21 Electric Charge and Electric Field

21.1 Unlike charges attract, like charges repel

Law of conservation of electric charge: net amount of electric charge produced is 0

21.2 Atom has positively charged nucleus w/ protons and neutrons, electrons surrounding

Becomes an ion if loses or gains an electron

21.3 Conductors - electrons are bound loosely, charge transfers easily

Insulators - electrons bound tightly to nucleus, charge does not transfer easily

Semiconductor - intermediate category, fewer free electrons

21.4 Charging by conduction - using charged object to make neutral object charged by contact

Induced charge - caused neutral object to be charged without contact

21.5 Coulomb's Law:

$$\boxed{\underbrace{F}_{\text{Magnitude of Electric Force}} = \underbrace{k}_{k=8.99\times10^{9}\ \frac{N\cdot m^2}{C^2}} \frac{Q_1 Q_2}{r^2} = \frac{1}{4\pi \underbrace{\varepsilon_0}_{\text{permitivity of free space}}} \frac{Q_1 Q_2}{r^2}}$$

$k = \frac{1}{4\pi\varepsilon_0}$

Used for Point charges

$\varepsilon_0 = 8.85\times10^{-12}\ \frac{C^2}{N\cdot m^2}$

Charge (Q) measured in Coulomb [C]

elemtary charge: $e = 1.602\times10^{-19}$ C

Principle of superposition - net force on object w/ multiple charges is vector sum of forces due to each of others

21.6 Each object radiates Electric field, use small positive test charge to measure field

Electric Field

$$\boxed{\vec{E} = \frac{\vec{F}}{q} \qquad \vec{E} = k\frac{Q}{r^2} \qquad \vec{F} = q\vec{E}}$$

magnitude of test charge (q); Force at q

Positive charge: E field points away, Negative: points toward

If multiple charges: Superposition Principle: $\vec{E} = \vec{E}_1 + \vec{E}_2 + \ldots$

draw diagram, find mag w/ Coulombs, add vector forces

21.7 Continous Charge Distribution problems

1) Choose Coordinate System (Cartesian, Polar, Spherical, Cylindrical)

$\frac{Q}{x} = \lambda \qquad \frac{Q}{A} = \sigma \qquad \frac{Q}{V} = \rho$

2) find dq

$dq = \lambda dx$, λ : linear charge density

$dq = \sigma dA$, σ : surface charge density

$dq = \rho dV$, ρ : volume charge density

$dA = dx\,dy = r\,dr\,d\theta$

$dV = dx\,dy\,dz = r^2 \sin\theta\, d\theta\, d\phi\, dr = r\,dr\,d\phi\,dz$

3) find dE

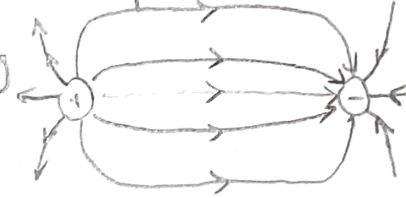
$$\boxed{dE = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{r^2}} \qquad d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{r^2}\hat{r}$$

4) find E

$$\boxed{\vec{E} = \int d\vec{E}}$$

21.8 Electric field lines indicate the direction of electric field at various points.

The closer the lines, the stronger, lines start on pos end on neg



21.9 Electric Fields and Conductors:

- Electric field inside conductors is 0
- Electric field is always perpendicular to surface outside conductor

21.10 Magnitude of electron acceleration $a = \frac{F}{m} = \frac{qE}{m}$

21.11 Electric Dipole - two equal charges w/ opposite signs separated by distance l

Dipole moment: $\vec{p} = Ql$

Torque: $\vec{\tau} = \vec{p} \times \vec{E} = pE\sin\theta$

Work: $W = \int_{\theta_1}^{\theta_2} \tau \, d\theta = pE(\cos\theta_2 - \cos\theta_1)$

22 Gauss's Law

22.1 Electric flux: electric field passing through area

For uniform electric field $\vec{E}$ through A

Flux → $\boxed{\Phi_E = EA\cos\theta}$

For Not Uniform

$\boxed{\Phi_E = \oint \vec{E} \cdot d\vec{A}}$

flux entering enclosed volume is negative, leaving is positive, nonzero when enclosed charge

