EECS 16A Midterm 1 Sheet — Jeffrey Shen

(1A)(1B) Systems of Linear Eq and Gaussian Elim (1A)(1B)

Vector - ordered list of numbers $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_n \end{bmatrix}$

Matrix - rectangular array of numbers $\begin{bmatrix} A_{11} & \cdots & A_{1n} \\ A_{m1} & & A_{mn} \end{bmatrix}$

Gaussian Elim - algorithm that reduces lin eq (can obtain unique sol, infinite sol, no sols)

(2A) Vector Addition: Commu: $\vec{x} + \vec{y} = \vec{y} + \vec{x}$ addit $\vec{x} + \vec{0} = \vec{x}$

Assoc: $(\vec{x} + \vec{y}) + \vec{z} = \vec{x} + (\vec{y} + \vec{z})$ add inv $\vec{x} + (-\vec{x}) = \vec{0}$

For water pump: rows - represent proportion of water a reservior will draw from other reservoirs

columns - how water will be distributed between other reservoirs the next day

Matrix vector form: $A\vec{x} = \vec{b}$

(3) **Linear dependence** - redundancy - one vector can be written as linear comb of another

Def: vectors $\{\vec{v}_1, \vec{v}_2 \ldots, \vec{v}_n\}$ are linearly dependent if $\alpha_1 \ldots \alpha_n$ such that $\alpha_1 \vec{v}_1, \alpha_2 \vec{v}_2 \ldots \alpha_n \vec{v}_n = 0$ and $\alpha_n \neq 0$

Linear independence - not linearly dependent $\alpha_n = 0$

Span - span of vectors $\{v_1 \ldots v_n\}$ is set of all linear comb of $\{v_1 \ldots v_n\}$

(2B) Matrix Mult $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11} b_{11} + a_{12} b_{21} & a_{11} b_{12} + a_{12} b_{22} \\ a_{21} b_{11} + a_{22} b_{21} & a_{21} b_{12} + a_{22} b_{22} \end{bmatrix}$

Not Commutative $AB \neq BA$ Is associative $(AB)C = A(BC)$

(6) Equivalent for $n \times n$ A

a) A is invertible

b) $\Leftrightarrow$ eq $A\vec{x} = \vec{b}$ has unique sol for $\vec{b}$

c) $\Leftrightarrow$ A has linearly independent cols ("full rank")

d) $\Leftrightarrow$ A has trivial null space $(\text{null}(A) = \{\vec{0}\})$

e) $\Leftrightarrow$ $\det(A) \neq 0$

Provable

a) A is invertible $\Rightarrow$ $A\vec{x} = \vec{b}$ unique sol

b) A is invertible $\Rightarrow$ linearly independent

c) A is invertible $\Rightarrow$ trivial null space

(7) **Vector Spaces** ($\mathbb{V}$) set of vectors and two operators satisfy

Vector addition
- Associative $\vec{u} + (\vec{v} + \vec{w}) = (\vec{u} + \vec{v}) + \vec{w}$
- Comm $\vec{u} + \vec{v} = \vec{v} + \vec{u}$
- additive $\vec{v} + \vec{0} = \vec{v}$ and $\vec{v} + (-\vec{v}) = 0$
- Closure sum $\vec{v} + \vec{u}$ must be in $\mathbb{V}$

Scalar Mult
- Associ $\alpha(\beta\vec{v}) = (\alpha\beta)\vec{v}$
- Multi Identity $1 \cdot \vec{v} = \vec{v}$
- Distributive $\alpha(\vec{u} + \vec{v}) = \alpha\vec{u} + \alpha\vec{v} = (\alpha + \beta)\vec{v} =$
- Closure any vector $\vec{v}$ scalar α, $\alpha\vec{v}$ must be in $\mathbb{V}$

Basis - of vector space $\mathbb{F}$ (min set of vects to represent all vectors).
1) $\vec{v}_1, \vec{v}_2 \ldots \vec{v}_n$ linerly independent
2) scalars such that $\vec{v} = \alpha_1 \vec{v}_1 + \alpha_2 \vec{v}_2 \ldots$

Dimension of a vector space is number of basis vectors.

