

CS 184: Computer Graphics and Imaging Lecture Notes

Week 1: Lecture 1 Introduction (1/18)

Why study Computer Graphics?

Computer Graphics

- Def: Use of computers to synthesize and manipulate visual information

Computer Graphics use

- Movies vs stop motion previously
- Motion capture for suits that capture location of facial expressions and actions
- Product Design and Visualization
 - Interaction of light, Raytracing graphics, ray-tracing, path-tracing
- Typography
 - Control points, Bezier curve
- Digital Illustrated design
- Computer-Aided Design
- Architectural Design
- Visualization
- Graphical User Interfaces
- Imaging in Mapping
- Virtual Reality

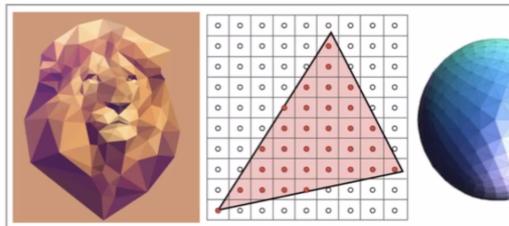
Foundations of Graphics and Imaging

- Applications require sophisticated theory and systems
- Science and Math
 - Physics of light, color optics
 - Math of curves, surfaces, geometry, perspective
- Technology and systems
 - Input devices, GPUs, displays
 - Cameras, lenses, sensors
- Art and Psychology
 - Perception: color, stereo, motion, image quality
 - Art and design: composition, form, lighting

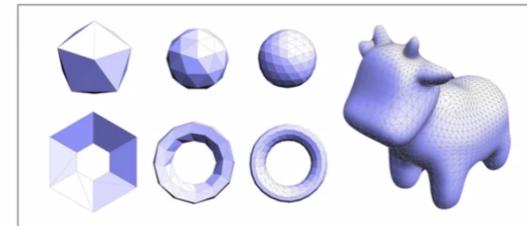
Course Overview

- Overview of core ideas in graphics and imaging
 - Modeling the world, image synthesis
 - 3D graphics: geometry, rendering, animation
 - Image capture, manipulation and display
- Acquire core concepts and skills
 - Representations (geometry)

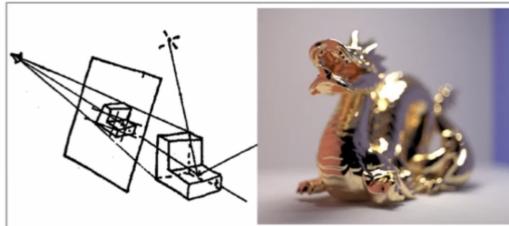
- Algorithms (sampling, subdivision, ray-tracing)
- Technology (GPUs, displays, cameras)
- Drawing digital Images (rasterization)
- Filtering and sampling
- Modeling geometry
- Modeling material properties
- Modeling lighting
- Light transport and Image synthesis (photograph CCD) vs computer rendering
- How do cameras work
- VR



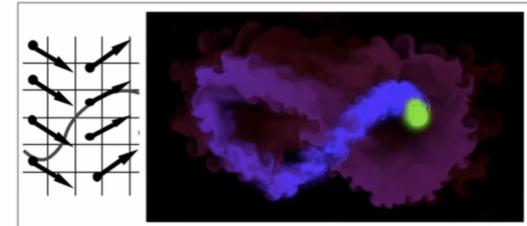
1. Digital Drawing (2 weeks)



2. Geometry (2 weeks)



3. Ray-Tracing (4 weeks)



4. Animation (2 weeks)

Project Competition

- **4 weeks, let your creativity take flight!**
(we will have suggested projects)
- **Proposal; checkpoint; presentation, video, report**

Logistics

Week 1: Lecture 2 Digital Drawing (1/20)

Today: Drawing Triangles to the Screen by Sampling

Drawing Machines

- CNC Drawing Machine, laser cutters

Frame buffer: memory for a raster display

- For every pixel what color should be
- Digital to analog convertors to emit color we want

A sampling of different raster displays

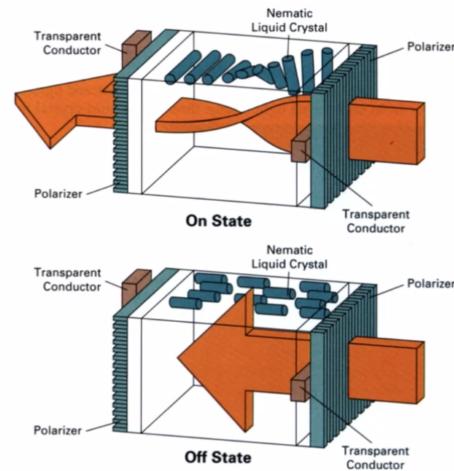
1. Flat Panel displays

- a. Los res LCD, Color LCD, OLED
- b. LCD (liquid crystal display)

Principle: block or transmit light by twisting polarization

**Illumination from backlight
(e.g. fluorescent or LED)**

Intermediate intensity levels by partial twist

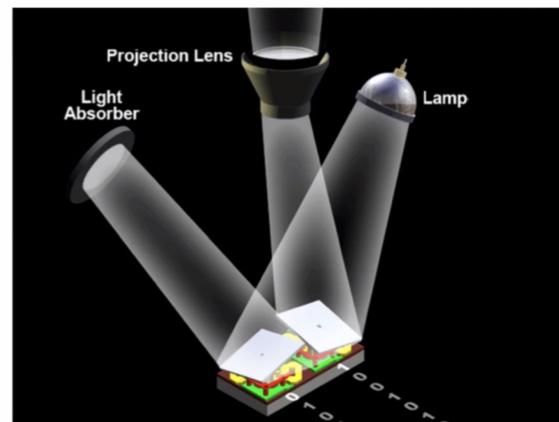
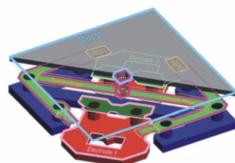


c. LED Array Display

- i. Light emitting diode array, each individual pixel

d. DMD Projection (Digital Micro Mirror Device)

- i. Pixels are mirrors and reflect light or not toward mirror



e. E-ink displays

f. Smartphon screen pixels

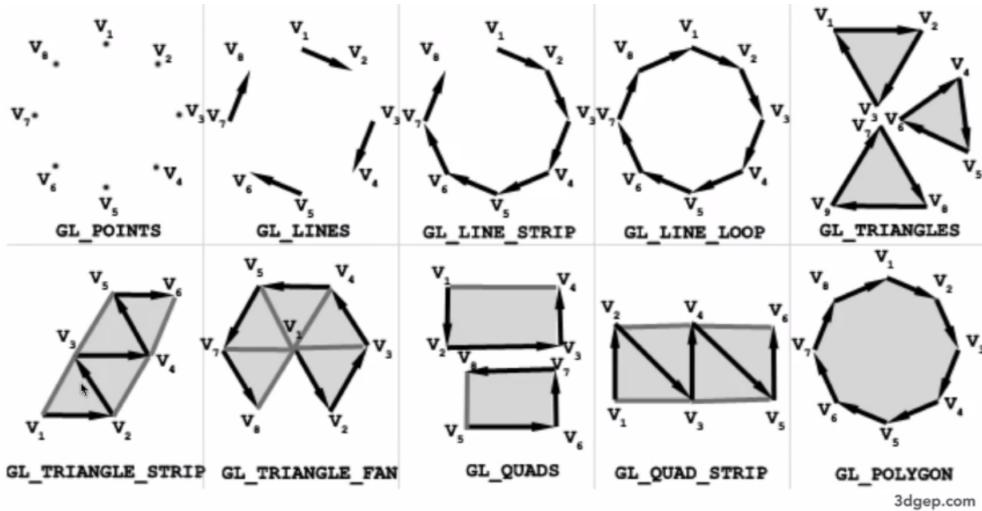
- i. Array of light emitors

Drawing to Raster Displays

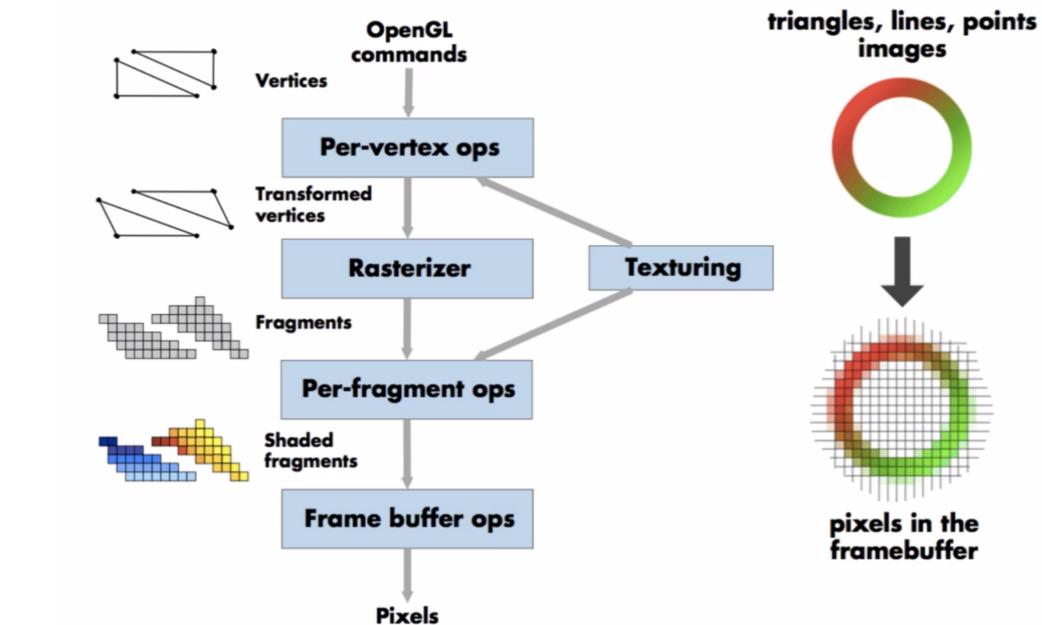
Polygon Meshes

Shape Primitives

- OpenGL: API for raster drawing machines



Graphics Pipeline = Abstract Drawing Machine



- Rasterization: transform triangles to fragments or pixels

Triangles - Fundamental Area Primitive

- Why Triangles?

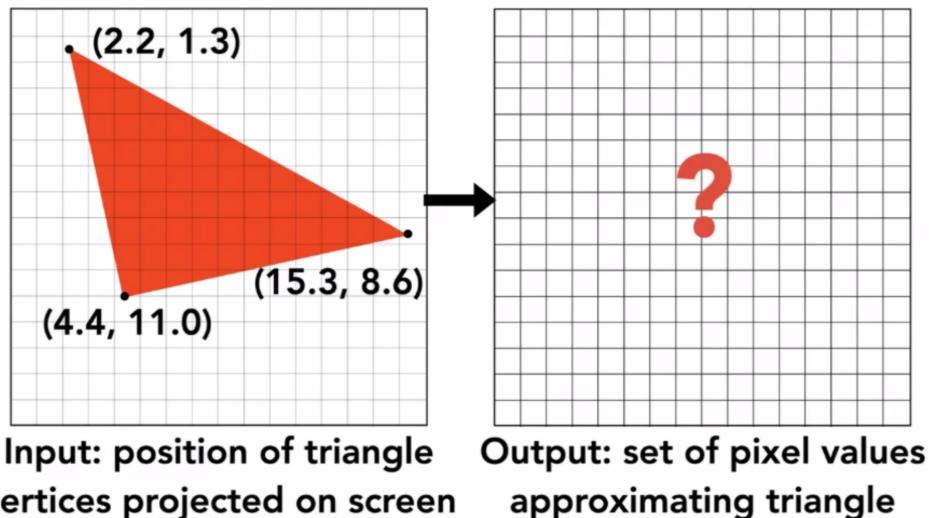
- Most basic polygon
 - Break up other polygons
 - Optimize one implementation
- Triangles have unique properties
 - Guaranteed to be planar
 - Well defined interior

- Well-defined method for interpolating values at vertices over triangle

Drawing a Triangle to the Framebuffer ("Rasterization")

Rasterization : sampling a 2d indicator function

What Pixel Values approximate a Triangle

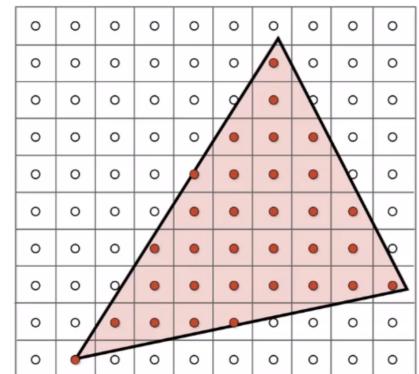


Sampling a Function

- Evaluating a function at a point is sampling
- Discretize a function by periodic sampling
 - Sample time (1d), area (2d), angle (2d), volume (3d)
- 1. Sample if each Pixel center is inside triangle

$$\text{inside}(t, x, y) = \begin{cases} 1 & (x, y) \text{ in triangle } t \\ 0 & \text{otherwise} \end{cases}$$

Triangle = Intersection of three half planes



Line Tangent Vector



$$T = P_1 - P_0 = (x_1 - x_0, y_1 - y_0)$$

$$\text{Perp}(x, y) = (-y, x)$$

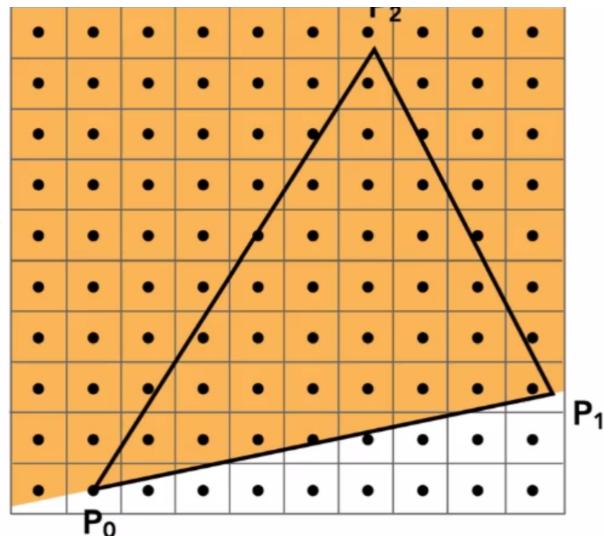
$$P_i = (X_i, Y_i)$$

$$dX_i = X_{i+1} - X_i$$

$$dY_i = Y_{i+1} - Y_i$$

$$\begin{aligned} L_i(x, y) &= -(x - X_i) dY_i + (y - Y_i) dX_i \\ &= A_i x + B_i y + C_i \end{aligned}$$

$L_i(x, y) = 0$: point on edge
 < 0 : outside edge
 > 0 : inside edge



$$L_0(x, y) > 0$$

Some Details

- Sample point covered by triangle 1, triangle 2 or both
- Modern approach: tiled Triangle Traversal
 - Parallelism

Signal Reconstruction on Real Displays

Assume Display Pixels Emit Square of light

- Send the display
- Jagged edges

Potential topics for your pair discussion:

- Ideas for “higher quality” pixel formula?
- What are all the relevant factors?
- What’s right/wrong about point sampling?
- Why do jagged edges look “wrong”?

Week 2: Lecture 3 Intro to Signal Processing: Sampling, Aliasing, Antialiasing (1/25)

Photograph = sample Image Sensor Plane

Ray tracing = sample rays

Sampling Artifacts in Graphics and Imaging

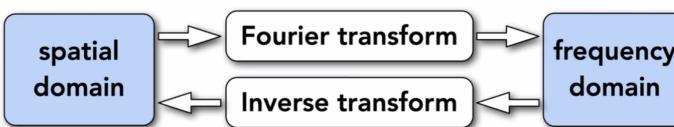
- Jaggies (staircase pattern)
 - Wagon wheel effect – sampling in time
 - Fast-changing signals, when we sample too slowly
- Antialiasing Idea: Filter out high frequencies before sampling
- Sharp is a fast varying signal, blurry is low frequency
 - Antialiasing filter out the higher frequencies

This Lecture

- Fundamental reasons why this works
- Look at how to implement antialiased rasterization

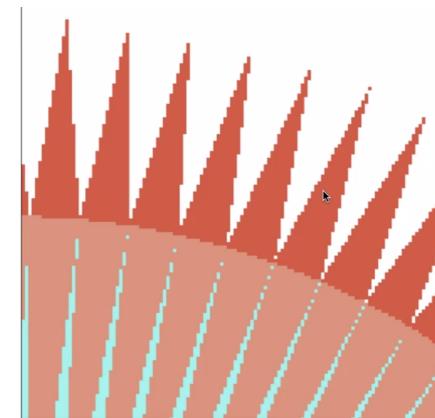
Frequency Space

Fourier Transform

$$f(x) \quad F(\omega) = \int_{-\infty}^{\infty} f(x)e^{-2\pi i \omega x} dx \quad F(\omega)$$


$$f(x) = \int_{-\infty}^{\infty} F(\omega)e^{2\pi i \omega x} d\omega$$

Higher Frequencies Need Faster Sampling

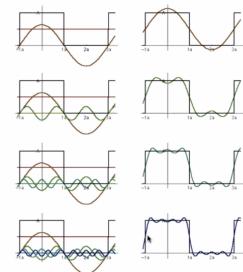


Point in Time
1/4000 sec exposure

Motion Blurred
1/60 sec exposure

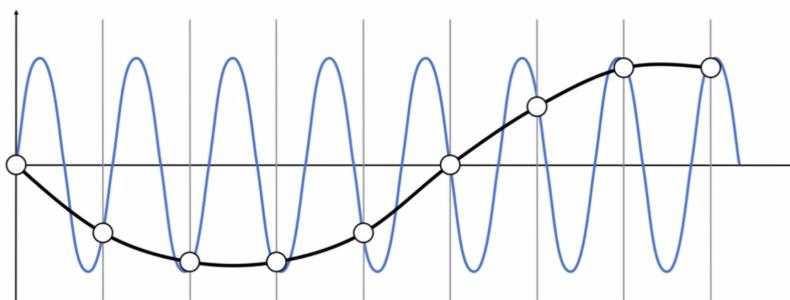
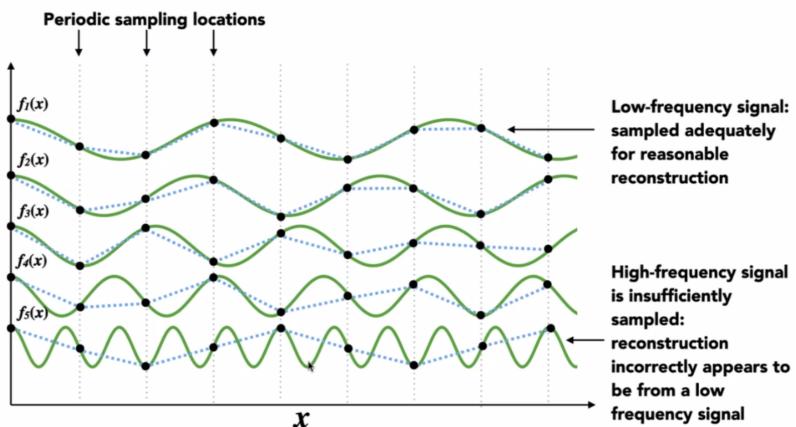
Fourier Transform

Represent a function as a weighted sum of sines and cosines



$$f(x) = \frac{A}{2} + \frac{2A \cos(t\omega)}{\pi} - \frac{2A \cos(3t\omega)}{3\pi} + \frac{2A \cos(5t\omega)}{5\pi} - \frac{2A \cos(7t\omega)}{7\pi} + \dots$$

Higher Frequencies Need Faster Sampling



High-frequency signal is insufficiently sampled: samples erroneously appear to be from a low-frequency signal

Two frequencies that are indistinguishable at a given sampling rate are called "aliases"

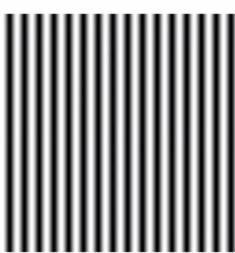
- Aliases are false identities

2D frequency domain

- Low frequency means it's varying very slowly, closer to the middle
- More it varies the f
- Sin correspond to different points on the frequency domain

