

# 17 Temperature, Thermal Expansion, and Ideal Gas Law

(17.1) atomic mass (molecular mass) [u]: relative masses of atoms/molecules, numerically same as molar mass

(17.2) Temperature [C°/F°/K]: how hot or cold something is

Freezing	Boiling
0°C	100°C
32°F	212°F

## Conversions

$$1u = 1.6605 \times 10^{-27} \text{ kg}$$

$$C = \frac{5}{9}(F - 32)$$

$$F = \frac{9}{5}(C) + 32$$

$$K = C + 273.15$$

(17.3) Zeroth Law of Thermodynamics: If two systems are in thermal equilibrium w/ a third system, they are in thermal equilibrium w/ each other

(17.4)

Linear Expansion:

$$\Delta L = \alpha L_0 \Delta T$$

$$L = L_0(1 + \alpha \Delta T)$$

(NOTE) ring expands heavily by diameter

coefficient of linear expansion  $[\frac{1}{C}]$   
coefficient of volume expansion  $[\frac{1}{C}]$

Volume Expansion:

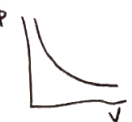
$$\Delta V = \beta V_0 \Delta T$$

for isotropic:  $\Delta V \approx (3\alpha) V_0 \Delta T$

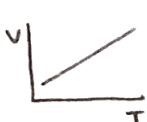
$$V = L_0(1 + \alpha \Delta T) W_0(1 + \alpha \Delta T) H_0(1 + \alpha \Delta T)$$

(17.6)

Boyle's Law

$$V \propto \frac{1}{P}$$


Charles's Law

$$V \propto T$$


Gay-Lussac's Law

$$P \propto T$$


(17.7)

mole [n]: amount of substance containing same atoms in 12g of C

$$n (\text{moles}) = \frac{\text{mass (g)}}{\text{molecular mass (g/mol)}}$$

Ideal Gas Law:

$$PV = nRT$$

← temperature [K]

moles ↑ universal gas constant:  $8.314 \frac{J}{mol \cdot K} = 1.99 \frac{cal}{mol \cdot K}$

Standard Temperature Pressure (STP):  $T = 273 \text{ K}$   $P = 1 \text{ atm} = 101.3 \text{ kPa}$

(17.9)

Avogadro's number: number of molecules in one mole

$$N_A = 6.022 \times 10^{23}$$

$$PV = n N_A k T = N k T = \frac{N}{N_A} R T$$

Boltzmann Constant  $(\frac{R}{N_A}) = 1.38 \times 10^{-23} \frac{J}{K}$

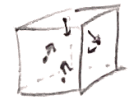
$$N = n N_A$$

# 18 Kinetic Theory of Gases

(18.1) Ques 7

Ideal Gas Law assumptions:

1. Large number of molecules  $N$ , mass  $m$ , moving in random directions different speeds
2. Molecules on average far apart from each other
3. Obey classical mechanics, interact when they collide
4. Collisions w/ wall or molecules elastic.



Average translational kinetic energy is proportional to temperature

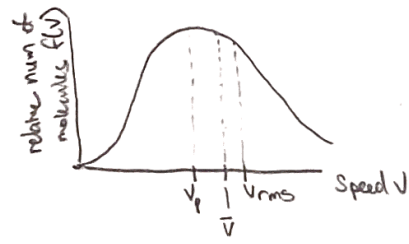
$$\bar{K} = \frac{1}{2} m \bar{V}^2 = \frac{3}{2} kT$$

Root-mean-square speed: how fast molecules move on avg

$$V_{rms} = \sqrt{\bar{V}^2} = \sqrt{\frac{3kT}{m}}$$

(18.2)

