“Shock-ABIS” Method: Shock Analysis Based on Imaging Spectrograph

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1 Introduction

The shocks have been identified in the solar atmosphere. However, a quantitative diagnosis of the shocks in the solar atmosphere is still lacking. Ruan, Yan, He et al. (2018) proposed a new method to realize the goal of the shock quantitative diagnosis, based on Rankine–Hugoniot equations and taking the advantages of simultaneous imaging and spectroscopic observations from, e.g., IRIS (Interface Region Imaging Spectrograph). By invoking this method, one can obtain the comprehensive features of the observed shocks: the propagating speed and direction of shocks, and the bulk velocities, temperatures and Mach numbers of plasmas in the upstream and downstream. Here, we introduce how to use this method to diagnose the observed shocks in detail. The introduction of this method refer to Ruan, Yan, He et al. (ApJ, 860, 99, 2018). The method is named as “Shock-ABIS”, which stands for “Shock Analysis Based on Imaging Spectrograph”.

To diagnose the observed shocks with this method, following information is needed: (1) intensities in upstream and downstream of the involved emission line; (2) Doppler shifts in upstream and downstream of the involved emission line; (3) propagation speed of the shock front in the plane of sky (POS). Figure 1 shows the sketch of this method.

2 Equations

Suppose that the oscillation propagates in the solar atmosphere and is observed by a telescope from a certain viewing angle. The angle between the LOS direction and the wave propagating direction is θ, and the propagating speed is v_p. We define

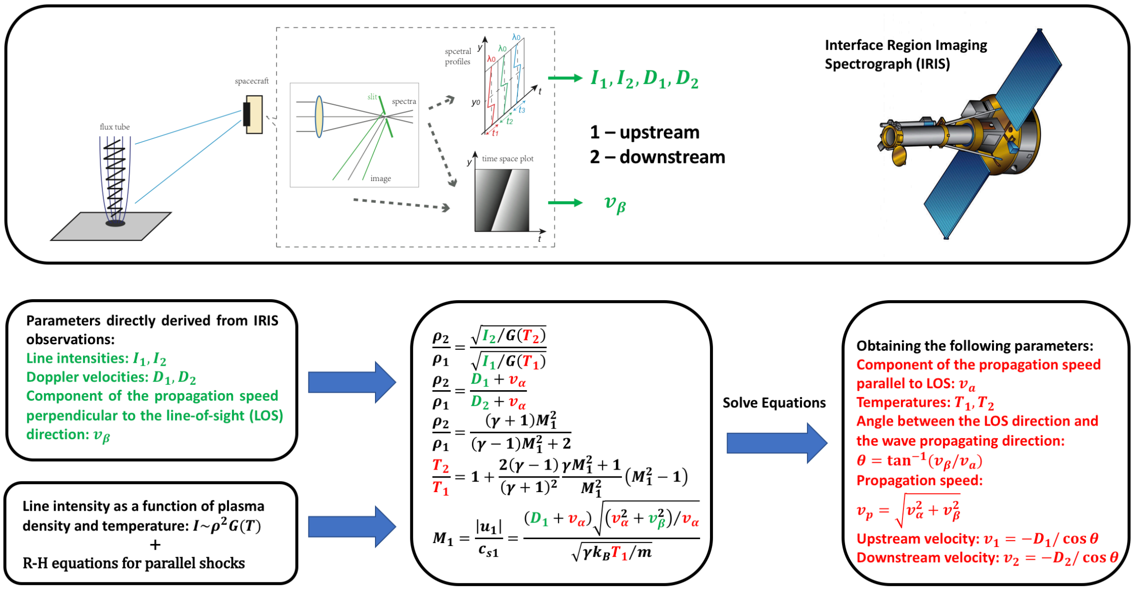

Figure 1: Sketch of the shock diagnose method.

that $v_\alpha = v_p \cos\theta$ is the component of the propagating speed v_p parallel to the LOS direction, $v_\beta = v_p \sin\theta$ is the component perpendicular to the LOS direction (i.e., projected onto the plane of sky). The R-H equations for parallel shock read

$$\rho_1 u_1 = \rho_2 u_2, \tag{1}$$
$$\rho_1 u_1^2 + p_1 = \rho_2 u_2^2 + p_2, \tag{2}$$
$$\rho_1 u_1 \epsilon_1 + p_1 u_1 + \rho_1 u_1^3/2 = \rho_2 u_2 \epsilon_2 + p_2 u_2 + \rho_2 u_2^3/2, \tag{3}$$

where ρ is the density of plasma, ϵ is internal energy density, u is the bulk velocity of plasma in the shock rest frame and p is the pressure of plasma. The subscript numbers 1 and 2 stand for the upstream and downstream of the shock respectively.

In the "laboratory" reference frame, R-H equations can be converted to

$$\frac{\rho_2}{\rho_1} = \frac{D_1 + v_\alpha}{D_2 + v_\alpha} \tag{4}$$
$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2+2}, \tag{5}$$
$$\frac{T_2}{T_1} = 1 + \frac{2(\gamma-1)}{(\gamma+1)^2}\frac{\gamma M_1^2+1}{M_1^2}(M_1^2-1), \tag{6}$$

where D, T and M represent Doppler velocity, temperature and Mach number, respectively. The ratio of density is also given by

$$\frac{\rho_2}{\rho_1} = \frac{\sqrt{I_2/G(T_2)}}{\sqrt{I_1/G(T_1)}}. \tag{7}$$

where I is the intensity of an emission line, and $G(T)$ is the contribution function of the emission line. The Mach number in upstream is given by

$$M_1 = \frac{|u_1|}{c_{s1}} = \frac{|v_1 - v_p|}{c_{s1}} = \frac{(D_1 + v_\alpha)\sqrt{(v_\alpha^2 + v_\beta^2)}/v_\alpha}{\sqrt{\gamma k_B T_1/m}}, \tag{8}$$

where local sound speed c_{s1} is a function of upstream temperature T_1, $v_1 = -D_1/\cos\theta$ is upstream velocity in the "laboratory" reference frame, $\gamma = 5/3$ is the ratio of specific heating, k_B is Boltzmann's constant, m is the estimated average mass of particles. For the plasma consist of protons and electrons, the value of m is $m_p/2$, where m_p is the mass of a proton.