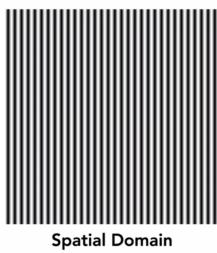


$\sin(2\pi/32)x$ — frequency 1/32; 32 pixels per cycle

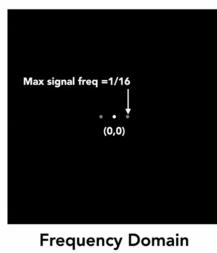


Spatial Domain

$\sin(2\pi/16)x$ — frequency 1/16; 16 pixels per cycle

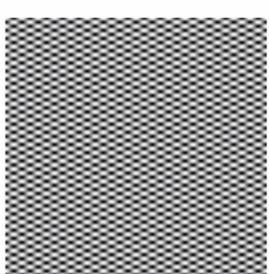


Spatial Domain

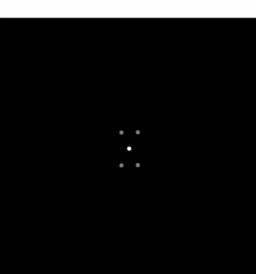


Frequency Domain

$\sin(2\pi/32)x \times \sin(2\pi/16)y$

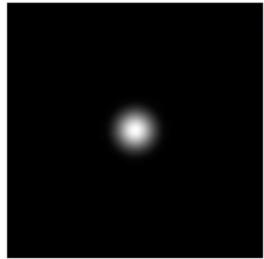


Spatial Domain



Frequency Domain

$$\exp(-r^2/32^2)$$

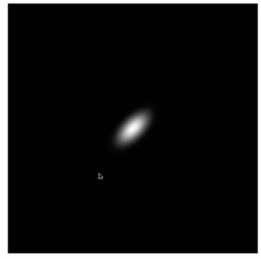


Spatial Domain



Frequency Domain

$$\textbf{Rotate 45} \quad \exp(-x^2/32^2) \times \exp(-y^2/16^2)$$



Spatial Domain



Frequency Domain

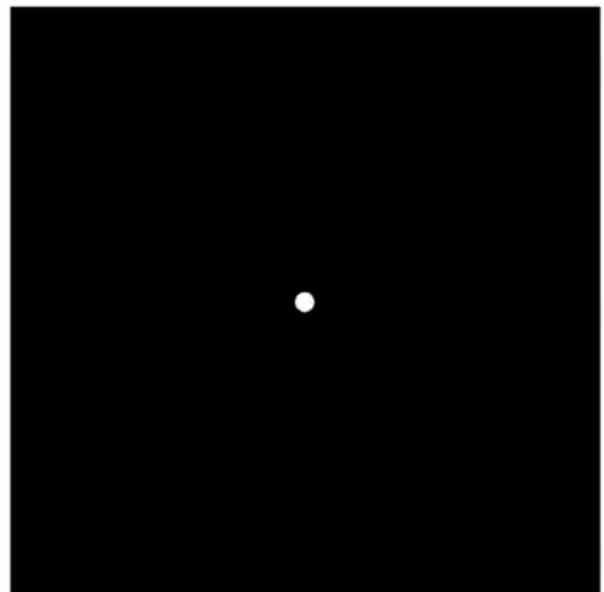
Filtering

- Filter out

high frequencies



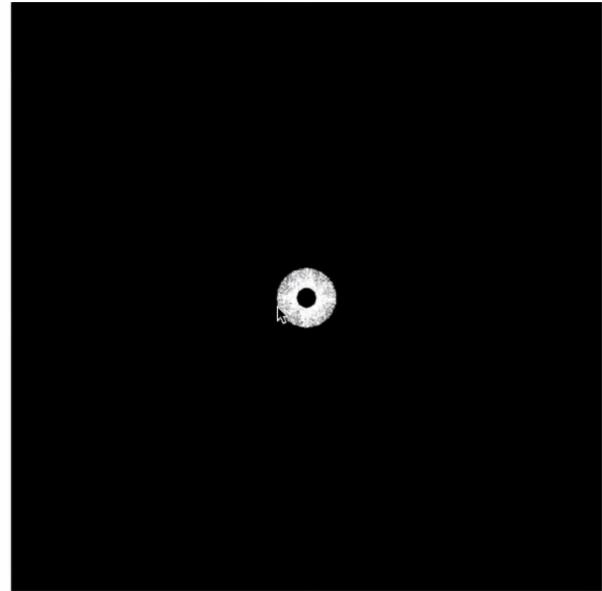
Spatial Domain



Frequency Domain



Spatial Domain

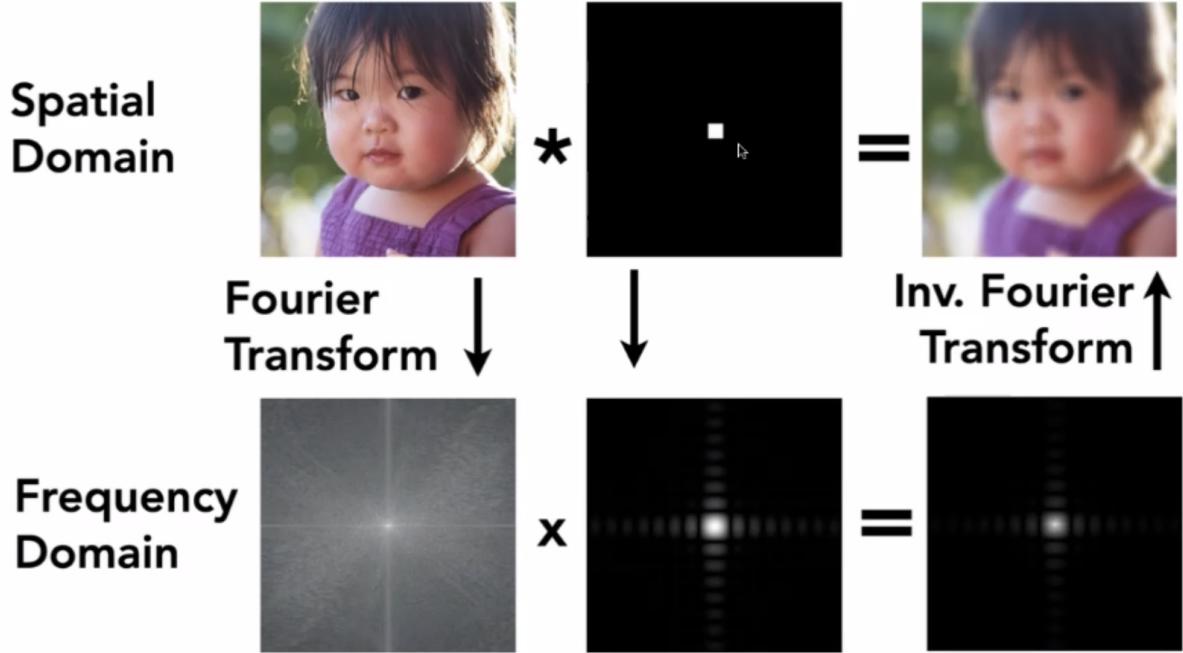


Frequency Domain

Further away the finer the rate of fluctuations

Filtering = Convolution

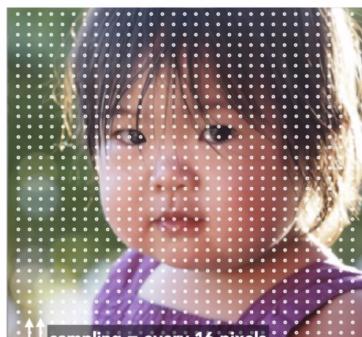
- Sliding weighted filter across the signal
- Convolution Theorem
 - Convolution in the spatial domain is equal to multiplication in the frequency domain and vice versa
 - Option 1
 - Filter by convolution in the spatial domain
 - Option 2
 - Transform to frequency domain
 - Multiply by fourier transform of convolution kernel
 - Transfrom back to spatial domain



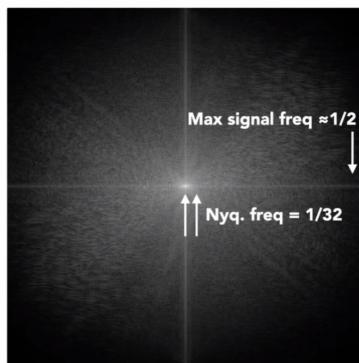
Nyquist Frequency & Antialiasing

- Nyquist Theorem: We get no aliasing from frequencies in the signal that are less than the Nyquist frequency (defined as half the sampling frequency)
- Half of sampling frequency

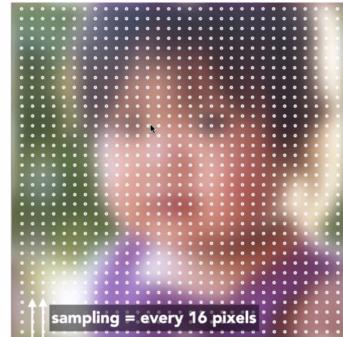
Visual Example: Image Frequency



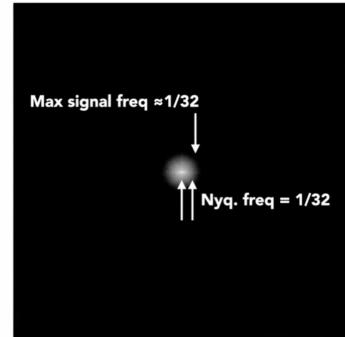
Spatial Domain



Frequency Domain



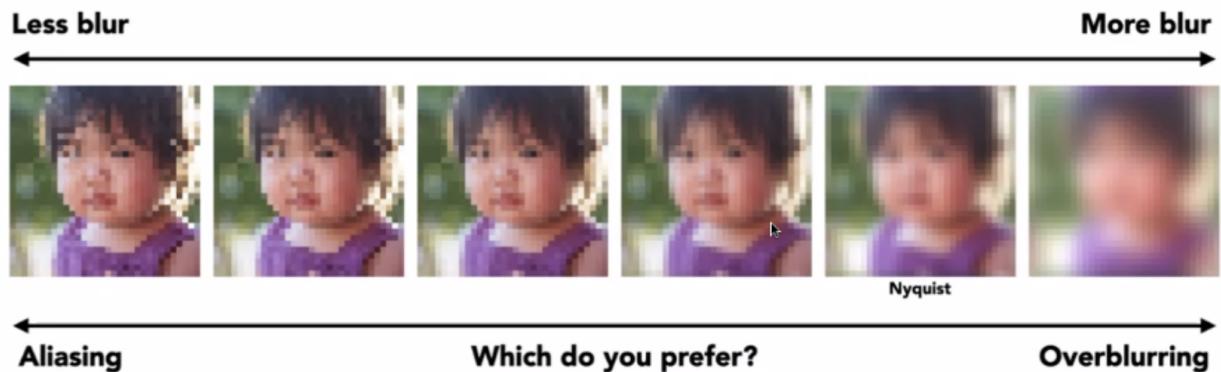
Spatial Domain



Frequency Domain

Recap:

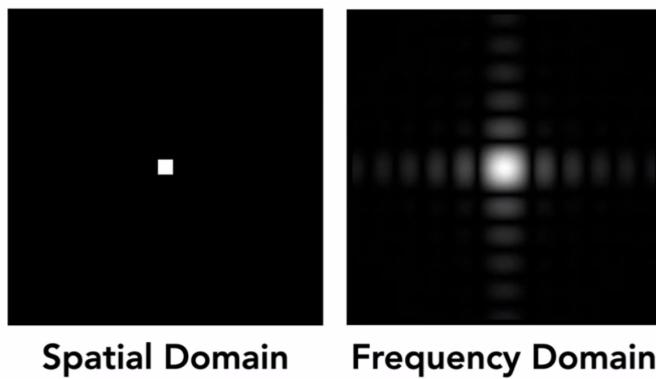
- Filter (blur) original image to reduce maximum signal frequency
- Create low-resolution image by sampling only every 16 pixels
 - (Sampling frequency is 1/16, and Nyquist frequency is 1/32)



How Can we reduce Aliasing error

- Increase sampling rate (increase Nyquist Frequency)
 - Higher resolution displays, sensors, framebuffers
 - But: costly & may need very high resolution
- Antialiasing
 - Simple idea: remove signal frequencies that are high
 -

Box Filter



AntiAliasing By Averaging Vlaues in Pixel Area

- Option 1
 - Convolve $f(x,y)$ by a 1-pixel box-blur
 - Then sample at every pixel
- Option 2
 - Compute the average value of $f(x, y)$ in the pixel

Week 2: Lecture 4 Transforms (1/27)

Rotation, Scale

Transforms are functions acting on points

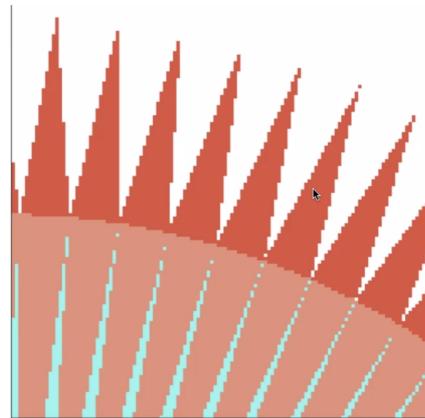
- $(x', y', z') = F(x, y, z)$

Project Polygons in 3D to 2D screen

- Depends on the camera angle

Why Study Transforms

- Modeling
 - Define shapes in convenient coordinates
 - Enable multiple copies of the same object
 - Represent hierarchical scenes
- Viewing
 - World coordinates to camera coordinates
 - Parallel / perspective projections from 3D to 2D



Lecture Outline

- Think about transformations
 - Types: rotate, translate, scale
 - Coordinate frames
 - Composing multiple transformations
 - Hierarchical transforms
 - Perspective projection
- How to implement
 - Represent transforms as matrices
 - Homogeneous coordinates

Linear Transforms = Matrices

- Scale matrix, reflection matrix, shear matrix

Linear Transforms = Matrices

$$x' = ax + by$$

$$y' = cx + dy$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\mathbf{x}' = \mathbf{M} \mathbf{x}$$

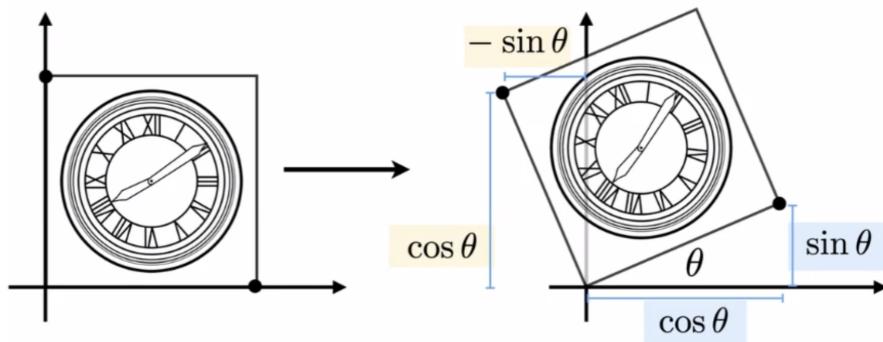
2D Coordinate Systems

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Rotation Matrix



$$\mathbf{R}_\theta = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

Translation

$$x' = x + t_x$$

$$y' = y + t_y$$

Homogenous Coordinates

- Add a third coordinate (w-coordinate)
- 2D point, $(x, y, 1)^T$ 2D vector $(x, y, 0)^T$
- Express translation as a matrix

$$\begin{pmatrix} x' \\ y' \\ w' \end{pmatrix} = \begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} x + t_x \\ y + t_y \\ 1 \end{pmatrix}$$

Affine Transformations

- Affine map = linear map + translation

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} a & b & t_x \\ c & d & t_y \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

2D Transformations

Scale

$$\mathbf{S}(s_x, s_y) = \begin{pmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

"Similarity Transform"

Rotation

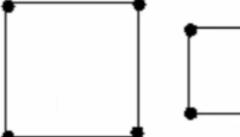
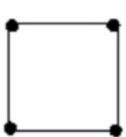
$$\mathbf{R}(\alpha) = \begin{pmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

"Rigid Transform"

Translation

$$\mathbf{T}(t_x, t_y) = \begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix}$$

- Transform Ordering matters

Transformation	Before	After
Affine		
Similarity		
Euclidean		

- Matrix multiplication is not commutative

$$R_{45} \cdot T_{(1,0)} \neq T_{(1,0)} \cdot R_{45}$$

- Composing transformations
 - Compose by matrix multiplication

$$A_n(\dots A_2(A_1(\mathbf{x}))) = \mathbf{A}_n \cdots \mathbf{A}_2 \cdot \mathbf{A}_1 \cdot \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

Pre-multiply n matrices to obtain a single matrix representing combined transform

Decomposing Complex Tranforms

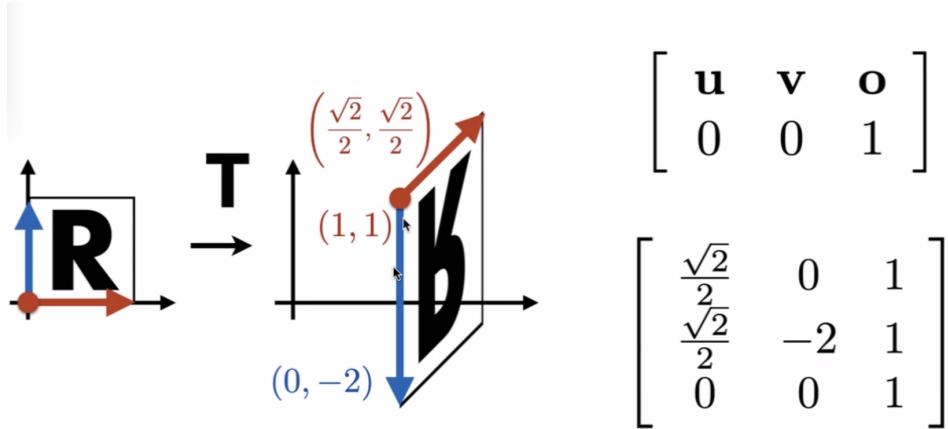
Coordinate Systems

- New coordinate frame is defined by origin (point) and two unit axes (vectors)
- Given coordinates in (o, u, v) reference frame, transform to coordinates in the (x, y)

$$F = \begin{bmatrix} \mathbf{u} & \mathbf{v} & \mathbf{o} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} u_x & v_x & o_x \\ u_y & v_y & o_y \\ 0 & 0 & 1 \end{bmatrix}$$

frame

Coordinate system transform

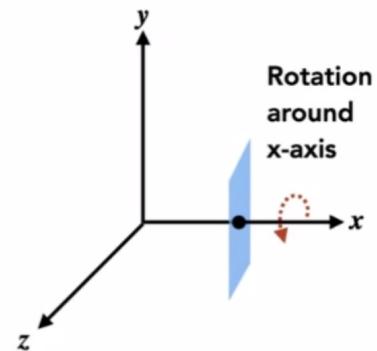


Rotation around x-, y-, or z-axis

$$\mathbf{R}_x(\alpha) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha & 0 \\ 0 & \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

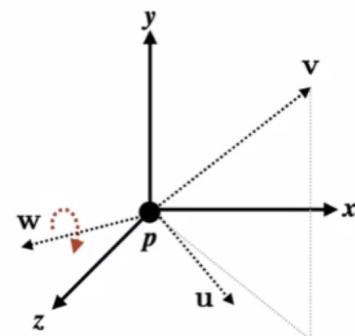
$$\mathbf{R}_y(\alpha) = \begin{pmatrix} \cos \alpha & 0 & \sin \alpha & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \alpha & 0 & \cos \alpha & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{R}_z(\alpha) = \begin{pmatrix} \cos \alpha & -\sin \alpha & 0 & 0 \\ \sin \alpha & \cos \alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



Construct orthonormal frame transformation \mathbf{F} with \mathbf{p} , \mathbf{u} , \mathbf{v} , \mathbf{w} , where \mathbf{p} and \mathbf{w} match the rotation axis

Apply the transform $(\mathbf{F} \mathbf{R}_z(\theta) \mathbf{F}^{-1})$



Rodrigues' Rotation Formula

Rotation by angle α around axis n

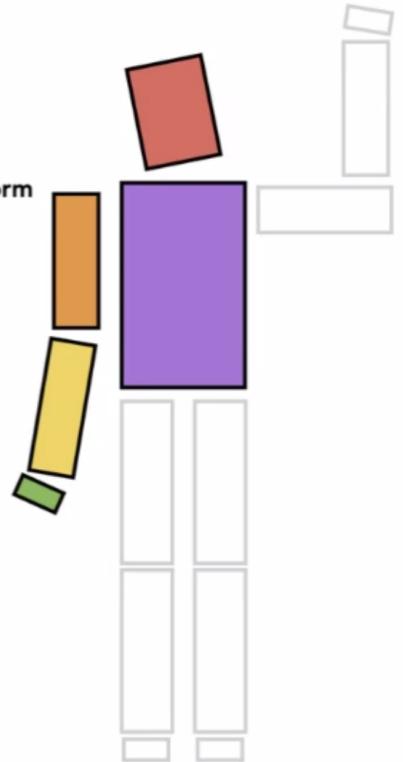
$$\mathbf{R}(\mathbf{n}, \alpha) = \cos(\alpha) \mathbf{I} + (1 - \cos(\alpha)) \mathbf{n}\mathbf{n}^T + \sin(\alpha) \underbrace{\begin{pmatrix} 0 & -n_z & n_y \\ n_z & 0 & -n_x \\ -n_y & n_x & 0 \end{pmatrix}}_{\mathbf{N}}$$

- Hierarchical Representation

- Grouped representation (tree)

```

translate(0, 10);
drawTorso();
pushmatrix(); // push a copy of transform onto stack
    translate(0, 5); // right-multiply onto current transform
    rotate(headRotation); // right-multiply onto current transform
    drawHead();
popmatrix(); // pop current transform off stack
pushmatrix();
    translate(-2, 3);
    rotate(rightShoulderRotation);
    drawUpperArm();
pushmatrix();
    translate(0, -3);
    rotate(elbowRotation);
    drawLowerArm();
pushmatrix();
    translate(0, -3);
    rotate(wristRotation);
    drawHand();
popmatrix();
popmatrix();
popmatrix();
....
```



- **CS184/284A**

Ren Ng

Viewing and Perspective for Transforms

-

Camera Space

- Camera located at the origin
- Looking down negative z-axis
- Vertical vector is y-axis
- (x-axis) orthogonal to y & z
- U = up vector
- V = view direction
- E = eye point (position of camera)
- Input: e, u & v given in world space coordinates

- Output: transform matrix from world space to standard camera space
- Camera "look-at" transformations
-

Intro to C++

Why C++

- Graphics = Computation-heavy
- Faster than Java, more control over computer resources, statically typed,
- Used frequently in games/animation software

Namespaces

- Provide additional scope for variables,

Namespaces Example

```

namespace mynamespace {
    int x = 5; // variable inside namespace
}

int main(){
    int x = 213; // local variable
    std::cout << x << '\n'; // prints "213"
    std::cout << mynamespace::x << '\n'; // prints "5"
}

```

Using namespaces, we can declare two x variables! One inside the namespace and one outside.

:: is how we signify that we're using a namespace variable

Classes and Objects

- All methods/attributes of C++ class are private, unless explicitly public
- Structs

```

#include <iostream>
using namespace std;

class Rectangle {
    int width, height;
public:
    void set_values (int,int); // These are private variables!
    int area(); // We declare these functions outside of the constructor
};

void Rectangle::set_values (int x, int y){
    width = x;
    height = y;
}
int Rectangle::area() {
    return width * height;
}

int main () {
    Rectangle rect;
    rect.set_values (3,4);
    cout << "Area: " << rect.area();
    return 0;
}

```

Output:
Area: 12

Memory: Pointers & Addresses

- C++ assigned an address in memory

Using * and &

- & (reference) operator gives you address occupied by a variable
- Values stored in memory address, use dereference operator (*)

Pointers to Objects

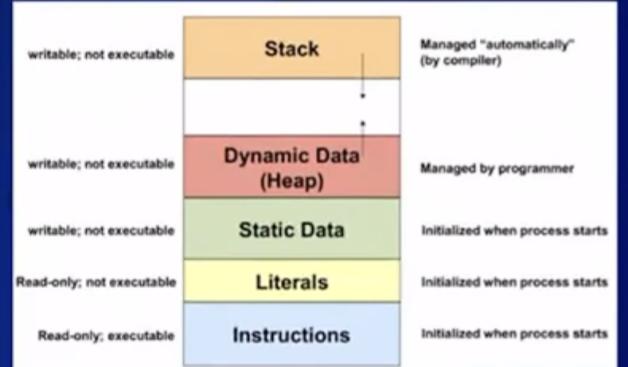
```
Rectangle *x = new Rectangle(3, 4);  
int w = x->width;  
// this is the same as (*x).width  
x->area();  
// also used to call methods
```

Vs

```
Rectangle x = Rectangle(3, 4);  
int w = x.width;  
x.area();
```

We can also have pointers to more complex variables, like objects or functions! To access their attributes and methods, we have to use slightly different notation.

- Memory in your program can be allocated on the **stack** or the **heap**
 - In the stack, the allocation and deallocation is automatically done.
 - Whereas, in the heap, it needs to be done by the programmer manually.
- The **new** operator denotes a request for memory allocation on the Heap
 - Like malloc in C, C++ has malloc but it's less user-friendly



Stack vs Heap

- Stack Rectangle rect
 - No need to delete, use rect.width
- Heap: Rectangle *rp = new Rectangle();
 - Must call delete rp or will exist until your program ends
 - To get attributes use (rp->width)
 - If constructor has arguments, Rectangle *rp = new Rectangle(3, 4)

Passing Arguments

- Pass by value
- Pass by pointer
 - Pass in a pointer to a variable, instead of a copy of the variable value

- Directly change the value stored at the passed in address
- Pass by Reference
 - Pass in a reference to a variable, instead of copy, directly change, cleaner than pass by pointer

```
void square_pointer(int *a) {
    *a = (*a) * (*a);
}

void square_reference(int &a) {
    a = a * a;
}

int square_value(int a) {
    return a * a;
}

int main() {
    int x = 2;
    square_pointer(&x); // After this line, x = 4
    square_reference(x); // After this line, x = 16
    x = square_value(x); // After this line, x = 256
}
```

Vectors

- `Std::vector`

- Ordered list of items, similar to a Java ArrayList

```
int main() {
    // We initialize a vector with an initializer list
    std::vector<int> nums = {2, 4, 6, 0, 1};

    // Access by index
    std::cout << "nums[2] is " << nums[2] << std::endl;

    // Index-based iteration
    for (int i = 0; i < nums.size(); i++) {
        // do something
    }

    // Range for loop
    for (int num : nums) {
        //do something
    }
}
```

- Not efficient to copy over the image each time

Range For Loops Warning

```
int main() {
    // We initialize a vector with an initializer list
    std::vector<Image> images = std::vector<Image>(5);

    // Range for loop
    for (Image image : images) {
        //do something
    }
}
```

For the range based for-loops, note that each item in the vector is copied over into the loop variable, which is fine for primitives, but problematic for objects

Range For Loops Warning

```
int main() {
    // We initialize a vector with an initializer list
    std::vector<Image> images = std::vector<Image>(5)

    // Range for loop
    for (Image &image : images) {
        //do something
    }
}
```

Looping over references fixes the problem by not copying over each image!