Maxwell Distribution of speeds: probable distribution of speeds in gas w/  $N$  molecules



$$f(v) = 4\pi N \left( \frac{m}{2\pi kT} \right)^{\frac{3}{2}} v^2 e^{-\frac{1}{2} \frac{mv^2}{kT}}$$

$$\int_0^{\infty} f(v) dv = N$$

most probable  $V_p = \sqrt{2 \frac{kT}{m}} \approx 1.41 \sqrt{\frac{kT}{m}}$

avg speed  $\bar{V} = \sqrt{\frac{8}{\pi} \frac{kT}{m}} \approx 1.60 \sqrt{\frac{kT}{m}}$

rms speed  $V_{rms} = \sqrt{3 \frac{kT}{m}} \approx 1.73 \sqrt{\frac{kT}{m}}$

Expected Value of  $F$

$$\langle F(x) \rangle = \int F(x) p(x) dx$$

(18.3)

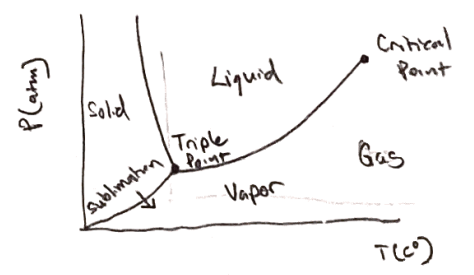
Liquid crystals between liquid and solid

Sublimation: solid  $\rightarrow$  vapor

Evaporation: liquid  $\rightarrow$  gas

Condensation: gas  $\rightarrow$  liquid

Phase diagram:



(18.4)

saturated vapor pressure: equilibrium between liquid, vapor

Boiling when saturated vapor pressure = external pressure

relative humidity:  $\frac{\text{partial pressure } H_2O}{\text{saturated vapor pressure } H_2O} \times 100$

Van der Waals

(18.5)

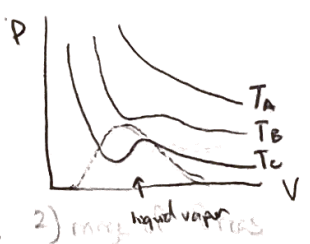
Van der Waals takes into account

- 1) finite size of molecules,
- 2) range of forces between molecules > size molecules

$$P = \frac{RT}{(\frac{V}{n}) - b} - \frac{a}{(\frac{V}{n})^2}$$

$$\left( P + \frac{a}{(\frac{V}{n})^2} \right) \left( \frac{V}{n} - b \right) = RT$$

$a, b$  different for different gases



# 19 Heat and First Law of Thermodynamics

19.1 Unit of heat: calorie (cal), kilocalorie (kcal): amount of heat necessary to raise 1g of water by 1°C

Conversions

$$1000 \text{ cal} = 1 \text{ kcal} = 4184 \text{ J}$$

$$1 \text{ L} \cdot \text{atm} = 101.33 \text{ J}$$

$$F = PA$$

heat: energy transferred from one object to another bc difference in temperature

19.2

Internal Energy: sum total of all energy of all molecules in an object

Temperature  
avg kinetic energy of individual molecules

vs.

Internal Energy  
total energy of all the molecules

vs.

Heat  
transfer of energy from one object to another

Internal Energy ( $E_{int}$ ):

$$E_{int} = \frac{3}{2} nRT$$

for monatomic gas

19.3

Heat  $Q$  to change temp of material:

$$Q = mc \Delta T$$

specific heat  $\left[ \frac{\text{J}}{\text{kg} \cdot \text{C}} \right]$ : specific to material

19.4

Systems:

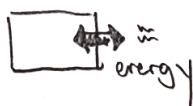
Open System

mass can be transferred  
energy can be transferred



Closed System

mass cannot be transferred  
energy can be transferred



Closed (Isolated)

mass cannot be transferred  
energy cannot be transferred



In isolated environments: <sup>heat</sup>

heat lost = heat gained

$$Q_1 = Q_2$$

19.5

Energy is involved in change of phase

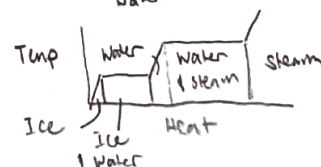
Heat of fusion ( $L_F$ ): heat to change 1kg substance