The parameters I_1, I_2, D_1, D_2 and v_β can be derived from the observational data directly. The parameters T_1, T_2 and v_α can be obtained via solving equations (4) to (8). Thereafter, we can obtain the parameters v_p, θ, u_1 and u_2 from the following equations:

$$\theta = \arctan(v_\beta/v_\alpha), \tag{9}$$

$$v_p = \sqrt{v_\alpha^2 + v_\beta^2}, \tag{10}$$

$$v_1 = -D_1/\cos\theta, \tag{11}$$

$$v_2 = -D_2/\cos\theta. \tag{12}$$

We can also estimate the upstream and downstream Mach numbers while the temperatures, bulk velocities of plasma and propagation velocity of shock front are known.

3 Solving the equations

This section provides a method to numerical solving the equations (4) to (8) to derive the parameters T_1, T_2 and v_α. These equations can be solved with iteration, where v_α is the iteration variable. According to equations (4) and (5), the upstream Mach number M_1 can be converted to a function of v_α:

$$M_1^2(v_\alpha) = \frac{2(D_1 + v_\alpha)}{(\gamma+1)(D_2 + v_\alpha) - (\gamma-1)(D_1 + v_\alpha)}. \tag{13}$$

According to equations (8) and (13), T_1 can be converted to a function of v_α:

$$T_1(v_\alpha) = \frac{(D_1 + v_\alpha)^2 (v_\alpha^2 + v_\beta^2) m}{\gamma k_B v_\alpha^2 M_1^2}. \tag{14}$$

According to equations (6) and (14), T_2 can be converted to a function of v_α:

$$T_2(v_\alpha) = [1 + \frac{2(\gamma - 1)}{(\gamma + 1)^2} \frac{\gamma M_1^2 + 1}{M_1^2} (M_1^2 - 1)] T_1(v_\alpha). \tag{15}$$

Here the gradient descent iterative method is adpoted. The iterative relationship comes from equations (4) and (7) reads:

$$f(v_\alpha) = \frac{\sqrt{I_2/G(T_2)}}{\sqrt{I_1/G(T_1)}} - \frac{D_1 + v_\alpha}{D_2 + v_\alpha}. \tag{16}$$

The pseudocode below is an example of the gradient descent iteration:

va = 50000.0 #initial value of v_α is 50 km/s
step = 500.0 #step of iteration is 500 m/s
f_current = f(va) #current value of $f(v_\alpha)$
while (f_current > 1.0e-4): #iteration stop when $f(v_\alpha) <= 10^{-4}$
    va = va - step * grad #new value of v_α, where grad is the gradient of $f(v_\alpha)$
    f_current = f(va) # current value of $f(v_\alpha)$

4 Diagnosing observed shock with the provided tool

A python code is provided to diagnose shocks observed in the solar atmosphere. Ones just need input the information of the observed shocks in the parameters input region (see figure 2) and run the code. The output of the code see figure 3.

```
#----------------------------------------------------------------------#
#-----------------------Parameters input begin------------------------#
#----------------------------------------------------------------------#
#Input
I1=2321          #upstream line intensity
D1=12.2e3        #[m/s],upstream Doppler velocity
I2=3481          #downstream line intensity
D2=-3.4e3        #[m/s], downstream Doppler velocity
vbeta=2.1e4      #[m/s], component of propagation speed perpendicular
                 #to the LOS direction


#Assumptions
mpr=1.1*mp/2.0    #average mass of charged particles (ions + electrons)
                  #local acoustic speed is assumped to be
                  #sqrt(gamma*kB*T/mpr)

#Emission line
Line='SiIV'

#Temperature of the emission line
T0=80000         #[K]

#Step for iteration. A small step lead to accurate results but a long
#running time
step=500.0      #[m/s]

#----------------------------------------------------------------------#
#-----------------------Parameters input end--------------------------#
#----------------------------------------------------------------------#
```

Figure 2: Input information of the observed shock.

```
#-----------------------------------------------------------------#
#---------------------------  INPUT  ----------------------------#
#-----------------------------------------------------------------#
Emission line:
SiIV
Upstream emission intensity:
I1 = 2321
Upstream Doppler shift:
D1 = 12.2 km/s
Downstream emission intensity:
I1 = 3481
Downstream Doppler shift:
D1 = -3.4 km/s
Component of propagation speed perpendicular to the LOS direction:
v_beta = 21 km/s


#-----------------------------------------------------------------#
#---------------------------  OUTPUT  ---------------------------#
#-----------------------------------------------------------------#
Component of propagation speed parallel to the LOS direction:
v_alpha = 37 km/s
Upstream temperature:
T1 = 7.3x10^4 K
Downstream temperature:
T2 = 9.6x10^4 K

Propagation speed:
vp = 43 km/s
Angle between the LOS direction and the propogation direction:
theta = 30 degree

Upstream velocity:
v1 = -14.1 km/s
Downstream velocity:
v2 = 3.9 km/s

Upstream Mach number:
M1 = 1.3
Downstream Mach number:
M2 = 0.8
```

Figure 3: Output of the shock diagnosing tool.