.cpp vs .h

- .cpp: Source code
 - Write code and logic in .cpp files
- .h: header file, included in .cpp files
 - Describe functions or classes/methhods using function declarations

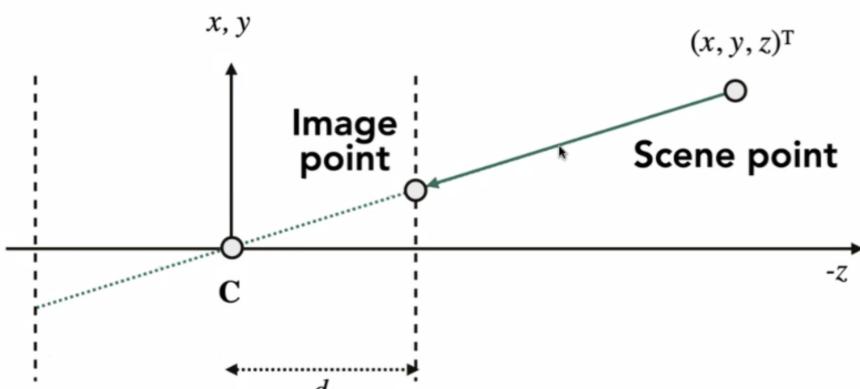
Week 3: Lecture 5 Texture Mapping (2/1)

Perspective in Art

- Learned converging lines and vanishing point

Pinhole Camera Model

Pinhole Camera Projective Transform



Upright image

Homogenous Coordinates (3D)

$$\mathbf{p} = \begin{pmatrix} wx \\ wy \\ wz \\ w \end{pmatrix} \leftrightarrow \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

$$\mathbf{M} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{pmatrix}$$

$$\mathbf{q} = \mathbf{M} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \\ z/d \end{pmatrix} \leftrightarrow \begin{pmatrix} xd/z \\ yd/z \\ d \\ 1 \end{pmatrix}$$

Note non-zero term in final row.
First time we have seen this.



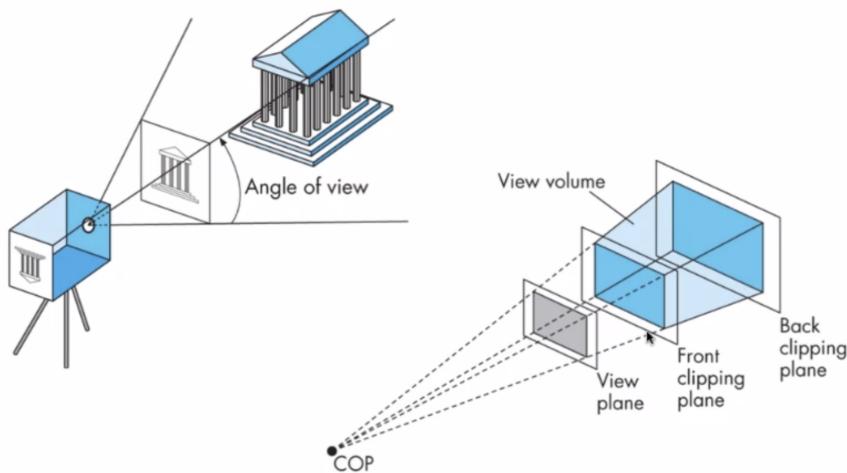
Produces all linear perspective effects,

- Converging lines + vanishing points
- Closer objects appear larger in images

Specifying real-world parameters

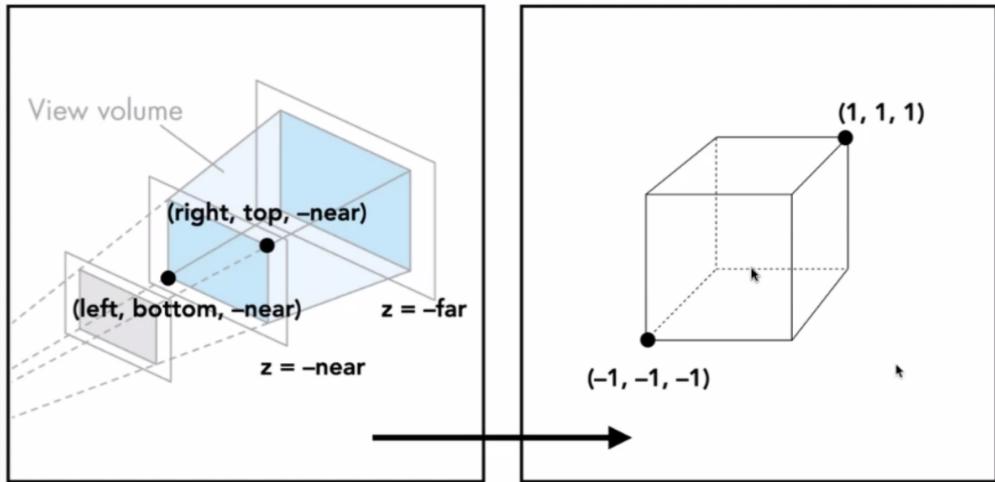
- Perspective composition = camera position + angle of view

Specifying Perspective Projection



From Angel and Shreiner, Interactive Computer Graphics

Perspective Projection Transform



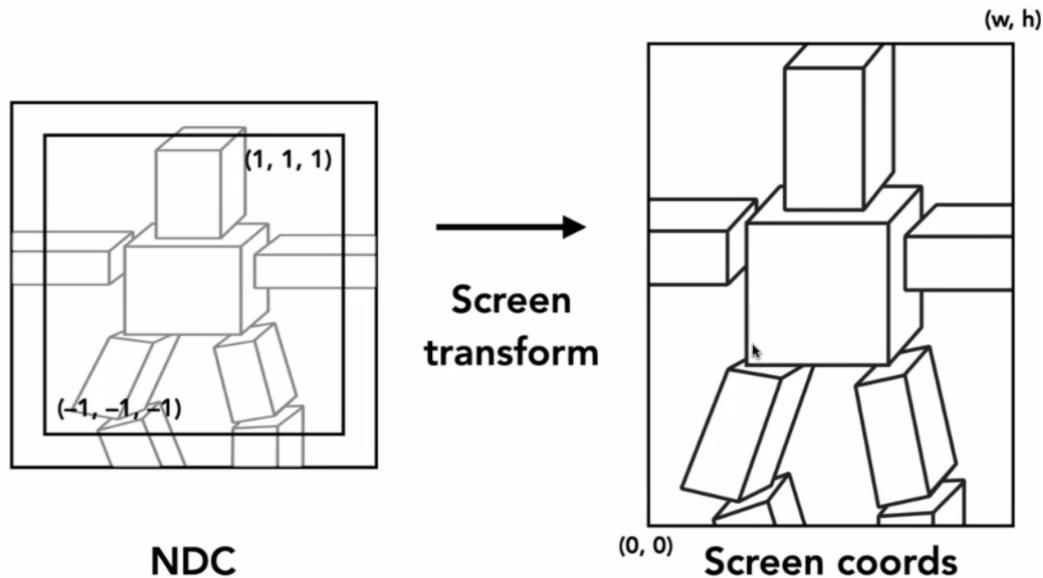
Camera Coordinates

**Normalized Device Coords
"NDC"**

Later we will "flatten and scale" NDC to get framebuffer coordinates

Coordinate Systems

- Object coordinates
 - Apply modeling transforms...
- World (scene) coordinates
 - Apply viewing transform...
- Camera (eye) coordinates
 - Apply perspective transform + homog. divide...
- Normalized device coordinates
 - Apply 2D screen transform...
- Screen coordinates

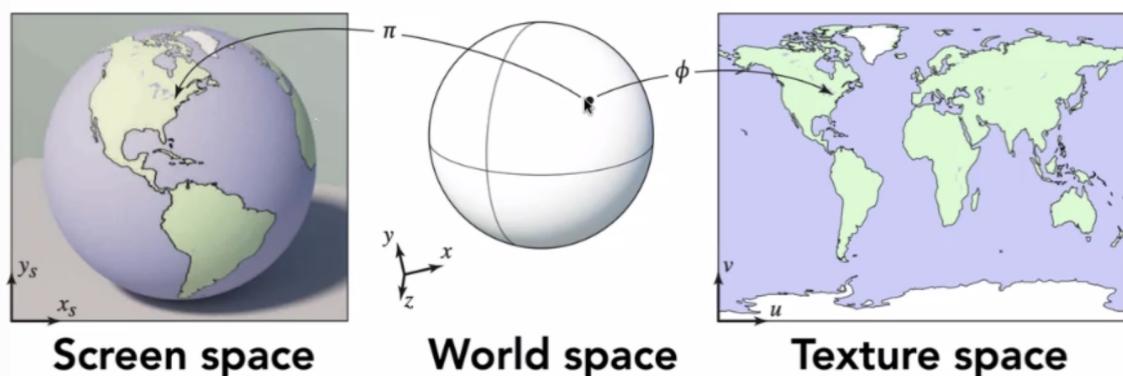


Texture Mapping

Describe Surface Material Properties

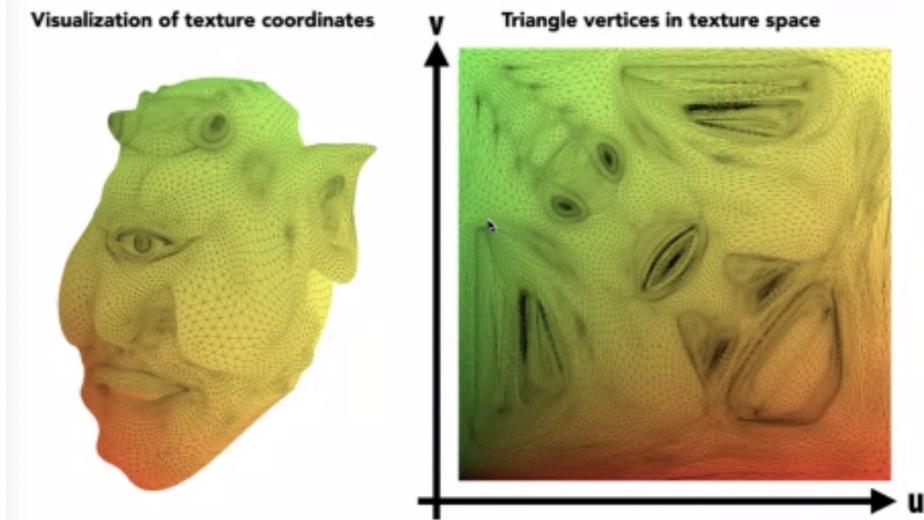
- Add details without raising geometric complexity
- Paste image onto geometry or define procedurally

Texture coordinate mapping



Each surface point was assigned a texture coordinate (u, v)

Each surface point is assigned a texture coordinate (u,v)

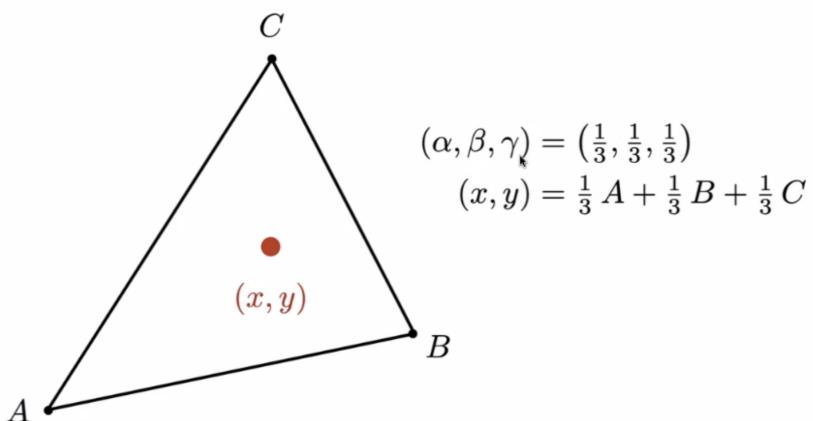


Interpolation across triangles

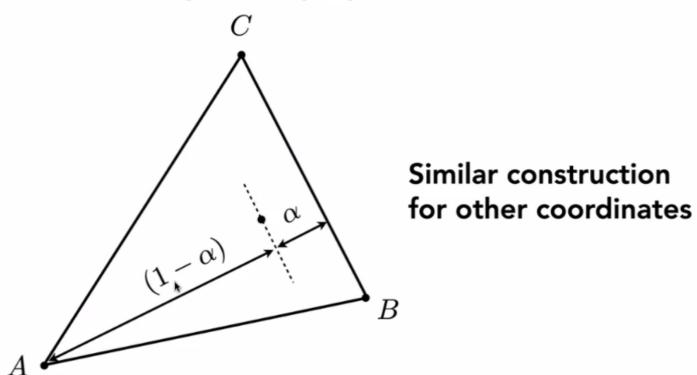
- Barycentric Coordinates
- Specify values at vertices and obtain smoothly varying values across surfaces
- What do we want to interpolate
 - Texture coordinates, colors, normal vectors

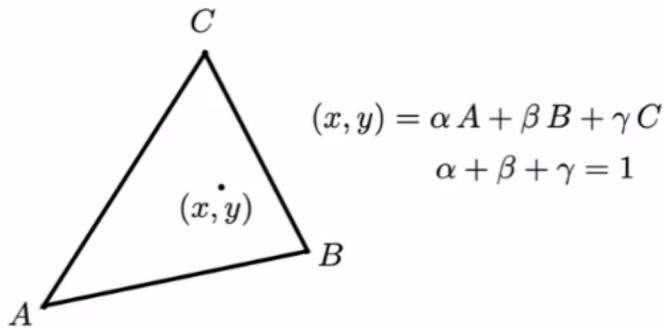
Barycentric Coordinates

- Coordinate system for triangles α, β, γ



Geometric viewpoint — proportional distances





$$\alpha = \frac{-(x - x_B)(y_C - y_B) + (y - y_B)(x_C - x_B)}{-(x_A - x_B)(y_C - y_B) + (y_A - y_B)(x_C - x_B)}$$

$$\beta = \frac{-(x - x_C)(y_A - y_C) + (y - y_C)(x_A - x_C)}{-(x_B - x_C)(y_A - y_C) + (y_B - y_C)(x_A - x_C)}$$

$$\gamma = 1 - \alpha - \beta$$

Perspective Projection and Interpolation

- Linear interpolation in world coordinates yields nonlinear interpolation in screen coordinates

Applying Textures is Sampling

Simple Texture Mapping Operation

for each rasterized screen sample (x,y):

(u,v) = evaluate texcoord value at (x,y)

float3 texcolor = texture.sample(u,v);

set sample's color to texcolor;

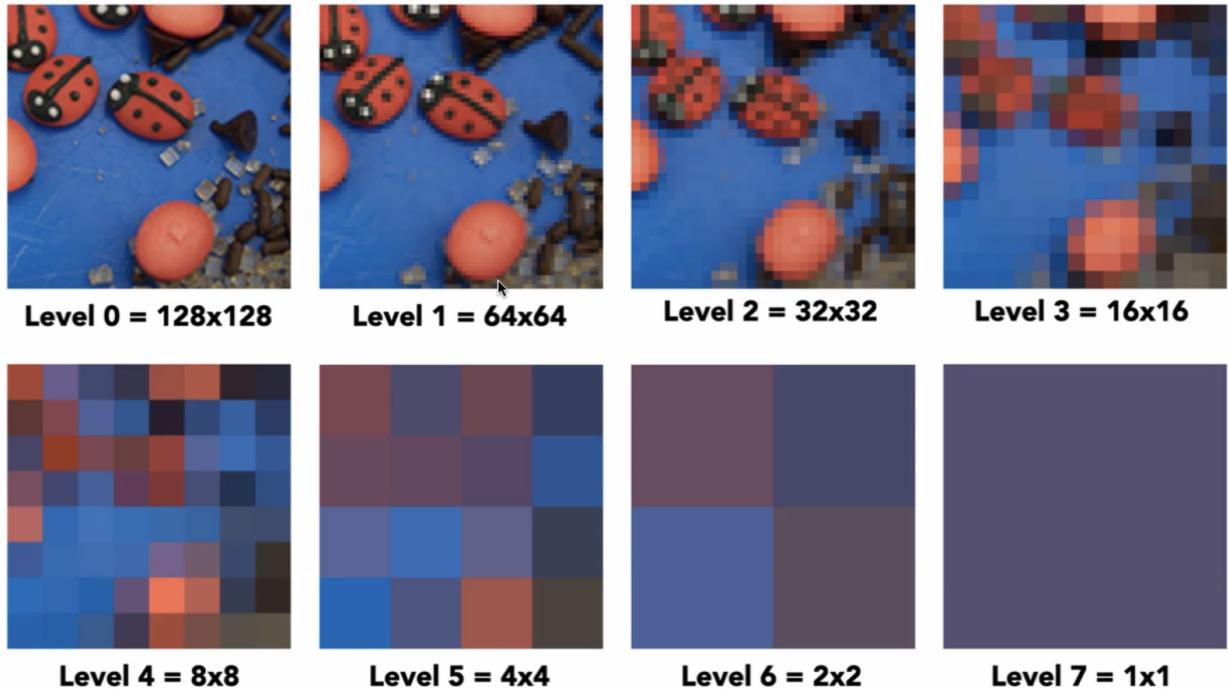
Week 3: Lecture 6 Texture Mapping (2/3)

Texture Minification - Hard Case

- Many texels can tributo pixel footprint
- Idea;
 - Take texture image file, low-pass filter it
 - Fewer pixels

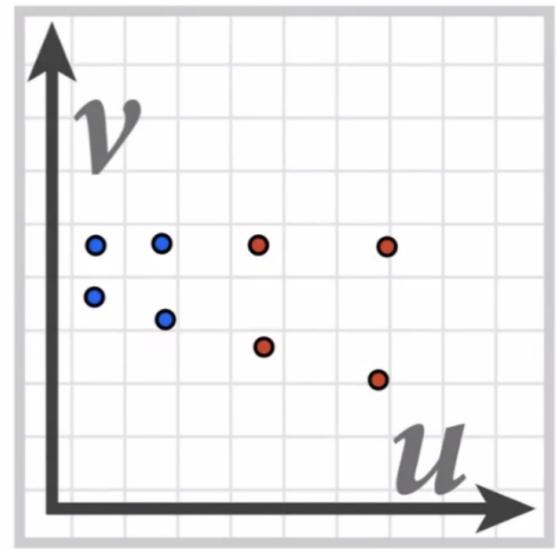
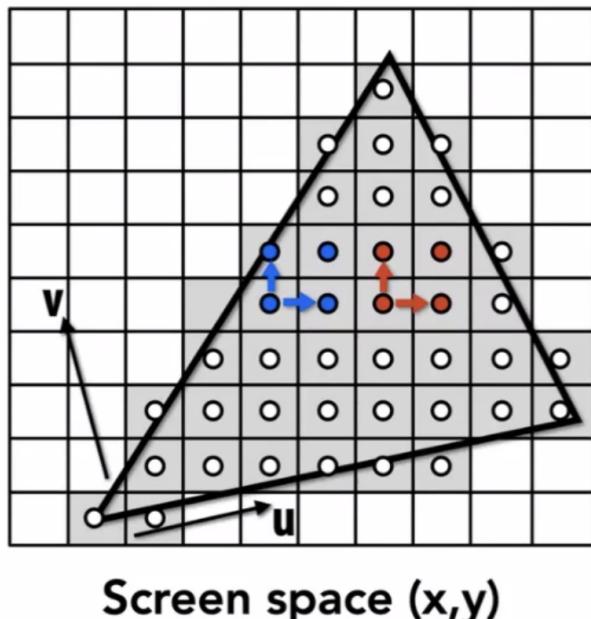
- Blur in the front, but background no aliasing

Mipmap (L. Williams 83)



Computing Mipmap level D

- Screen space (x, y) , Texture space (u, v)

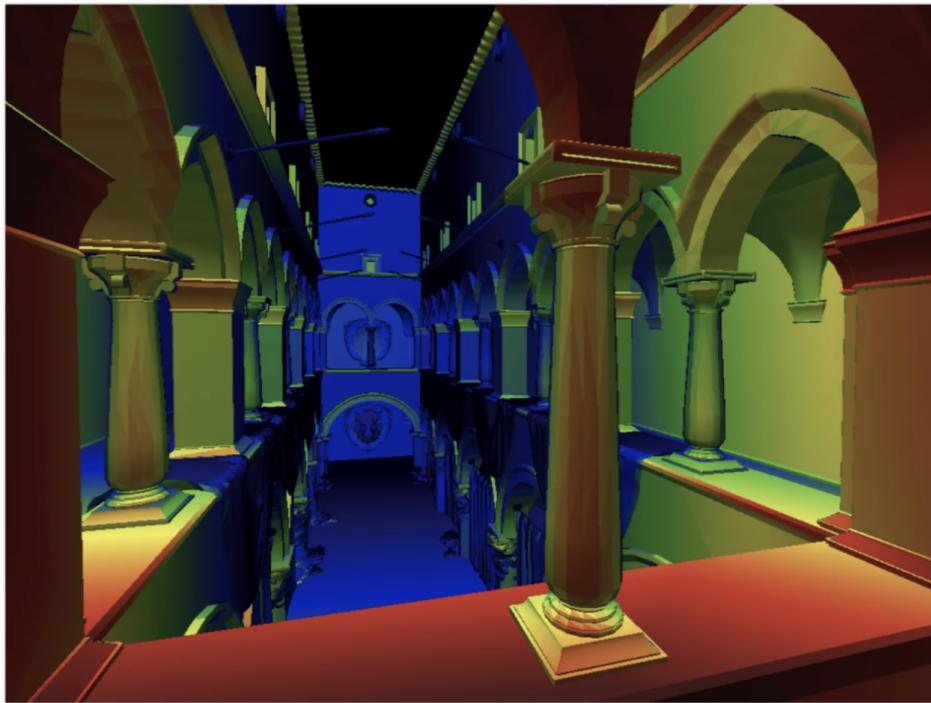


$$D = \log_2 L$$

$$L = \max \left(\sqrt{\left(\frac{du}{dx} \right)^2 + \left(\frac{dv}{dx} \right)^2}, \sqrt{\left(\frac{du}{dy} \right)^2 + \left(\frac{dv}{dy} \right)^2} \right)$$

Visualization of Mipmap level

- Continuous D value

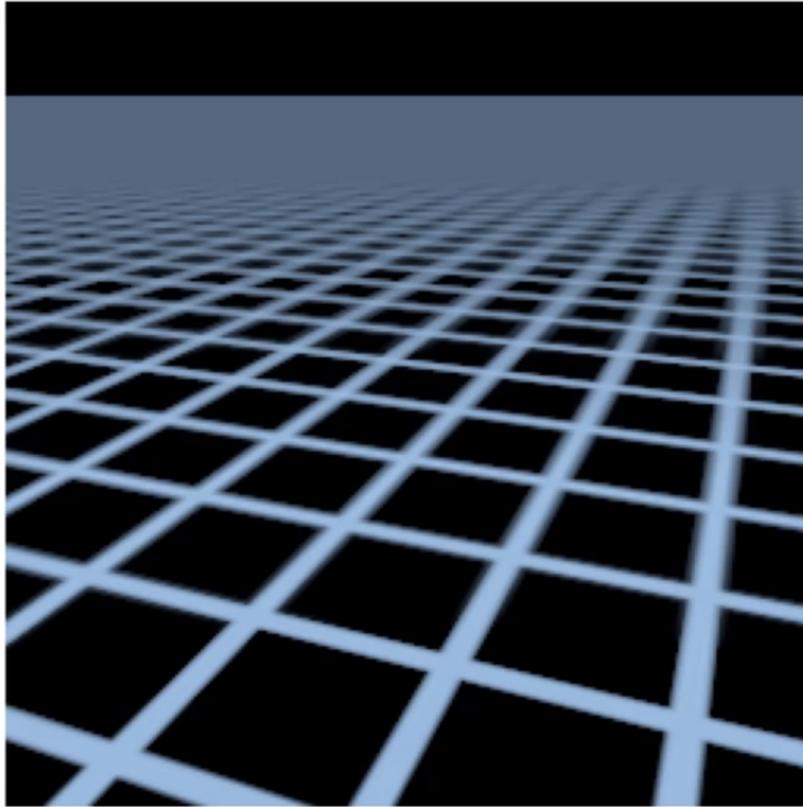


Trilinear filtering: visualization of continuous D

- Interpolate between them
 - Bilinear vs Trilinear Filtering Cost
 - Bilinear resampling
 - 4 texel reads
 - 3 lerps (3 mul + 6 add)
 - Trilinear resampling
 - 8 texel reads
 - 7 lerps (7 mul + 14 add)

Mipmap Limitations

- Overblur in mipmap trilinear sampling



Uses for texturing

- GPUs, texture = memory + filtering

Environment Map

- Function from sphere to colors, stored as a texture

Cube Map

- Vector maps to cube points along that direction, textured with 6 square texture maps
- Give a displacement map to give non perfect
- Bump mapping to put displacement mapping, faster

Things to remember texture mapping

Many uses of texturing

- Bring high-resolution data to fragment calculations
- Colors, normals, lighting on sphere, volumetric data, ...