Heat of vaporization ( $L_v$ ): heat to change 1kg substance

Solid  $\rightarrow$  Liquid  
liquid  $\rightarrow$  vapor  $\left\{ \left[ \frac{\text{kJ}}{\text{kg}} \right] \right.$

$$Q = mL$$

Latent heat



energy is needed to break attractive forces

19.6

# First Law of Thermodynamics:

$$\Delta E_{int} = Q - W$$

net heat added to system

net work done by system

Heat added : +  
Heat lost : -  
Work on System : -  
Work by System : +

Thermodynamics:

$$\Delta E_{int} = \frac{1}{2} n R \Delta T$$

degrees of freedom

$$W = \int_{V_1}^{V_2} P dV$$

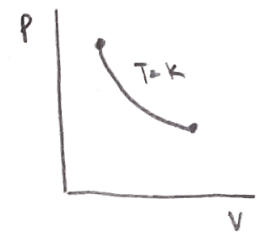
$$\Delta E_{int} = Q - W$$

19.7

## 1) Isothermal ( $\Delta T = 0$ )

$$\Delta E_{int} = 0, \text{ so } Q = W$$

$$W = nRT \ln\left(\frac{V_2}{V_1}\right)$$

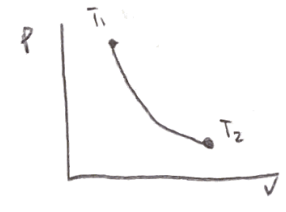


## 2) Adiabatic ( $Q = 0$ )

$$\Delta E_{int} = -W = \frac{1}{2} n R \Delta T$$

$$PV^\gamma \Rightarrow \frac{P_1}{P_2} = \left[\frac{V_2}{V_1}\right]^\gamma = \left[\frac{T_1}{T_2}\right]^{\frac{\gamma}{\gamma-1}}$$

Monatomic:  $\gamma = 5/3$   $d=3$   
Diatomic:  $\gamma = 1.4$   $d=5$   
Triatomic:  $\gamma \approx 1.31$   $d=7$

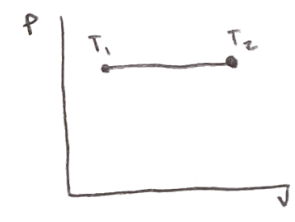


## 3) Isobaric ( $\Delta P = 0$ )

$$\text{if ideal: } W = nRT_2 \left(1 - \frac{V_1}{V_2}\right)$$

$$\text{else: } W = P \Delta V$$

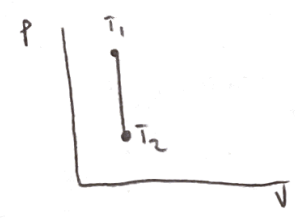
$$Q = \Delta E_{int} + P \Delta V$$



## 4) Isovolumetric ( $\Delta V = 0$ )

$$W = 0$$

$$\Delta E_{int} = Q = \frac{1}{2} n R \Delta T$$



x) For problems, set up table

	a	b	c
P			
V			
T			
$\Delta E_{int}$	ab	bc	ca
Q			
W			

19.8

Molar Specific Heats ( $C_v, C_p$ ): heat required to raise 1mol of gas by 1C° const volume, temp

Constant Volume

$$Q = n C_v \Delta T$$

molecular mass  $\left(\frac{g}{mol}\right)$   
 $C_v = M C_v \leftarrow \text{specific heat @ const volume}$

$$\Delta E_{int} = Q$$

$$C_v = \frac{3}{2} R$$

Constant Pressure

$$Q = n C_p \Delta T$$

$$C_p = M C_p$$

- More heat is required, need work  
 $Q_p - Q_v = P \Delta V$   
Work is done

$$C_p - C_v = R$$



19.13

Heat transfer via:

1) Conduction: hot to cold via molecular collisions

$$\frac{dQ}{dt} = -kA \frac{dT}{dx}$$

↑ thermal conductivity constant (specific to metal)

2) Convection: heat flows by mass movement of molecules

3) Radiation: heat by electromagnetic waves

Stephan-Boltzmann eq:  $\frac{\Delta Q}{\Delta t} = \epsilon \sigma A T^4$

emissivity:  $[0,1]$   
characteristic of surface

↑ area of emitting object  
Stephan-Boltzmann constant:  
 $\sigma = 5.67 \times 10^{-8} \frac{W}{m^2 \cdot K^4}$