22.2 Gauss's Law: $\boxed{\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}}$ ← net charge enclosed in surface

Solving w/ Gauss:

1) Find surface S that respects symmetry
2) $\int E \cdot dA = |E| \cdot$ Surface Area of S
3) $Q_{encl} \Rightarrow \int dq = \lambda dx, \sigma dA, \rho dV$

22.3 If conductor w/ charge Q and inside cavity has charge +q

Must be -q charge on surface of cavity and outer surface with Q+q

23 Electric Potential

23.1 Electric Potential Energy (U) - conservative force for electrostatic

$$\Delta U = -W = -qEd \qquad [\text{Uniform } \vec{E}]$$

Electric Potential (V) - electric potential energy per unit charge

$$V_a = \frac{U_a}{q} \qquad \Delta V = \frac{U_b - U_a}{q} = \frac{-W_b}{q}$$

Voltage (V) - Potential Difference $\quad 1V = 1\,J/C \quad$ measures how much work a given charge can do

$\Delta U = qV_{ba}$

$\Delta V = -\int \vec{E}\cdot d\vec{l}$ since E is force per unit charge $E = \frac{F}{q}$

(23.3) Electric Potential at distance r away

$$\Delta V = -\int_{r_a}^{r_b} \vec{E}\cdot d\vec{l} = -\frac{Q}{4\pi\varepsilon_0}\int_{r_a}^{r_b} \frac{1}{r^2}\,dr = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{r_b} - \frac{Q}{r_a}\right)$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$ [single pt charge, $V=0$ at $r=\infty$]

(23.4) with continuous distribution

$\boxed{V = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r}}$ $\boxed{V = -\int E\,dr}$ can add together voltages since scalar

(23.5) Equipotential lines with same potential, perpendicular to electric field

(23.6) Electric Dipole Potential

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Ql\cos\theta}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$ [dipole, $r \gg l$]

(23.7)

$$E_l = -\frac{dV}{dl} \qquad E_x = -\frac{\partial V}{\partial x},\ E_y = -\frac{\partial V}{\partial y},\ E_z = -\frac{\partial V}{\partial z}$$

(23.8) Charges moved from $V=0$ $r=\infty$

$$U = Q_2 V = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r_{12}} \qquad U = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1Q_2}{r_{12}} + \frac{Q_1Q_3}{r_{13}} + \frac{Q_2Q_3}{r_{23}}\right)$$

electron Volt (eV) $1\,eV = 1.6\times10^{-19}\,J$

24 Capacitance, Dielectrics, Electric Energy Storage

(24.1) Capacitors - store electric charge by using two conducting objects

capacitor [-||-] battery [+|⊢-]

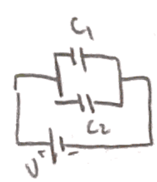
amount of charge acquired by plate $\boxed{Q = CV}$

↑ capacitance Farad [F] $\frac{C}{V}$

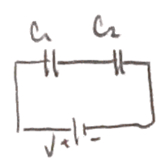
(24.2) Capacitance $\boxed{C = \varepsilon_0 \frac{A}{d}}$ ← area of plates

↑ distance between plates

(24.3) Parallel: $Q = C_{eq}V$ $\boxed{C_{eq} = C_1 + C_2 \ldots}$



Series: $Q = C_{eq}V$ $\boxed{C_{eq} = \frac{C_1C_2}{C_1+C_2}}$



24.4 Harder to charge capacitor the more energy it has

Work to charge $W = \int_0^Q V\,dq = \frac{1}{C}\int_0^Q q\,dq = \frac{1}{2}\frac{Q^2}{C}$

Energy stored $U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV = \frac{1}{2}CV^2$

Energy Density (u) $\frac{\text{energy}}{\text{volume}}$: $u = \frac{1}{2}\varepsilon_0 E^2$ $E = \frac{Q}{\varepsilon_0 A}$

24.5 Dielectric: piece of insulating sheet of material in between plates

$\boxed{C = K C_0}$ ← capacitance if space is vacuum

↑ Dielectric constant

$C = K\varepsilon_0 \frac{A}{d}$ $\varepsilon = K\varepsilon_0$ ← permitivity of dielectric

