Nanyang Technological University
School of Computer Science and Engineering

CZ 2003 - Lab 3:

Parametric Surfaces and Solids

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Lab Group: SSR1

1. 3D Plane

Points on plane:
P1 = (0,1,2)
P2 = (1,3,4)
P3 = (1,-2,10)

P = P1+ u(P2-P1) + v(P3-P1)
=(0,1,2) + u(1,2,2) + v(1,-3,8)

x = u+v
y = 1+2u-3v
z = 2+2u+8v

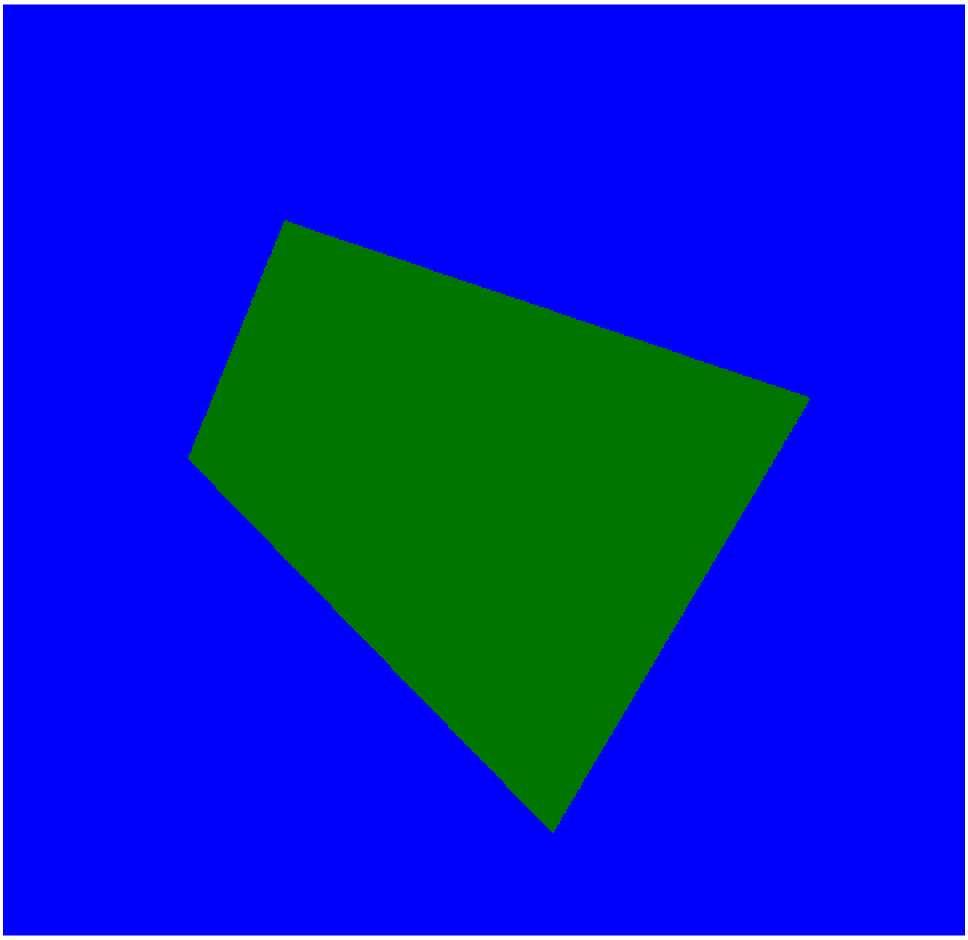


Figure 1: A 3D plane defined parametrically. The plane does not extend infinitely as the range for u and v are set to [-10,10]. Source code: 3DPlane.wrl

2. 3D Triangle

P1	P2	P3	P4=P3
X1=-1	X2=1	X3=0	X4=0
Y1=0	Y2=0	Y3=1	Y4=1
Z1=1	Z2=0	Z3=0	Z4=0

$x(u,v)=x1+u\cdot(x2-x1)+v\cdot(x3-x1+u\cdot(x4-x3-x2+x1)) = -1+2u+v-2uv$
$y(u,v)=y1+u\cdot(y2-y1)+v\cdot(y3-y1+u\cdot(y4-y3-y2+y1)) = v$
$z(u,v)=z1+u\cdot(z2-z1)+v\cdot(z3-z1+u\cdot(z4-z3-z2+z1)) = 1-u-v+uv$

