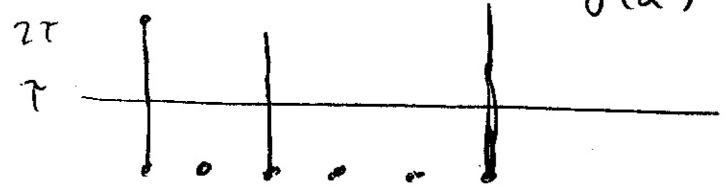
Assume $\mu \geq 2\sqrt{2\log\left(\frac{2n}{\alpha}\right)} = 2\tau$



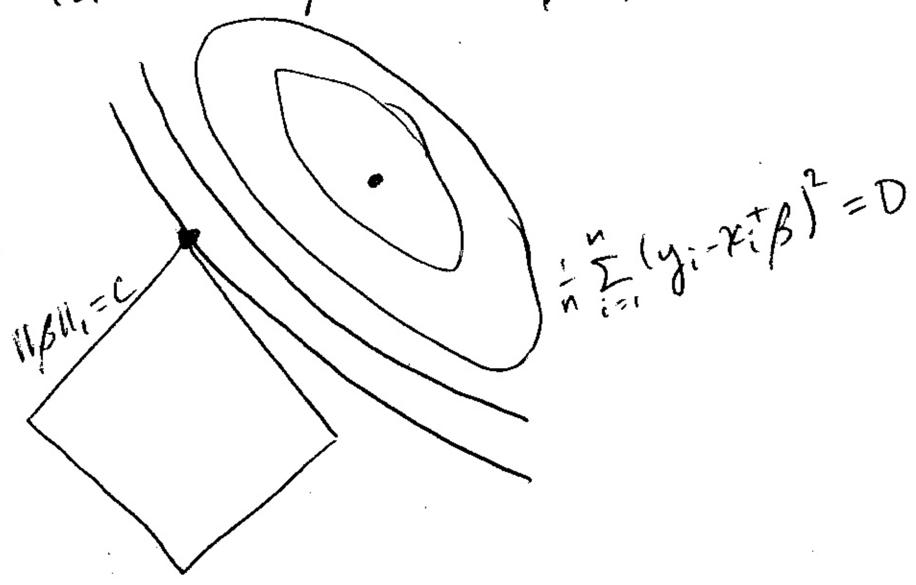
mistake $= \left\{ \begin{array}{l} |y_i| < \tau, \ i \in S, \\ |y_i| > \tau, \ i \notin S \end{array} \right\}$

$$\text{MSE} \leq \left(\frac{1}{n}\sum_{i=1}^{n} y_i^2\right) \cdot \mathbb{P}\{\text{mistake}\} = \left(\frac{1}{n}\sum_{i=1}^{n} y_i^2\right) \cdot \alpha$$

$\alpha \to 0$ like $\frac{1}{n}$ is sufficient

Lasso

$$\min_{\beta \in \mathbb{R}^d} \frac{1}{n}\sum_{i=1}^{n} (y_i - x_i^T\beta)^2 + \lambda \|\beta\|_1$$



Suppose that $x_i = e_i = (0, \dots, 0, \overset{i}{1}, 0, \dots, 0)$

$$\min_{\beta} \frac{1}{n}\sum_{i=1}^{n} (y_i - \beta_i)^2 + \lambda \|\beta\|_1$$

minimized at $\hat{\beta}_i = \text{sign}(y_i) \cdot \underbrace{(|y_i| - \lambda)_+}_{\max\{0,\, |y_i| - \lambda\}}$