How does texturing work?

- Texture coordinate parameterization
- Barycentric interpolation of coordinates
- Texture sampling pattern and frequency
- Mipmaps: texture filtering hierarchy, level calculation, trilinear interpolation
- Anisotropic sampling

The Rasterization Pipeline

Surface representations

Lighting and materials

Rasterization Pipeline

Core Concepts

- Sampling
- Antialiasing
- Transforms

Intro

Rasterization

Transforms & Projection

Texture Mapping

Today: Visibility, Shading, Overall Pipeline

Geometric Modeling



Lighting & Materials



Cameras & Imaging

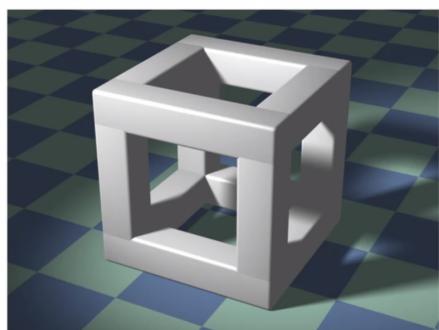


Painter's Algorithm

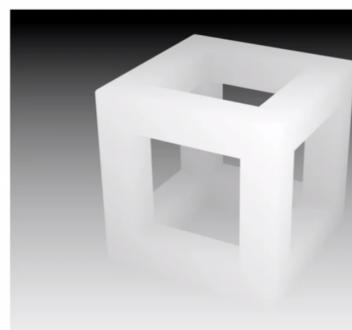
- Back to front, overwrite in the frame buffer

Z-Buffer

- *Hidden surface removal* algorithm that eventually won
- Store current min z value for each sample position



Rendering



Depth buffer

Image credit: Dominic Alves, flickr.

0-1 for the pixel depth close/far from camera

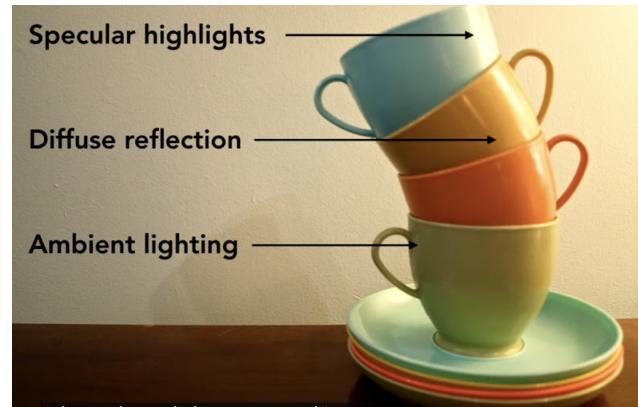
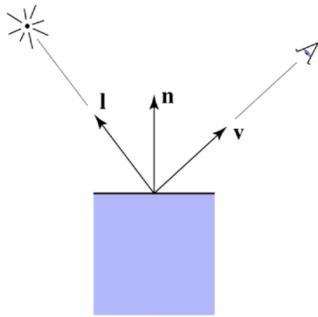
Week 4: Lecture 7 Shading, Geometry, Splines (2/8)

Shading usually takes a lot of computation
Local Shading

Compute light reflected toward camera

Inputs:

- Viewer direction, v
- Surface normal, n
- Light direction, l
(for each of many lights)
- Surface parameters
(color, shininess, ...)



Diffuse Reflection

- Light is scattered uniformly in all directions
- Surface color is the same for all viewing directions

Light Falloff

- Intensity = $\frac{I}{r^2}$

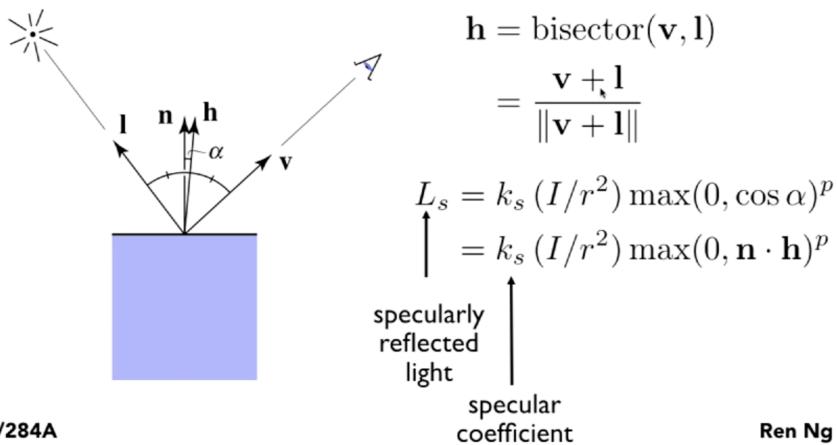
Lambertian Shading

- $L_d = k_d (I/r^2) \max(0, n \cdot 1)$
- k_d : diffuse coefficient
- L_d : diffusely reflected light

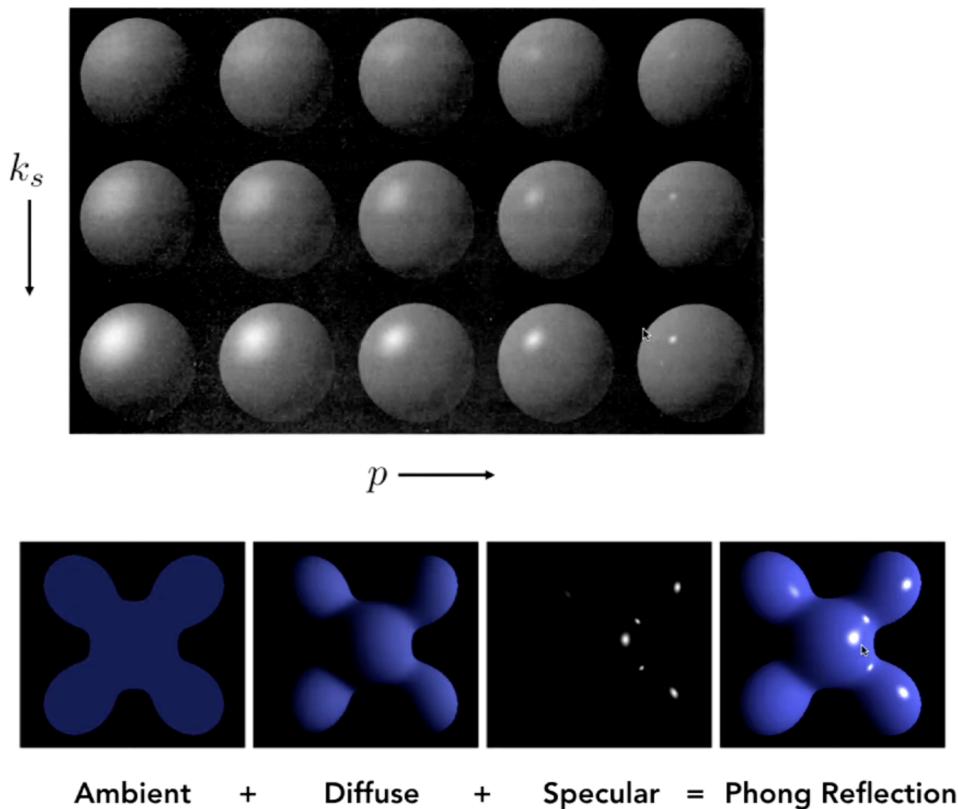
Specular Shading (Blinn-Phong)

Close to mirror direction \Leftrightarrow half vector near normal

- Measure "near" by dot product of unit vectors



$$L_s = k_s (I/r^2) \max(0, \mathbf{n} \cdot \mathbf{h})^p$$



$$\begin{aligned}
 L &= L_a + L_d + L_s \\
 &= k_a I_a + k_d (I/r^2) \max(0, \mathbf{n} \cdot \mathbf{l}) + k_s (I/r^2) \max(0, \mathbf{n} \cdot \mathbf{h})^p
 \end{aligned}$$

Shade each pixel ("Phong" shading)

- Can get normal vector

Shader Programs

- Program vertex and fragment processing stages
- Describe operation on a single vertex

GPU: Heterogeneous, Multi-Core Processor

- Tera-Op's fixed function compute capability over here

Geometry Bezier Curve Splines

Many Ways to represent geometry

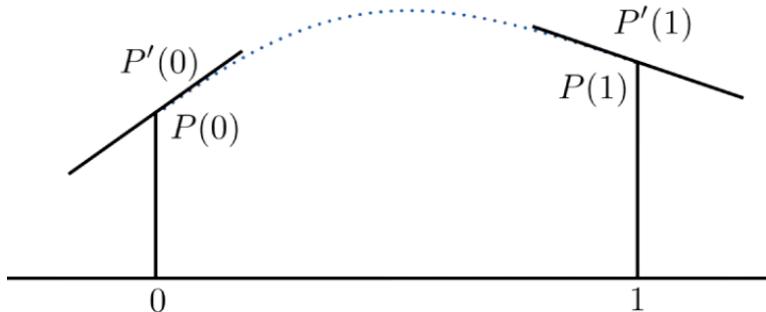
- Explicit
 - Point cloud
 - Polygon mesh
 - subdivision , NURBS

- Implicit
 - Level sets, algebraic surface, distance functions

Spline Topics

- Interpolation
 - Cubic hermite interpolation
 - Catmull rom interpolation

Cubic Hermite Interpolation



Inputs: values and derivatives at endpoints

- Cubic Polynomial Interpolation: $P(t) = at^3 + bt^2 + ct + d$

$$h_0 = d$$

$$h_1 = a + b + c + d$$

$$h_2 = c$$

$$h_3 = 3a + 2b + c$$

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 3 & 2 & 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} h_0 \\ h_1 \\ h_2 \\ h_3 \end{bmatrix}$$

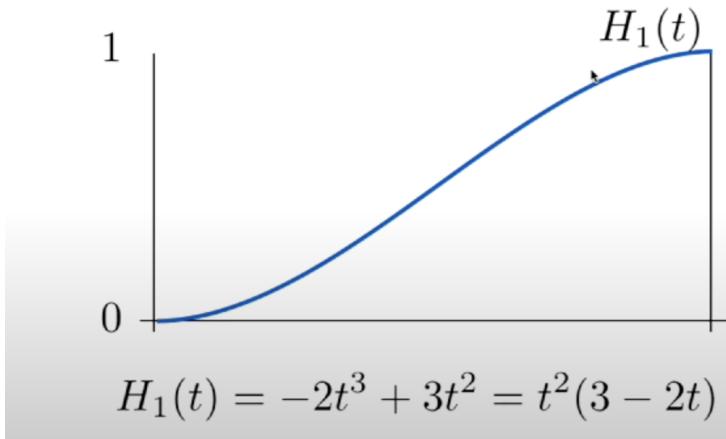
$$\begin{bmatrix} h_0 \\ h_1 \\ h_2 \\ h_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 3 & 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} h_0 \\ h_1 \\ h_2 \\ h_3 \end{bmatrix}$$

Matrix form of hermite function

- Either basis can represent any cubic polynomial through linear combination

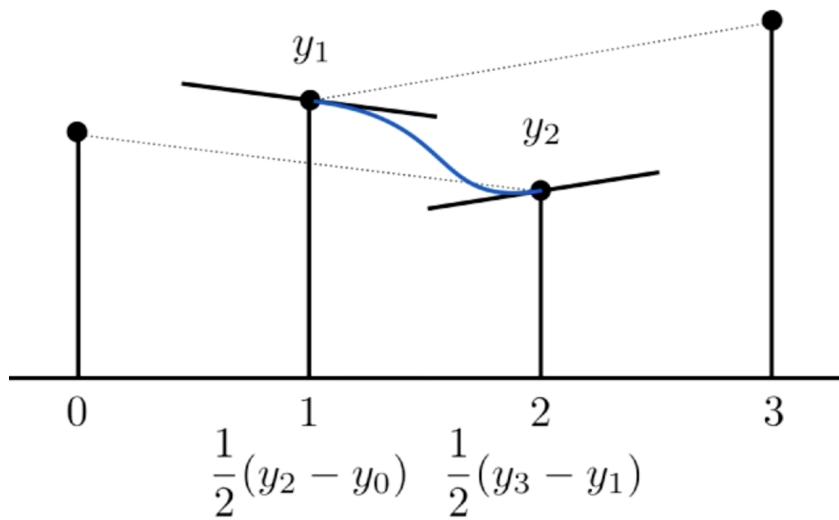
Ease function

- Very useful function, start and stop gently (zero velocity)



Catmull-Rom Interpolation

- Rule for derivatives : match slope between previous and next values

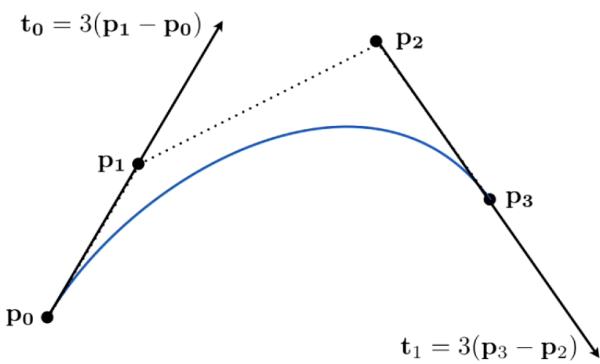


- Catmull-Rom matrix form

$$\begin{aligned}
 P(t) &= \begin{bmatrix} t^3 \\ t^2 \\ t \\ 1 \end{bmatrix}^T \begin{bmatrix} -\frac{1}{2} & \frac{3}{2} & \frac{3}{2} & \frac{1}{2} \\ 1 & -\frac{5}{2} & 2 & -\frac{1}{2} \\ -\frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{p}_0 \\ \mathbf{p}_1 \\ \mathbf{p}_2 \\ \mathbf{p}_3 \end{bmatrix} \\
 &= C_0(t) \mathbf{p}_0 + C_1(t) \mathbf{p}_1 + C_2(t) \mathbf{p}_2 + C_3(t) \mathbf{p}_3
 \end{aligned}$$

Matrix columns = Catmull-Rom basis functions

Bézier curves with tangents



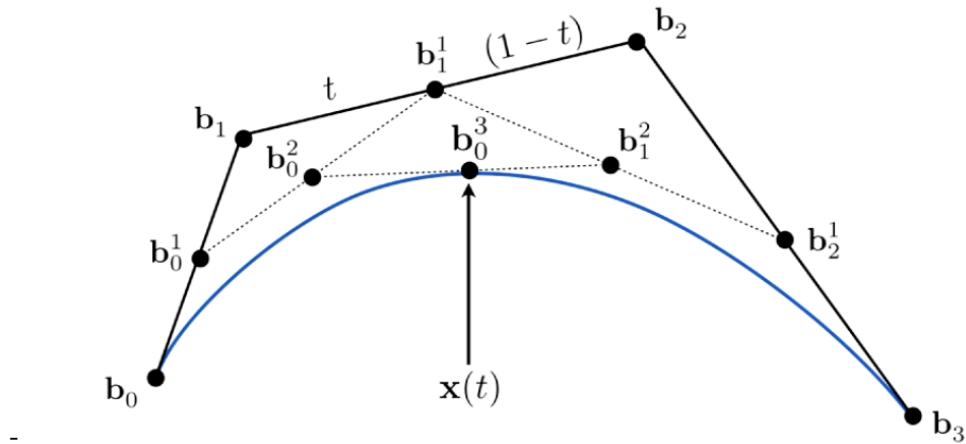
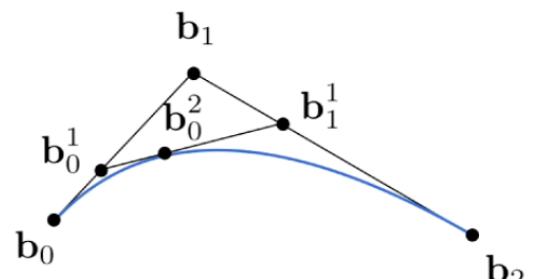
4 points used to define tangent and create Bezier curve

De Casteljau Algorithm

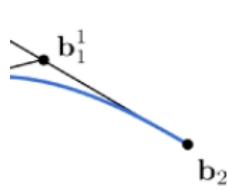
- Insert a point using linear interpolation
- "Corner cutting" recursive subdivision
- Successive linear interpolation

Consider four points

Same recursive linear interpolations



- Can apply in different dimensions



$$\mathbf{b}_0^1(t) = (1-t)\mathbf{b}_0 + t\mathbf{b}_1$$

$$\mathbf{b}_1^1(t) = (1-t)\mathbf{b}_1 + t\mathbf{b}_2$$

—
1

$$\mathbf{b}_0^2(t) = (1-t)^2\mathbf{b}_0 + 2t(1-t)\mathbf{b}_1 + t^2\mathbf{b}_2$$

- Final point on bezier curve

Bernstein form of a Bézier curve of order n:

$$\mathbf{b}^n(t) = \mathbf{b}_0^n(t) = \sum_{j=0}^n \mathbf{b}_j B_j^n(t)$$

Bezier surfaces

- Bicubic bezier patches
- 2D de casteljau algorithm

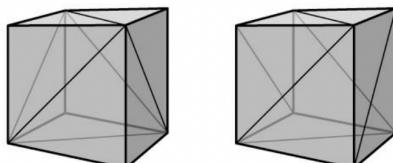
Week 4: Lecture 8 Mesh Representations & Geometry Processing (2/10)

Mesh Representations

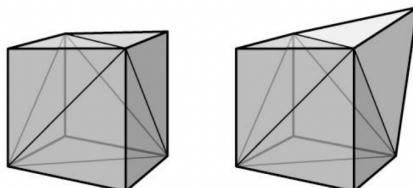
- Triangles, points + triangles

Topology vs Geometry

Same geometry, different mesh topology



Same mesh topology, different geometry



Topological Validity: manifold

- Def: 2D manifold is surface when cut with a small sphere always yields a disk
 - Mesh if manifold, useful properties
 - Edge connects two faces
 - Edge connects two vertices
 - Face consists of a ring of edges and vertices
 - Vertex consists of a ring of edge and faces
 - Euler: $F - E + V = 2$

Triangle-Neighbor Data Structure

- Struct Tri {
 - Vert *v[3]
 - Tri *t[3]
- }

Half-Edge Facilitates Mesh Traversal

- Process vertex, edge, face points to go through a face

Half-Edge : Edge Flip

- Long list of pointer reassignments

Subdivision Surfaces

- Coarse control to manipulate a smooth curve
- Start with coarse polygon mesh
 - Subdivide each element
- Interpolating or approximating
- Continuity at vertices

Core Idea: Let subdivision define the surface

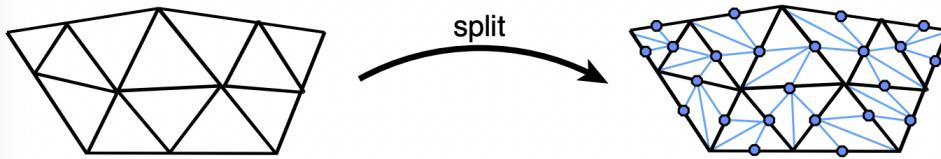
- Evaluation by subdivision
- Evaluation by algebra
- Insight that leads to subdivision surfaces

Loop Subdivision

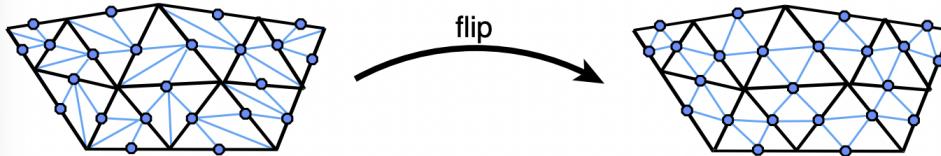
- C2 smoothness away from extraordinary vertices

- Move all edges that touch a new & old vertex and reset it

First, split edges of original mesh in any order:



Next, flip new edges that touch a new & old vertex:



(Don't forget to update vertex positions!)