Sun radiation heat:  $\frac{\Delta Q}{\Delta t} = (1000 \frac{W}{m^2}) \epsilon A \cos \theta$



## 20. Second Law of Thermodynamics

20.2

Heat Engines produce work from thermal energy

$$Q_H = W + Q_L$$

$$\Delta E_{int} = 0$$

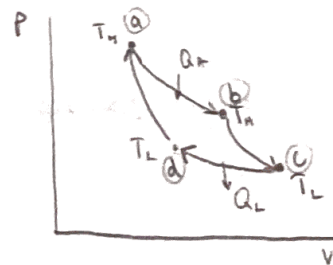
Efficiency ( $e$ ): ratio of work done to heat input

$$e = \frac{W}{Q_H} = 1 - \frac{Q_L}{Q_H}$$

20.3

Carnot's Engine is an idealized reversible cycle

- ab 1) expanded isothermally,  $Q_H$  added
- bc 2) expanded adiabatically, temperature reduced to  $T_L$
- cd 3) compressed isothermally,  $Q_L$  removed
- da 4) compressed adiabatically, temperature raised to  $T_H$



$$\frac{Q_L}{Q_H} = \frac{T_L}{T_H}$$

$$e_{ideal} = 1 - \frac{T_L}{T_H} \quad [K]$$

(20.4)

Coefficient of Performance (COP): heat removed for work done refrigerator

$$COP = \frac{Q_L}{W} = \frac{Q_L}{Q_H - Q_L}$$

$$COP_{ideal} = \frac{T_L}{T_H - T_L}$$

(20.5)

Entropy [S] is a state variable, measure of order or disorder

(20.6)

$$\Delta S = \Delta S_H + \Delta S_L = -\frac{Q}{T_{HM}} + \frac{Q}{T_{LM}} \quad \text{For hot} \rightarrow \text{cold, } \Delta S > 0$$

$$\Delta S = \frac{Q}{T} = \int_{T_1}^{T_2} \frac{mc \, dT}{T} = mc \ln\left(\frac{T_2}{T_1}\right)$$

Entropy of isolated system never decreases.

$$\Delta S = \Delta S_{syst} + \Delta S_{env} > 0$$

(20.7)

Second Law of Thermodynamics: Natural Processes tend to move toward a state of greater disorder

(20.8)

Energy eventually becomes degraded and unavailable to do useful work

Tips

\* Remember to change T to Kelvins

### Units/Conversions

Force Newton [N]

$$\frac{m \cdot kg}{s^2}$$

Pressure Pascal [Pa]

$$\frac{kg}{m \cdot s^2}$$

Energy Joule [J]

$$\frac{m^2 \cdot kg}{s^2}$$

Energy Calorie [cal]

$$\frac{m^2 \cdot kg}{s^2}$$

Power Watt [W]

$$\frac{J}{s} = \frac{kg \cdot m^2}{s^3}$$

Gas constant [R]

Pressure

$$F = PA$$

momentum  $p = mv$

$$F = \frac{p}{t}$$

$$101,325 \text{ Pa} = 1 \text{ atm} = 760 \text{ mmHg} = 14.7 \text{ psi}$$

$$1000 \text{ J} = 1 \text{ kJ}, \quad 101.33 \text{ J} = 1 \text{ L} \cdot \text{atm}$$

$$1000 \text{ cal} = 1 \text{ kcal}, \quad 1 \text{ cal} = 4.184 \text{ J}$$

$$745.7 \text{ W} = 1 \text{ hp}, \quad \text{power} \rightarrow P = \frac{W}{\Delta t}$$

$$8.314 \frac{J}{mol \cdot K} = 1.99 \frac{cal}{mol \cdot K}$$

$$KE = \frac{1}{2} mv^2$$

$$\Delta K + \Delta U + \Delta E_{int} = Q - W$$

# 21 Electric Charge and Electric Field

21.1 Unlike charges attract, like charges repel

21.2 Law of conservation of electric charge: net amount of electric charge produced is 0

Atom has positively charged nucleus w/ protons and neutrons, electrons surrounding

