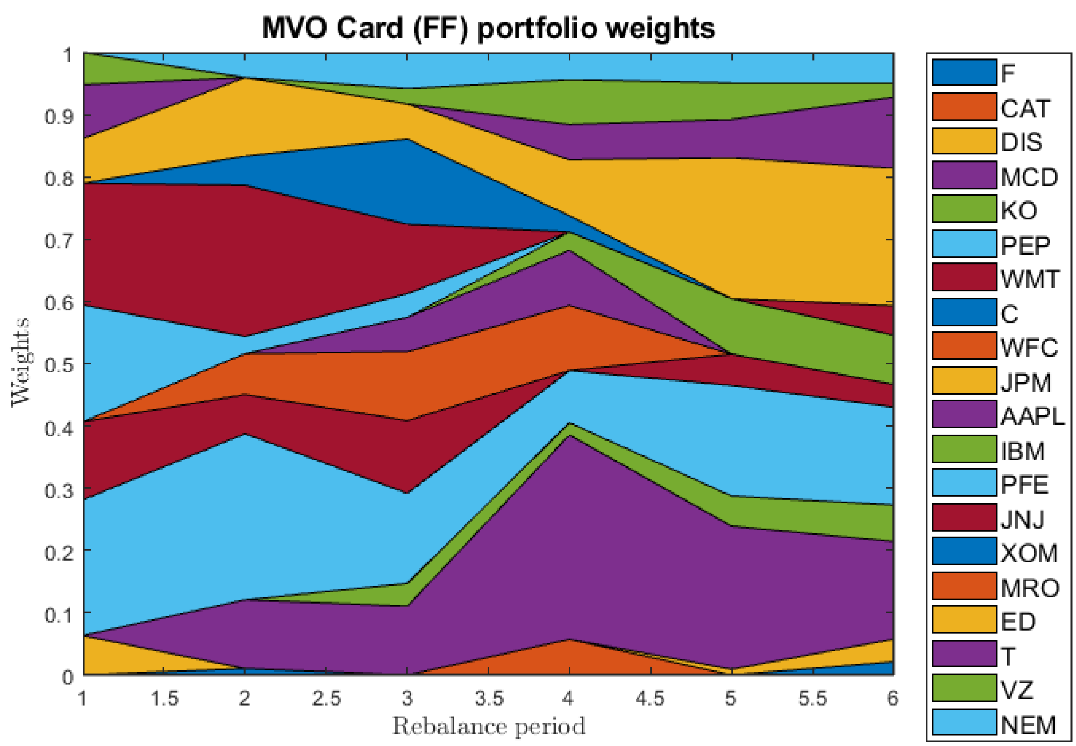


Figure 6: Portfolio asset weights for portfolio created using PCA model and MVO investment strategy

