# EECS 16A Midterm 1 Sheet

Jeffrey Shen
3034555577

(1A) (1B) Systems of Linear Eq and Gaussian Elim

**Vector** - ordered list of numbers $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_n \end{bmatrix}$

**Matrix** - rectangular array of numbers $\begin{bmatrix} A_{11} & \cdots & A_{1n} \\ A_{m1} & & A_{mn} \end{bmatrix}$

**Gaussian Elim** - algorithm that reduces lin eq (can obtain unique sol, infinite sol, no sols)

(2A) Vector Addition: Comm: $\vec{x} + \vec{y} = \vec{y} + \vec{x}$ Assoc: $(\vec{x} + \vec{y}) + \vec{z} = \vec{x} + (\vec{y} + \vec{z})$ addiv $\vec{x} + \vec{0} = \vec{x}$ add inv $\vec{x} + (-\vec{x}) = \vec{0}$

For water pump: rows - represent proportion of water a reservior will draw from other reservoirs
columns - how water will be distributed between our reservoirs the next day

**Matrix vector form:** $A\vec{x} = \vec{b}$

(3) **Linear dependence** - redundancy - one vector can be written as linear comb of another

Def: vectors $\{\vec{v}_1, \vec{v}_2 \ldots \vec{v}_n\}$ are linearly dependent if $\alpha_1 \ldots \alpha_n$ such that $\alpha_1 \vec{v}_1 + \alpha_2 \vec{v}_2 \ldots \alpha_n \vec{v}_n = 0$ and $\alpha_n \neq 0$

**Linear independence** - not linearl dependent $\alpha_n = 0$

**Span** - span of vectors $\{v_1 \ldots v_n\}$ is set of all linear comb of $\{v_1 \ldots v_n\}$

(2B) Matrix Mult: $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$

Not Commutative $AB \neq BA$ Is associative $(AB)C = A(BC)$

(6) Equivalent for $n \times n$ A

a) A is invertible
b) $\leftrightarrow$ eq $A\vec{x} = \vec{b}$ has unique sol for $\vec{b}$
c) $\Leftrightarrow$ A has linearly independent cols (full rank)
d) $\Leftrightarrow$ A has trivial null space $(\text{null}(A) = \{\vec{0}\})$
e) $\Leftrightarrow$ $\det(A) \neq 0$

Proofs:
a) A is invertible $\Rightarrow$ $A\vec{x} = \vec{b}$ unique sol
b) A is invertible $\Rightarrow$ linearly independent
c) A is invertible $\Rightarrow$ trivial null space

(7) **Vector spaces** (V) set of vectors and two operators satisfy

Vector addition
- Associative $\vec{u} + (\vec{v} + \vec{w}) = (\vec{u} + \vec{v}) + \vec{w}$
- Comm $\vec{u} + \vec{v} = \vec{v} + \vec{u}$
- additive $\vec{v} + \vec{0} = \vec{v}$ and $\vec{v} + (-\vec{v}) = 0$
- Closure sum $\vec{v} + \vec{v}$ must be in V

Scalar mult
- Associ $\alpha(\beta\vec{v}) = (\alpha\beta)\vec{v}$
- Multi Identity $1 \cdot \vec{v} = \vec{v}$
- D. property $\alpha(\vec{u} + \vec{v}) = \alpha\vec{u} + \alpha\vec{v} = (\alpha + \beta)\vec{v} =$
- Closure Any vector $\vec{v}$ scalar $\alpha$ $\alpha\vec{v}$ must be in V

**Basis** - of vectorspace $\mathbb{V}$ (min set of vects to represent all vectors)
1) $\vec{v}_1, \vec{v}_2 \ldots \vec{v}_n$ lnerly independent
2) scalars such that $\vec{v} = \alpha_1 \vec{v}_1 + \alpha_2 \vec{v}_2 \ldots$

**Dimension of** a vector space is number of basis vectors

(3) **Subspace** - $(\mathbb{W}, \mathbb{F})$ is a subspace if $\mathbb{W}$ is a subset of $\mathbb{V}$ and $(\mathbb{W}, \mathbb{F})$ is a vector space

**Nullspace(A)** = solutns to $A\vec{x} = \vec{0}$

**Column space of (A)** = Span of the columns of A

**rank** - max num of lnerly independnt columns

**Proofs**

(1) Write what you know and what you want to prove
(2) - Try smal simple exampe to see if you find patterns
(3) manpulate both sides and simplify Justify each step
(4) 1) Constructe proofs (generate example/ solutn)
2) Proofs by contradiction (

**Know:** $A\vec{v} = \lambda\vec{v}$ Subset $A \subseteq B$

**Ex)** Does inverse exist? matrx column's are lnerly independnt, invertble, excts same pivots as columns

Lnerdependence: $R_4/k = (R_2 + R_3)$

Basis: The basis for null(A) is $\left\{\begin{bmatrix} 1 \\ -1 \\ 0.5 \end{bmatrix}\right\}$

non trivial nullspace $\Rightarrow$ lnerly dependnt $\Rightarrow$ no unique sol.