Becomes an ion if loses or gains an electron

21.3 Conductors - electrons are bound loosely, charge transfers easily

Insulators - electrons bound tightly to nucleus, charge does not transfer easily

Semiconductor - intermediate category, fewer free electrons

21.4 Charging by conduction - using charged object to make neutral object charged by contact

21.5 Induced charge - caused neutral object to be charged without contact

Coulomb's Law:

$$\vec{F} = k \frac{Q_1 Q_2}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{r^2}$$

Magnitude of Electric Force  $k = 8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}$   $\epsilon_0 = 8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}$   $k = \frac{1}{4\pi\epsilon_0}$

used for Point charges

Charge (Q) measured in Coulomb [C]

elementary charge:  $e = 1.602 \times 10^{-19} \text{ C}$

Principle of superposition - net force on object w/ multiple charges is vector sum of forces due to each of others

21.6 Each object radiates Electric field, use small positive test charge to measure field

Electric Field  $\rightarrow$

$$\vec{E} = \frac{\vec{F}}{q} \quad \vec{F} = k \frac{Q}{r^2} \quad \vec{F} = q\vec{E}$$

Magnitude of test charge  $\uparrow$  Force at  $q$

Positive charge: E field points away, Negative: points toward

If multiple charges: Superposition Principle:  $\vec{E} = \vec{E}_1 + \vec{E}_2 + \dots$

draw diagram, find mag w/ Coulombs, add vector forces

21.7 Continuous Charge Distribution problems

1) Choose Coordinate System (Cartesian, Polar, Spherical, Cylindrical)

2) find  $dq$

$dq = \lambda dx$ ,  $\lambda$ : linear charge density  
 $dq = \sigma dA$ ,  $\sigma$ : surface charge density  
 $dq = \rho dV$ ,  $\rho$ : volume charge density

$$\frac{Q}{x} = \lambda \quad \frac{Q}{A} = \sigma \quad \frac{Q}{V} = \rho$$

$$dA = dx dy = r dr d\theta$$

$$dV = dx dy dz = r^2 \sin\theta d\theta d\phi dr = r dr dy dz$$

3) find  $dE$

$$dE = \frac{1}{4\pi\epsilon_0} \frac{dQ}{r^2}$$

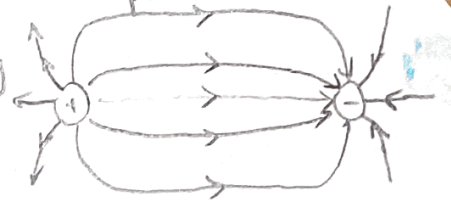
$$dE = \frac{1}{4\pi\epsilon_0} \frac{dQ}{r^2} \hat{r}$$

4) find  $E$

$$\vec{E} = \int d\vec{E}$$

21.8 Electric field lines indicate the direction of electric field at various points.

The closer the lines, the stronger, lines start on pos end on neg



21.9 Electric Fields and Conductors:

- Electric field inside conductors is 0
- Electric field is always perpendicular to surface outside conductor

21.10 Magnitude of electron acceleration  $a = \frac{E}{m} = \frac{qE}{m}$

21.11 Electric Dipole - two equal charges w/ opposite signs separated by distance  $l$

Dipole moment:  $\vec{p} = Ql$

Torque:  $\vec{\tau} = \vec{p} \times \vec{E} = pE \sin \theta$

Work:  $W = \int_{\theta_1}^{\theta_2} \tau d\theta = pE(\cos \theta_2 - \cos \theta_1)$

## 22 Gauss's Law

22.1 Electric Flux: electric field passing through area

For uniform electric field  $\vec{E}$  through  $A$

For Not uniform

Flux  $\rightarrow \boxed{\Phi_E = EA \cos \theta}$

$\boxed{\Phi_E = \oint \vec{E} \cdot d\vec{A}}$

22.2 flux entering enclosed volume is negative, leaving is positive, nonzero when enclosed charge

Gauss's Law:

$\boxed{\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}}$  ← net charge enclosed in surface

Solving w/ Gauss:

- 1) Find surface  $S$  that respects symmetry
- 2)  $\oint \vec{E} \cdot d\vec{A} = |\vec{E}| \cdot \text{Surface Area of } S$
- 3)  $Q_{\text{enc}} \Rightarrow \int dq = \lambda dx, \sigma dA, \rho dV$

22.3

If conductor w/ charge  $Q$  and inside cavity has charge  $+q$   
must be  $-q$  charge on surface of cavity and outer surface with  $Q+q$

## 23 Electric Potential

23.1 Electric Potential Energy ( $U$ ) - conservative force for electrostatic

$\Delta U = -W = -qEd$  [Uniform  $\vec{E}$ ]

Electric Potential ( $V$ ) - electric potential energy per unit charge

$V_a = \frac{U_a}{q}$   $\Delta V = \frac{U_b - U_a}{q} = -\frac{W_i}{q}$

Voltage ( $V$ ) - Potential Difference

$\Delta U = qV_{ba}$

$1V = 1 \frac{J}{C}$

measures how much work a given charge can do



$$\Delta V = - \int \vec{E} \cdot d\vec{r} \quad \text{since } E \text{ is force per unit charge } E = \frac{F}{q}$$

23.3 Electric Potential at distance  $r$  away

$$\Delta V = - \int_{r_a}^{r_b} \vec{E} \cdot d\vec{r} = - \frac{Q}{4\pi\epsilon_0} \int_{r_a}^{r_b} \frac{1}{r^2} dr = \frac{1}{4\pi\epsilon_0} \left( \frac{Q}{r_a} - \frac{Q}{r_b} \right)$$

$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r} \quad \left[ \begin{array}{l} \text{single pt charge} \\ V=0 \text{ at } r=\infty \end{array} \right]$$

23.4

with continuous distribution

$$\left| V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r} \right| \quad \left| V = - \int \vec{E} \cdot d\vec{r} \right| \text{ can add together Voltages since scalar}$$

23.5

Equipotential lines with same potential, perpendicular to electric field

23.6

Electric Dipole Potential

$$V = \frac{1}{4\pi\epsilon_0} \frac{ql \cos\theta}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{p \cos\theta}{r^2} \quad [\text{dipole, } r \gg l]$$



23.7

$$E_r = - \frac{\partial V}{\partial r}$$

$$E_x = - \frac{\partial V}{\partial x}, E_y = - \frac{\partial V}{\partial y}, E_z = - \frac{\partial V}{\partial z}$$

23.8

Charges moved from  $V=0$   $r=\infty$

$$U = Q_2 V = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{r_{12}}$$

$$U = \frac{1}{4\pi\epsilon_0} \left( \frac{Q_1 Q_2}{r_{12}} + \frac{Q_1 Q_3}{r_{13}} + \frac{Q_2 Q_3}{r_{23}} \right)$$

electron Volt (eV)

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

## 24 Capacitance, Dielectrics, Electric Energy Storage

24.1

Capacitors - store electric charge by using two conducting objects

capacitor  $[-+]$  battery  $[+|-]$

amount of charge  
acquired by plate

$$Q = CV$$

Capacitance  $[F] \frac{C}{V}$

24.2

Capacitance

$$C = \epsilon_0 \frac{A}{d}$$

area of plates

distance between plates

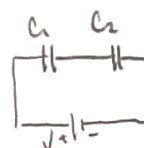
24.3

Parallel:  $Q = C_{eq} V$

$$C_{eq} = C_1 + C_2$$

Series:  $Q = C_{eq} V$

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$



24.4 Harder to charge capacitor the more energy it has

Work to charge  $W = \int_0^Q V dq = \frac{1}{C} \int_0^Q q dq = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} QV$

Energy stored  $U = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} QV = \frac{1}{2} CV^2$

Energy Density (u)  $\frac{\text{energy}}{\text{volume}} : u = \frac{1}{2} \epsilon_0 E^2 \quad E = \frac{Q}{\epsilon_0 A}$