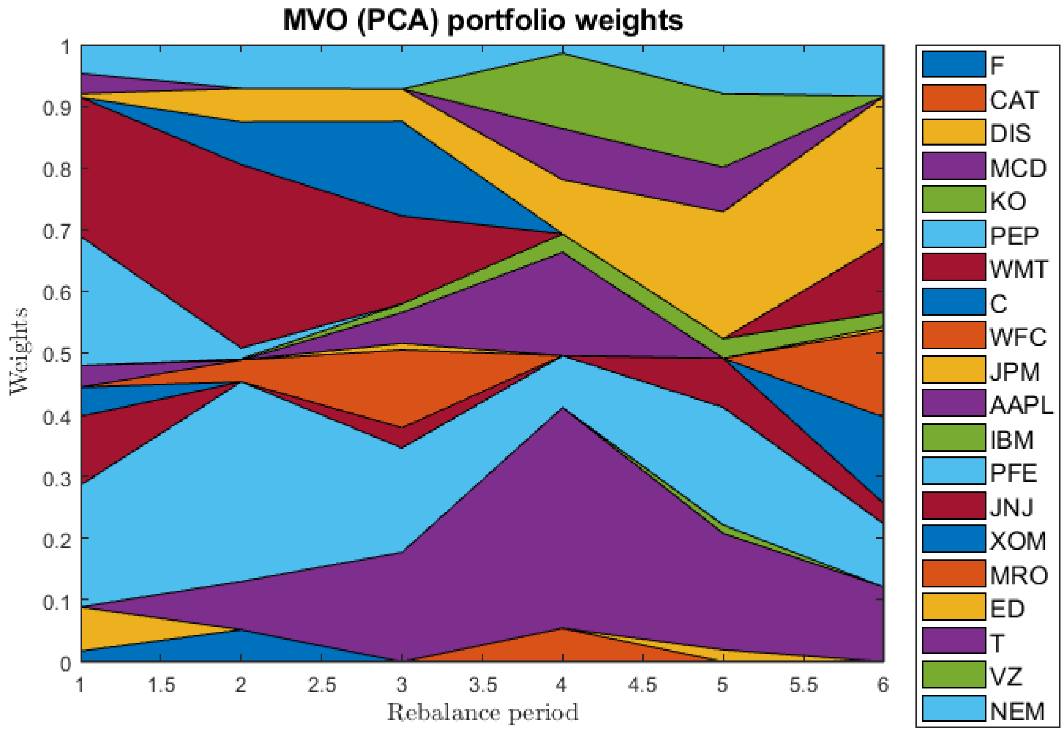
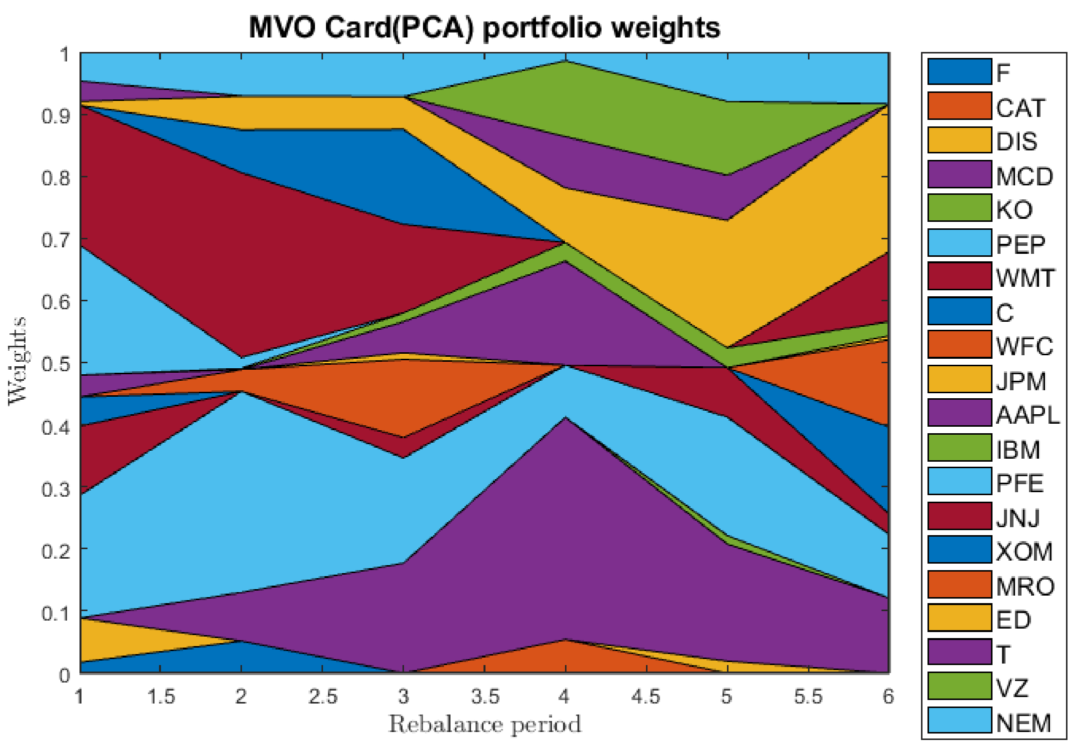


Figure 7: Portfolio asset weights for portfolio created using PCA model and Cardinality ConstrainedMVO investment strategy



Analysis of Results

The objective when creating the portfolios was to minimize risk. Table 9 displays the portfolios in order of lowest to highest variance. The PCA factor model has a relatively lower standard deviation when compared to CAPM and FF.