(8) Subspace - $(\mathbb{W}, \mathbb{F})$ is a subspace if $\mathbb{W}$ is a subset of $\mathbb{V}$ are
$(\mathbb{W}, \mathbb{F})$'s a vector space

Nullspace(A) := solutions to $A\vec{x} = \vec{0}$

Column space of (A) = span of the columns of A

rank - max num of linerly independent columns

Proofs
1. Write what you know and what you want to prove
2. - Try small simple example to see if you find patterns
3. manipulate both sides and simplify Justify each step
4. 1) Constructive proofs (generate example / solution)
   2) Proofs by contradiction (

Know: $A\vec{v} = \lambda \vec{v}$ Subset $A \subseteq B$

Ex) Does inverse exist? matrix column's are linearly independent, invertible, exc13
same pivots as columns

Linear dependence: $R_4/k = (R_2 + R_3) l$

Basis: The basis for null(A) is $\left\{ \begin{bmatrix} -1 \\ 1 \\ 0.5 \end{bmatrix} \right\}$

non trivial nullspace $\Rightarrow$ linearly dependent $\Rightarrow$ no unique sol.

Thm: If $A\vec{x} = \vec{b}$ has two or more sol, then col of A are linerly dependent

let $\vec{x}_1, \vec{x}_2$ be two sol

$A\vec{x}_1 = \vec{b}$ $A\vec{x}_2 = \vec{b}$

$A(\vec{x} - \vec{x}_2) = 0$

$\vec{y} = \vec{x}_1 - \vec{x}_2 \neq 0$

$A \begin{bmatrix} y_1 \\ y_1 \end{bmatrix} = 0$

$y_1 \vec{a}_1 + y_2 \vec{a}_2 \ldots y_n = 0$

WLOG let $y_1 \neq 0$

$\vec{a}_1 = \frac{-y_2}{y_1} \vec{a}_2 + \ldots \frac{-y_n}{y_1} \vec{a}_n$ QED

Show $c_1 \vec{a} + c_2 \vec{a} + \ldots c_n \vec{a}_n = 0$

Given $H\vec{\psi}_1 = e\vec{\psi}_1$ $H\vec{\psi}_2 = e\vec{\psi}_2$

$\psi = \alpha_1 \psi_1 + \alpha_2 \psi_2$

$H\psi = H(\alpha \psi_1 + \alpha_2 \psi_2) = H\alpha\psi_1 + H\alpha_2\psi_2 = \alpha e \psi_1 \alpha H e \psi_2 =$
$e(\alpha_1 \psi_1 + \alpha_2 \psi_2) = e\psi$

EECS 16A MT 2 Formula Sheet

Jeffrey Shen

Circuits : Resistors

- Kirchhoff Current Law (KCL) - All currents entering a node equal the sum of all currents exiting that node
- Kirchhoff Voltage Law (KVL) - Voltage around all loops sum to 0

Ohm's Law

$V = IR$ $\quad R = V/I$ $\quad I = \frac{V}{R}$

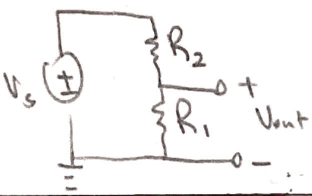
Resistance

$R = \rho \frac{L}{A}$

ρ ← resistivity; L ← Length; A ← area of cross section

Voltage Divider

$V_{out} = V_s \left(\frac{R_1}{R_1 + R_2} \right)$

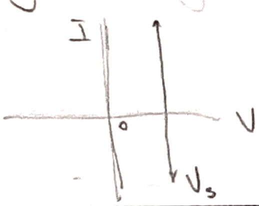


Current Divider

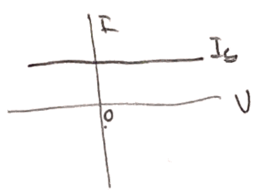
$I_n = I_s \frac{R_{rest}}{R_n + R_{rest}}$

I/V

Voltage Source



Current Source



Passive Sign Convention:
+ to −

Open Circuit means $i = 0$

- Can combine $3+$ in series but not $3+$ in parallel

Power / Energy

$P = \frac{dE}{dt} = V \cdot I = \frac{V^2}{R}$

Power [W]

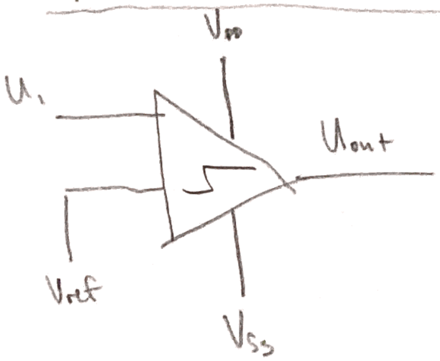
$E = P \cdot \Delta t = \int_0^t P(t)\, dt$

Energy [J] or [W·s]

