Statistical Power

PSY 300



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- Significance level α : Probability of rejecting the null hypothesis H_0 given that the null hypothesis is true
 - ► Or: Probability of a "false positive" or "Type I" error

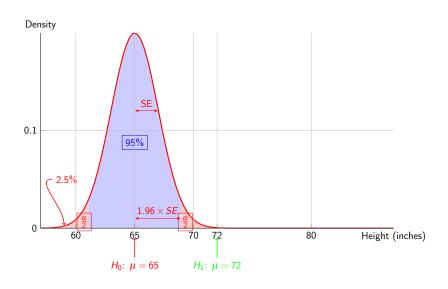
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- Statistical power 1β : Probability of rejecting the null hypothesis H_0 given that the null hypothesis is false

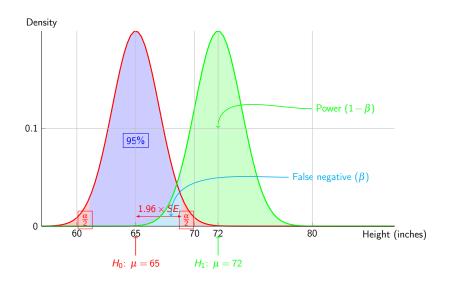
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- Statistical power $1-\beta$: Probability of rejecting the null hypothesis H_0 given that the null hypothesis is false Alternative ways of saying this:
 - Power = $Pr(reject H_0 | H_0 is false)$.
 - Power is the probability of correctly rejecting the null hypothesis
 - Probability of not obtaining a "false negative" or "Type II" error.
 - ► The probability of making a Type II error is β , so the probability of *not* doing that is 1β .

"Distribution under the Null"



Distributions under the Null and an Alternative



Probability of a false negative (Type II error, β)

Power for a specific alternative hypothesis is the probability of rejecting the null when the alternative hypothesis is true.

Probability of a false negative (Type II error, $oldsymbol{eta}$)

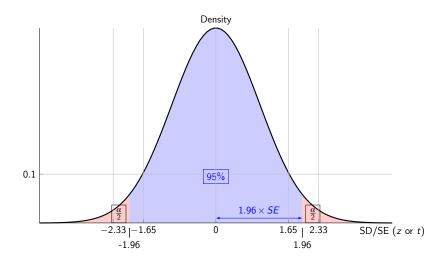
- Power for a specific alternative hypothesis is the probability of rejecting the null when the alternative hypothesis is true.
- Consider an alternative hypothesis that claims that $\mu=72$. What is the probability that we will reject the null of $\mu=65$ when the alternative hypothesis is true?

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- ▶ Power for a specific alternative hypothesis is the probability of rejecting the null when the alternative hypothesis is true.
- Consider an alternative hypothesis that claims that $\mu=72$. What is the probability that we will reject the null of $\mu=65$ when the alternative hypothesis is true?
- ▶ It's the probability of drawing from a sampling distribution centered at 72 a sample that lies in the left-most 2.5% of the distribution under the null, plus the probability of drawing from that sampling distribution centered at 72 a sample that lies in the right-most 2.5% of the distribution under the null.

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- So we need to figure out those areas. How do we calculate the area under a normal distribution?



 Areas under the normal distribution are best obtained by standardizing the distribution first

- ► Areas under the normal distribution are best obtained by standardizing the distribution first
- For an entire variable, this is achieved by subtracting the mean of the variable, and dividing by the standard deviation: $x_{std} = \frac{x \bar{x}}{SD}$. The resulting standardized variable will have mean 0 and standard deviation 1. It is called a "normalized" or "standardized" or "z-scored" variable.

▶ We can also calculate what a particular value of the variable would turn into if the distribution were normalized. This amounts to simply expressing the distance between that value and the mean of the distribution in terms of standard deviations. Example: where does 72 lie in a normal distribution centered at 65 with SD=2 when that distribution is normalized?

$$z = \frac{72 - 65}{2} = 3.5$$

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- ▶ This is also called z-scoring. So we can say "72 has a z-score of 3.5".
- ▶ If this looks familiar to the way we calculated *t* last time: Note that *z* is the asymptotic equivalent of *t*; *t* is for finite samples. It's usually fine to use either, and in most applications you don't have to worry about which one is correct because the computer will make that choice for you.

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- ► In *R*:
 - Using the z-distribution (asymptotic):
 - > pnorm(-1.96, mean = 0, sd = 1)
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- ▶ When R does a power calculation, it uses the z-distribution.