Table 9: Portfolio standard deviation displayed in ascending order

1	MVO (PCA)	0.0107
	MVO Card (PCA)	0.0107
3	MVO (FF)	0.0109
4	MVO Card (FF)	0.0110
5	MVO Card (CAPM)	0.0114
6	MVO (CAPM)	0.0115

It is also in the investor's best interest to evaluate the returns of each portfolio. Table 10 ranks the portfolio returns of each portfolio. The PCA factor model again, performs the best. When comparing the cardinality constrained portfolios with their respective non-constrained portfolios, the cardinality constrained portfolios consistently perform better or just as well as in comparison to the non-constrained portfolios.

Table 10: Portfolio returns displayed in descending order

1	MVO Card (PCA)	0.1267
2	MVO (PCA)	0.1263
3	MVO Card (FF)	0.1258
	MVO (FF)	0.1258
5	MVO Card (CAPM)	0.1172
6	MVO (CAPM)	0.1171

Additionally, transaction costs should also be considered. The portfolio returns from above (~12%) are much higher than the transaction costs (~0.2%). Although transaction costs can have a large impact the net return on an investment, it is negligible for this scenario.

Discussion

Motivation Behind Factor Models

The purpose of using a single factor model such as the CAPM is to reduce the number of parameter estimations that are required. In the absence of factor models, there are n expected values, n variances and n*(n-1)/2 covariances that need to be estimated. Thus the number of estimations is 2n + n*(n-1)/2 which has complexity of O(n^2). By using a single factor model, only n asset intercepts, 1 factor variance, n residual variances, 1 factor loading and 1 factor geometric mean need to be estimated. Thus the number of estimations is 3n+ 2 which has O(n) complexity.

Capital Asset Pricing Model (CAPM)

The CAPM is limited due to its assumption of a linear relationship between market beta and expected stock returns, investor behaviours and the absence of other factors, besides an asset's volatility that affect the asset's expected return.

The CAPM assumes that there is a linearly positive relation between market beta and the expected stock returns through the beta coefficient that relates the asset's risk premium above the market to the expected return of the asset. Fama and French concluded however, that after adjusting for size effects, the beta coefficient is irrelevant for explaining the differences of expected return between assets [1].

The CAPM also makes a series of assumptions with regards to the behaviour of investors in determining that a portfolio using the CAPM will be the most efficient such that there will not be another portfolio with lower risk and a higher return than it. These assumptions are that all investors are permitted to short-sell all securities without restriction, all investors agree about risk and the expected return for all securities, the investment opportunity set for all investors holding any security in the index is restricted to the securities in the cap-weighted index and no investor's return is exposed to federal or state income tax liability [2]. Portfolios constructed in accordance with CAPM will only be efficient if it is assumed that all investors have the same efficient frontier such that the market portfolio will be perceived as efficient by all investors. [2] Thus CAPM will not makes the most optimal investment strategy when any of the following occurs: investment income is taxed, investors disagree about risk and expected returns, short-selling is restricted, foreign investors are in the domestic capital market and investment alternatives are not included in the target index [2]. When the above scenarios occurs, there will be alternatives to cap-weighted portfolios that will have a higher expected return, but lower volatility.

As the CAPM only uses one factor, the covariance of an asset's volatility to the market, the CAPM fails to explain the small firm effect noted by Banz and the value effect noted by Stattman [3] [4].

