Introduction to Linear Regression

PSY 300

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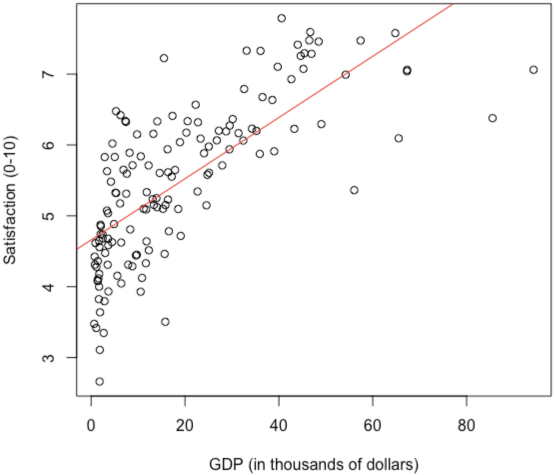
  - So: each observation is a country

Life satisfaction and GDP

country	sat	gdp1000
Afghanistan	2.66	1.80
Albania	4.63	11.80
Algeria	5.24	13.91
...	...	...

Life satisfaction and GDP



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- The property of the line is that it *minimizes the sum of the squared distances* of each datapoint to the line. It's the *line of best fit in the least-squares sense*.
- This kind of regression is also called "Ordinary Least Squares" (OLS).

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- x is our *independent variable* or *regressor*, also called "right-hand side variable". In our example, it measures GDP in 1000s of dollars.
- In our example, we are "regressing life satisfaction on GDP", or we are "running a regression of life satisfaction on GDP".

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  - β_1 is "the coefficient on GDP". It measures the slope of the relationship between x and y.

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- The index "i" denotes our *units of observation*; in this example, i denotes countries. In other analyses, i can denote individuals or trials.

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fit <- lm(satisfaction ~ gdp, data=data) # fit model
summary(fit) # show results
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 4.660279   0.092296   50.49   <2e-16 ***
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- (Note that R includes the intercept or constant term automatically.)

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  - Estimates of the coefficients β_0 and β_1. We denote these estimates with ˆ to indicate that they are estimates rather than the true population parameters: $\hat{\beta}_0 = 4.66$; $\hat{\beta}_1 = 0.04$. In the R output, they are listed as "(Intercept)" for $\hat{\beta}_0$, and "gdp" for $\hat{\beta}_1$.

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  - Estimates of the standard errors for both coefficients: $\widehat{SE(\beta_0)} = 0.09$; $\widehat{SE(\beta_1)} = 0.04$.

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- This derivative asks what we asked in prose above: "How much does y change for a one-unit change in x?" And it gives us the answer: β_1.
- (Note that both x and y are still in their own units: x is measured in thousands of dollars, y in points on the Cantril Ladder. If you had standardized the variables, you would talk about them in standard deviation units.)

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- Suppose the GDP of a particular country is USD 75,000. We are measuring GDP in 1000s, so $x = 75$.
- What is the predicted value of y given this level of GDP? We simply plug our coefficients and the value of x into the model:

$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$$

$$\hat{y}_i = 4.66 + 0.04 \times 75 = 7.66$$

Prediction

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- Suppose the GDP of a particular country is USD 75,000. We are measuring GDP in 1000s, so $x = 75$.
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- So the predicted average life satisfaction in a country with a GDP of USD 75,000 is 7.66 on the Cantril Ladder.

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  - Is GDP significantly associated with life satisfaction?
  - We observe a positive coefficient on GDP (0.04), suggesting that an increase in GDP is associated with an increase in life satisfaction. Is this positive relationship statistically significantly different from zero?
  - In the regression of life satisfaction on GDP, is the coefficient on GDP statistically significantly different from zero?

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- How do we test for the statistical significance of a coefficient relative to zero? Recall the R output:

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- Remember what we said about obtaining the t-statistic: it's defined as $t = \frac{estimator}{SE}$. Here, the estimator is our coefficient estimate, which is given to us by R, minus our comparison value, zero: $\hat{\beta} - 0$. The standard error is also given to us by R. So we can calculate t as $t = \frac{\hat{\beta} - 0}{SE}$.

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- R actually does this for us! You can check yourself that the t-value given above is just the ratio of the coefficient estimate and the standard error (e.g. $\frac{0.043182}{0.003439} = 12.56$).

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- "We find that the coefficient on GDP is significantly different from zero ($t = 12.56$, $p < 0.001$), suggesting that increases in GDP are associated with increases in life satisfaction."