$u \in [0,1]$, $v \in [0,1]$

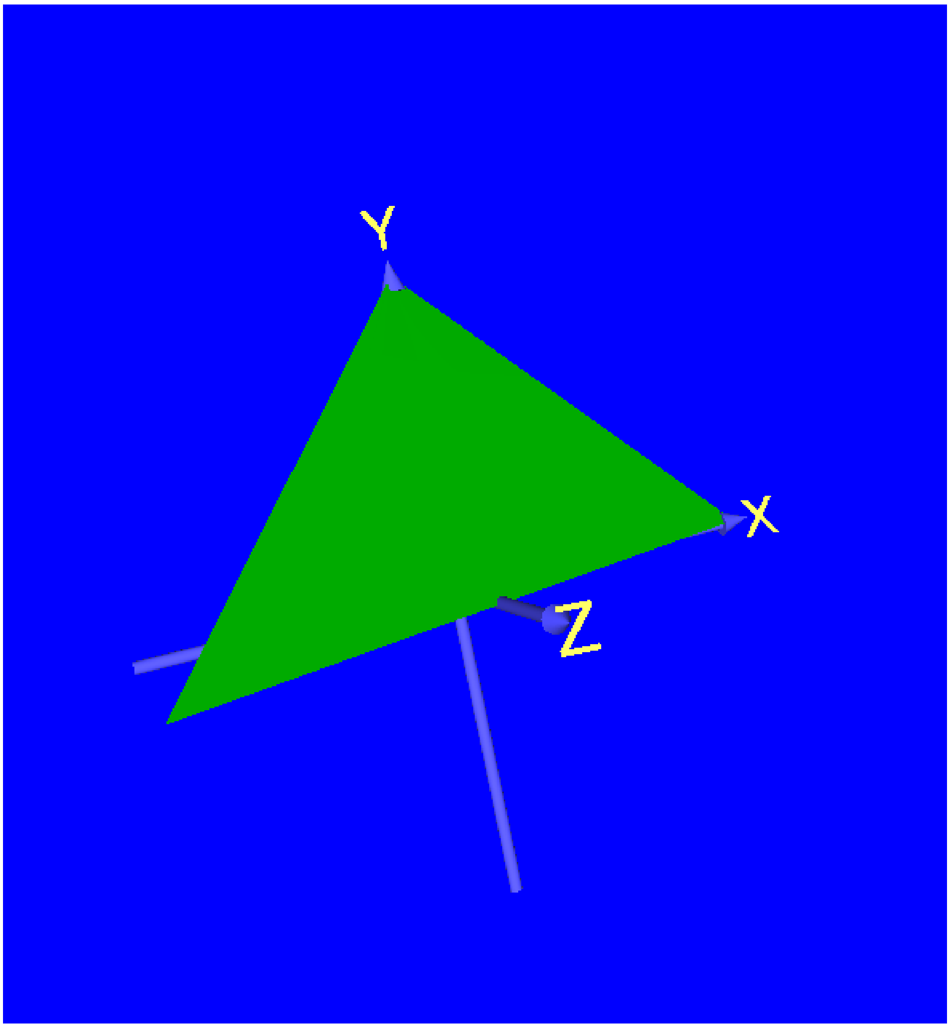


Figure 2: 3D Triangle defined parametrically through the use of equation for bilinear surfaces. Source code: 3DTriangle.wrl

3. Bilinear Surface

P1	P2	P3	P4
X1=-1	X2=1	X3=0	X4=3
Y1=0	Y2=0	Y3=1	Y4=1
Z1=1	Z2=0	Z3=0	Z4=2

$x(u,v)=x1+u\cdot(x2-x1)+v\cdot(x3-x1+u\cdot(x4-x3-x2+x1)) = -1+2u+v+uv$
$y(u,v)=y1+u\cdot(y2-y1)+v\cdot(y3-y1+u\cdot(y4-y3-y2+y1)) = v$
$z(u,v)=z1+u\cdot(z2-z1)+v\cdot(z3-z1+u\cdot(z4-z3-z2+z1)) = 1-u-v+3uv$

$u \in [0,1]$, $v \in [0,1]$

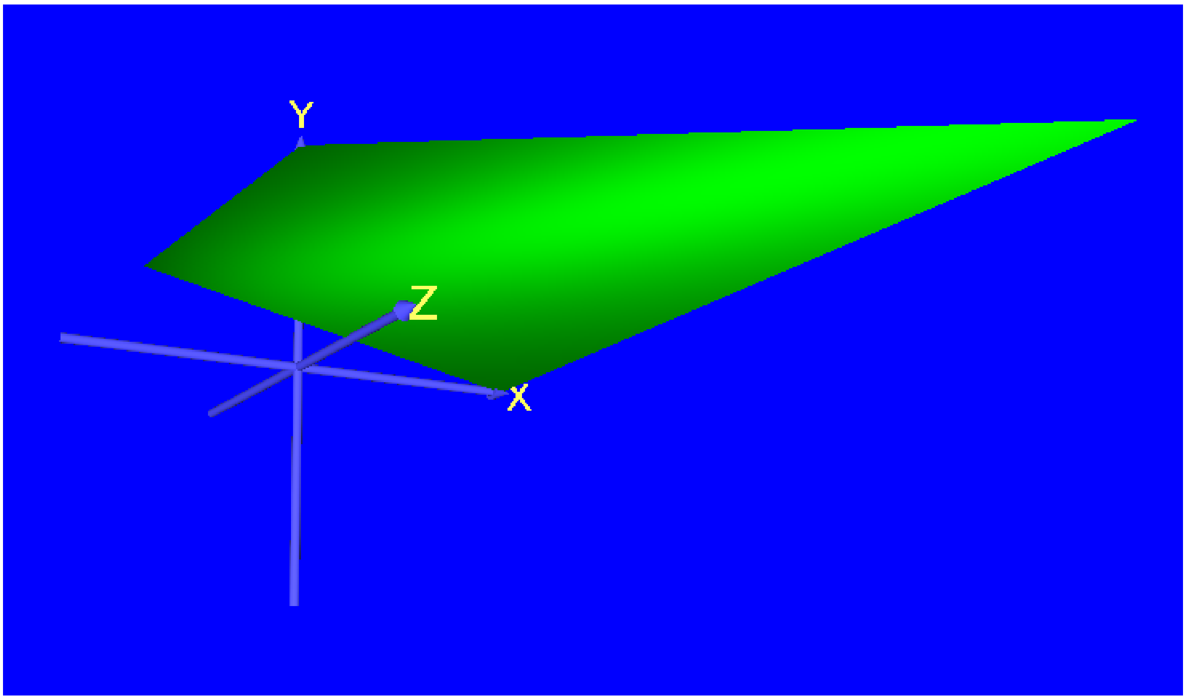


Figure 3: Bilinear Surface defined parametrically. Source code: BLSurface.wrl

4. Sphere (surface)

Equations:

x = cos(u)*sin(v)
y = sin(u)
z =

u ε [-π/2, π/2], v ε [-π, π]