Catmull-Clark Subdivision

- Each subdivision step
- Add vertex in each face
- Add midpoint on each edge
- Connect all new vertices

Catmull-Clark Vertex Update Rules (General Mesh)

- $f = \text{average of surrounding vertices}$

Sharp Creases

- Make some edges always high frequency

Mesh Simplification

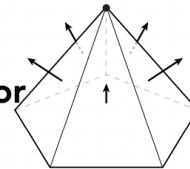
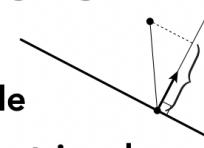
- Goal: reduce number of mesh elements without maintain overall shape

How do We resample meshes

- Iteratively collapse edges
- Edge split is local upsampling
- Assign score with quadric error metric
- Approx distance to surface as sum of distance to planes containing triangles
- Iteratively collapse edge with smallest score
- Greedy algorithm
- Score: surface displacement, symmetric matrix encodes distance to plane
- Quadric Error at vertex: approx distance to vertex triangles as sum of distances to each triangles plane. Encode this as a sinngle quadric matrix for the vertex

Quadratic Error Simplification: Algorithm

- Compute quadratic error matrix Q for each triangle
- Set Q at each vertex to sum of Q s from neighbor triangles
- Set Q at each edge to sum of Q s at endpoints
- Find point at each edge minimizing quadratic error
- Until we reach target # of triangles:
 - collapse edge (i,j) with smallest cost to get new vertex m
 - add Q_i and Q_j to get quadratic Q_m at vertex m
 - update cost of edges touching vertex m



What makes a good triangle mesh

- Delaunay
- Circumcircle interiors contain no vertices

Make triangle more round

- Center vertex
-

Week 5: Lecture 9 Intro to Ray-Tracing (2/15)

Towards Photorealistic rendering

Basic Ray tracing

1. Generate an image by casting one ray per pixel
2. Check for shadows by sending a ray to the light

Generating Eye Rays

- Eye ray \rightarrow light source

Recursive Ray Tracing

- Go from light source to all the other objects
- Specular reflection is mirror ray
- Refractive rays (specular transmission)

Building Eye Rays

- Ray Equation
 - $R(t) = E + t(P - E)$
 - $t \in [1... + \infty]$
 - Through eye at $t = 0$

Shadow Rays

- Test for occluder

- Shade normally
- Skip light
- Self shadowing
- Recursion depth, truncate at fixed number of bounces

Ray Equation

- Ray defined by its origin and direction vector

Ray equation:

$$\mathbf{r}(t) = \mathbf{o} + t \mathbf{d}, \quad 0 \leq t < \infty$$

Plane equation:

$$\mathbf{p} : (\mathbf{p} - \mathbf{p}') \cdot \mathbf{N} = 0$$

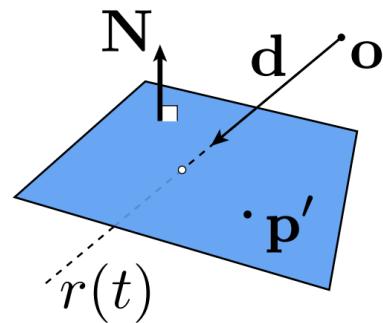
Solve for intersection

Set $\mathbf{p} = \mathbf{r}(t)$ and solve for t

$$(\mathbf{p} - \mathbf{p}') \cdot \mathbf{N} = (\mathbf{o} + t \mathbf{d} - \mathbf{p}') \cdot \mathbf{N} = 0$$

$$t = \frac{(\mathbf{p}' - \mathbf{o}) \cdot \mathbf{N}}{\mathbf{d} \cdot \mathbf{N}}$$

Check: $0 \leq t < \infty$



Can Optimize: e.g. Möller Trumbore Algorithm

$$\vec{\mathbf{O}} + t\vec{\mathbf{D}} = (1 - b_1 - b_2)\vec{\mathbf{P}}_0 + b_1\vec{\mathbf{P}}_1 + b_2\vec{\mathbf{P}}_2$$

Where:

$$\begin{bmatrix} t \\ b_1 \\ b_2 \end{bmatrix} = \frac{1}{\vec{\mathbf{S}}_1 \cdot \vec{\mathbf{E}}_1} \begin{bmatrix} \vec{\mathbf{S}}_2 \cdot \vec{\mathbf{E}}_2 \\ \vec{\mathbf{S}}_1 \cdot \vec{\mathbf{S}} \\ \vec{\mathbf{S}}_2 \cdot \vec{\mathbf{D}} \end{bmatrix}$$

$$\begin{aligned} \vec{\mathbf{E}}_1 &= \vec{\mathbf{P}}_1 - \vec{\mathbf{P}}_0 \\ \vec{\mathbf{E}}_2 &= \vec{\mathbf{P}}_2 - \vec{\mathbf{P}}_0 \\ \vec{\mathbf{S}} &= \vec{\mathbf{O}} - \vec{\mathbf{P}}_0 \end{aligned}$$

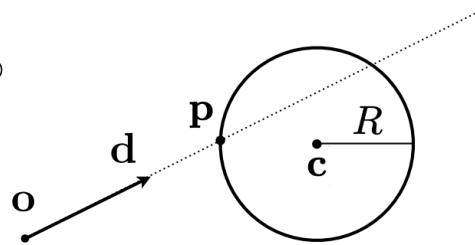
Cost = (1 div, 27 mul, 17 add)

$$\begin{aligned} \vec{\mathbf{S}}_1 &= \vec{\mathbf{D}} \times \vec{\mathbf{E}}_2 \\ \vec{\mathbf{S}}_2 &= \vec{\mathbf{S}} \times \vec{\mathbf{E}}_1 \end{aligned}$$

Ray Intersection With Sphere

Ray: $\mathbf{r}(t) = \mathbf{o} + t \mathbf{d}$, $0 \leq t < \infty$

Sphere: $\mathbf{p} : (\mathbf{p} - \mathbf{c})^2 - R^2 = 0$



Solve for intersection:

$$(\mathbf{o} + t \mathbf{d} - \mathbf{c})^2 - R^2 = 0$$

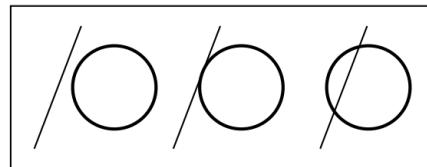
$$at^2 + bt + c = 0, \text{ where}$$

$$a = \mathbf{d} \cdot \mathbf{d}$$

$$b = 2(\mathbf{o} - \mathbf{c}) \cdot \mathbf{d}$$

$$c = (\mathbf{o} - \mathbf{c}) \cdot (\mathbf{o} - \mathbf{c}) - R^2$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



Bound each object with a bounding box and test bvol first, then test object if it hits

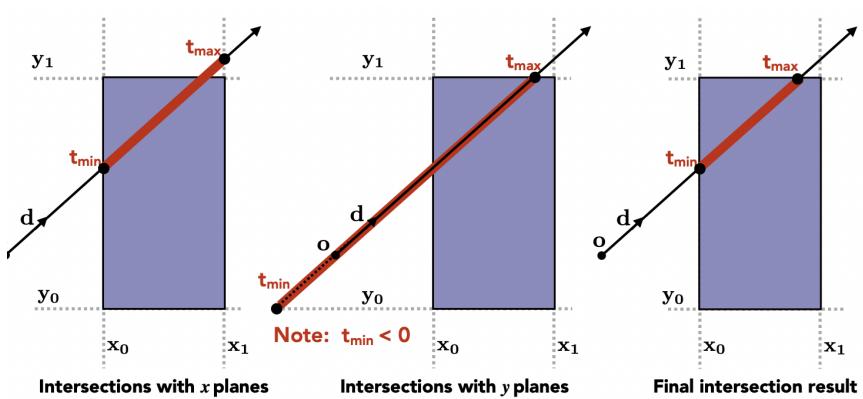
Week 5: Lecture 10 Ray-Tracing (2/17)

Ray Tracing - Acceleration

- Pixels * $\log(\# \text{ objects})$

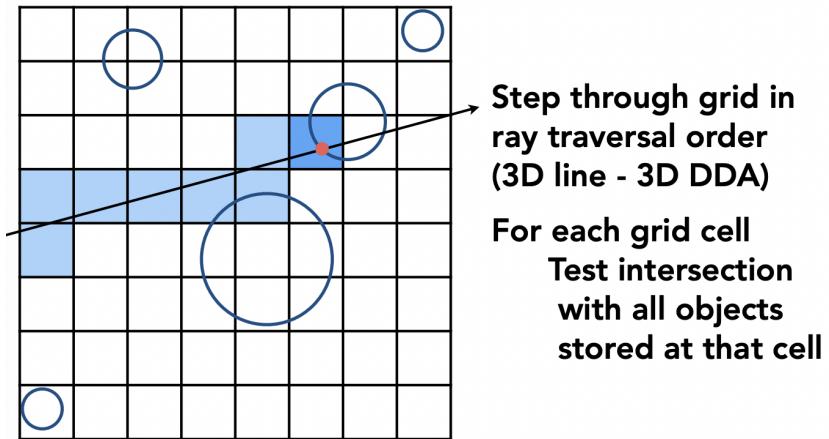
Bounding Volumes

- Axis aligned bounding box
- Find



- Ray intersection with Axis aligned base
- When it enters and exits the box

Uniform Spatial Partitions (Grids)

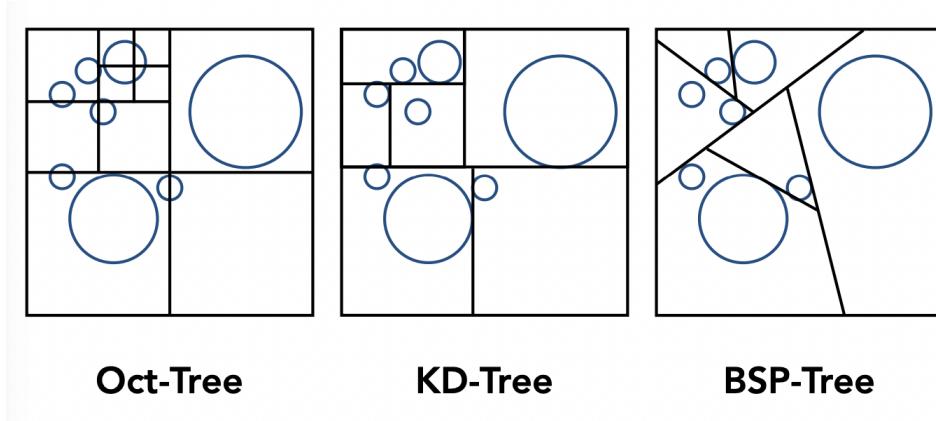


- Check if intersection is inside the cell

Uniform Grids - when they fail

- Teapot in a stadium

Non-Uniform Spatial Partition: Spatial Hierarchies



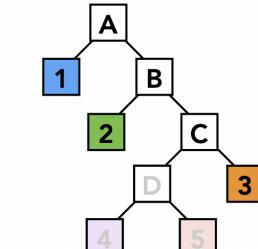
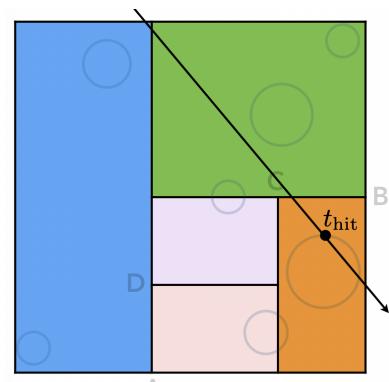
Oct-Tree

KD-Tree

BSP-Tree

KD Trees

- Internal nodes store
 - Split axis: x-, y-, z-
 - KD Tree Preprocessing
- Pre-processing
 - Choosing the split plane
 - Simple: midpoint, median split
 - Ideal: split to minimize expected cost of ray intersection
- Termination Criteria
 - Simple: common to prescribe maximum tree depth
 - Ideal: stop when splitting does not reduce expected cost of ray intersection



Intersection found

Object Partitions & Bounding Volume Hierarchy (BVH)

Spatial Partition (KD-tree)

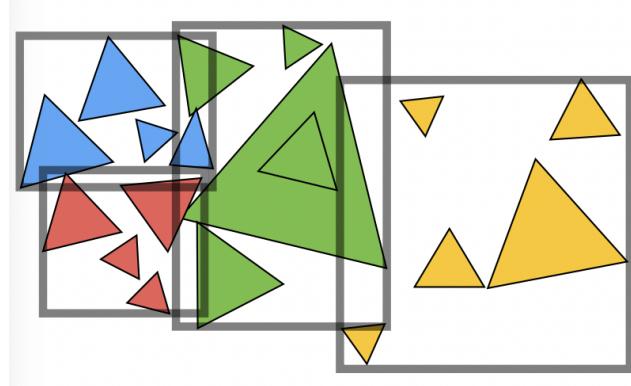
- Partition space into non-overlapping regions
- Objects can be contained in multiple regions

Object partition (BVH)

- Partition set of objects into disjoint subsets
- Bounding boxes for heaps

BVH

- BVH Preprocessing
 - Find bounding box
 - Recursively split into set of objects in two subsets
- BVH Recursive Traversal
 - If ray misses node.bbox : return
 - If node is leaf node
 - Test intersection with all objs
 - Return closest intersection
 - Hit1 = intersect(ray, node.child1)
 - Hit2 = intersect(ray, node.child2)
 - Return closer of hit1, hit2



How to split into two sets (BVH)

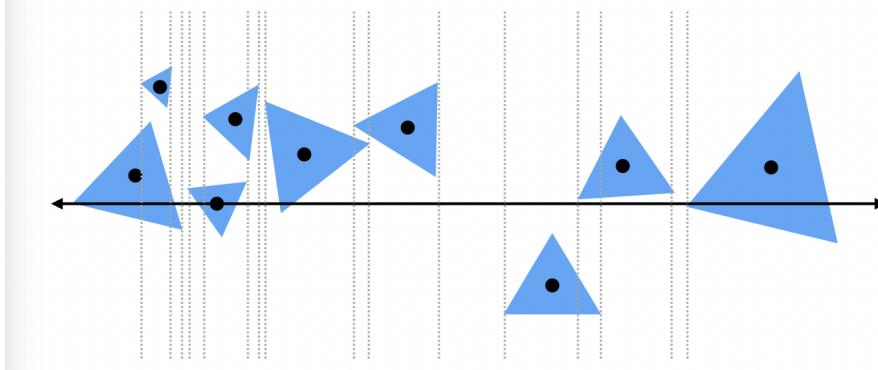
- Want a split that works best spatial, sometimes almost all triangles in one side sometimes half half

Which Hierarchy is fastest

- A good partition minimizes the average cost of tracing a ray
 - csot(node)
- Surface Area Estimating the cost with a heuristic

Constrain search to axis-aligned spatial partitions

- Choose an axis
- Choose a split plane on that axis
- Partition objects into two halves by centroid
- **2N-2 candidate split planes for node with N primitives. (Why?)**



Things to remember

- Linear vs log ray-intersection techniques
- Many techniques for accelerating ray-intersection

Week 6: Lecture 11 Radiometry/Photometry (2/22)

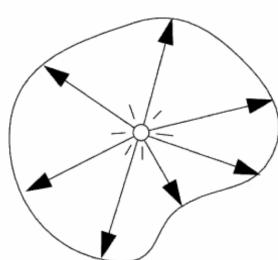
Lights

Radiant Energy and Flux (Power)

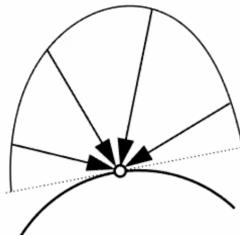
- Radiant energy is the energy of electromagnetic radiation in Joules
 - Q [J = Joule]
- Radiant flux is energy emitted, reflected, transmitted or received, per unit time

Photometry

- Accounts for the response of the human visual system



**Light Emitted
From A Source**



**Light Falling
On A Surface**



**Light Traveling
Along A Ray**

Radiant Intensity

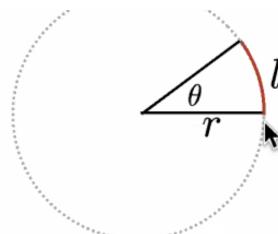
- Radiant intensity is power per unit solid angle emitted by a point light source
- $I(\omega) = \frac{d\phi}{d\omega}$

Angles and Solid Angles

Angle: ratio of subtended arc length on circle to radius

$$\bullet \theta = \frac{l}{r}$$

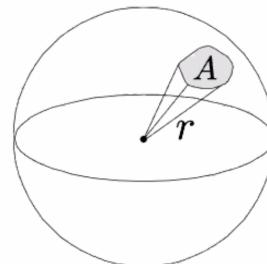
• Circle has 2π radians



Solid angle: ratio of subtended area on sphere to radius squared

$$\bullet \Omega = \frac{A}{r^2}$$

• Sphere has 4π steradians



- $d\omega = \frac{dA}{r^2} = \sin \theta d\theta d\phi$
- Very important for rendering when you are far away

Radiance

1. Fundamental field quantity that describes the distribution

Surface Radiance

- Reflected radiance is power emitted, reflected, transmitted or received by a surface, per unit solid angle, per unit projected area

$$L(p, \omega) \equiv \frac{d^2\Phi(p, \omega)}{d\omega dA \cos \theta}$$

$\cos \theta$ accounts for projected surface area

$$\left[\frac{W}{m^2} \right] \left[\frac{cd}{m^2} = \frac{lm}{sr m^2} = nit \right]$$

Incident & Exiting surface radiance differ

- Distinguish between incident radiance and exitant radiance functions

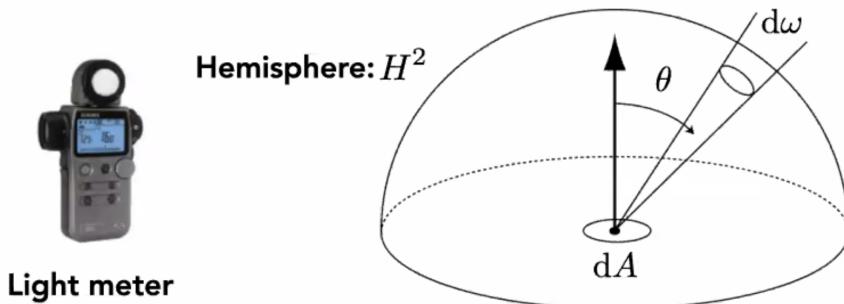
Irradiance from the Environment

Computing flux per unit area on surface, due to incoming light from all directions.

$$dE(p, \omega) = L_i(p, \omega) \cos \theta d\omega$$

Contribution to irradiance from light arriving from direction ω

$$E(p) = \int_{H^2} L_i(p, \omega) \cos \theta d\omega$$



Multi-Camera Array -> 4D light field

Spherical Gantry -> 4D light field

- Take photographs of an object from all points on an enclosing sphere
- Captures all light leaving an object like a hologram

Week 6: Lecture 12 Monte Carlo Integration (2/24)

High Dimensional Integration

Complete set of samples: $N = \underbrace{n \times n \times \cdots \times n}_d = n^d$

- “Curse of dimensionality”

Numerical integration error: Error $\sim \frac{1}{n} = \frac{1}{N^{1/d}}$

Random sampling error: Error = Variance $^{1/2} \sim \frac{1}{\sqrt{N}}$

In high dimensions, Monte Carlo integration requires fewer samples than quadrature-based numerical integration

Monte Carlo Integration

- Idea: estimate integral based on random sampling of function
- Advantages
 - General and relatively simple method
 - Requires only function evaluation at any point
 - Good for general functions
 - Efficient for high-dimensional integrals
- Disadvantages
 - Noise: integral estimate is random, only correct “on average”
 - Can be slow to converge need a lot of samples

Random Variables

- X

Probability Distribution Function

- N discrete values x_i with probability p_i
- $\sum_{i=1}^n p_i = 1, p_i \geq 0$

Continuous Probability Distribution Function

- $X \sim p(x)$
- Expected value of X $E[X] = \int xP(x) dx$

Monte Carlo Integration

- Estimate the integral of a function by averaging random samples of the function’s value
- Define the monte carlo estimator for the definite integral of given function $f(x)$

Definite integral

$$\int_a^b f(x)dx$$

Note: $p(x)$ must
be nonzero for
all x where
 $f(x)$ is nonzero

Random variable

$$X_i \sim p(x)$$

Monte Carlo estimator

$$F_N = \frac{1}{N} \sum_{i=1}^N \frac{f(X_i)}{p(X_i)}$$

Basic Monte Carlo Estimator

- Sampel with a uniform random variable
- Uniform random variable
 - $X_i \sim p(x) = C$

Unbiased Estimator

- A randomized integral estimator is unbiased if its expected value is the desired integral

Proof Monte carlo unbiased

$$\begin{aligned} E[F_N] &= E \left[\frac{1}{N} \sum_{i=1}^N \frac{f(X_i)}{p(X_i)} \right] \\ &= \frac{1}{N} \sum_{i=1}^N E \left[\frac{f(X_i)}{p(X_i)} \right] \\ &= \frac{1}{N} \sum_{i=1}^N \int_a^b \frac{f(x)}{p(x)} p(x) dx \\ &= \frac{1}{N} \sum_{i=1}^N \int_a^b f(x) dx \\ &= \int_a^b f(x) dx \end{aligned}$$

**Properties of
expected values:**

$$\begin{aligned} E \left[\sum_i Y_i \right] &= \sum_i E[Y_i] \\ E[aY] &= aE[Y] \end{aligned}$$

Expected value of monte carlo is the desired integral

Variance of a RV

Definition

$$\begin{aligned} V[Y] &= E[(Y - E[Y])^2] \\ &= E[Y^2] - E[Y]^2 \end{aligned}$$

Variance decreases linearly with number of samples

$$V\left[\frac{1}{N} \sum_{i=1}^N Y_i\right] = \frac{1}{N^2} \sum_{i=1}^N V[Y_i] = \frac{1}{N^2} N V[Y] = \frac{1}{N} V[Y]$$

Properties of variance

$$V\left[\sum_{i=1}^N Y_i\right] = \sum_{i=1}^N V[Y_i] \quad V[aY] = a^2 V[Y]$$

Direct Lighting Estimate

- Idea: sample directions over hemisphere uniformly in solid angle
- $E(p) = \int L(p, \omega) \cos \theta d\omega$

Given surface point p

Initialize Monte Carlo estimator F_N to 0

For each of N samples:

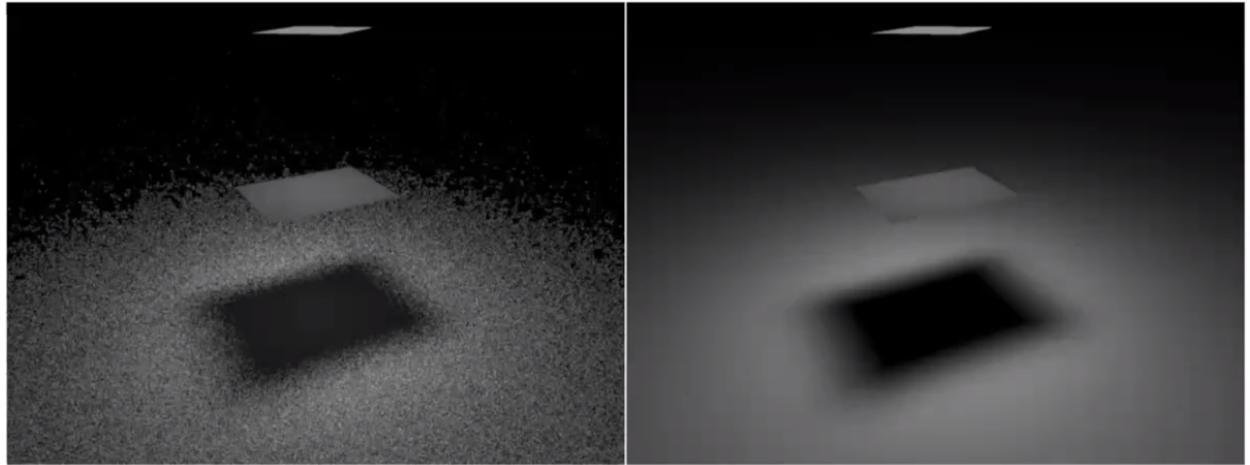
A ray tracer evaluates radiance along a ray

Generate random direction: ω_i

Compute incoming radiance L_i arriving at p from direction ω_i

Increment the Monte Carlo estimator: $F_N := F_N + \frac{2\pi}{N} L_i \cos \theta_i$

- MC estimator uses different random directions at each pixels, only some directions point towards the light
- Important sampling

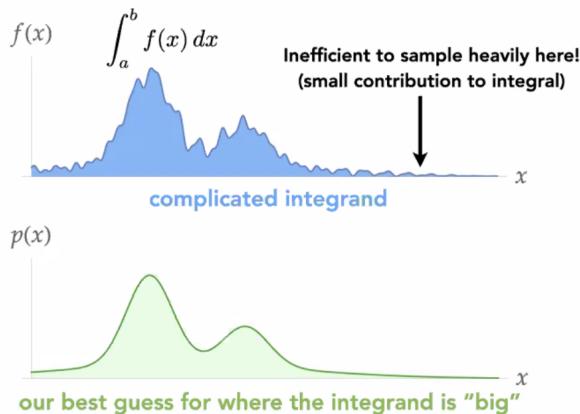


Sampling solid angle

100 random directions on hemisphere

Importance Sampling

- Sample the integrand according to how much we expect it to contribute to the integral



Sampling light source area

100 random points on area of light source

Basic Monte Carlo:

$$\frac{b-a}{N} \sum_{i=1}^N f(X_i)$$

(X_i are sampled uniformly)

Importance-Sampled Monte Carlo:

$$\frac{1}{n} \sum_{i=1}^n \frac{f(x_i)}{p(x_i)}$$

(x_i are sampled proportional to p)

Changing Basis of Integration: sampling hemisphere \rightarrow sampling light source area

$$E(p) = \int_{A'} L_o(p', \omega') V(p, p') \frac{\cos \theta \cos \theta'}{|p - p'|^2} dA' \leftarrow \text{Change of variables to integral over area of light}$$

$d\omega = \frac{dA' \cos \theta'}{|p' - p|^2}$

Randomly sample light source area A' (assume uniformly over area)

$$\int_{A'} p(p') dA' = 1$$

$$p(p') = \frac{1}{A'}$$

Monte Carlo Estimator

$$F_N = \frac{A'}{N} \sum_{i=1}^N Y_i$$

$$Y_i = L_o(p'_i, \omega'_i) V(p, p'_i) \frac{\cos \theta_i \cos \theta'_i}{|p - p'_i|^2}$$

How to draw samples from a desired probability Distribution: Inversion Method

- Task: Draw a RV from a given PDF
 - Draw a random value X from this PDF
1. Calculate cumulative pdf
 2. Given a uniform RV, choose $X = x_i$ st $P_{i-1} < \zeta \leq P_i$, calculate inverse
 - 3.