25 Electric Currents and Resistance

25.2 Current only flows with complete circuit

$I = \frac{dQ}{dt}$ Current [I] measured in Amperes [A] $1A = 1\frac{C}{s}$

25.3

Ohm's Law: $\boxed{V = IR}$

← Resistance of a wire [Ω] Ohms $1\Omega = 1\frac{V}{A}$

25.4

Resistivity $\boxed{R = \rho\frac{l}{A}}$ $\boxed{dR = \rho\frac{dl}{A}}$ $\sigma = \frac{1}{\rho}$

← wire length l; ← cross sectional area; ← resistivity $[\Omega \cdot m]$; ← conductivity $\left[\frac{1}{\Omega \cdot m}\right]$

resistivity can vary based on temperature

$\rho_T = \rho_0[1 + \alpha[T - T_0]]$

resistivity Temp T; ← resistivity at temp T_0

25.5 $\boxed{P = IV = I^2R = \frac{V^2}{R}}$ Can be measured in kilo watt-hour (kWh) $1\,kWh = 3.6 \times 10^6\,J$

← applies to resistors

Power [W] Watt $1W = 1\,J/s$

25.7

$V = V_0 \sin(\omega t)$ $\begin{matrix}+V_0\\-V\end{matrix}$ peak voltage

$I = I_0 \sin(\omega t)$ $I_0 = V_0/R$ peak current

$P = I^2R = I_0^2 R \sin^2 \omega t$

$$I_{rms} = \sqrt{\overline{I^2}} = \frac{I_0}{\sqrt{2}} = 0.707 I_0 \qquad V_{rms} = \sqrt{\overline{V^2}} = \frac{V_0}{\sqrt{2}} = 0.707\ V_0$$

$$\overline{P} = I_{rms} V_{rms} = \frac{1}{2} I_0^2 R = I_{rms}^2 R = \frac{1}{2}\frac{V_0^2}{R} = \frac{V_{rms}^2}{R}$$

25.8 $\vec{j}$ current per unit cross-sectional area

$$\Delta Q = (\#\ \text{charges}, N) \times (\text{charges per particle})$$
$$= (nV)(-e) = -(n A v_d \Delta t)e$$

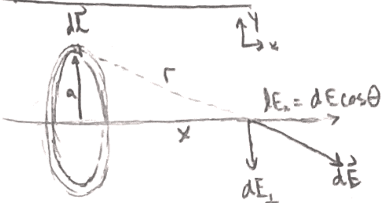
$$I = \frac{\Delta Q}{\Delta t} = -ne A v_d$$

$$j = \frac{I}{A} \qquad \vec{j} = -ne\vec{v_d}$$

Constant / Example — Reference

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Qx}{(x^2+a^2)^{3/2}}$$

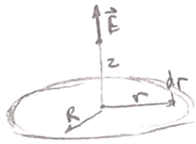
Coulomb's



$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{(x^2+a^2)^{1/2}}$$

$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{(z^2+R^2)^{\frac{1}{2}}}\right]$$

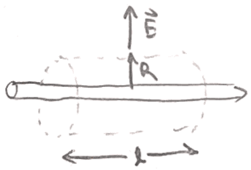
Coulomb's



$$V = \frac{Q}{2\pi\varepsilon_0 R^2}\left[(z^2+R^2)^{1/2} - z\right]$$

$$E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{R}$$

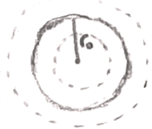
Coulombs
Gauss



$$r > r_0 : E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

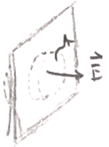
$$r < r_0 : E = \frac{Qr}{4\pi\varepsilon_0 r_0^3}$$

Gauss



$$r > r_0 : \quad V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

Gauss



$$E = \frac{\sigma}{2\varepsilon_0}$$

Constants

$k = 8.99 \times 10^9\ \frac{N \cdot m^2}{C^2}$

$\varepsilon_0 = 8.85 \times 10^{-12}\ \frac{C^2}{N \cdot m^2}$

$e = 1.602 \times 10^{-19}\ C$

Surface Area of Sphere: $4\pi r^2$