Neg Power dissipates Energy

$V = I^2 R$

Op-Amp (Comparator)



if $u_1 > V_{ref} \Rightarrow U_{out} = V_{DD}$

if $u_1 < V_{ref} \Rightarrow V_{out} = V_{SS}$

$V_{out} = A(V^+ - V^-)$

$i = 0$

Superposition

- Set other independent sources to null
- Find each $V_{out,k}$
- Sum all V_{out}, sources to find V_{out}

Capacitance

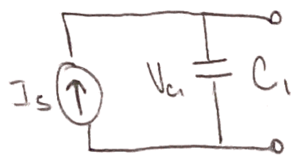
$Q_c = C \cdot V_c$ $\qquad C = \frac{A}{d} \varepsilon$ $\qquad E = \frac{1}{2} C V_c^2$

Charge [C] $\quad$ Capacitance [F] $\quad$ permitivity

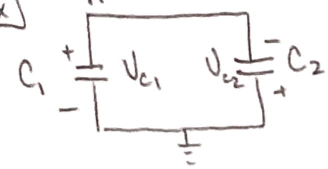
Charging: $I(t) = C \frac{dV}{dt}$

Discharging: $-I(t) = C \frac{dV}{dt}$

$V_c(t) = \frac{I_c}{C} \cdot T + V_0$



Ex) Flipped Signs



$Q_{total} = Q_{c1} + Q_{c2}$

$Q = C_1 V_{c1} + (-C_2 V_{c2})$

Conservation of Charge

$Q[t] = Q[t+1]$ - Charge in system remains same after time step

Thevenin

$V_{Th} = V_{oc}$ $\quad$ Thevenin is open circuit voltage between out ports

$R_{Th} = R_{Nor}$

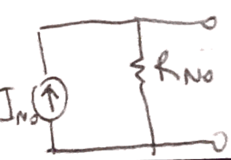
$V_{Th} = I_N \cdot R_{Th}$

Way to solve:
- find V at A
- find V at B w/o taking into account A
- Subtract

- Null Independent Sources
- Calc R_{Th}

Norton

$I_{No} = I_{meas}$ $\quad$ Current flowing through output ports when ports are shorted



Way to solve:
- Connect A & B by wire and ignore inbetween

Node Voltage Analysis (NVA)

1) Label Voltages at all Nodes
2) Label Element Voltages and Currents & Passive sign conv.
3) Write KCL Equations of Element Currents
4) Expressions for Element Currents using KVL
5) Subsitute Expressions for Element Currents into KCL
6) Solve

Tips

- Redraw circuits if necessary
- Pay attention to units
- See what variables/terms answer is supposed to be in

EECS 16A Final Formula Sheet Jeffrey Shen

Cross-Coorelation

Def: measure of the similarity between two signals

$$\text{Corr}_{\vec{x}}[\vec{y}][\vec{k}] = \sum_{i=-\infty}^{\infty} x[i]\, y[i-k]$$

Because of noise we want $A\vec{x} = \vec{b}$ but to minimize noise we use $\vec{e} = \vec{b} - A\vec{x}$ minimize $\|\vec{e}\|^2$ find smallest error

Least Squares

Def: Solves approx. systems in the presence of noise

$$\hat{\vec{x}} = (A^T A)^{-1} A^T \vec{b}$$ where A is independent

$\boxed{\text{Note:}}$ Use for tall and thin / overdetermined matrixes

Orthogonal Matching Pursuit (OMP)

Def: unmixes signals from multiple beacons, uncover messages

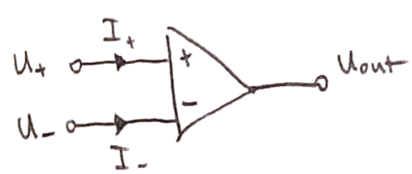
Known: matrix M, vector $\vec{y}$, sparsity level k or threshold for norm

1. Find inner product for each vector, $\vec{m}_i$ with $\vec{e}$ (first iter: $\vec{e} = \vec{y}$)
2. Add vector that had max inner prod to matrix A
3. Use proj / least squares to compute $\vec{x} = (A^T A)^{-1} A^T \vec{y}$
4. Update Error vector $\vec{e} = \vec{y} - A\vec{x}$

↻ Repeat until reached sparsity level

- Linearize distance formula eqs

Op Amp



Golden Rules:

Ideal Op Amp

1. $I_+ = I_- = 0$
2. $A \to \infty$

Ideal & Negative Feedback

1. $U_+ = U_-$

Unity Gain Buffer

1. $V_{in} = V_{out}$

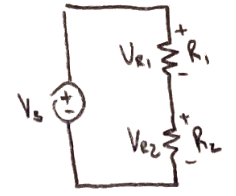
loading - small resistance causes it to draw alot of current causing large voltage drop, make it harder bc it will behave differently depending on what it's connected to

- Buffers allow us to split into blocks to analyze separately

Design Problem

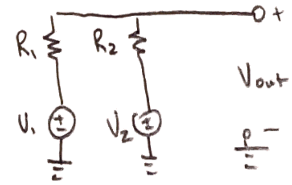
1. Restate important goals for design
2. Describe strategy using block diagram
3. Implement
4. Check implementation with Step 1

Voltage Divider



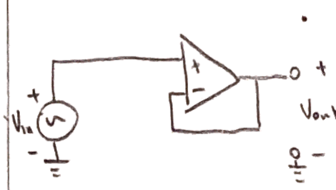
$$V_{R_2} = V_s\left(\frac{R_2}{R_1+R_2}\right)$$

Voltage Summer



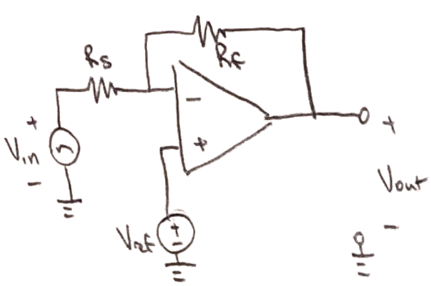
$$V_{out} = V_1\left(\frac{R_2}{R_1+R_2}\right) + V_2\left(\frac{R_1}{R_1+R_2}\right)$$

Unity Gain Buffer



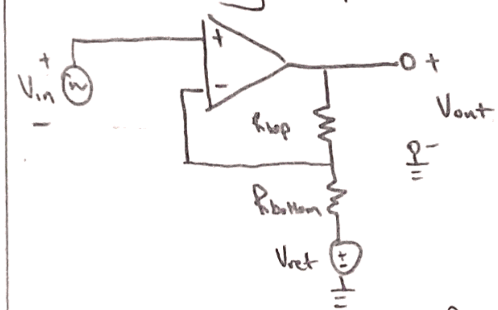
$$\frac{V_{out}}{V_{in}} = 1$$

Inverting Amplifier



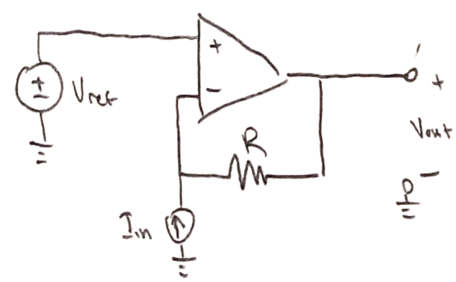
$$V_{out} = V_{in}\left(-\frac{R_f}{R_s}\right) + V_{ref}\left(\frac{R_f}{R_s}+1\right)$$

Non-inverting Amplifier



$$V_{out} = V_{in}\left(1+\frac{R_{top}}{R_{bottom}}\right) - V_{ref}\left(\frac{R_{top}}{R_{bottom}}\right)$$

Transresistance Amplifier



$$V_{out} = i_{in}(-R) + V_{ref}$$

Orthonormal Matrix

Colums $\vec{a}_i$ are
1. Orthogonal $\langle a_i, a_j \rangle = 0$
2. Normalized $\|a_i\| = 1$

Eigenvalues

$$A\vec{x} = \lambda\vec{x}$$

$$\det(A - \lambda I) = 0$$

If A squared, Eigenvectors same, Eigenvalues squared

$$\begin{bmatrix} -3 & -3 \\ 4 & 4 \end{bmatrix}\vec{a} = 0 \qquad a_1 + a_2 = 0 \qquad \begin{aligned} a_1 &= -a_2 \\ a_2 &= -a_1 \end{aligned} \qquad \begin{bmatrix} 1 \\ -1 \end{bmatrix} a_1$$

$$\begin{bmatrix} -4 & -3 \\ 4 & 3 \end{bmatrix}\vec{a} = 0 \qquad -4a_1 - 3a_2 = 0 \qquad \begin{aligned} -4a_1 &= 3a_2 \\ a_1 &= -\frac{3}{4}a_2 \end{aligned} \qquad \begin{bmatrix} -\frac{3}{4} \\ 1 \end{bmatrix} a_2$$