Week 7: Lecture 13 Global Illumination & Path Tracing (3/1)

Material Reflection

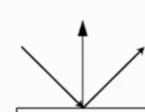
- Ideal Specular
 - Perfect mirror reflection
- Ideal Diffuse
 - Equal reflection in all directions
- Glossy Specular
 - Majority of light reflected near mirror direction
- Retro Reflective

Reflection at a point

- Light integrated

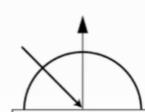
Ideal specular

- Perfect mirror reflection



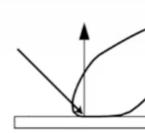
Ideal diffuse

- Equal reflection in all directions



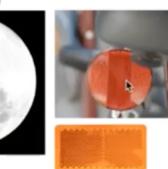
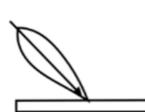
Glossy specular

- Majority of light reflected near mirror direction



Retro-reflective

- Light reflected back towards light source



Diagrams illustrate how light from

- Solving the reflection equation

$$L_r(p, \omega_r) = \int_{H^2} f_r(p, \omega_i \rightarrow \omega_r) L_i(p, \omega_i) \cos \theta_i d\omega_i$$

- Use monte carlo integration
- Direct lighting, uniform RV

Global Illumination: deriving the rendering equation

- Reflected radiance depends on incoming radiance
- Rewrite as transport function:

- **Emitted radiance function**
(all surface points & outgoing directions) $L_e(p, \omega)$

- **Incoming/outgoing reflected radiance**
(all surface points & in/out directions) $L_i(p, \omega), L_o(p, \omega)$

- **Transport function - returns the first scene intersection point along given ray** $tr(p, \omega)$

- **Reflection operator:**

$$R(g)(p, \omega_o) \equiv \int_{H^2} f_r(p, \omega_i \rightarrow \omega_o) g(p, \omega_i) \cos \theta_i d\omega_i$$

$$R(L_i) = L_o$$

- **Transport operator:** $T(f)(p, \omega_o) \equiv f(tr(p, \omega), -\omega)$

$$T(L_o) = L_i$$

- Natural scenes alot of the light comes from other sources of lights, bouncing lights
- 1 bounce path connecting ray to light, can bounce back to the light source
- Sum over all paths of all lengths

Try Monte Carlo sum over paths

Russian Roulette- Unbiased random termination

- Evaluate original estimator with probability p reweighted and has the same expected value of the original estimator

Week 7: Lecture 14 Path Tracing (3/3)

Path Tracing Overview

- Terminate paths randomly with Russian Roulette
- Partition the recursive radiance evaluation.
 - Direct lighting, indirect lighting
- Monte Carlo estimate for each partition separately,

Path Tracing Code

- At least one bounce will get the bounce after one bounce

```

AtLeastOneBounceRadiance(p, wo)           // out at p
    L = OneBounceRadiance(p, wo);           // direct il

    wi, pdf = p.brdf.sampleDirection(wo);      // Imp. sa
    p' = intersectScene(p, wi);
    cpdf = continuationProbability(p.brdf, wi, wo);
    if (random01() < cpdf)                   // Russ. Rou
        L += AtLeastOneBounceRadiance(p', -wi) // Recursive
            * p.brdf(wi, wo) * costheta / pdf / cpdf; // indirect
    return L;

OneBounceRadiance(p, wo)           // out at p
    return DirectLightingSampleLights(p, wo); // direct

```

Global illumination

- Multiple all bounces of light

Multiple Light sources

- Consider multiple lights in direct lighting estimate
- Loop over all n lights, sum Monte-carlo estimates

Summary Intuition on Global Illumination and Path Tracing

- Trace N paths through a pixel, sample radiance
- Build paths by recursively tracing to next surface points and choosing a random reflection direction, use reussion roulette to kill probabilities, use improtance sampling to reduce noise

Point lights / Ideal Specular materials

Floating Point Precision Remedies

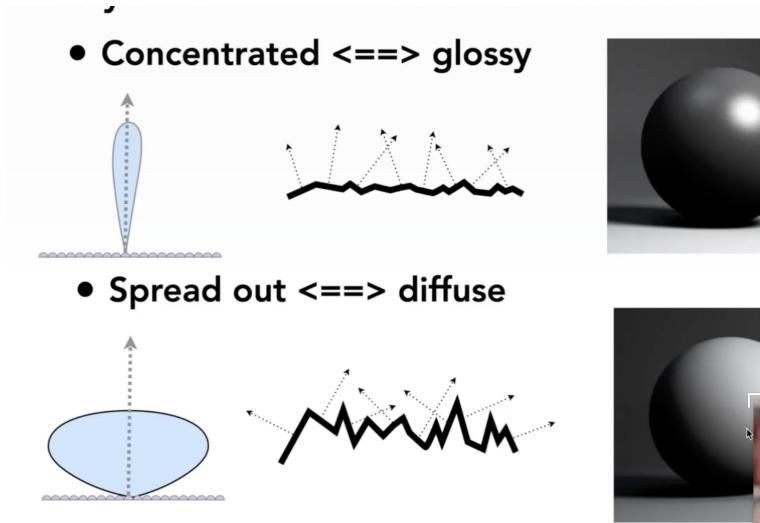
- Double rather than float
- Ignore reintersection with the last object hit
- Offset origin along ray to ignore
- Preject intersection point to surface

Introduction to Material Modeling

Perfect specular reflection

Microfacet Theory

- Macroscale: flat& rough
- Micro scale: bumpy & specular

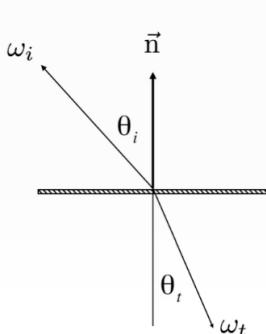


- Individual elements of surfaces look like mirrors
- Fresnel Reflection Term
- Reflectance increases with grazing angle

Microfacet BRDF

- Distribution of microfacet's normals

Law of Refraction



$$\begin{aligned} \eta_i \sin \theta_i &= \eta_t \sin \theta_t \\ \cos \theta_t &= \sqrt{1 - \sin^2 \theta_t} \\ &= \sqrt{1 - \left(\frac{\eta_i}{\eta_t}\right)^2 \sin^2 \theta_i} \\ &= \sqrt{1 - \left(\frac{\eta_i}{\eta_t}\right)^2 (1 - \cos^2 \theta_i)} \end{aligned}$$

Anisotropic BRDFs

- Point light + Metal = Rough / Elliptical highlight

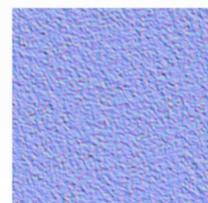
Isotropic : directionality of underlying surface

Create glint with surface normal distribution

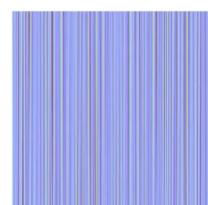
Reflection off fibers

Kamiya-Kay Model

Isotropic



Anisotropic

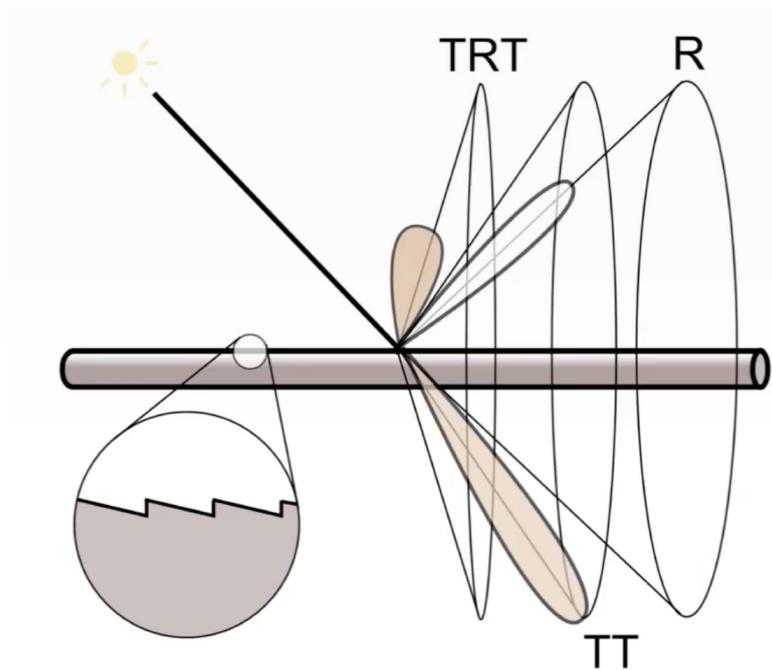


Surface (normals)



BRDF (fix wi, vary wo)

- If tiny bounces different when bouncing in a cylinder



- Reflect into multiple cones

Fur appearance

- fur fibers have a more different rendering object, need to have a different model with a bigger medulla
- Fog must be partially absorbed and scattered

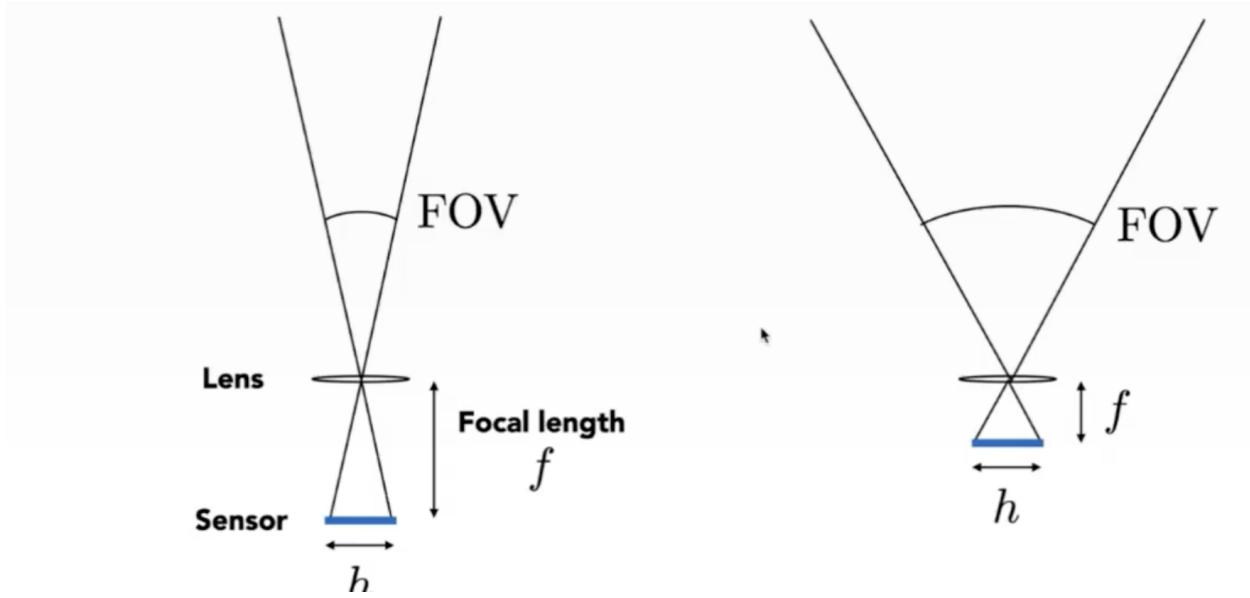
Week 8: Lecture 15/16 Cameras & Lenses (3/8)

Image Capture Overview

- DSLR: opens up lense for sensor

Optics of Image Formation: Field of View

- Effect of Focal Length on FOV



Effect of Sensor size of FOV

- full frame sensor, larger picture

Focal Length v Field of View

- Changing focal length on smartphones

- 17mm is wide angle 104 degree
- 50mm is a "normal" lens 47
- 200mm is telephoto lens 12 "degree"

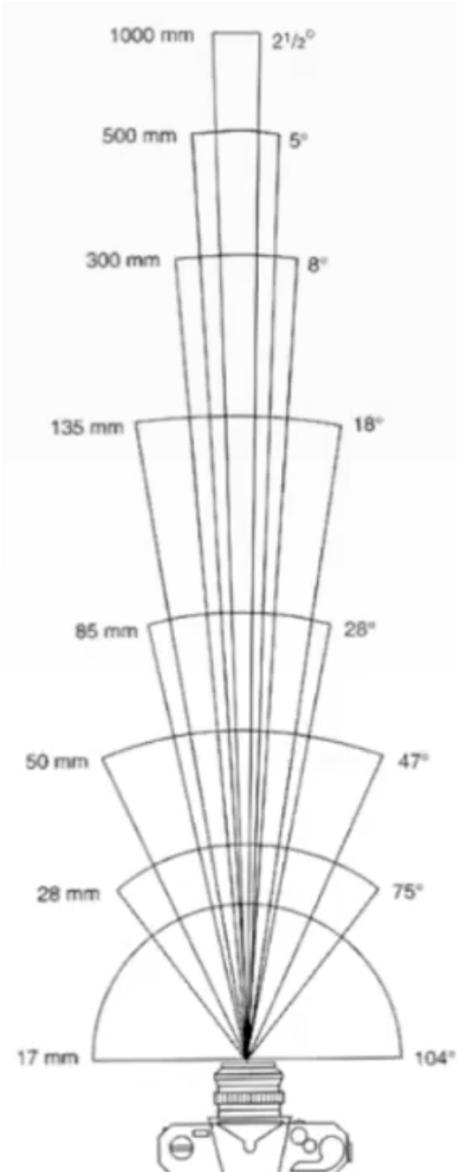
"Choose your perspective before you choose yours lens"

Improve your Own Photography

1. Make sure you have a strong subject, $\frac{1}{3}$ of the image
2. Choose a good perspective relationship (relative size) between subject and background
 - a. Complement don't compete with the subject
3. Change the zoom and camera distance to your subject
 - a. Actively zoom, and move your camera

Exposure: Fast and slow photography

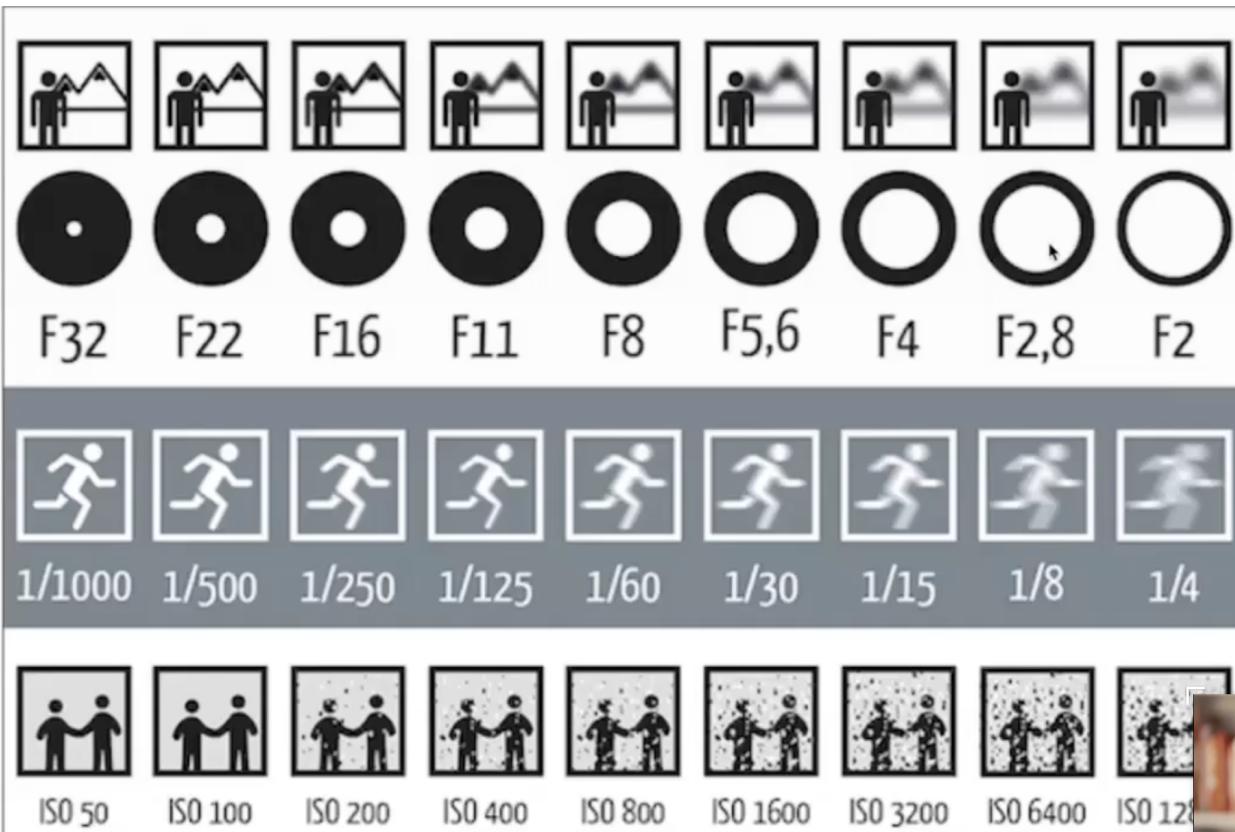
- High-Speed photography
 - Objects you can't see
- Long exposure
 - Leave light on
 - $Q = T \times E$
 - Exposure = time x irradiance
 - Exposure time (T)
 - Controlled by shutter



- Irradiance (E)
 - Power of light falling on a unit area of sensor
 - Controlled by lens aperture and focal length

Exposure levels

- 1 "stop" = 2x exposure
- Bracketing with +/- 1 stop exposure



F-Number of a lens

- defined as the focal length divided by the diameter of the aperture
- Common f-stop on real lenses: 1.4, 2, 2.8, f
- 1 stop doubles exposure

Exposure controls; Aperature, shutter, gain (ISO)

- Aperture size: change the f-stop by opening/ closing the Aperature
- Shutter Speed: Change the duration the sensor pixels integrate light
- ISO: change the amplification between sensor values and digital image value s

Constant exposure: f-stop vs shutter speed

- pairs of aperture and shutter speed give equivalent exposure
- Too bright/dark can adjust one of them

Lower iso has less noise

- more iso has more noise

Electornic Shutter

- Pixel is electronically reset to start exposure
- Fills with photoelectrons as light falls on sensor
- Reading out pixel electronically "ends" exposure
- Most sensors read out pixels sequentially to read entire sensor
- Rolling shutter artifact
 - Mechanical don't suffer from the artifact as much

Lenses

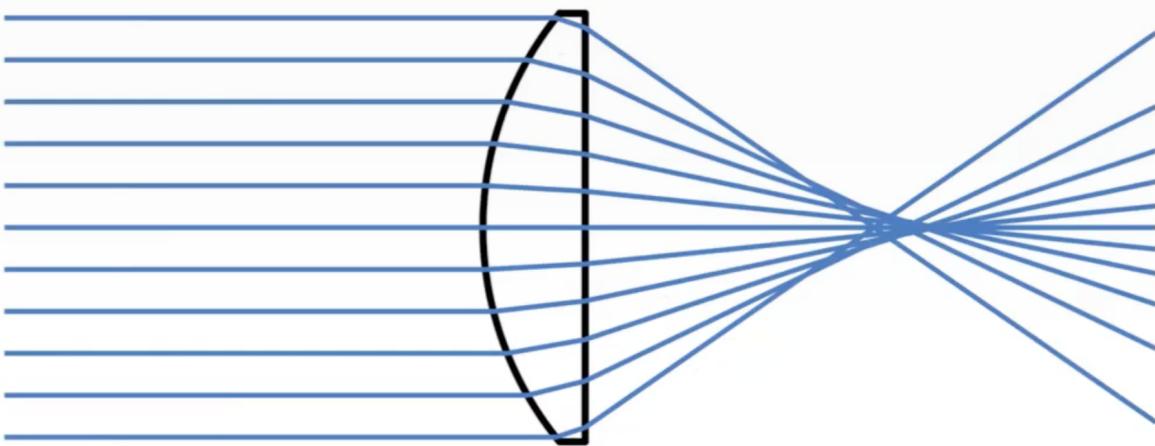
- Thin lens approximation
- Assume all parallel rays entering a lens pass through its focal point

Week 8: Lecture 16 Cameras & Lenses (3/10)