**Thm**: if $A\vec{x} = \vec{b}$ has two or more sol, then col of A are lnerly dipndt

Let $\vec{x}_1, \vec{x}_2$ be two sol

$A\vec{x}_1 = \vec{b}$ $A\vec{x}_2 = \vec{b}$

$A(\vec{x}_1 - \vec{x}_2) = 0$

$\vec{y} = \vec{x}_1 - \vec{x}_2 \neq 0$

$A\begin{bmatrix} y_1 \\ y_1 \end{bmatrix} = 0$

$y_1\vec{a}_1 + y_2\vec{a}_2 \ldots y_n = 0$

WLOG let $y_1 \neq 0$

$\vec{a}_1 = \frac{-y_2}{y_1}\vec{a}_2 + \ldots \frac{-y_n}{y_1}\vec{a}_n$

Show $c_1\vec{a} + c_2\vec{a} + \ldots c_n\vec{a}_n = 0$

Given $H\vec{\psi}_1 = e\vec{\psi}_1$ $H\vec{\psi}_2 = e\vec{\psi}_2$

$\psi = \alpha_1\psi_1 + \alpha_2\psi_2$

$H\psi = H(\alpha\psi_1 + \alpha_2\psi_2) = H\alpha\psi_1 + H\alpha_2\psi_2 = \alpha e\psi_1 + \alpha H e\psi_2 =$

$e(\alpha_1\psi_1 + \alpha_2\psi_2) = e\psi$

∎ QED

# EECS 16A MT 2 Formula Sheet

Jeffrey Shen
3034555577

## Circuits : Resistors

- **Kirchhoff Current Law (KCL)** - All currents entering a node equal the sum of all currents exiting that node
- **Kirchhoff Voltage Law (KVL)** - Voltage around all loops sum to 0

### Ohm's Law

$$V = IR \qquad R = \frac{V}{I} \qquad I = \frac{V}{R}$$

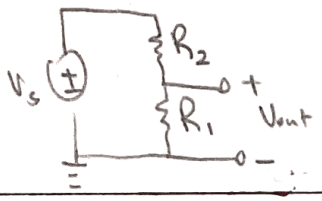
### Resistance

$$R = \rho \frac{L}{A}$$

$\rho$: resistivity; $L$: length; $A$: area of cross section

### Voltage Divider

$$V_{out} = V_s \left( \frac{R_1}{R_1 + R_2} \right)$$

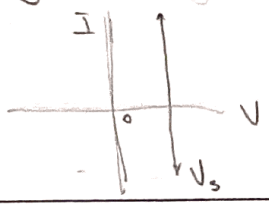



### Current Divider

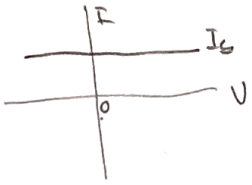
$$I_n = I_S \frac{R_{rest}}{R_n + R_{rest}}$$


### I/V

Voltage Source




Current Source




Passive Sign Convention: + to −

Open Circuit means $i = 0$

- Can combine 3+ in series but not 3+ in parallel

### Power / Energy

$$P = \frac{dE}{dt} = V \cdot I = \frac{V^2}{R}$$

Power [W]

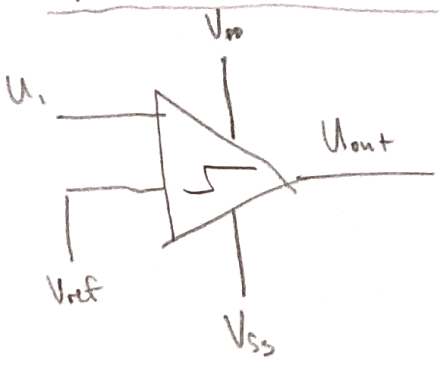
$$E = P \cdot \Delta t = \int_0^t P(t)\, dt$$