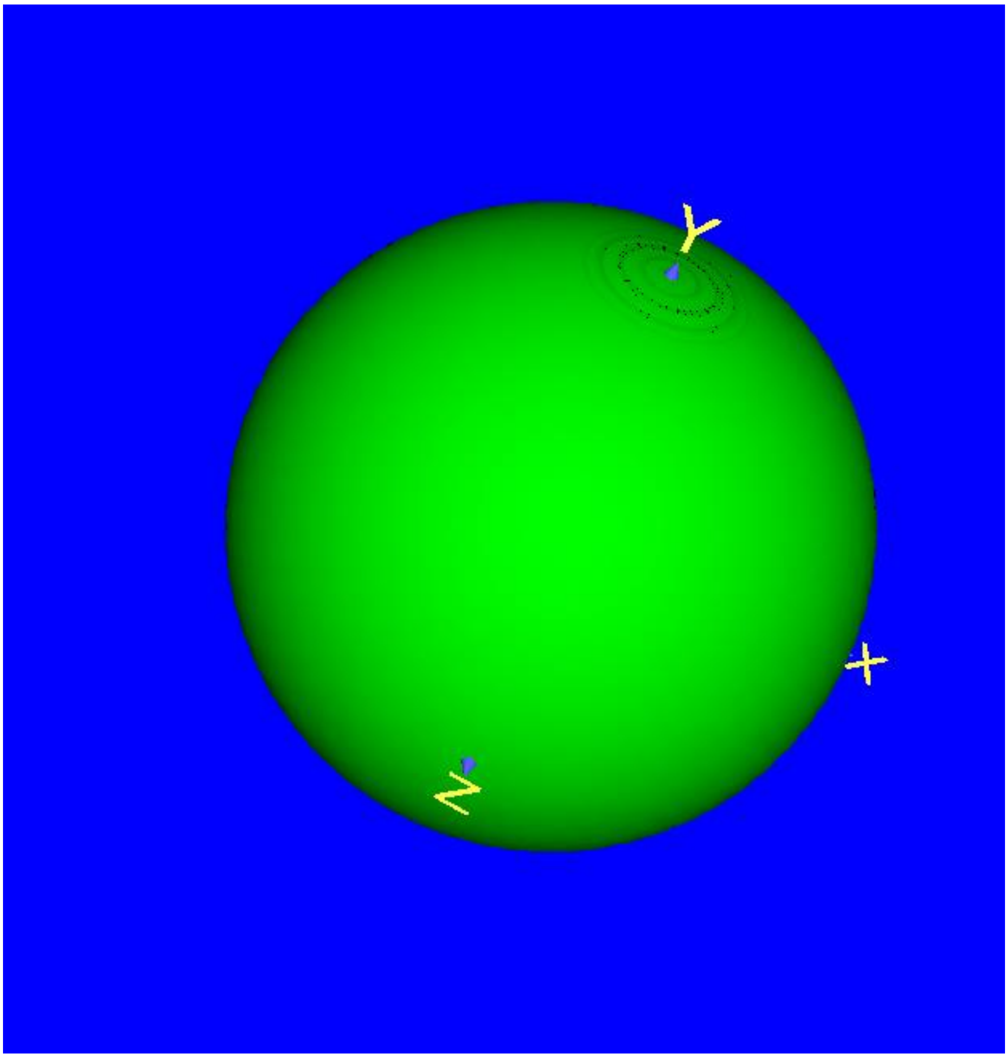


Figure 4: Sphere surface centered at (0,0,0) with radius 1. Source code: sphereSurface.wrl

5. Ellipsoid

Equations:

x = cos(u)*sin(v)
y = 1/3*sin(u)
z = 2/3*cos(u)*cos(v)

u ε [-π/2, π/2], v ε [-π, π]