24.5

Dielectric: piece of insulating sheet of material in between plates

$C = K C_0$  ← capacitance of space is vacuum

↑ Dielectric constant

↑ permittivity of dielectric

$C = K \epsilon_0 \frac{A}{d}$

$\epsilon = K \epsilon_0$

## 25 Electric Currents and Resistance

25.2 Current only flows with complete circuit

$I = \frac{dQ}{dt}$

Current [I] measured in Amperes [A]  $1A = 1 \frac{C}{s}$

25.3

Ohm's Law:

$V = IR$

↑ Resistance of a wire [R] Ohms  $1\Omega = 1 \frac{V}{A}$

25.4

Resistivity

$R = \rho \frac{l}{A}$  wire length  $l$   
 $dR = \rho \frac{dl}{A}$   
↑ cross sectional area  
↑ resistivity [ $\Omega \cdot m$ ]

$\sigma = \frac{1}{\rho}$

↑ conductivity [ $\frac{1}{\Omega \cdot m}$ ]

Resistivity can vary based on temperature

$\rho_T = \rho_0 [1 + \alpha [T - T_0]]$

↑ resistivity at temp  $T_0$   
resistivity Temp T

25.5

$P = IV = I^2 R = \frac{V^2}{R}$

Power [W] Watt  $1W = 1 \frac{J}{s}$  applies to resistors

can be measured in  
kilo watt-hour (kWh)  $1kWh = 3.6 \times 10^6 J$

25.7

$V = V_0 \sin(\omega t)$

$+V_0$  peak voltage  
 $-V$

$I = I_0 \sin(\omega t)$

$I_0 = \frac{V_0}{R}$  peak current

$P = I^2 R = I_0^2 R \sin^2 \omega t$

$$I_{rms} = \sqrt{I^2} = \frac{I_0}{\sqrt{2}} = 0.707 I_0$$

$$V_{rms} = \sqrt{V^2} = \frac{V_0}{\sqrt{2}} = 0.707 V_0$$

$$P = I_{rms} V_{rms} = \frac{1}{2} I_0^2 R = I_{rms}^2 R = \frac{1}{2} \frac{V_0^2}{R} = \frac{V_{rms}^2}{R}$$

25.8

$\vec{j}$  current per unit cross-sectional area

$$\Delta Q = (\# \text{ charges, } N) \times (\text{charges per particle})$$

$$= (nV)(e) = -(nA v_d \Delta t)e$$

Current

$$I = \frac{\Delta Q}{\Delta t} = -neAv_d$$

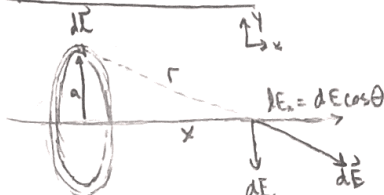
$$v = \frac{I}{A}$$

$$\vec{j} = -ne\vec{v}_d$$

Constant / Example Reference

$$E = \frac{1}{4\pi\epsilon_0} \frac{Qx}{(x^2 + a^2)^{3/2}}$$

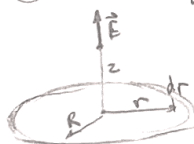
Coulombs



$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{(x^2 + a^2)^{1/2}}$$

$$E = \frac{\sigma}{2\epsilon_0} \left[ 1 - \frac{z}{(z^2 + R^2)^{1/2}} \right]$$

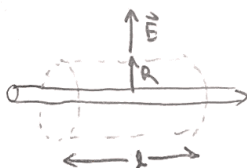
Coulombs



$$V = \frac{Q}{2\pi\epsilon_0 R^2} \left[ (z^2 + R^2)^{1/2} - z \right]$$

$$E = \frac{1}{2\pi\epsilon_0} \frac{\lambda}{R}$$

Coulombs  
Gauss



$$r > r_0: E = \frac{Q}{4\pi\epsilon_0 r^2}$$

Gauss



$$r < r_0: E = \frac{Qr}{4\pi\epsilon_0 r_0^3}$$

$$r > r_0: V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$$

$$E = \frac{\sigma}{2\epsilon_0}$$

Gauss



Constants

$$k = 8.99 \times 10^9 \frac{N \cdot m^2}{C^2}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{N \cdot m^2}$$

$$e = 1.602 \times 10^{-19} C$$

$$\text{Surface Area of Sphere: } 4\pi r^2$$