Fama-French Three Factor Model (FF)

The FF model attempt to augment the CAPM with size (SMB) and value(HML) factors that attempt to address the limitations of the CAPM not accounting for Banz's small-firm effect and Strattman's value effect [5]. Thus the FF model allows the expected returns of an asset to also be characterized by the size of its capitalization and its book-to-market equity value. Fama and French noted that their model's size (SMB) and book-to-market equity (HML) factors act as proxies for sensitivities to common risk factors used to determine expected returns. When used in combinations, the size (SMB) and book-to-market factor (HML) seemed to absorb the role of leverage and earnings to price ratio (E/P) in determining average returns [6]. Thus, as the size and book-to-market factor are not independent of each other, the FF model does not abide by the ideal factor environment in which asset return factors are considered uncorrelated to each other. Fama and French observed that the use of the FF model had regressions with intercepts close to 0, thereby suggesting that market factor, size and book-to-market equity seem to characterize the differences between the average returns across various stocks [6].

As the FF model is an augmentation of the CAPM, it still determines that expected return of a stock using the covariance of an asset to the market. Fama and French noted, themselves, that covariance between an asset and the market was not relevant in determining the expected return of an asset [1].

Principal Component Analysis (PCA)

In addition to reducing the number of parameter estimations, when working in higher dimensions, it is difficult to extract important features without losing information without incurring high computational costs. Hence, it is advantageous to use PCA to reduce dimensionality.

The PCA factor method reduces dimensionality by creating artificial features composed of linear combinations of the original underlying features [7] [8]. The top most factors of the PCA method contain majority of the information from the data. Thus, with only the top-most factors one can create an optimal portfolio. Furthermore, the factors are independent which helps to brace against collinearity amongst the observed data [9]. Reduction in dimensionality reduces computational costs and since majority of the data is conserved, the resulting portfolio is optimal.

In comparison to the other factor models, PCA uses much less observed data to produce an optimal portfolio. Whereas the other factor models use factor returns to determine the optimal portfolio, PCA uses only the observed asset returns and further discards information when reducing dimensionality.

The objective in creating the portfolios was to reduce variance. PCA best satisfies this objective in comparison to the other factor models as the standard deviations are relatively lower than that of the other factor models. Additionally, the returns of the PCA portfolios in this project are relatively higher. Hence, PCA seems to satisfy the objective best.

MVO vs Cardinality Constrained MVO

The two investment strategies don't largely affect the portfolio variance or have a noticeable impact on the portfolio returns. The cardinality constrained MVO performs slightly better than MVO, although the difference between the two is not very significant. It is important to note the advantage of limiting the number of assets in the portfolio for investors. It is beneficial to limit the number of assets in the portfolio as it allows investors to track assets more efficiently.

Statistical Analysis

The adjust R^2 values indicate that Table 7 range from 0.3098 to 0.5548. The low values indicate that the factor models are a poor representation of the true asset values. Although, the F-test values suggest that most of the assets, >80%, are statistically significant. That is, most assets have an associated factor that is not zero. Each factor in the model is calculated by minimizing the residuals and so, the provided R value is theoretically the best possible value. Hence, the factor models do not represent the asset returns well.

Additionally, the expected portfolio returns are consistently higher than the realized portfolio returns and the expected portfolio standard deviation varies strongly from the realized portfolio standard deviation. If the regressed model had a more accurate fit, the returns and standard deviations would not be very different. Since the relationship is weak, it should be expected that expectation calculations do not provide a good estimate of the realized outcomes.

Other Considerations

The analysis above was conducted using only 20 different assets. Hence, the results are likely to vary should it be conducted with a larger group of assets. Furthermore, Figure 1 shows changes in the portfolio value. It can be observed that the highest valued portfolio changes many times before the end of the 3 year period. Thus, results may become more distinct and improve the analysis if it is conducted for a longer period of time. Lastly, investors must be aware that these investment strategies do not consider the true value of the underlying assets or other major market indicators that may largely impact portfolio returns. Hence, it is important from an investor's perspective to validate the portfolio weights recommended by the different investment strategies observed above by conducting background research about each company.

Conclusion

PCA has a standard deviation of 0.0107 – the lowest amongst the different factor models. Furthermore, PCA factor models requires less information than other factor models since it generates its own factor returns.

Additionally, investment strategies using MVO with cardinality constraints have a slightly higher portfolio return than their MVO counterparts that do not have cardinality constraints. Conveniently, investment strategies involving PCA have the highest returns as well – 12.67% for PCA with cardinality constrained MVO and 12.63% for PCA without cardinality constrains. Cardinality constrained MVO is also more beneficial as it has less assets and thus, is more efficient to track for the investor.

In conclusion, although portfolios balanced using PCA have the highest transaction costs, the transaction costs are negligible and the investment strategy composed of cardinality constrained MVO and PCA factor models is best for minimizing variance and maximizing returns.

References

[1] K. Fama and E. Fama, "The cross-section of expected stock returns," *Journal of Finance,* vol. 47, no. 2, pp. 427-465, 1992.

[2] R. A. Haugen and N. L. Baker, "Commonality in the determinants of expected stock returns," *Journal of Financial Economics,* vol. 41, no. 3, pp. 401-439, 1996.

[3] R. W. Banz, "The Relationship Between Return and Market Value of Common Stocks," *Journal of Financial Economics,* vol. 9, no. 1, pp. 3-18, 1980.

[4] D. Strattman, "Book values and stock returns," *The Chicago MBA,* vol. 4, pp. 25-40, 1980.

[5] D. Blitz, "Agency-Based Asset Pricing and the Beta Anomaly," *European Financial Management,* vol. 20, no. 4, pp. 770-801, 2014.

[6] E. Fama and K. French, "Common risk factors in the returns on stocks and bonds," *Journal fo Financial Economics,* vol. 33, no. 1, pp. 3-56, 1993.

[7] P. Kempthorne, "Lecture 15: Factor Models," October 2013. [Online]. [Accessed 6 March 2019].

[8] E. Zivot, "Factor Models for Asset Returns," BlackRock Alternative Advisors, Seattle, 2011.

[9] M. Ebden and A. Gibbs, "STA302 Files," 29 November 2018. [Online]. [Accessed 6 March 2019].

Appendix A: Portfolio weights at each period

MVO CAPM

Asset	1	2	3	4	5	6
F	1.05E-06	2.46E-09	2.28E-10	2.57E-09	1.87E-10	4.99E-07
CAT	1.71E-07	2.99E-08	6.96E-10	0.063037	1.04E-10	1.98E-09
DIS	0.067917	7.54E-09	9.37E-11	4.51E-09	9.35E-10	0.015209
MCD	8.61E-07	0.088329	0.10731	0.30362	0.21165	0.12683
KO	1.66E-05	0.050343	0.060577	0.04046	0.068671	0.082604
PEP	0.20333	0.25363	0.15066	0.098791	0.17951	0.19491
WMT	0.10733	0.082722	0.12774	0.0082073	0.07868	0.07582
C	6.90E-07	2.51E-09	5.97E-11	7.49E-11	2.21E-11	8.29E-08
WFC	8.00E-07	1.62E-07	0.047619	0.061648	5.26E-10	7.62E-07
JPM	5.60E-07	2.28E-09	1.73E-09	2.36E-10	3.09E-11	4.17E-08
AAPL	2.00E-05	1.36E-07	0.023515	0.086678	0.00029103	0.00907
IBM	3.61E-06	8.27E-08	4.21E-09	0.031578	0.08572	0.071593
PFE	0.19292	0.084769	0.049285	4.83E-10	1.01E-10	0.0042846
JNJ	0.2057	0.22424	0.11704	0.025024	9.24E-10	0.020279
XOM	7.96E-07	0.051272	0.12194	0.040847	6.17E-11	4.80E-09
MRO	1.54E-07	5.27E-09	1.73E-10	0.0036699	1.22E-11	1.39E-09
ED	0.077554	0.15918	0.07886	0.078098	0.19559	0.21254
T	0.097472	2.80E-06	0.029952	0.069849	0.070289	0.096454
VZ	0.047739	9.05E-07	0.04462	0.054699	0.064648	0.030271
NEM	5.87E-07	0.0055093	0.040885	0.033792	0.044959	0.060134

MVO Card CAPM

Asset	1	2	3	4	5	6
F	0	0	0	0	0	0
CAT	0	0	0	0.065416	0	0
DIS	0.067928	0	0	0	0	0.016367
MCD	0	0.08832	0.11336	0.30545	0.21171	0.12702
KO	0	0.050357	0.063373	0.044302	0.068697	0.082698
PEP	0.20333	0.2536	0.15268	0.1025	0.17957	0.19564
WMT	0.10733	0.082729	0.13159	0	0.078716	0.075931
C	0	0	0	0	0	0
WFC	0	0	0.047967	0.075082	0	0
JPM	0	0	0	0	0	0
AAPL	0	0	0.023981	0.087757	0	0.0098456
IBM	0	0	0	0.032778	0.085734	0.071591
PFE	0.19293	0.084748	0.050312	0	0	0
JNJ	0.20571	0.22416	0.1177	0	0	0.020717
XOM	0	0.051312	0.12601	0.044966	0	0
MRO	0	0	0	0	0	0
ED	0.077556	0.15923	0.083545	0.079552	0.1956	0.21265
T	0.097479	0	0	0.071557	0.070321	0.096875
VZ	0.047746	0	0.046529	0.057118	0.064699	0.030554
NEM	0	0.0055404	0.042959	0.033522	0.044956	0.060112

MVO FF

Asset	1	2	3	4	5	6
F	1.54E-10	0.010547	1.19E-10	3.24E-06	0.0010903	0.020056
CAT	2.48E-11	1.13E-07	4.39E-10	0.057366	1.46E-09	2.00E-08
DIS	0.063384	1.79E-09	1.88E-11	6.99E-08	0.0090922	0.030761
MCD	4.37E-11	0.11025	0.11051	0.32826	0.22932	0.15602
KO	4.08E-10	2.82E-06	0.036686	0.019439	0.048418	0.058548
PEP	0.2179	0.2669	0.14474	0.083745	0.17758	0.15424
WMT	0.12516	0.062607	0.11655	1.52E-07	0.05021	0.036784
C	3.14E-11	7.22E-10	1.38E-11	5.02E-10	3.09E-10	0.0047685
WFC	4.87E-11	0.065874	0.11084	0.10471	1.84E-08	0.012079
JPM	2.57E-11	5.57E-10	1.03E-08	1.52E-09	2.38E-10	4.33E-07
AAPL	9.65E-05	1.56E-05	0.054893	0.088562	9.29E-07	6.33E-08
IBM	2.37E-10	5.97E-08	5.84E-09	0.029568	0.089087	0.078741
PFE	0.18807	0.027496	0.038139	4.69E-09	6.36E-10	2.06E-06
JNJ	0.19504	0.24282	0.11145	2.46E-06	1.67E-08	0.045992
XOM	4.43E-11	0.046916	0.13698	0.025724	3.30E-10	4.32E-08
MRO	1.83E-11	2.27E-09	3.15E-11	9.23E-06	1.21E-10	1.52E-08
ED	0.071806	0.126	0.056504	0.090043	0.2261	0.22008
T	0.08647	4.88E-09	8.53E-07	0.056387	0.061517	0.11161
VZ	0.052066	6.63E-09	0.024372	0.072183	0.058242	0.020323
NEM	4.15E-11	0.040575	0.058329	0.043996	0.049338	0.049998

MVO Card FF

Asset	1	2	3	4	5	6
F	0	0.010546	0	0	0	0.021396
CAT	0	0	0	0.057367	0	0
DIS	0.06343	0	0	0	0.0094595	0.036355
MCD	0	0.11026	0.11051	0.32826	0.22948	0.15666
KO	0	0	0.036686	0.01944	0.048463	0.05874
PEP	0.21791	0.2669	0.14474	0.083747	0.1777	0.15721
WMT	0.12517	0.062609	0.11655	0	0.05024	0.036195
C	0	0	0	0	0	0
WFC	0	0.065872	0.11084	0.10472	0	0
JPM	0	0	0	0	0	0
AAPL	0	0	0.054893	0.088562	0	0
IBM	0	0	0	0.029569	0.089158	0.079147
PFE	0.1881	0.027493	0.038139	0	0	0
JNJ	0.19504	0.24281	0.11145	0	0	0.047439
XOM	0	0.046935	0.13698	0.025725	0	0
MRO	0	0	0	0	0	0
ED	0.071786	0.126	0.056504	0.090043	0.22617	0.22031
T	0.086489	0	0	0.056388	0.061621	0.11436
VZ	0.052072	0	0.024373	0.072184	0.058374	0.022642
NEM	0	0.04058	0.058329	0.043996	0.049338	0.049545

MVO PCA

Asset	1	2	3	4	5	6
F	0.017408	0.051717	1.44E-05	1.12E-06	4.15E-09	6.05E-09
CAT	3.58E-09	3.16E-11	2.09E-06	0.05415	6.14E-09	1.54E-09
DIS	0.071415	2.29E-11	2.20E-07	7.03E-05	0.019435	3.90E-08
MCD	1.70E-09	0.078361	0.17704	0.35752	0.18843	0.12087
KO	1.72E-09	5.07E-11	3.72E-06	1.41E-05	0.013624	3.98E-08
PEP	0.19725	0.32332	0.16929	0.083914	0.19066	0.10242
WMT	0.11147	1.90E-10	0.032639	7.41E-05	0.079245	0.032854
C	0.046923	2.02E-11	1.16E-07	2.43E-08	3.08E-09	0.13989
WFC	7.21E-10	0.035205	0.12606	8.79E-06	4.83E-08	0.14052
JPM	7.29E-10	4.65E-11	0.01069	7.81E-08	3.29E-09	0.0063155
AAPL	0.034923	3.78E-11	0.049814	0.16726	7.21E-09	7.23E-09
IBM	9.17E-09	0.0019217	0.014149	0.029841	0.031973	0.023216
PFE	0.20973	0.017597	4.54E-07	2.34E-07	7.31E-09	4.16E-08
JNJ	0.22488	0.29705	0.14233	1.85E-07	1.39E-08	0.11235
XOM	1.35E-09	0.069124	0.15301	3.73E-05	2.99E-09	2.77E-09
MRO	9.48E-10	3.76E-10	4.18E-07	6.03E-07	9.74E-10	7.81E-10
ED	0.005903	0.054365	0.052749	0.088029	0.20547	0.23737
T	0.032897	2.03E-11	9.43E-07	0.082374	0.072113	2.82E-07
VZ	9.30E-08	2.00E-11	1.52E-07	0.12217	0.11913	2.79E-09
NEM	0.047196	0.071331	0.072207	0.014543	0.079925	0.084182

MVO Card PCA

Asset	1	2	3	4	5	6
F	0.017408	0.051718	0	0	0	0
CAT	0	0	0	0.054174	0	0
DIS	0.07141	0	0	0	0.019435	0
MCD	0	0.078362	0.17706	0.35753	0.18843	0.12088
KO	0	0	0	0	0.013624	0
PEP	0.19727	0.32332	0.16928	0.083985	0.19066	0.10243
WMT	0.11148	0	0.032645	0	0.079245	0.032847
C	0.046913	0	0	0	0	0.13993
WFC	0	0.035208	0.12609	0	0	0.14055
JPM	0	0	0.010624	0	0	0.0062306
AAPL	0.034915	0	0.049826	0.16729	0	0
IBM	0	0.0018987	0.014162	0.029857	0.031973	0.023221
PFE	0.20974	0.017596	0	0	0	0
JNJ	0.22491	0.29705	0.14235	0	0	0.11237
XOM	0	0.069135	0.15302	0	0	0
MRO	0	0	0	0	0	0
ED	0.0058426	0.054368	0.052743	0.088053	0.20547	0.23737
T	0.032906	0	0	0.082397	0.072113	0
VZ	0	0	0	0.12219	0.11913	0
NEM	0.04721	0.071336	0.072201	0.014529	0.079925	0.08418