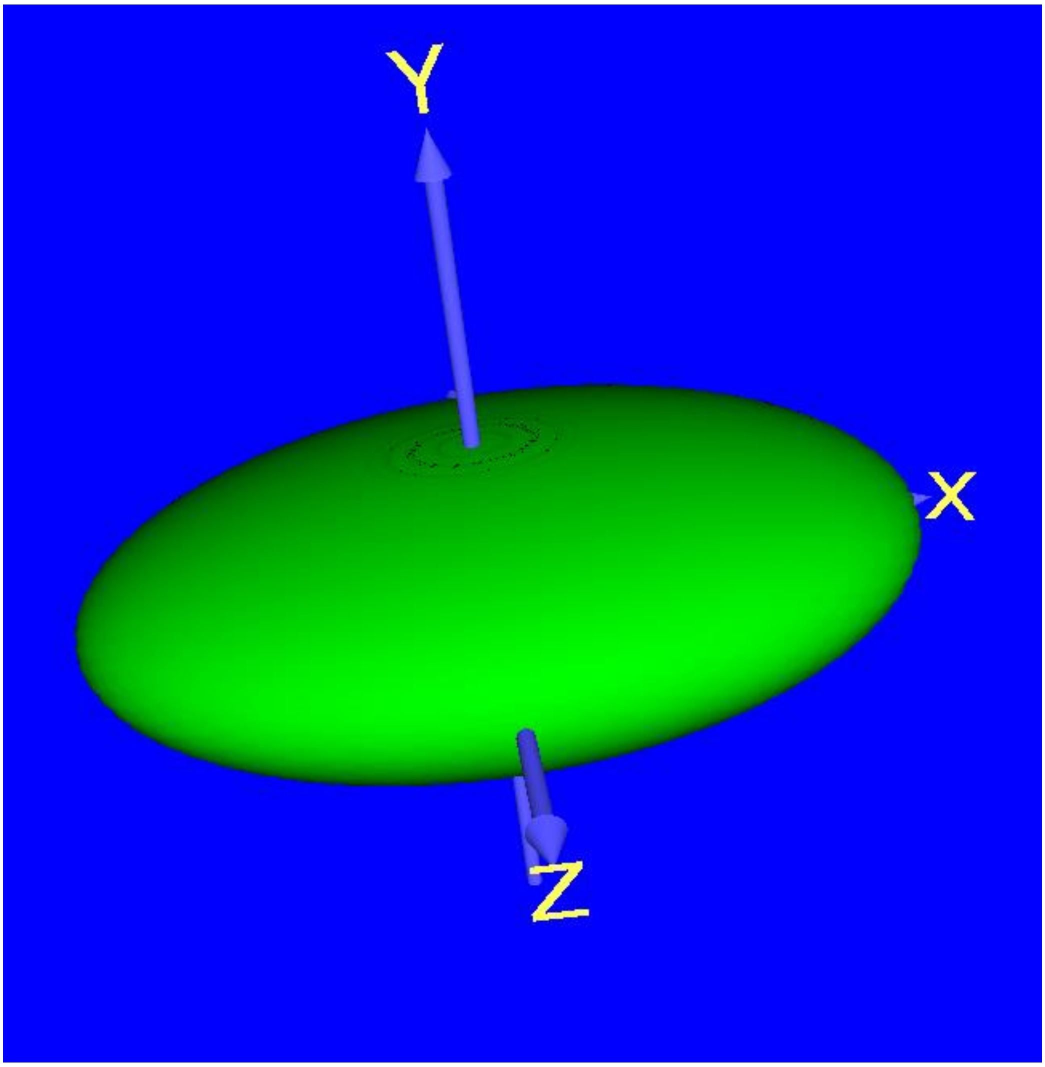


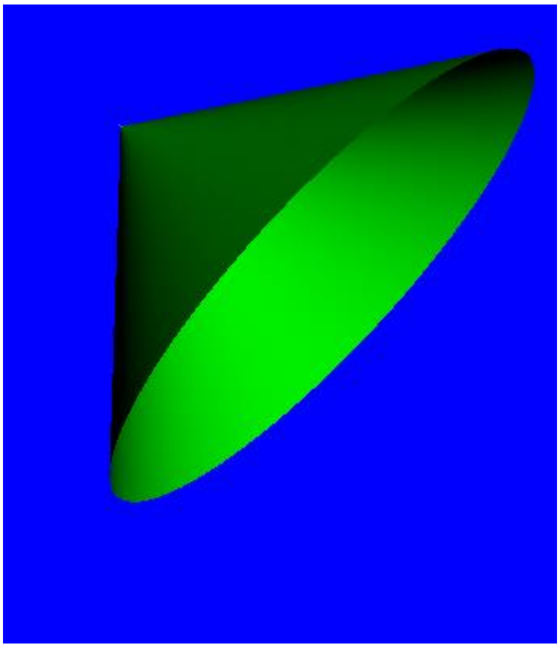
Figure 5: Ellipsoid surface. Source code: ellipsoidSurface.wrl

Cone (surface)

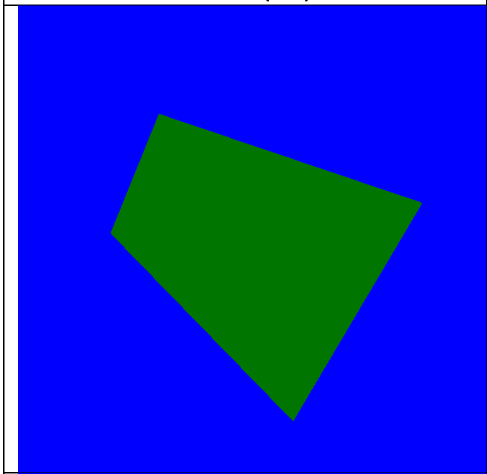
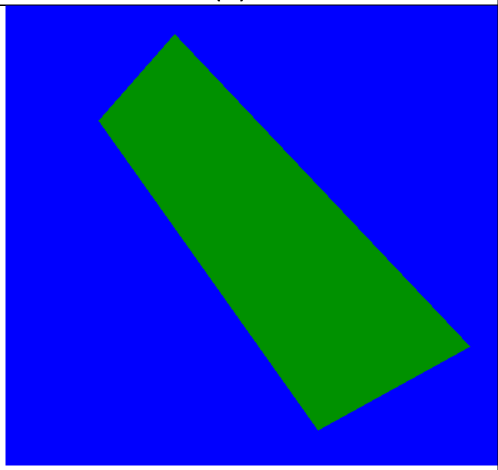
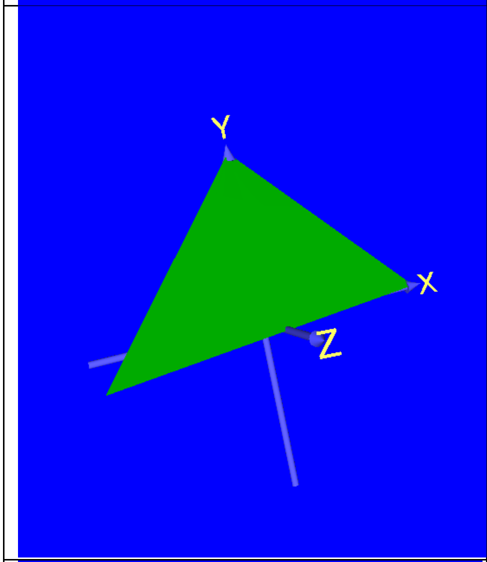
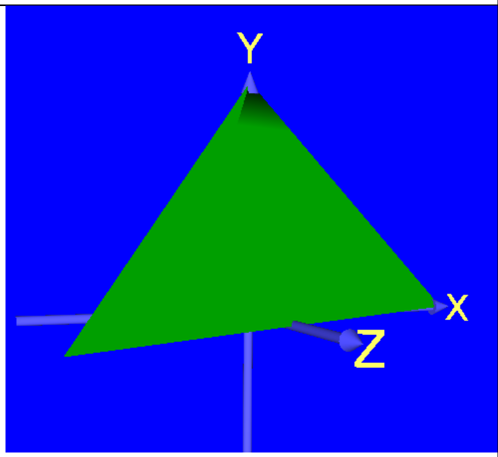
Equations:

x = u
y = u*cos(v)
z = u*sin(v)

u ε [0,1/√2], v ε [0,2π]



