CSSE 461 – Computer Vision
Rose-Hulman Institute of Technology
Computer Science and Software Engineering Department

Problem Set 4

This problem set is due 15 April 2016.

This document contains hyperlinks and is best viewed as html.

1 Calibrated Reconstruction

Using at least one pair of images from Myers and the internal camera calibration recover the rotation and translation between the images.

Note: A set of correspondences and solution (including intermediate results) for the shed demo are posted under the handouts link (Day16). Testing on the shed data can help identify errors in your code.

1. Select a set of corresponding points $\{< \mathbf{x}_i, \mathbf{x'}_i >\}$ from a pair of images. The absolute minimum is 8, but you will want more than a dozen. In general, better results are obtained if the correspondences are distributed throughout the image.

   There are a number of ways that you can do this. In matlab, you can use `cpselect` (you may be able to refine the correspondences using `cpcorr`) or `ginput`.

   If `x1` is a $2 \times n$ matrix of points, in matlab you can use something like `figure(1), imshow(im1), hold on, plot(x1(1,:), x1(2,:), 'rx'), hold off` to display the points. Alternately, you can use `showPoints.m` to view the correspondences.

   You can save data using `save` <*filename*> <*list of variables to save*>. Once you get a good set of correspondences, it would be a good idea to save them.

Turn in: When you have a good set of correspondences, save the corresponding images points as "`input_points`" and "`base_points`" in a matlab file named "`correspondences.mat`". In addition, save a set of images with the correspondences marked and labeled.

2. Convert the points to homogeneous form and normalize them so that their mean is zero and their RMS[1] distance from the origin is $\sqrt{2}$. Be sure to save the transformation you

[1] Root mean squared distance, i.e. $\sqrt{\frac{\sum_{i=1}^{N} \text{distance}_i^2}{N}}$.

used to normalize the points – you'll need it later. You may find the matlab functions `mean(x')` and `var(x',1)` useful.

$$\hat{\mathbf{x}}_{\mathbf{i}} = \mathbf{T}_{\text{norm}}\mathbf{x}_i \quad \text{and} \quad \hat{\mathbf{x}'}_{\mathbf{i}} = \mathbf{T}'_{\text{norm}}\mathbf{x}'_i$$

where $\mathbf{T}$ and $\mathbf{T}'$ are normalizing transformations consisting of a translation and and scaling.

Turn in: Save your $\mathbf{T}$ and $\mathbf{T}'$ matrices.

3. Use the relationship $\hat{\mathbf{x}'}^{\top}\mathbf{F}_{\mathbf{norm}}\hat{\mathbf{x}} = 0$ to solve for $\mathbf{F}_{\text{norm}}$. Recall that

$$\begin{bmatrix} x'_1x_1 & x'_1y_1 & x'_1 & y'_1x_1 & y'_1y_1 & y'_1 & x_1 & y_1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x'_nx_n & x'_ny_n & x'_n & y'_nx_n & y'_ny_n & y'_n & x_n & y_n & 1 \end{bmatrix} \mathbf{f} = 0$$

   You may find the matlab function `reshape()` useful.

   You should be able verify that $\hat{\mathbf{x}'}^{\top}\mathbf{F}_{\text{norm}}\hat{\mathbf{x}} \approx 0$ for each pair of normalized points.

4. Replace $\mathbf{F}_{\text{norm}}$ with $\mathbf{F}'_{\text{norm}}$ such that $\det(\mathbf{F}'_{\text{norm}}) = 0$.

   Recall that SVD decomposes a matrix $\mathbf{M}$ into three matrices $\mathbf{U}$, $\mathbf{D}$ and $\mathbf{V}$ such that $\mathbf{M} = \mathbf{UDV}^{\top}$. $\mathbf{D}$ is diagonal matrix of the singular values in sorted order.

$$\mathbf{F}'_{\text{norm}} = \mathbf{U} \begin{bmatrix} \mathrm{D}(1) & & \\ & \mathrm{D}(2) & \\ & & 0 \end{bmatrix} \mathbf{V}^{\top}$$

   where $\mathbf{U}$, $\mathbf{D}$ and $\mathbf{V}$ are the SVD of $\mathbf{F}_{\text{norm}}$.