Lenses

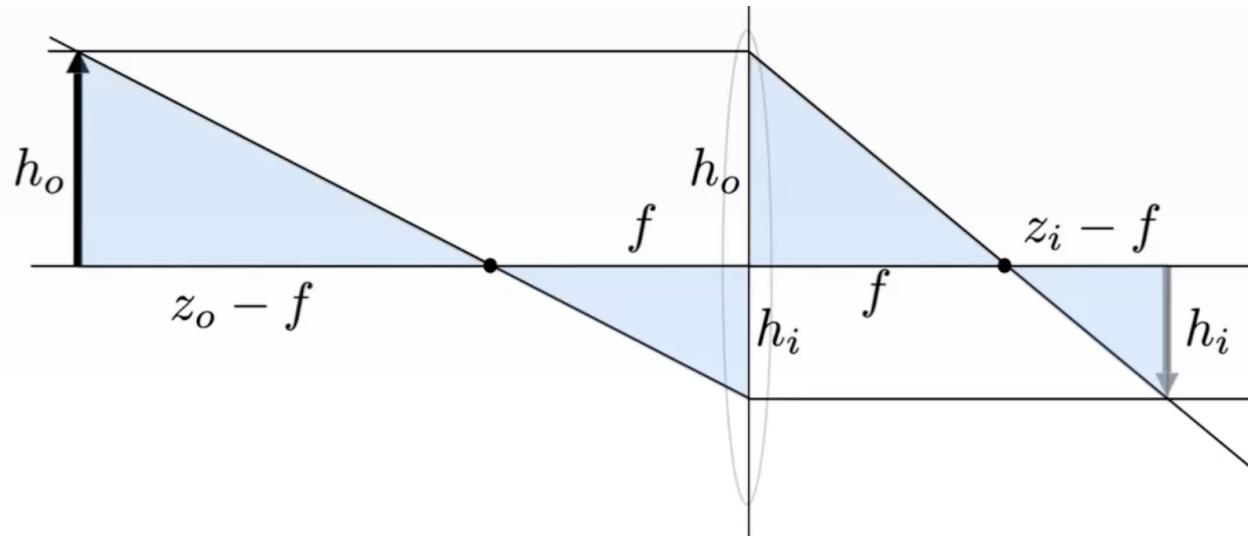
Aberrations

- not ideal convergence



Ideal Thin Lens: parallel rays entering lens pass through its focal point

Gauss' Ray Diagrams



$$\frac{h_o}{z_o - f} = \frac{h_i}{f}$$

$$\frac{h_o}{f} = \frac{h_i}{z_i - f}$$

$$\frac{z_o - f}{f} = \frac{f}{z_i - f}$$

Object / image height factor out - applies to

$$(z_o - f)(z_i - f) = f^2$$

Newtonian Thin Lens

$$z_o z_i - (z_o + z_i) f + f^2 = f^2$$

$$z_o z_i = (z_o + z_i) f$$

$$\frac{1}{f} = \frac{1}{z_i} + \frac{1}{z_o}$$

Gaussian Thin Len

Z_i and Z_o are conjugate points

Magnification example - focus at infinity

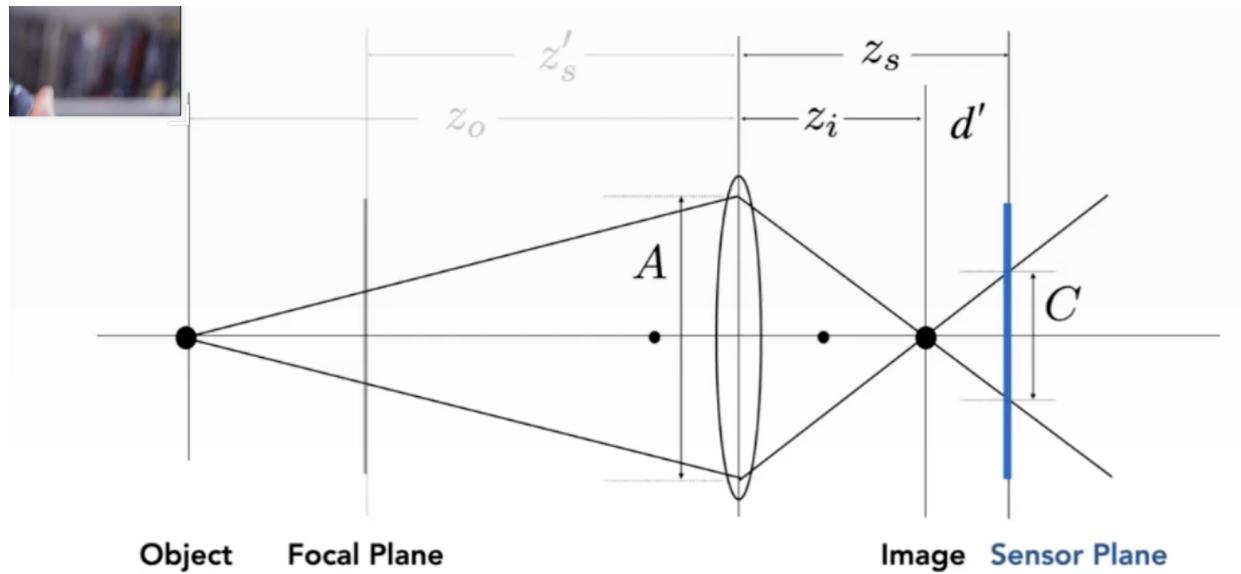
Thin Lens

- compressed in depth for low magnification
- 1:1 in 4d for unit
- Stretched for high magnification

Defocus blur

- size of blur kernel depends on depth from focal plane

- Only see the blur kernel itself if you have a point light



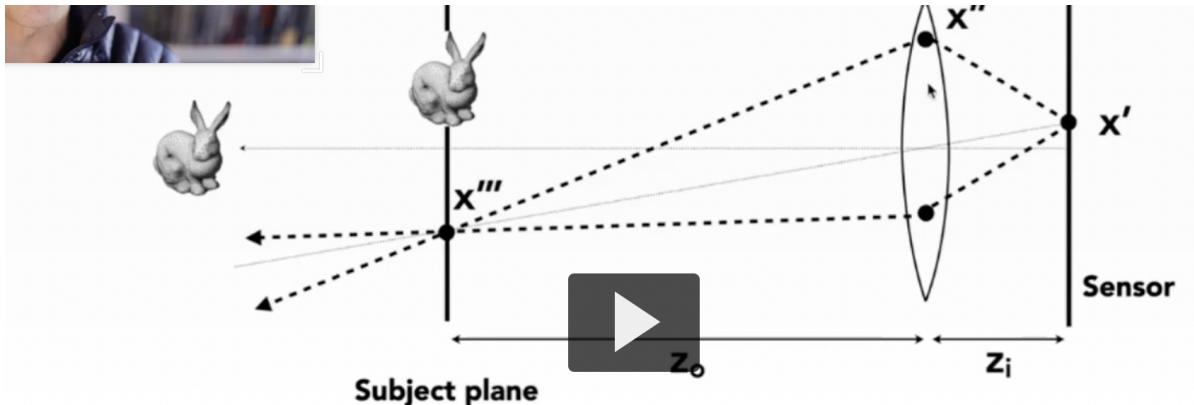
Circle of confusion is proportional to the size of the aperture

$$\frac{C}{A} = \frac{d'}{z_i} = \frac{|z_s - z_i|}{z_i}$$

Exposure Tradeoffs

- Depth of Field vs Shutter speed
- Larger blur when larger f stop

Ray Tracing for Defocus Blur (thin Lens)



To compute value of pixel at position x' by Monte Carlo integration:

- Select random points x'' on lens plane
- Rays pass from point x' on image plane z_i through points x'' on lens
- Each ray passes through conjugate point x''' on the plane of focus z_o
 - Can determine x''' from Gauss' ray diagram
 - So just trace ray from x'' to x'''
- Estimate radiance on rays using path-tracing, and sum over all points x''

Bokeh

- Shape and quality of the out-of-focus blur

Modern Lens Designs are highly complex

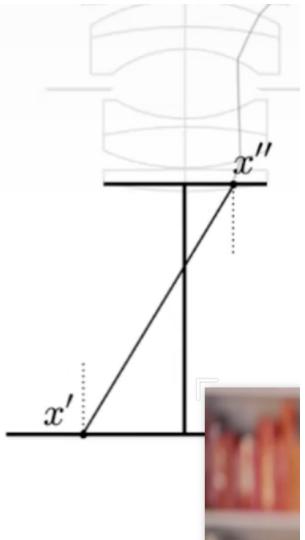
- many pieces of glass
- Snails law
 - Designed to provide optical lens point
 - Ray tracing through real lens designs

Ray Tracing Real Lens Designs

- Monte Carlo approach: compute integral of rays incident on pixel area arriving from all paths through the lens

Algorithm (for a pixel)

- Choose N random positions in pixel
- For each position x' , choose a random position on the back element of the lens x''
- Trace a ray through from x' to x'' , trace refractions through lens elements until it misses the next element (kill ray) or exits the lens (path trace through the scene)
- Weight each ray according to radiometric calculation on next slide to estimate irradiance $E(x')$



Week 9: Lecture 17 Intro to Animation (3/15)

Rigging skeleton structure, skimming: putting skin on Animation

- Bring things to life, aesthetic issues often dominate technical issues
- An extension of modeling
- Output: sequence of images provide a sense of motion

First Film

- Scientific tool rather than entertainment
- Development of animation

Animation Principles

- Squash and Stretch:
 - defining the rigidity and mass of an object by distorting its shape during an action, shape of object changes during movement not its volume
- Anticipation:
 - prepare for each movement for physical realism, direct audience attention
- Staging:
 - Picture is 2D, make situation clear, audience looking in right place, action clear in silhouette
- Follow Through:
 - overlapping motion, motion doesn't stop suddenly, continue at different rates, one motion starts while previous is finishing
- Ease-In and Ease-Out:
 - movement doesn't start & stop abruptly
- Arcs:
 - Move in curves, not in straight lines
- Secondary Action:

- Motion that results from some other action, needed for interest and realism
- Timing
 - Rate of acceleration conveys weight, speed and acceleration of character's movements convey emotion
- Exaggeration
 - Helps make actions clear, emphasize story points and emotion, balance with non-exaggerated parts
- Appeal
 - Attractive to the eye, strong design, avoid symmetries
- Personality
 - Action of character is result of its thoughts, know purpose & mood before animating each action, no two characters move the same way

Keyframe Animation

- Keyframes done by lead animator, tweens done by computer or other animators



Frame as vector of parameter values

Keyframe Interpolation of each parameter

- need smooth/controllable kinematics

Forward Kinematics

- Articulated skeleton: topology (what connected to what), geometric relations from joints, tree structure
- Joint types: Pin (1d rotation), ball (2d rotation), prismatic joint (translation)

Inverse Kinematics

- Given the end effector position, find the joint angles
- Goals: keep end of limb fixed while body moves
 - Position end of limb by direct manipulation

Inverse Kinematics

- multiple solutions separated in configuration space
- Multiple solutions connected in configuration space
- Solutions may not always exist
- Numerical solution

- Choose an initial configuration
- Define an error metric
- Compute gradient of error as a function of configuration
- Apply grad descent

Kinematics Pros and Cons

- Direct control is convenient
- Implementation is straightforward
- May be inconsistent with real life

Skinning

- Move the surface along with assigned bones or "handles"
- Transform each vertex with each bone rigidly
- Blend the results using weights, or assignments

Common Approach: Linear Blend Skinning (LBS)

$$\mathbf{v}' = \sum_{j \in H} w_j(\mathbf{v}) \mathbf{T}_j \begin{pmatrix} \mathbf{v} \\ 1 \end{pmatrix}$$

Diagram illustrating the LBS formula:

- New vertex**: The resulting vertex position.
- How much influence this bone has on v (often sparse)**: The weight $w_j(\mathbf{v})$.
- Bone j transformation**: The transformation matrix \mathbf{T}_j .
- Original**: The original vertex position.

Blend Shapes

- Not all deformation is from bones
- Interpolate surfaces between key shapes
- Create triangular mesh model, linearly blend the faces

Week 9: Lecture 18 Animation, Physical Simulation (3/17)

Rigging

- Augment character with controls to easily change its pose, create facial expressions,

Motion Capture

- Data-Driven approach to creating animation sequences
- Record real-world performances

Optical Motion Capture

- positions by triangulations from multiple cameras, 8+ cameras, 240 Hz, markers on subject
- Capture large amounts of real data quickly
- Realism can be high
- Complex and costly set-ups, not need artistic needs,

Physical Simulation

Newton's Law: $F = ma$

Particle Systems

- Single particles are very simple
- Large groups can have interesting effects
 - Gravity, friction, collisions, force fields

Mass and Spring systems

- Mass Spring mesh
- Can be used to model cloths

A simple Spring

- Force pulls points together

$$\mathbf{f}_{a \rightarrow b} = k_s \frac{\mathbf{b} - \mathbf{a}}{||\mathbf{b} - \mathbf{a}||} (||\mathbf{b} - \mathbf{a}|| - l)$$

↑
Rest length

- Non-Zero Length Spring
- Dot notation for derivatives

Simple Motion Damping

- Behaves like viscous drag on motion, slows down motion in the direction of motion, k_d is a damping coefficient

Internal damping for Spring

$$\mathbf{f}_a = -k_d \frac{\mathbf{b} - \mathbf{a}}{||\mathbf{b} - \mathbf{a}||} (\dot{\mathbf{b}} - \dot{\mathbf{a}}) \cdot \frac{\mathbf{b} - \mathbf{a}}{||\mathbf{b} - \mathbf{a}||}$$

- Damp only the internal, spring-driven motion

Spring Constants

- Consider the strain = change in length as a fraction of original length

Structures from Springs

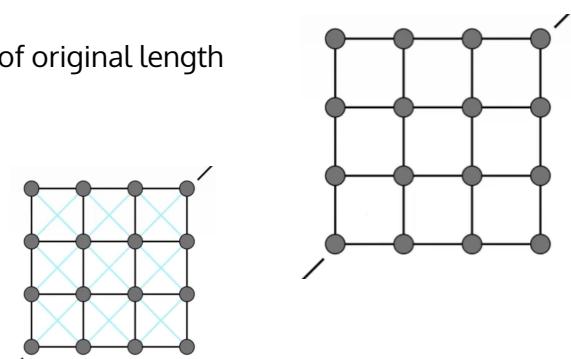
- Sheet: structure linkages, will not resist shearing
- Will resist shearing but not out of plane

Particle Simulation

- Euler's Method
 - Simple, commonly used method

$$\mathbf{x}^{t+\Delta t} = \mathbf{x}^t + \Delta t \dot{\mathbf{x}}^t$$

$$\dot{\mathbf{x}}^{t+\Delta t} = \dot{\mathbf{x}}^t + \Delta t \ddot{\mathbf{x}}^t$$



Errors and Instability

- Solving by numerical integration with finite differences leads to two problems
- Errors at each time step accumulate,
- Instability: errors can compound, cause the simulation to go to infinity, diverge

Methods to combat instability

- Modified euler
 - Avg velocities at start and endpoint
- Adaptive step size, compare one step and two half-steps
- Implicit methods
 - Use the velocity of the next time step

$$\mathbf{x}^{t+\Delta t} = \mathbf{x}^t + \Delta t \dot{\mathbf{x}}^{t+\Delta t}$$

$$\dot{\mathbf{x}}^{t+\Delta t} = \dot{\mathbf{x}}^t + \Delta t \ddot{\mathbf{x}}^{t+\Delta t}$$

$$\dot{\mathbf{x}}^{t+\Delta t} = \mathbf{V}(\mathbf{x}^{t+\Delta t}, \dot{\mathbf{x}}^{t+\Delta t}, t + \Delta t)$$

$$\ddot{\mathbf{x}}^{t+\Delta t} = \mathbf{A}(\mathbf{x}^{t+\Delta t}, \dot{\mathbf{x}}^{t+\Delta t}, t + \Delta t)$$

- Position-based/Verlet integration
 - Constrain positions and velocities of particles after time step
 - Idea:
 - After modified Euler forward-step, constrain positions of particles to prevent divergent, unstable behavior, use constrained positions to calculate velocity, will dissipate
 - Fast, highly recommended

$$\mathbf{x}^{t+\Delta t} = \mathbf{x}^t + \frac{\Delta t}{2} (\dot{\mathbf{x}}^t + \dot{\mathbf{x}}^{t+\Delta t})$$

$$\dot{\mathbf{x}}^{t+\Delta t} = \dot{\mathbf{x}}^t + \Delta t \ddot{\mathbf{x}}^t$$

$$\mathbf{x}^{t+\Delta t} = \mathbf{x}^t + \Delta t \dot{\mathbf{x}}^t + \frac{(\Delta t)^2}{2} \ddot{\mathbf{x}}^t$$

Algorithm 1 Position-based dynamics

```
1: for all vertices  $i$  do
2:   initialize  $\mathbf{x}_i = \mathbf{x}_i^0$ ,  $\mathbf{v}_i = \mathbf{v}_i^0$ ,  $w_i = 1/m_i$ 
3: end for
4: loop
5:   for all vertices  $i$  do  $\mathbf{v}_i \leftarrow \mathbf{v}_i + \Delta t w_i \mathbf{f}_{\text{ext}}(\mathbf{x}_i)$ 
6:   for all vertices  $i$  do  $\mathbf{p}_i \leftarrow \mathbf{x}_i + \Delta t \mathbf{v}_i$ 
7:   for all vertices  $i$  do genCollConstraints( $\mathbf{x}_i \rightarrow \mathbf{p}_i$ )
8:   loop solverIteration times
9:     projectConstraints( $C_1, \dots, C_{M+M_{\text{Coll}}}, \mathbf{p}_1, \dots, \mathbf{p}_N$ )
10:  end loop
11:  for all vertices  $i$  do
12:     $\mathbf{v}_i \leftarrow (\mathbf{p}_i - \mathbf{x}_i)/\Delta t$ 
13:     $\mathbf{x}_i \leftarrow \mathbf{p}_i$ 
14:  end for
15:  velocityUpdate( $\mathbf{v}_1, \dots, \mathbf{v}_N$ )
16: end loop
```

Particle Systems

- Model dynamical systems as collection of large numbers of particles,
- Each motion is defined by a set of physical forces

Particle System Animations

- For each frame in animation
 - Create new particles
 - Calculate forces on each particle
 - Update each particles position and velocity
- Gravitational Attraction
- Model flocking each bird as a particle
- Attraction to center of neighbors, repulsions from individual neighbors, alignment toward avg trajectory of neighbors

Week 11: Lecture 19 Intro to Color Science (3/29)

Automatic White Balance

- Divide by r, b, g

Color Perception is Highly Adaptive

- Surrounding colors have influence

Perception operates on "opponent" color axes

Color Reproduction Problem We will study

- Goal: At each pixel, choose R, G, B values for display so that the output color matches the appearance of the colors in the real world

What is Color?

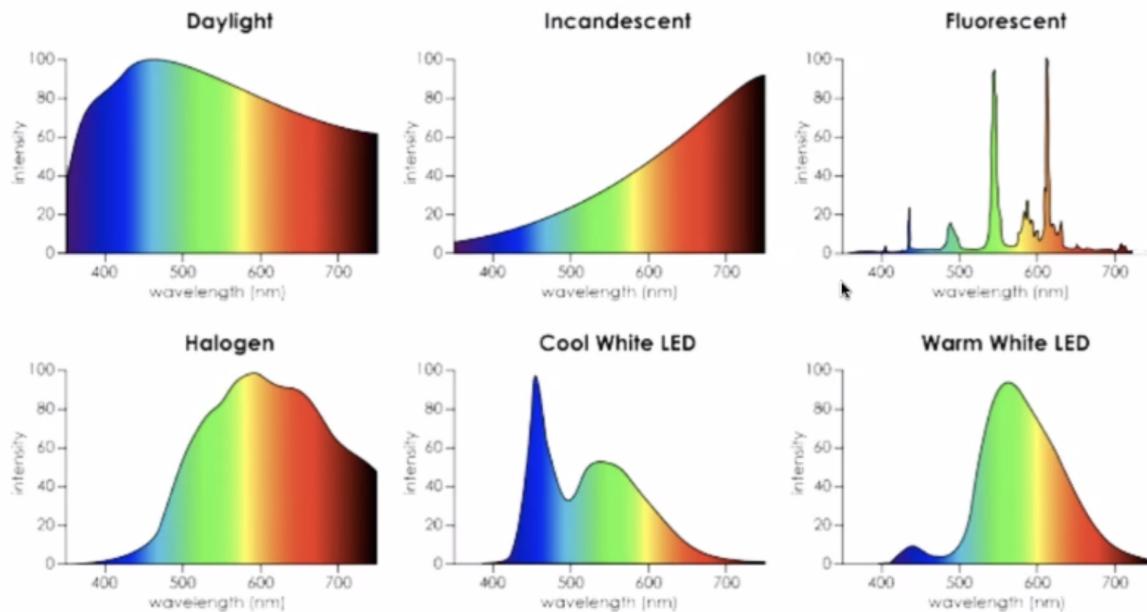
- Color is a human perception, not a universal property of light
- Colors are visual sensations that arise

Monochromatic

Spectral Power Distribution (SPD)

- Salient property in measuring light
- Amount of light present at each wavelength
- Radiometric units / nanometer
- Spectral Power distributions vary

Describes distribution of energy by wavelength



A simple model of a light detector

- produces a scalar value when photons land on it

$$X = \int n(\lambda)p(\lambda) d\lambda$$

Mathematics of Light Detection

- Light entering the detector has its spectral power distribution
- Detector has its spectral sensitivity or spectral response

$$X = \int s(\lambda) r(\lambda) d\lambda$$

| | |
 measured signal input spectrum detector's sensitivity

Sampled representations rather than continuous functions

Tristimulus Theory of Color

Week 11: Lecture 20 Intro to Color Science II (3/31)

Tristimulus Theory of Color

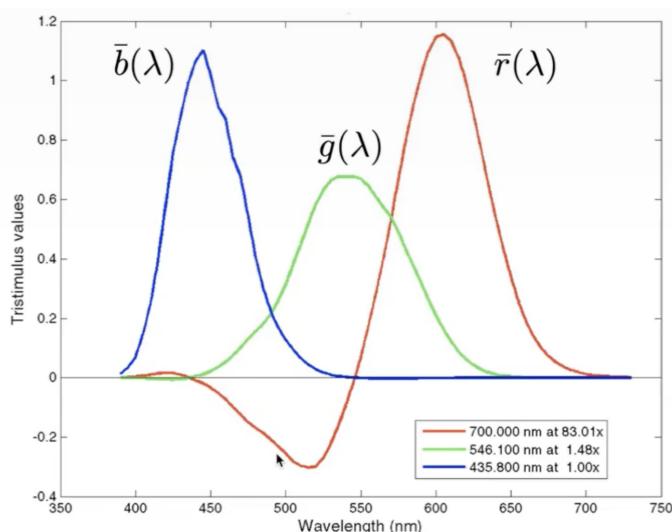
Experiment

Out of gamut of light using 3 lights

- Can adjust it on the other side
 - Linearly of colors

Dimensionality of Human Color Perception

- "Dimension" equals the rank of a basis for the linear space
 - For subjects with normal color vision, 3 primary colors are necessary to match any test color
 - Primary: 700 nm red, 546 nm green, 435 nm blue
 - Color matching set fully characterizes set of all colors



Biological Basis of Color

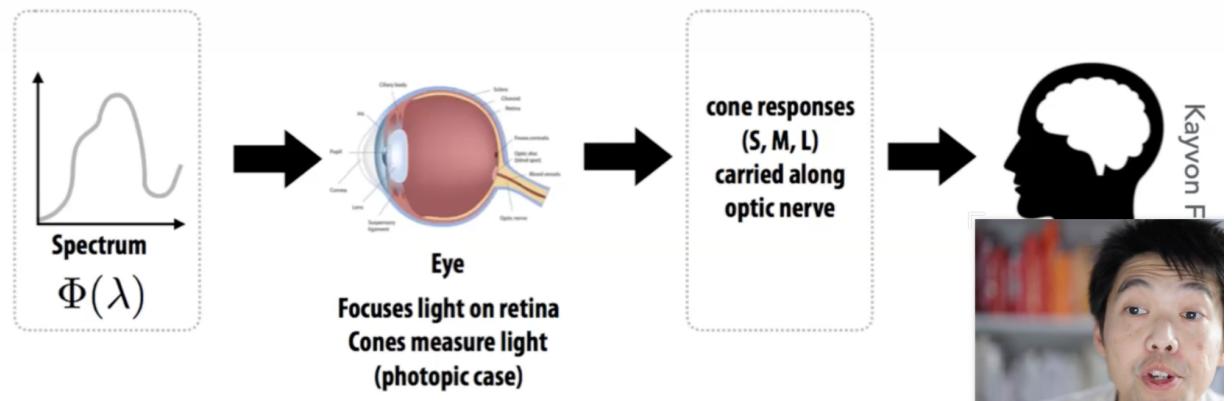
- Rods: 120 million rods in eye only shades of gray
- Cones: photopic: 6-7 million cones in eye
- Different spectral provide sensation of color

Dimensionality Reduction From ∞ to 3

- SPD is function of wavelength

Human Visual System

- Eye measures 3 response values only SML at each position in visual field and this is only spectral info available to brain
- Result of
-



Metamerism

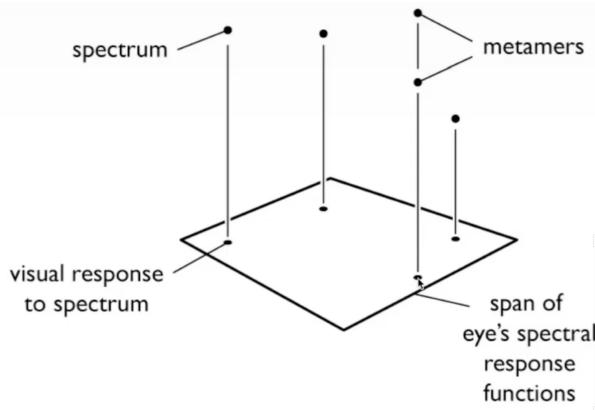
- Metamers: two different spectra that project to the same (SML) 3 dim response
- Will have the same color to a human
- Existence of metamer is critical to color reproduction