How shapes change in response to sampling resolution

<table>
<tr><th>Normal Resolution(75)</th><th>Low Resolution (5)</th><th>Observation</th></tr>
<tr><td></td><td></td><td>No significant change.</td></tr>
<tr><td>
</td><td>
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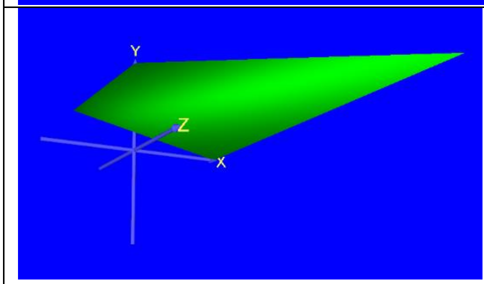
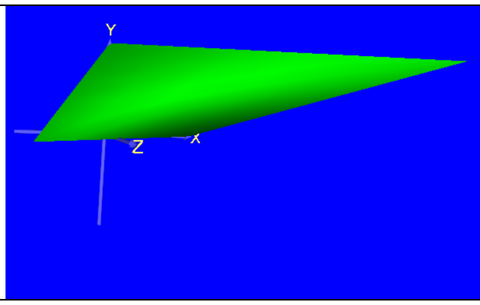
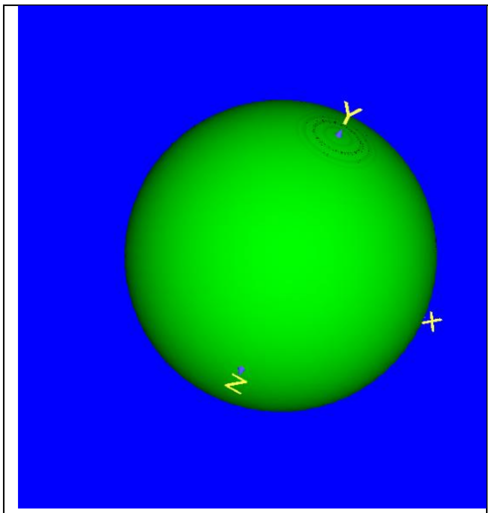
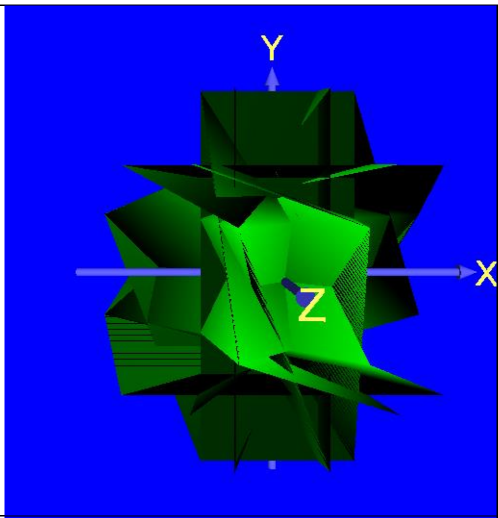
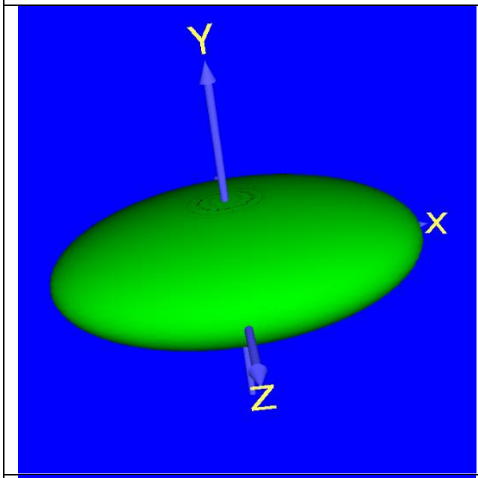
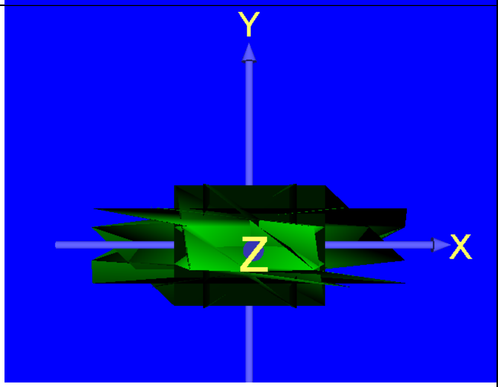
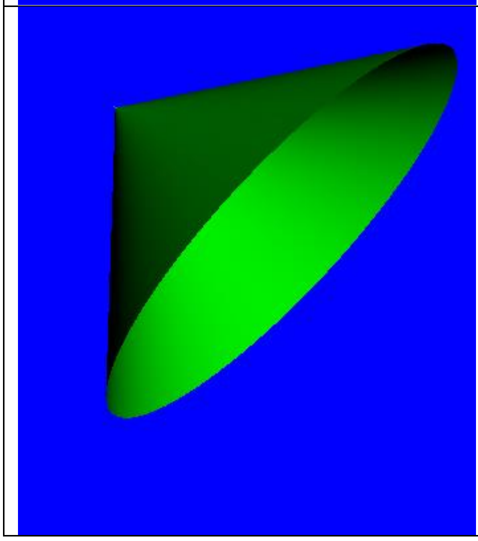
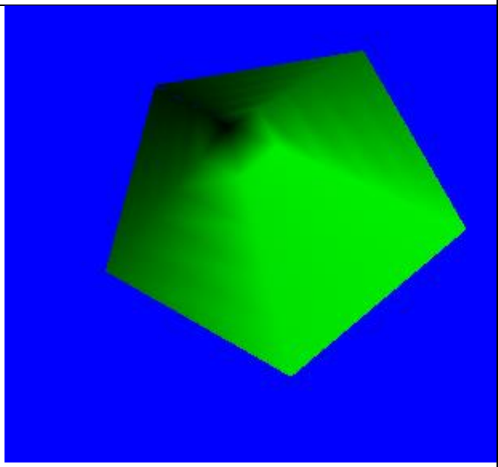
<tr><td>
</td><td>
</td><td>No significant change.</td></tr>
</table>