   You should be able verify that $\hat{\mathbf{x}'}^{\top}\mathbf{F}'_{\text{norm}}\hat{\mathbf{x}} \approx 0$ for each pair of normalized points.

Turn in: Save your $\mathbf{F}'_{\text{norm}}$ matrix.

5. Denormalize $\mathbf{F}'_{\text{norm}}$ using

$$\mathbf{F} = \mathbf{T}'^{\top}_{\text{norm}}\mathbf{F}'_{\text{norm}}\mathbf{T}_{\text{norm}}.$$

   You should be able verify that $\mathbf{x}'^{\top}\mathbf{F}\mathbf{x} \approx 0$ for each pair of normalized points.

Turn in: Save your $\mathbf{F}$ matrix.

6. Verify that your estimate of $\mathbf{F}$ is reasonable. A good way to do this is to take a pair of corresponding points $< \mathbf{x}, \mathbf{x}' >$ and confirm that $\mathbf{x}$ lies on the epipolar line $(\mathbf{x}'^{\top}\mathbf{F})^{\top}$ and $\mathbf{x}'$ lies on the epipolar line $\mathbf{F}\mathbf{x}$. You may find `drawLine.m` useful to add lines to a figure.

Turn in: Save a set of images with points and corresponding epipolar lines marked and labeled.

7. With $\mathbf{F}$, we can perform a projective reconstruction. However, if we have the camera calibration or have enough information about the images to recover the camera calibration, we can perform a metric reconstruction. We will assume that we have the camera calibration.

   For our images use a focal length of 1584 and a principle point of $[593 \quad 380]^\top$.

8. Using $\mathbf{K}$ and $\mathbf{K}'$, recover $\mathbf{E}'$ using
$$\mathbf{E}' = \mathbf{K}'^\top \mathbf{F}\mathbf{K}.$$

9. Replace $\mathbf{E}'$ with $\mathbf{E}$ such that the first two singular values of $\mathbf{E}$ are equal. Using a procedure similar to that used in Step 4
$$\mathbf{E} = \mathbf{U}\begin{bmatrix} \frac{\mathrm{D(1)+D(2)}}{2} & & \\ & \frac{\mathrm{D(1)+D(2)}}{2} & \\ & & 0 \end{bmatrix}\mathbf{V}^\top$$
   where $\mathbf{U}$, $\mathbf{D}$ and $\mathbf{V}$ are the SVD of $\mathbf{E}'$.

Note: For comparison purposes, $\mathrm{E}(3,3)$ should be greater than or equal to 0. If it is not multiply $\mathbf{E}$ by -1.

Turn in: Save your $\mathbf{E}$ matrix.

10. Recover $\mathbf{R}$ and $\mathbf{T}$ from $\mathbf{E}$. The best way is described in Section 9.6 of Hartley and Zisserman. Let $\mathbf{U}$, $\mathbf{D}$ and $\mathbf{V}$ be the SVD of $\mathbf{E}$ and let
$$\mathbf{W} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \text{and} \qquad \mathbf{Z} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$
    The possible rotations and translations are given by
$$\mathbf{R} = \mathbf{U}\mathbf{W}\mathbf{V}^\top \qquad \text{or} \qquad \mathbf{R} = \mathbf{U}\mathbf{W}^\top\mathbf{V}^\top$$
    and
$$\mathbf{T} = \mathbf{U}_3 \qquad \text{or} \qquad \mathbf{T} = -\mathbf{U}_3$$

Turn in: Save your $\mathbf{R}$ and $\mathbf{T}$ matrices.

11. Construct the four possible pairs of camera matrices $\mathbf{P}$ and $\mathbf{P}'$.

2 Turning it in

Turn in the items requested above, and a brief discussion of your experiences in electronic form using svn. Your materials should be placed in the `ProblemSet4` directory of your class repository (`http://svn.csse.rose-hulman.edu/repos/1516c-csse461-<your username>`).