Color Reproduction Problem

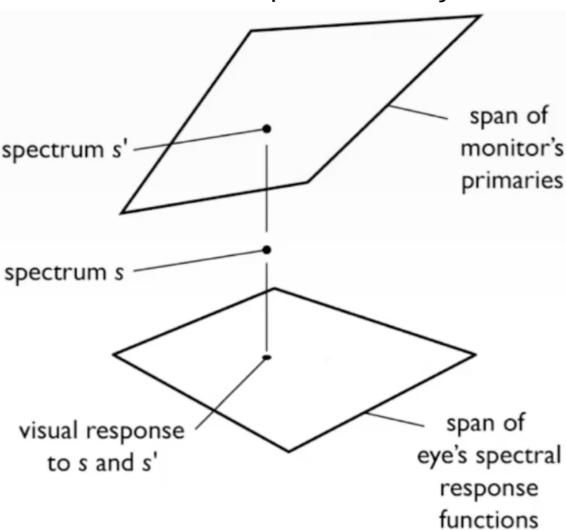
- At each pixel, choose RGB values for display so that the output color matches the appearance of the target color in the real world

Pseudo-Geometric Interpretation

- Projecting a high dimensional vector onto a low-dimensional subspace
- Differences that are perpendicular to the basis vectors of the low-dimensional space are not detectable



- Display can only produce a low-dimensional subspace of all possible spectra
- Given spectrum want to choose spectrum s' that s' and s project to the same low-dimensional subspace of the eyes SML response



Color reproduction as linear algebra

- Spectrum projected by display given values RGB
- What color do we perceive when we look at the display
- What the displayed spectrum as a metamer

Color perceived for display spectra with values R,G,B

$$\begin{bmatrix} S \\ M \\ L \end{bmatrix}_{\text{disp}} = \begin{bmatrix} \quad r_S \quad \quad \\ \quad r_M \quad \quad \\ \quad r_L \quad \quad \end{bmatrix} \begin{bmatrix} | & | & | \\ s_R & s_G & s_B \\ | & | & | \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

Color perceived for real scene spectra, s

$$\begin{bmatrix} S \\ M \\ L \end{bmatrix}_{\text{real}} = \begin{bmatrix} \quad r_S \quad \quad \\ \quad r_M \quad \quad \\ \quad r_L \quad \quad \end{bmatrix} \begin{bmatrix} | \\ s \\ | \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} R \\ G \\ B \end{bmatrix} = \left(\begin{bmatrix} r_S & & \\ r_M & & \\ r_L & & \end{bmatrix} \begin{bmatrix} s_R & & \\ s_G & & \\ s_B & & \end{bmatrix} \right)^{-1} \begin{bmatrix} r_S & & \\ r_M & & \\ r_L & & \end{bmatrix} \begin{bmatrix} s \\ | \\ s \end{bmatrix}$$

$$RGB = (\mathbf{M}_{SML} \mathbf{M}_{RGB})^{-1} \mathbf{M}_{SML} s$$

$$\begin{array}{ccccccccc} 3x1 & & 3xN & & Nx3 & & 3xN & & Nx1 \\ & & \underbrace{\hspace{10em}} & & & & & & \\ & & & & 3xN & & & & \end{array}$$

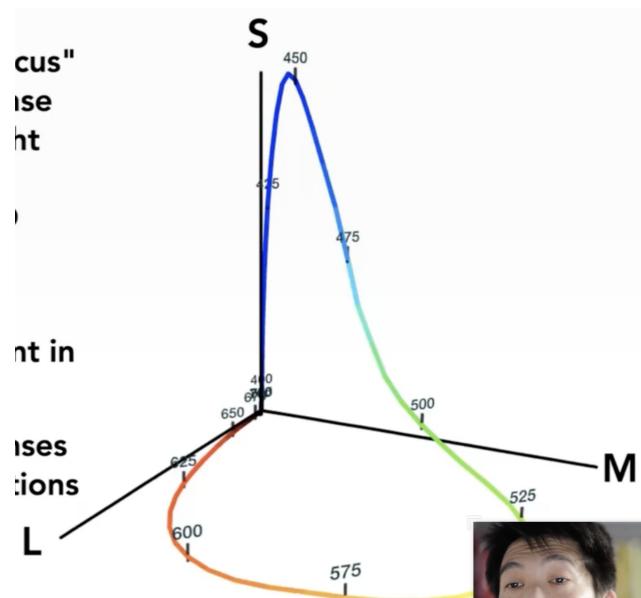
Week 12: Lecture 21 Image Sensors (4/5)

Gamut is a set of colors

- Visualization of "spectral locus" of human cone cells response to monochromatic light

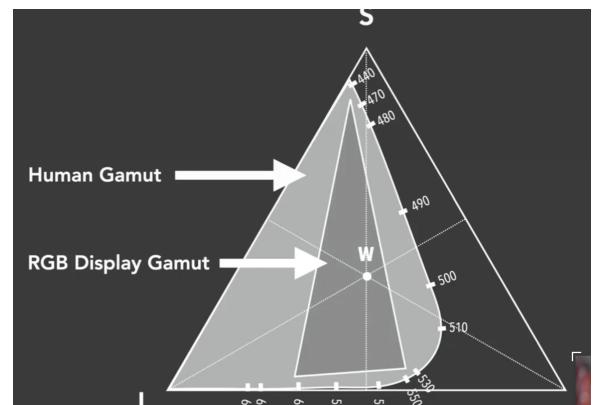
LMS responses plotted as 3D color space

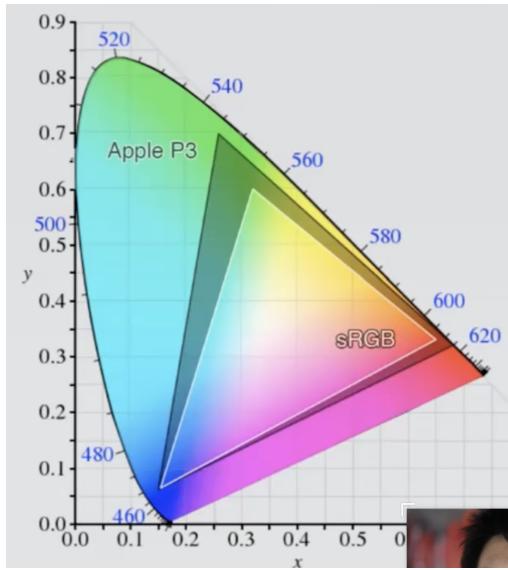
- Plot of SML as a point in 3D space
- Space of all possible responses are possible linear combinations of points on this curve



Chromaticity Diagram

- 3x3 basis change to x, y LMS to XYZ





Color Spaces

- Need 3 numbers to specify a color
- Color space is an answer to this question
- Define by what RGB scalar values will produce them on monitor
- Device dependent

Standardized RGB (sRGB)

- Makes a particular monitor RGB standard, usable as an interchange space: still widely used today, gamut is limited

Historical "Standard" Color Space: CIE XYZ

- Imaginary set of standard color primaries XYZ
- Designed such that XYZ span all observable colors
- Matching functions are strictly positive

Luminance (Lightness)

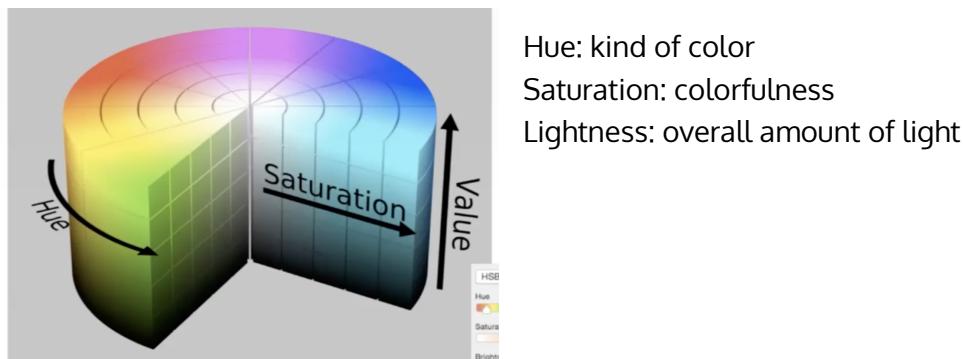
- Y is luminance
- $Y = \int \phi(\lambda) V(\lambda) d\lambda$

Chromaticity

CIELAB (aka L*a*b*)

- Chromatic adaptation (whitebalance)
- Perceptual uniformity

HSV Color Space (Hue s)



Additive Color

Image Sensors

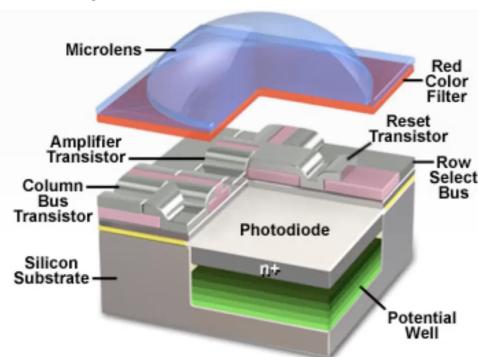
Photoelectric Effect

- If straight incident photons, it will eject electrons

CMOS APS (Active pixel sensor)

- Like memory laid out in 2d array

Anatomy of the Active Pixel Sensor

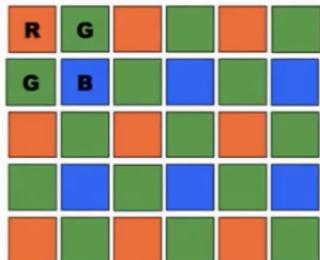


Quantum Efficiency

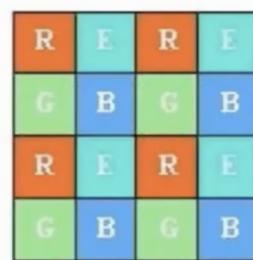
- Depends on quantum efficiency of the device
- $QE = \frac{\# \text{ electrons}}{\# \text{ photons}}$

Color Architectures

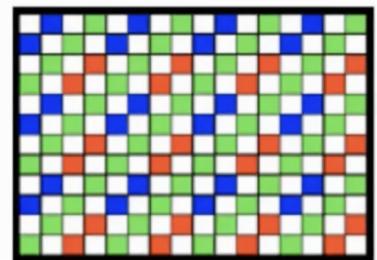
- Color Filter Arrays (mosaics)
- Bayer Pattern



**Bayer pattern
(most common)**



**Sony RGB+E
wider color gamut**



**Kodak RGB+W
higher dynamic range**

Demosaicing Algorithms

- Interpolate sparse color samples into RGB at every output image pixel
- Simple algorithm: bilinear interpolation
 - Avg 4 nearest neighbors of the same color
- Consumer cameras use more sophisticated techniques
 - Try to avoid

Week 12: Lecture 22 Image Sensors & Digital Image Processing (4/7)

Dynamic Range of the world is great

High Dynamic Range Image (HDR)

- Multiple exposures
- Synthetic Motion blur (8 bit image)

DIY HDR

- Sequence of pictures at different exposures automatically on phones
- Front-Side-illuminated CMOS
 - Photodiodes 50% fill factor, metal 1, metal 2, metal 3, metal 4
- Pixel Fill Factor: light to flow through to photodiode

Most CMOS Sensors are BSI

Pixel Aliasing Antialiasing

- Insufficient sampling rate

Antialiasing Filter

- Optical low-pass filter
- Layer of birefringent
- Antialiasing reduces details

Imagine

Signal-to-Noise Ratio (SNR)

- $SNR = \frac{\text{mean pixel value}}{\text{standard deviation of pixel value}} = \frac{\mu}{\sigma}$
- $SNR (\text{dB}) = 20 \log_{10}(\frac{\mu}{\sigma})$

Photon Shot Noise

- Number of photons arriving during an exposure varies from exposure to exposure and from pixel to pixel, even if the scene is completely uniform
- Number is governed by the Poisson distribution

Poisson Distribution

- Probability that a certain number of random events will occur during an interval of time
 - Known mean rate
 - Independent event s
- $\mu = \lambda$
- $\sigma^2 = \lambda$, $\sigma = \sqrt{\lambda}$
- Error grows slower than the mean
- $SNR = \sqrt{\lambda}$
- Opening aperture by 1 f/stop increases the photons by 2 so SNR increases by $\sqrt{2}$ or +3dB

Pixel Noise: Dark Current

- Electronics displaced increases linearly with exposure time

Pixel Noise: Hot Pixels

- Electrons leaking into well due to manufacturing defects

- Solution #1: Chill the sensor , dark frame subtraction

Pixel Noise: Read Noise

- Thermal noise in readout circuitry

Effect of Downsizing on Image Noise

- SNR increases as $\text{sqrt}(\# \text{ of frames})$ neglecting read noise

Things to remember

- Photoelectric effect

Image Processing

JPEG Compression: The big ideas

- Low-frequency content is predominant in images of the real world
- Human visual system is
 - Less sensitive to detail in chromaticity than in luminance
 - Less sensitive to high frequency sources of error
- Y'CbCr Color space (luma), Cb, Cr chroma channels
 - Compress in Y' channel is much worse than compression in CbCr channels
- 4:2:2 representation
 - Store Y' at full resolution
 - Store cb, cr at half resolution in horizontal
- 4:2:0
 - Store Y' at full resolution
 - Store cb, cr at half resolution in both directions

Transforms into Discrete Cosine transform (DCT)

JPEG Quantization: Prioritize Low Frequencies

- Changes by quantization Matrix
- Convert to Y'CbCr, for each color channel do DCT

Theme: Exploit Perception in Visual Computing

- JPEG is example of theme of exploiting characteristics of human perception to build efficient visual computing systems

Week 13: Lecture 23 Light Field Cameras (4/12)

Smarter Blue preserves Crisp Edges

- Denoising

Convolution

$$(f * g)(x) = \int_{-\infty}^{\infty} f(y)g(x - y)dy$$

output signal filter input signal

2D Convolution

$$(f * I)(x, y) = \sum_{i,j=-\infty}^{\infty} f(i, j)I(x - i, y - j)$$

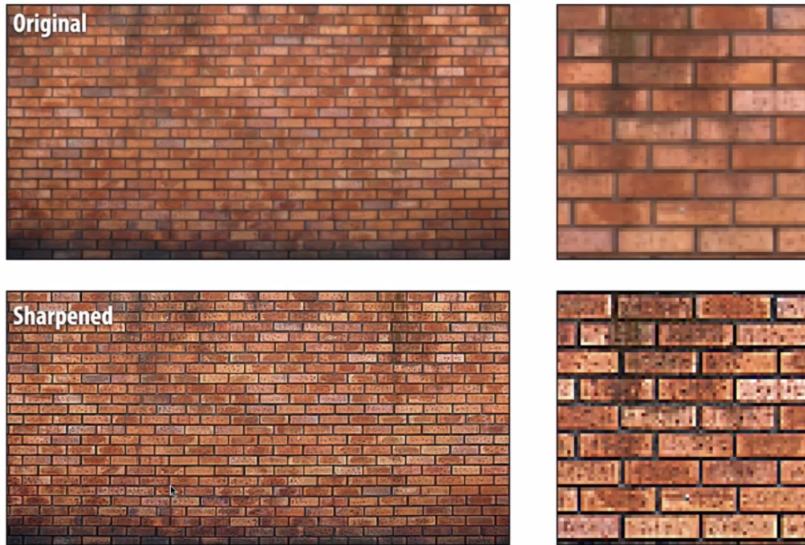
output image filter input image

Gaussian Blur

- Obtain filter coefficients from sampling 2D Gaussian
- $f(i, j) = \exp()$

Sharpen filter

3x3 Sharpen Filter



$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

Gradient Detection

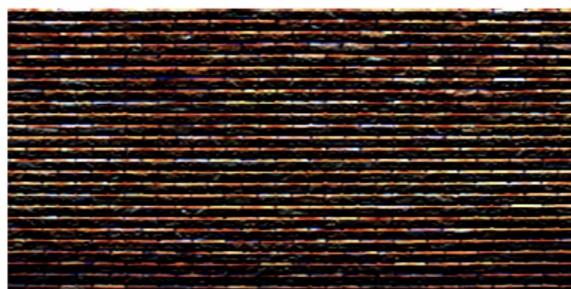
$$G_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} * I$$

$$G_y = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} * I$$

Gradient Detection Filters



Horizontal gradients



Vertical gradients

Note: you can think of a filter as a "detector" of a pattern, and the magnitude of a pixel in the output image as the "response" of the filter to the region surrounding each pixel in the input image (this is a common interpretation in computer vision)

Algorithmic Cost of Convolution-Based Image Processing

- Convolution: $N^2 * \text{width} * \text{height}$

Fast 2D Box Blur via two 1d Convolution

- $2N * \text{Width} * \text{Height}$

Fast Fourier is $n^2 \log n$ and multiplication is n^2

- Otherwise spatial domain is n^4

Efficiency

- When is it faster to implement by convolution and filter by multiplication in frequency domain

Data-Dependent Filters

- Median Filter
- Bilateral Filter

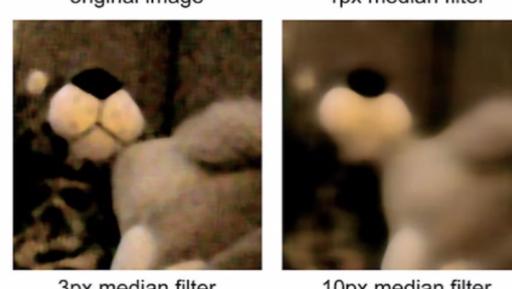
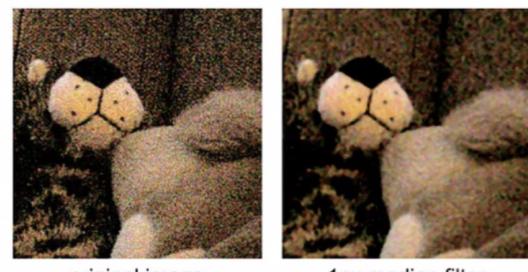
Isotropic Filters vs Anisotropic, data dependent filter

- Distinct flavors in different parts, bilateral fileter separation

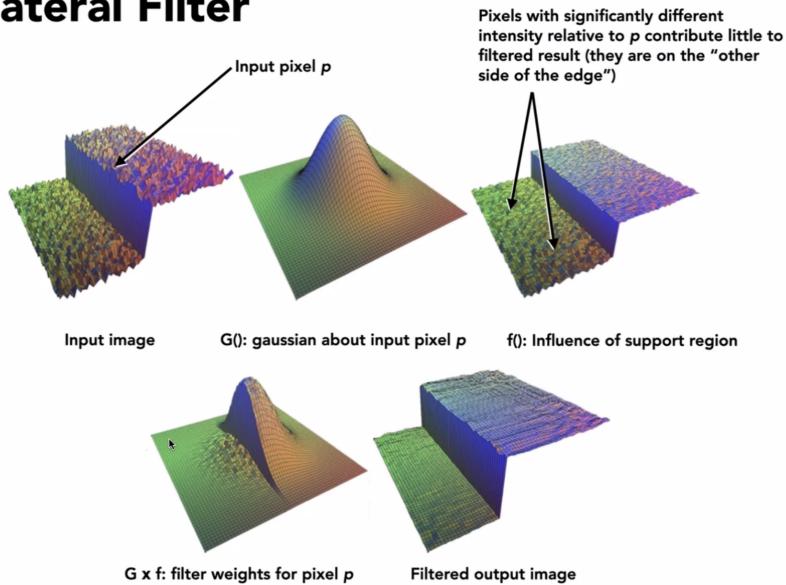
Bilateral Filter

$$\text{BF}[I](x, y) = \sum_{i,j} f(\|I(x-i, y-j) - I(x, y)\|) G(i, j) I(x-i, y-j)$$

Gaussian blur kernel Input image
 For all pixels in support region of Gaussian kernel Re-weight based on difference in input image pixel values



Bilateral Filter



Data-Driven Image Processing: "Image Manipulation by Example"

- Denoising with Non-Local Means (averaging)
- Instead of average over nearby pixels, search over image that look similar and using those data values to help

Denoising Using non-local means

- Non-parametric Texture Synthesis
- Find a probability distribution function for possible values of p , based on its neighboring pixels
- More texture synthesis examples

Things to remember

- JPEG as an example of exploiting

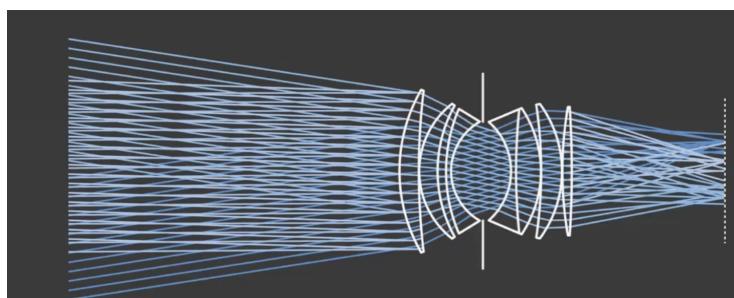
Week 13: Lecture 24 Light Field Cameras (4/14)

3 focus related problems in 2d photography

1. Must focus before taking picture
2. Trade off between depth of field and motion blur
3. Lens designs are complex due to optical aberrations

Capture full range of light fields

4D light fields



Photograph = irradiance at every pixel on plane (2)
Light field = radiance flowing along every ray (4)

How to record light field

- Plenoptic camera

Lens Aberration Example

- Real spherical lens does not converge to a single point
- Computationally redirect rays from physical trajectory to ideal location
- Computationally correct

4D light field: radiance along every ray

Light field camera

- Capture light field flowing into lens in every shot
- Light field sensor = microlens array in front of sensor

Computational refocusing

- Refocusing = reproject rays assuming new sensor depth
- Can think of this as shift-and-add of sub aperture images

Computational lens aberration correction with light fields

- Correction = reproject rays assuming no aberration

Week 14: Lecture 25 VR (4/14)

VR: virtual reality

- Completely immersed in virtual world

AR: augmented reality

- Overlay that augments normal world

VR Applications

- Gaming, video chat

VR Displays

- Displays attached to head
- Head orientation tracked physically, synchronized to head orientation in low latency

3D visual cues

- Occlusion, perspective, shading focus blue, z buffer
- Stereo in 3d cues

Oculus Quest 2

- QC Snapdragon XR2
 - 7M total pixels up to 120 Hz display
 - Role of eyepiece lenses
1. Create wide field of view
 2. Place focal plane at several meters away from eye

Human Visual Field of View

- About 160 degrees per eye and 200 degrees overall

Current VR headset field of view and resolution

- Approx 100 degrees per eye
- 6MP pixel display 24 pixels per degree

VR Display at Human Visual Acuity

- 160 degree
- 8k x 8k display per eye 50 ppd 128 MPixel, current at 24 ppd

Binocular Stereo and Eye Focus

- Passive
- Present each eye with perspective view corresponding to that eye's location relative to the other eye
- Eyes will converge by rotating physically in sockets in order to bring closer and further objects into physical alignment on retina
- At the same depth as optical lenses

Motion Parallax from eye motion

- Environment supported vision based tracking
- External camera find the xyz position and

Quest

- Uses SLAM to estimate 3d structure of the world and position/orientation of camera in the world
- Cameras also track the position and orientation of the controllers 15 infrared LEDs to aid tracking