		Sphere is completely distorted and unrecognizabl e.
		Ellipsoid is completely distorted and unrecognizabl e.
		Cone is distorted to look like pyramid with hexagonal base.

6. Solid Box

Equations:

$x = u$
$y = v$
$z = w$

$u \varepsilon [0,1]$, $v \varepsilon [0,1]$, $w \varepsilon [0,1]$

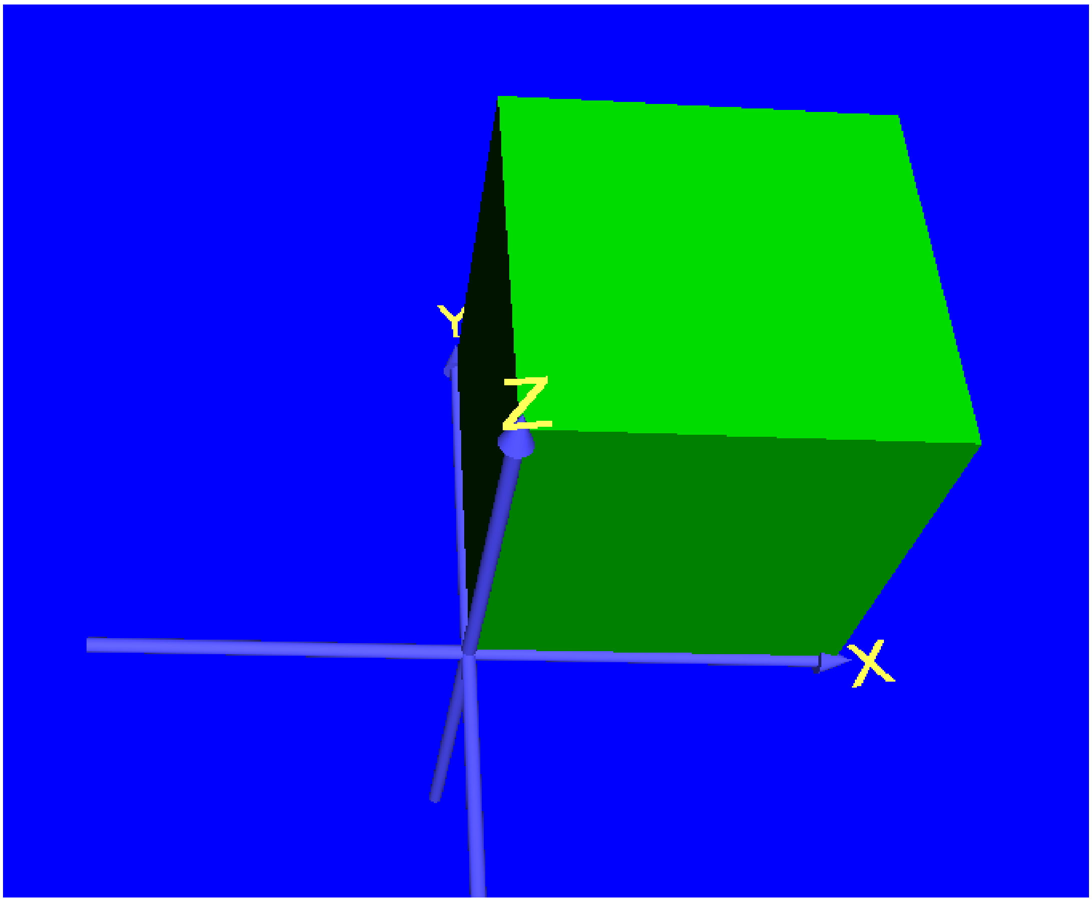


Figure 6: Solid Box. Source code: solid.wrl

7. Solid Sphere

Equations:
x = w*cos(u)*sin(v)
y = w*sin(u)
z = w*cos(u)*cos(v)

u ε [-π/2, π/2], v ε [-π, π], w ε [0,1]