Energy [J] or [W·s]

My Power dissipates Energy

$V = I^2 R$

### Op Amp (Comparator)




if $V_1 > V_{ref} \Rightarrow V_{out} = V_{DD}$

if $V_1 < V_{ref} \Rightarrow V_{out} = V_{SS}$

$$V_{out} = A(V^+ - V^-)$$

$i = 0$

### Superposition

- Set other independent Sources to null
- Find each $V_{out,x}$
- Sum all $V_{out}$, sources to find $V_{out}$

## Capacitance

$Q_c = C \cdot V_c$

↑ Charge [C] ↑ Capacitance [F]

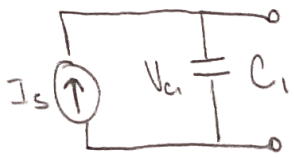
$C = \frac{A}{d}\varepsilon$ ↑ permitivity

$E = \frac{1}{2} C V_c^2$

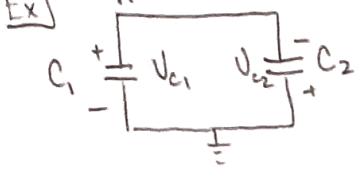
Charging: $I(t) = C\frac{dV}{dt}$

Discharging: $-I(t) = C\frac{dV}{dt}$

$V_c(t) = \frac{I_c}{C} \cdot T + V_o$




Ex] Flipped Signs




$Q_{total} = Q_{c1} + Q_{c2}$

$Q = C_1 V_{c1} + (-C_2 V_{c2})$

## Conservation of Charge

$Q[t] = Q[t+1]$ - Charge in system remains same after time step

## Thevenin

$V_{TH} = V_{OC}$

$R_{TH} = R_{Nor}$

$V_{TH} = I_N \cdot R_{TH}$

Thevenin is open circuit voltage between out ports

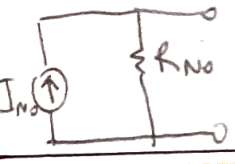
Way to solve:
- find V at A
- find V at B w/o taking into account A
- Subtract
- Null Independent Sources
- Calc $R_{TH}$

## Norton

$I_{No} = I_{meas}$

Current flowing through output ports when ports are shorted

Way to solve:
- connect A & B by wire and ignore inbetween




## Node Voltage Analysis (NVA)

1) Label Voltages at all Nodes
2) Label Element Voltages and Currents & Passive sign conv.
3) Write KCL Equations of Element Currents
4) Expressions for Element Currents using KVL
5) Substitute Expressions for Element Currents into KCL
6) Solve

## Tips

- Redraw circuits if necessary
- Pay attention to units
- See what variables terms answer is supposed to be in

# EECS 16A Final Formula Sheet

Jeffrey Shen
30345555 77

## Cross-Coorelation

Def: measure of the similarity between two signals

$$\text{Corr}_{\vec{x}}[\vec{y}][\vec{k}] = \sum_{i=-\infty}^{\infty} x[i]\, y[i-k]$$

Because of noise we want $A\vec{x} = \vec{b}$ but to minimize noise we use $\vec{e} = \vec{b} - A\vec{x}$ minimize $\|\vec{e}\|^2$ find smallest error

## Least Squares

Def: Solves approx. systems in the presence of noise

$$\hat{\vec{x}} = (A^T A)^{-1} A^T \vec{b}$$ where A is independent

Note: Use for tall and thin / overdetermined matrixes

## Orthogonal Matching Pursuit (OMP)

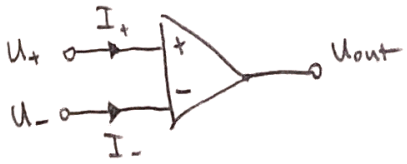
Def: unmixes signals from multiple beacons, uncover messages

Known: matrix M, vector $\vec{y}$, sparsity level k or threshold for norm

1. Find inner product for each vector, $\vec{m}_i$ with $\vec{e}$ (first iter: $\vec{e} = \vec{y}$)
2. Add vector that had max inner prod to matrix A
3. Use proj / least squares to compute $\hat{x} = (A^T A)^{-1} A^T \vec{y}$
4. Update Error vector $\vec{e} = \vec{y} - A\vec{x}$
5. Repeat until reached sparsity level

- Linearize distance formula eqs

## Op Amp




Golden Rules:

### Ideal op Amp

1. $I_+ = I_- = 0$
2. $A \to \infty$

### Ideal & Negative Feedback

1. $U_+ = U_-$

### Unity Gain Buffer

1. $V_{in} = V_{out}$

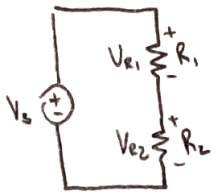
loading - small resistance causes it to draw alot of current causing large voltage drop, make it harder bc it will behave differently depending on what it's connected to

- Buffers allow us to split into blocks to analyze separately

## Design Problem

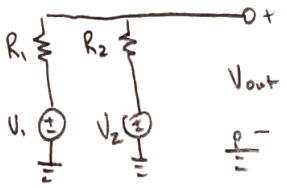
1. Restate important goals for design
2. Describe strategy using block diagram
3. Implement
4. Check implementation with Step 1

### Voltage Divider



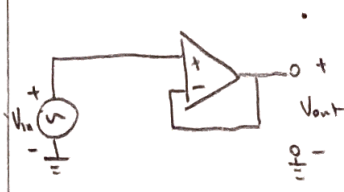

$$V_{R2} = V_s\left(\frac{R_2}{R_1+R_2}\right)$$

### Voltage Summer



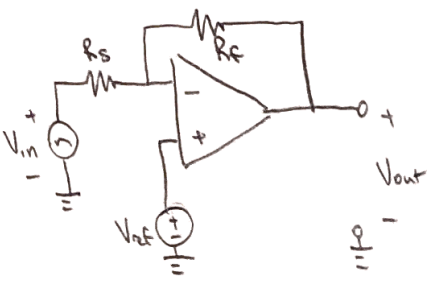

$$V_{out} = V_1\left(\frac{R_2}{R_1+R_2}\right) + V_2\left(\frac{R_1}{R_1+R_2}\right)$$

### Unity Gain Buffer



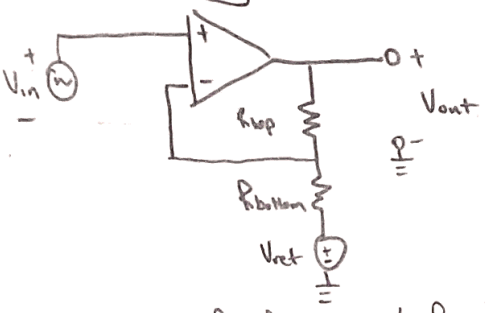

$$\frac{V_{out}}{V_{in}} = 1$$

### Inverting Amplifier



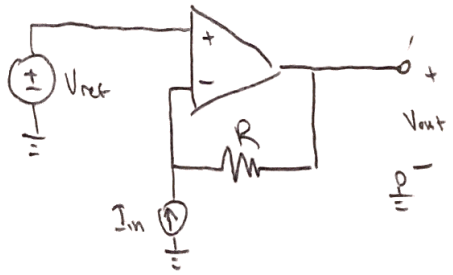

$$V_{out} = V_{in}\left(-\frac{R_f}{R_S}\right) + V_{ref}\left(\frac{R_f}{R_S}+1\right)$$

### Non-inverting Amplifier




$$V_{out} = V_{in}\left(1+\frac{R_{top}}{R_{bottom}}\right) - V_{ref}\left(\frac{R_{top}}{R_{bottom}}\right)$$

### Transresistance Amplifier




$$V_{out} = i_{in}(-R) + V_{ref}$$

## Orthonormal Matrix

Colums $\vec{a}_i$ are

1. Orthogonal $\langle a_i, a_j \rangle = 0$
2. Normalized $\|a_i\| = 1$

## Eigenvalues

$$A\vec{x} = \lambda\vec{x}$$

$$\det(A - \lambda I) = 0$$

If A squared, Eigenvectors same, Eigenvalues squared

$$\begin{bmatrix} -3 & -3 \\ 4 & 4 \end{bmatrix}\vec{a} = 0 \qquad a_1 + a_2 = 0 \qquad a_1 = -a_2 \qquad a_2 = -a_1 \qquad \begin{bmatrix} 1 \\ -1 \end{bmatrix} a_1$$

$$\begin{bmatrix} -4 & -3 \\ 4 & 3 \end{bmatrix}\vec{a} = 0 \qquad -4a_1 - 3a_2 = 0 \qquad -4a_1 = 3a_2 \qquad a_1 = -\frac{3}{4}a_2 \qquad \begin{bmatrix} -\frac{3}{4} \\ 1 \end{bmatrix} a_2$$