Some important z-scores

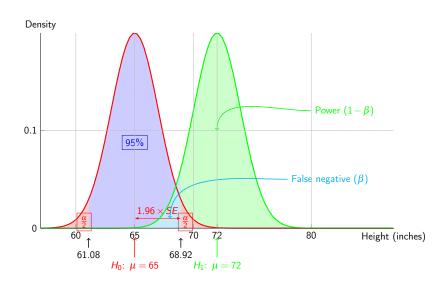
Left + right tail	Central area	Corresponding
area $(lpha)$	(1-lpha)	z-score
0.1	0.9	1.645
0.05	0.95	1.96
0.01	0.99	2.58

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 - ► This was the olden days—now we think even 0.2 SD is a perfectly respectable effect size (Funder & Ozer, 2018)



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- ▶ Probability of a sample to the left of 61.08:
 - ightharpoonup z-score: $z = \frac{61.08 72}{2} = -5.46$
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- ▶ Power is the sum of these: $Power = 2.4 \times 10^{-08} + 0.94 \approx 0.94$

Power calculations in R: One-sample t-test

Power calculations in R: Two-sample t-test

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What to write: "A power calculation determined that with 393 participants in each group (786 participants in total), the study had 80 percent power to detect effect sizes of 0.2 standard deviations at the 5 percent significance level."

Two-sample t-test: finding the detectable effect size

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What to write: "A power calculation determined that with 100 participants in each group (200 participants in total), the study had 80 percent power to detect effect sizes of 0.39 standard deviations at the 5 percent significance level."

Two-sample t-test: finding power

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What to write: "A power calculation determined that with 100 participants in each group (200 participants in total), the study had 29 percent power to detect effect sizes of 0.2 standard deviations at the 5 percent significance level."

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- \blacktriangleright We have some control over all of these; mostly n.

- ► Sometimes, studies are structured such that the observations are grouped:
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 - Students in classrooms
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 - Households in cities
- When there is correlation between units within a cluster, they are not fully independent observations. But our tests assume that they are!

- ► How to fix this problem? We adjust the standard errors for clustering during analysis. Will talk about this more later.
 - When the outcomes of units within a cluster are positively correlated, this will increase the standard errors, i.e. make our estimates less precise.
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 - Rarely, the outcomes of units in a cluster are negatively correlated. Then, standard errors decrease, i.e. our estimates become more precise.
- ▶ Intuitive way of thinking about it: when the outcomes of a group are correlated, the *effective number of observations* is smaller than the number of units in the group.

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- We can adjust for clustering during the power calculation

Power calculation with clusters in R

- > install.packages("CRTSize")
- > library(CRTSize)
- > n4means(delta=0.2, sigma=1, m=10, ICC=0.2, alpha=0.05, power=0.8, AR=1, two.tailed=TRUE, digits=3) The required sample size is a minimum of 110 clusters of size 10 in the Experimental Group and a minimum of 110 clusters (size 10) in the Control Group.
- delta: effect size; sigma: standard deviation (1 if using Cohen's d); m: cluster size; ICC: intra-cluster correlation (estimated from data, or a guess); alpha: significance level; AR: allocation ratio (1 if treatment and control group sizes equal); digits: rounding.

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- Note the sample size required here (2200) is much larger than when detecting the same effect size without clustering (786).

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- So what should we do?

- Power the study for "the smallest effect size you care about" based on theoretical or practical significance
 - ► E.g.: "The smallest difference in scores considered clinically important by psychiatrists is X. We therefore power the study to observe effect sizes of this magnitude."
 - Or: "Due to budget constraints, we the study was powered to detect effect sizes of 0.2 SD with 80% power at the 5 percent significance level. In our view, this detectable effect size is small enough to be theoretically and practically interesting."

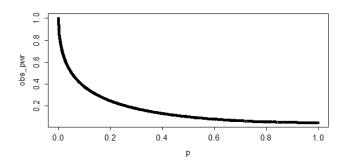
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- It's tempting to run an ex-post power calculation based on your observed effect in the study and your actual n: "How much power did we have to detect the effect we observe?"
- But: This is circular; ex-post power is a direct function of your p-value.
- More information: http://daniellakens.blogspot.com/2014/12/observed-powerand-what-to-do-if-your.html

Observed power is a direct function of the p-value

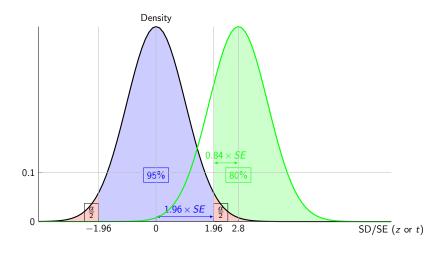


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 - Use the observed standard error to determine what effect size we had 80% power to detect



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- ➤ Example: "To determine which effect sizes we had 80% power to observe, we calculate the minimum detectable effect size (MDE) by multiplying the standard error of the estimate by 2.8. This gives an MDE of 0.05 SD."

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- ➤ Example: "To determine which effect sizes we had 80% power to observe, we calculate the minimum detectable effect size (MDE) by multiplying the standard error of the estimate by 2.8. This gives an MDE of 0.05 SD."
- More information: https://blogs.worldbank.org/impactevaluations/why-ex-post-power-using-estimated-effect-sizes-bad-ex-post-mde-not

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- "We observe no statistically significant effect of our experimental manipulation on the outcome. Because we were powered to detect very small effect sizes (MDE = 0.05 SD), we can rule out even small treatment effects of our manipulation."