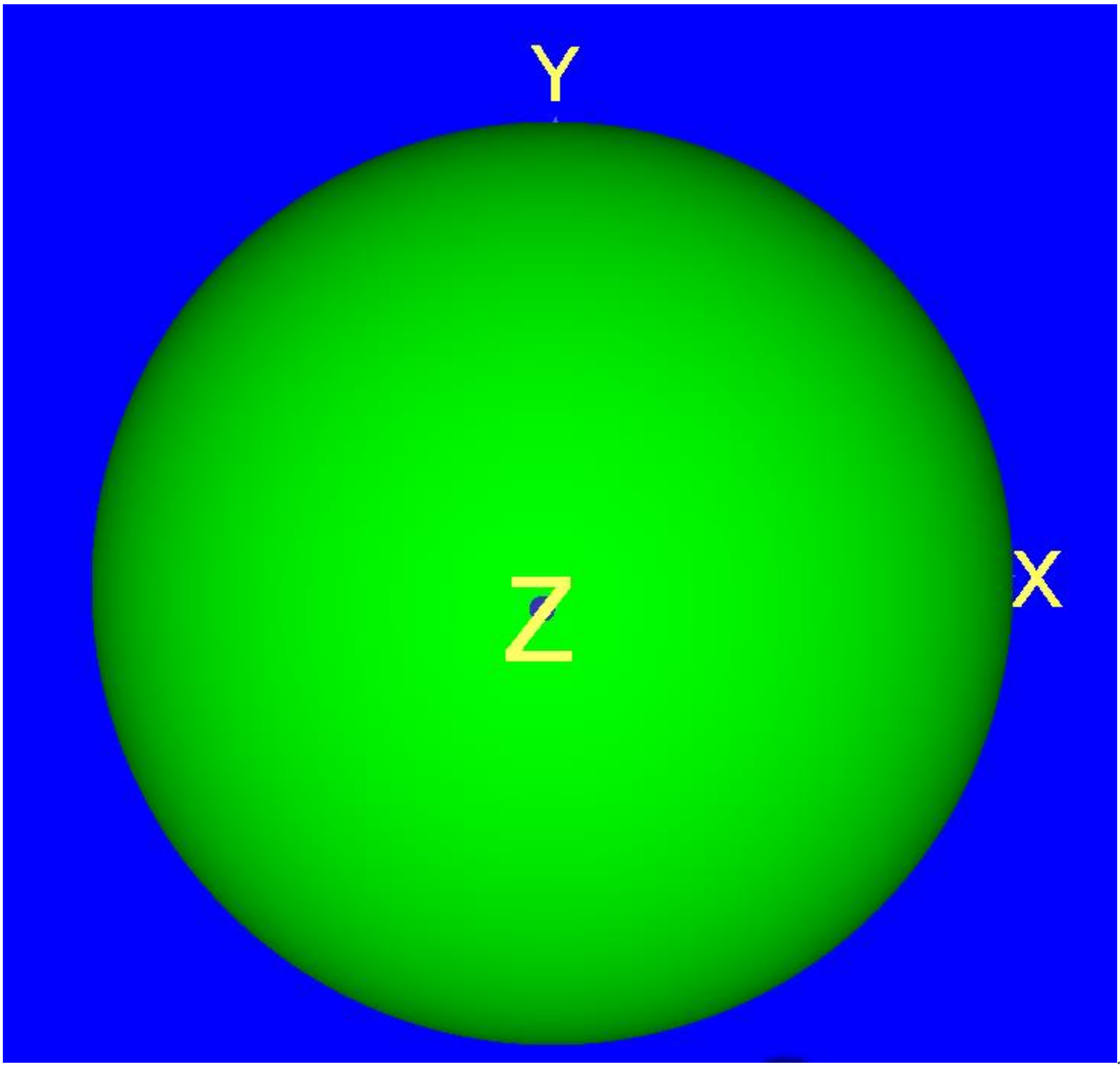


Figure 7: Solid sphere centered at (0,0,0) with radius 1. Source code: sphereSolid.wrl

8. Solid Cylinder

Equations:
x = u*sin(2π*v)
y = w*1.5
z = u*cos(2π*v)

u ε [0,1], v ε [0,1], w ε [0,1]

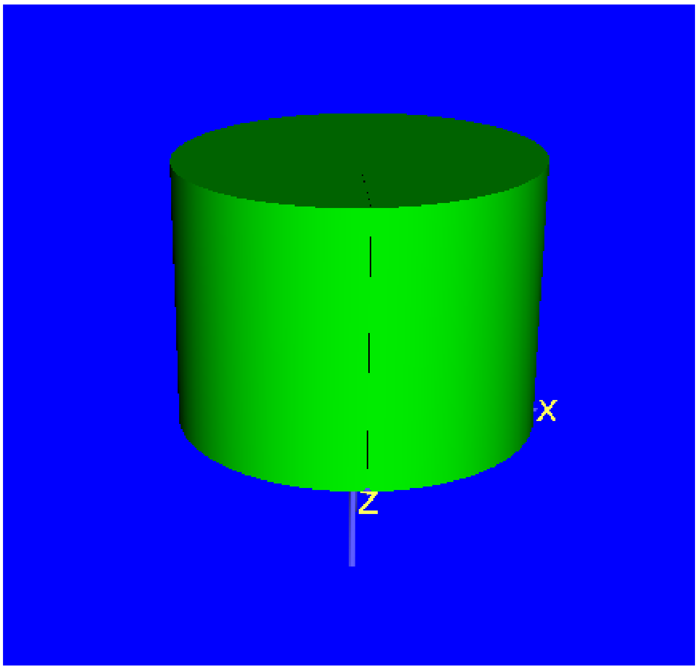


Figure 8: Cylinder with height 1.5 and radius 1, centered at (0,0,0). Source code: cylinderSolid.wrl

9. Solid Cone

Equations:
$x = u*(1-w)*\sin(2\pi*v)$
$y = w*1.5$
$z = u*(1-w)*\cos(2\pi*v)$

$u \in [0,1]$, $v \in [0,1]$, $w \in [0,1]$

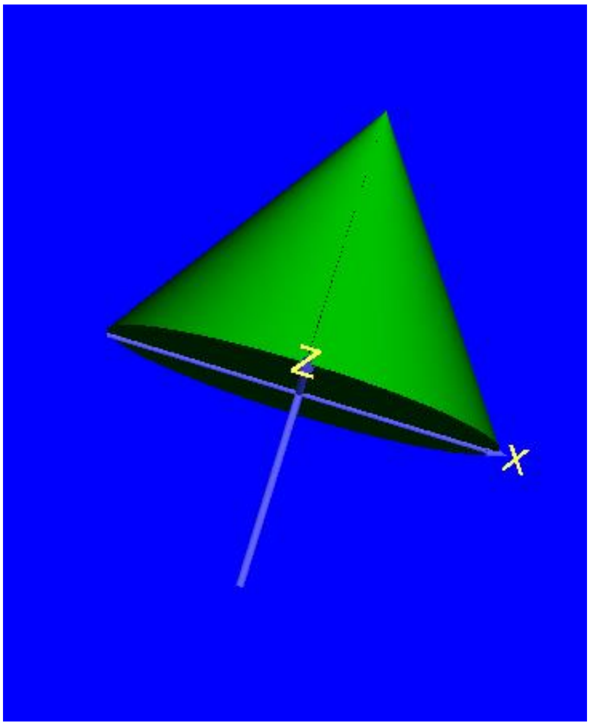


Figure 9: Solid cone with base of radius 1, centered at (0,0,0) and height of 1.5. Source code: coneSolid.wrl