Regression with dummy variables are t-tests

- In the previous example, x was continuous (a measure of GDP). Now we'll see what we can do with regression when the right-hand side variable is a dummy variable, i.e. only takes values 0 and 1. (Spoiler: they're t-tests!)

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- We also define a variable T that is 1 for people who were assigned to treatment, and 0 for control participants. We call this variable the "treatment indicator".

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- Because it only takes values 0 and 1, it is also called a "dummy variable" or an "indicator variable".

Regression with dummy variables

Here are the last few participants of the control group ($T = 0$) and the first few participants of the treatment group ($T = 1$), and their outcomes:

	y	T
493	3.644057	0
494	6.369840	0
495	4.874867	0
496	3.640351	0
497	2.861199	0
498	1.516264	0
499	3.120248	0
500	4.557885	0
501	6.832107	1
502	5.098828	1
503	6.809172	1
504	6.283809	1

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To get a feeling for our data, let's find the means of the two groups:

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- The treatment effect is $6.086 - 4.044 = 2.04$.

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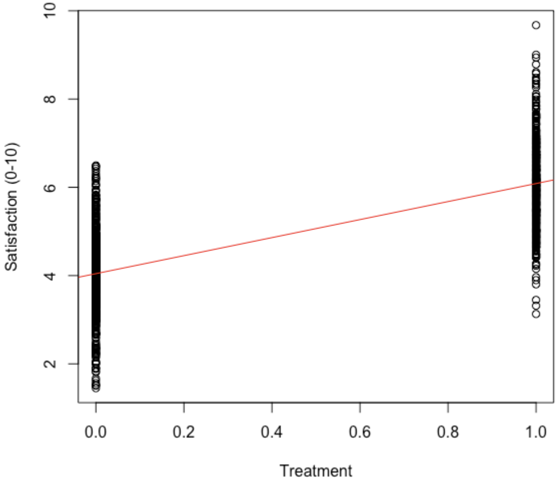
Let's plot the outcomes of both groups:



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  - A "one-unit increase in T" means going from $T = 0$ to $T = 1$; it's like switching treatment on.
  - **Fact 1: $\hat{\beta}_1$ gives us the treatment effect.**
  - Mathematically: $\frac{dy_i}{dT_i} = \beta_1$. "How much does the outcome change when treatment status changes?"

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  - The predicted value of y when $T = 1$ (treatment group) is $\hat{\beta}_0 + \hat{\beta}_1$. So $\hat{\beta}_0 + \hat{\beta}_1$ gives us the mean of the treatment group.

Regression with dummy variables

Summary:

$$y_i = \beta_0 + \beta_1 T_i + \varepsilon_i$$

- $\hat{\beta}_0$: Mean of the control group
- $\hat{\beta}_0 + \hat{\beta}_1$: Mean of the treatment group
- $\hat{\beta}_1$: Treatment effect (difference between treatment and control group)

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Let's check if R gets it right! Run the regression of y on T:

```
> fit <- lm(y ~ T, data=data)
> summary(fit)
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 4.04371  0.04375    92.43   <2e-16 ***
T           2.04231  0.06187    33.01   <2e-16 ***
```

We can directly read off the mean of the control group, and the treatment effect: remember from above that we calculated the treatment effect as 2.042, and the mean of the control group as 4.044.

Regression with dummy variables

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> predict(fit, data.frame(T = c(1)))
       1
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```

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```
> predict(fit, data.frame(T = c(1)))
       1
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```

This is also correct (remember from above that we calculated the mean of the treatment group as 6.086).

Inference

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  - The estimator is the difference between the treatment and control groups.
  - The standard error is the standard error of that difference.
  - Then we can run a t-test and get a p-value.

Inference

- R gives us all of this!
  - Remember that the coefficient on T, β_1, is the treatment effect.
  - R also spits out a standard error for that treatment effect, and the associated t-statistic and p-value

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```

- So we can write: "We observe a statistically significant effect of the treatment on life satisfaction; the treatment effect is 2.04 points on the life satisfaction scale, statistically significant at the 1 percent level ($t = 33.01$, $p = 2 \times 10^{-16}$)."

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- This is a general point: t-tests and ANOVAs are just special cases of regression. Using a regression framework to run these tests gives us more flexibility to include control variables and cluster standard errors. We will talk about that in future lectures.
- Question to think about: There was also a (slightly less interesting) one-sample *t*-test in the regression output! Where, and what does it test?