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Linear Algebra Review

3. General Matrix Properties

3.1) 1x1

3.2) 3x3

3.3) DNE

3.4) 1x2

Types of matrices:

Orthogonal: a sq. matrix whose columns are orthogonal vectors. Its transpose is also its inverse.

Orthonormal: An orthogonal matrix whose columns are vectors which are aligned with their respective axes. Aka, each row only has 1 non-zero entry.

Diagonal: An orthonormal matrix whose columns are aligned in descending order by row height.

Symmetric: A square matrix which is equal to its transpose.

Skew Symmetric: A square matrix whose transpose is also its negative.

Interpreting Matrices

1)

$$\begin{vmatrix} r_x \end{vmatrix} = \begin{vmatrix} r_{x1} \\ r_{x2} \\ r_{x3} \end{vmatrix} \qquad \begin{vmatrix} r_x^T \end{vmatrix} = \begin{vmatrix} r_{x1} & r_{x2} & r_{x3} \end{vmatrix}$$

$$R = \begin{vmatrix} r_x^T \\ r_y^T \\ r_z^T \end{vmatrix} \qquad \begin{vmatrix} r_y \end{vmatrix} = \begin{vmatrix} r_{y1} \\ r_{y2} \\ r_{y3} \end{vmatrix} \qquad \begin{vmatrix} r_y^T \end{vmatrix} = \begin{vmatrix} r_{y1} & r_{y2} & r_{y3} \end{vmatrix} \qquad R = \begin{vmatrix} r_{y1} & r_{y2} & r_{y3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{vmatrix}$$

$$\begin{vmatrix} r_z \end{vmatrix} = \begin{vmatrix} r_{z1} \\ r_{z2} \\ r_{z3} \end{vmatrix} \qquad \begin{vmatrix} r_z^T \end{vmatrix} = \begin{vmatrix} r_{z1} & r_{z2} & r_{z3} \end{vmatrix}$$

2) $\mathbf{x} = | x, y, 1 |^T$, $C = [[C_{1,1} , C_{2,1} , C_{3,1}]^T, [C_{1,2} , C_{2,2} , C_{3,2}]^T , [C_{1,3} , C_{2,3}, C_{3,3}]^T]$

if $x^TCx = 0$, then $x^TCx = ax^2 + bxy + cy^2 + dx + ey + f$

$x^T C = [C_{1,1}\, x + C_{2,1}\, y + C_{3,1}, C_{1,2}\, x + C_{2,2}\, y + C_{3,2}, C_{1,3}\, x + C_{2,3}\, y + C_{3,3}]$

thatx $= C_{1,1}\, x^2 + C_{2,1}\, yx + C_{3,1}x + C_{1,2}\, xy + C_{2,2}\, y^2 + C_{3,2}\, y + C_{1,3}\, x + C_{2,3}\, y + C_{3,3}$

$= C_{1,1}\, x^2 + (C_{2,1} + C_{1,2})\, yx + C_{2,2}\, y^2 + (C_{3,1} + C_{1,3}\,)\, x + (C_{3,2} + C_{2,3})\, y + C_{3,3}$

$= ax^2 \quad + \quad bxy + \quad cy^2 + \quad dx + \quad ey + \quad f$

We know: $C_{2,1} = C_{1,2}$, $C_{3,1} = C_{1,3}$, $C_{2,3} = C_{3,2}$,

$C = [[a , b/2 , d/2]^T, [b/2 , c , e/2]^T , [d/2 , e/2, f]^T]$

3) TODO

The rank of a matrix:

1) the rank of a matrix is the dimension in vector space spanned by the matrix.

2) either m or n, whichever is smaller.

3) n – 1

Determinant:

1) the determinant is a computed value from the entries in a matrix which can be used in order to solve systems of linear equasions.

2) $\det([[a, b]^T, [c, d]^T]) = ad – bc$

3) $\det([[a, b, c]^T, [d, e, f]^T, [g, h, i]^T] = a(ei – fh) – b (di – fg) + c (dh – eg)$

4) $\det([a]) = a$

The transpose of a matrix

1) It is the reorganization of a matrix so that the rows become the columns and the columns become the rows.

2) a) it's inverse

b) itself

c) itself

d) it's inverse

3) $N^T M^T$

The inverse of a Matrix

1) It is the reciprocal of the matrix, such that the product of matrix A and its inverse is 1

2) a) Its transverse

b) the inverse of each individual term, each still in the same locations

3) $N^{-1} M^{-1}$

The null space of a matrix

1) The null space of matrix A is the set of all solutions to the equation Ax = 0

2) The left null space is the null space of the transpose of the matrix

3) its nullity is 0, so it's null space is 0

4) 2

5) if x_0 is a solution to Ax = b and v_1, v_2, v_3, … form a basis for the null space of A, then any solution to Ax=b can be written as $x = x_0 + c_1v_1 + c_2v_2 + c_3v_3$ ...

The condition number of a matrix

1) The condition number is a measure of how "good" a matrix is, as in it is a way of measuring error propagation. A matrix is well conditioned if the condition number is small.

2) Taking an ill-conditioned system Ax = b and converting it to $M^{-1}Ax = M^{-1}b$ so that the matrix $M^{-1}Ax$ is better conditioned

3) Data normalization is when you scale all of the entries in a matrix in order to minimize error propogation

Solving systems of linear equations

1) a) if Ax = b, then $A^{-1}Ax = A^{-1}b$, so $I x = A^{-1}b$, or $x = A^{-1}b$

b) Used when A is not square or doesn't have an inverse. Used in the same way as the normal inverse. If the pseudo inverse of A = $A^{\$}$ and Ax = b, then $x = A^{\$}b$

c) set up the equations in the format so the coefficients of the variables are in A, the right hand side of the equation compose b, and then solve for x.

d) Decompose A into L and U such that A = LU, now you can find L and U and not have to perform the complex full matrix multiplication, you only have to multiply L y = b to solve for y and then Ux = y for x. These operations are easier because L and U are simpler for a computer to multiply the vectors b and y with.

If a system is overdetermined, we cannot use (a).

2) a) x is a member of the null space of A (most often the only member)

b) Used when A is ill-conditioned, for the matrix A you use the eigenvalues of $A^T A$ to sum up the components of x

if it is overdetermined or ill-conditioned then you need to use SVD. Otherwise the null space works the best.

Singular value decomposition

1) the singular value decomposition is the process of estimating an accurate real matrix from an incomplete set of points using the incomplete matrix's orthogonal matricies.

2) they are orthonormal left-singular vectors, each of wich the matrix M maps the basis of vector V_i using the unit vector σ_i U_i

3) I have no idea

4) orthonormal right-singular vectors