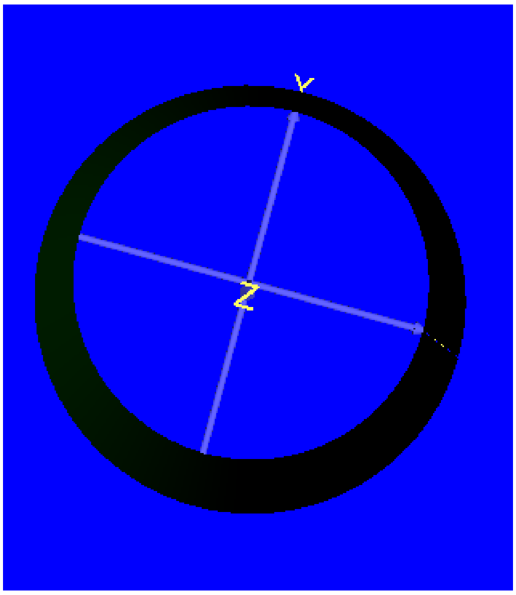
10. Conversion of closed surface to solid object

First, we start of with a cylindrical surface created by translational sweeping of a circle centered at (0,0,0) with radius 1. The translational sweeping takes place along the positive z-axis by 1.5 units.

$x = \cos(u*2\pi)$
$y = \sin(u*2\pi)$
$z = 1.5*v$

$u \in [0,1]$, $v \in [0,1]$

We get the following cylindrical surface:

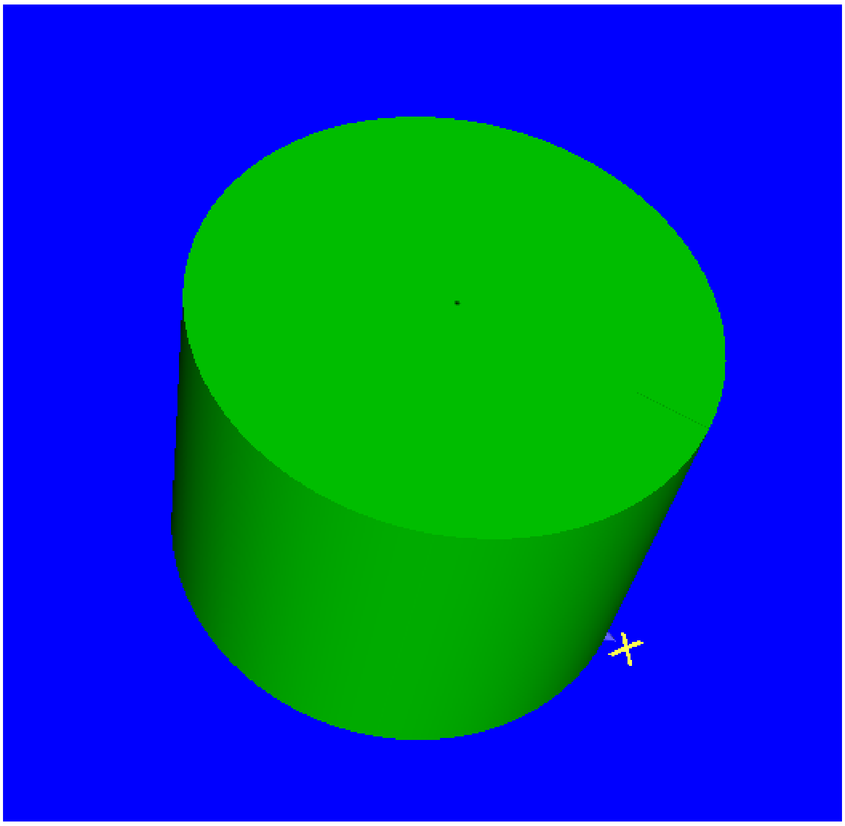


By multiplying a new parameter, w, to the radius of the circle before translational sweeping, we are effectively creating a circle surface made up of infinitely many cicle curves of varying radii (from 0 to the original radius, 1). Thus, when we are doing translational sweeping, we are transforming a surface into a solid, and not a curve into a surface.

New equations:
$x = w*\cos(u*2\pi)$
$y = w*\sin(u*2\pi)$
$z = 1.5*v$

$u \in [0,1]$, $v \in [0,1]$, $w \in [0,1]$

We then get the following solid cylinder:



11. Use curve y=sin(x) for making a solid by applying rotational and translational sweepings together.

We start with:

x = u
y = sin(u)
z = 0, u ε [0,2π]

We then perform rotational sweeping on the curve by π radians anticlockwise.

x = u*cos(v)
y = sin(u)
z = -sin(v)*u, u ε [0,2π], v ε [0, π]

Finally we perform translational sweeping on the curve by 0.5 units parallel to the negative y-axis.

x = u*cos(v)
y = sin(u)-0.5*w
z = -sin(v)*u, u ε [0,2π], v ε [0, π], w ε [0,1]

And we get this solid: