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Boundary Vibration Control of Variable Length Crane Systems in Two Dimensional Space with Output Constraints

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***Abstract*—A variable length crane system under the external disturbances and constraints is studied in the two-dimensional space. The dynamical analysis of the cable system considers the variable length, variable tension, variable speed and the coupled vibrations of the cable in the longitudinal-transverse directions. Considering the output constraint problems, boundary control algorithms with output signal barriers are designed and acted on the boundary of the cable to reduce the coupled vibrations of the flexible crane cable and to ensure the stability of the system in theory. Effectiveness and performance of the proposed control schemes are depicted via several simulation examples.**

***Index Terms*—Crane system, Longitudinal-transverse vibration, Variable length, Boundary control, Output constraints.**

I. Introduction

Crane systems have been extensively applied to material transportation in many industrial fields including offshore engineering [1], cable-driven parallel robots [2] and so on [3] for their unique advantages including light weight, easy assembly, lower cost and less energy consumption. However, considering the cable's flexible properties, the fast movement of the crane cable with external disturbance will generate the swing of the payload, which in turn deteriorates accurate position and precise transportation. In the dynamical modeling, the crane system is regarded as a classical flexible axially hoisting string system with a freely moving payload [4], [5], and should consider the coupled relationship between longitudinal vibrations and transverse vibrations for the longitudinal-moving of the flexible cable.

Many researches [6], [7], [8] investigated and analyzed the coupled vibrations of the axially-moving cable system with a set of nonlinear ODEs. In [6], the authors analyzed the model of a slowly hoisting cable system in the two-dimensional directions. Bao et al. [9] studied the model and control method of the longitudinal vibration on flexible hoisting system. In [8], Wang et al. investigated the control method to suppress transverse vibration for a hoisting cable by adjusting the payload. In addition, the neural network control is an effective method to handle the flexible system described with ODEs model [10] and this method has the development potential [11], [12]. However, these methods [13], [14], [15], [16], [17] are based on ODE model and maybe not applied to the infinite dimensional PDE systems [18], [19], [20], [21], [22], [23], [24].

Different from the common methods used to handle distributed parameter system, boundary control [5], [25], [26], [27], [28], [29], which is directly based on the infinite system described by PDEs and can overcome the aforementioned drawbacks, has been employed to control crane systems. In [30], the transverse vibration control problems and the control strategy for a moving cable-guided hoisting system with the varying tension were given and proposed. In [31], [32], [33], He et al. proposed an active control method to handle the vibration problem for a flexible string system. In [25], the vibration control laws for a constant-length longitudinally moving string system with hydraulic actuator were considered and studied in the longitudinal-transverse directions. Besides, there are other contributions to the similar problems [34], [35], [36], [37], [38], [39], [32]. However, few works apply the active boundary control to the researches for the length-varying hoisting cable with the longitudinal-transverse coupled vibrations. Therefore, we plan to propose an effective control algorithm to deal with this problem, namely, to suppress both longitudinal-transverse coupled vibrations for the hoisting cable with variable length.

Since the crane system studied in this paper move and vibrate freely, taking safety into consideration, it is essential to ensure the boundary vibrations of the payload constrain into a specified scope, as transgression of constraints would lead to performance deterioration or even severe hazards [5], [40], [41], [42], [43]. Output signal barrier function [44], [45] is brought to guarantee that the output states do not violate the desired limitations and we construct an appropriate barrier Lyapunov function to solve the output constraint problem.

In this paper, we aim at the active vibration control design for a crane system under unknown exterior boundary disturbances and constraints. Two sets of boundary controllers are implemented on the both ends of the cable to suppress vibrations from each direction respectively. Combining with dynamical model, the active boundary control laws with the barrier conditions are provided to guarantee the convergence and boundedness of the systemic states. Simultaneously, the output constrains at the bottom of the string are not violated

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in two directions.

The main innovations and contributions of this paper cover

(i) The dynamical characteristic is analyzed and the nonlinear coupled PDEs-ODEs model is established for the crane system considering the variable length, variable tension, variable speed and the longitudinal-transverse coupled relationship.

(ii) Boundary control is brought in the crane system to deal with the vibration suppression problem. The longitudinal-transverse coupled vibrations of the flexible cable are reduced to a small neighbourhood of the initial position in crane system.

(iii) Barrier functions are constructed and integrated into the boundary control laws to ensure the stability of the crane system and to guarantee the output signals within the given constraints based on barrier Lyapunov function.

In order to make the paper readable, we give the organization for the rest paper. Section II provides some lemmas and introduces the formulation of the dynamic model of the crane system. Four boundary controllers are proposed in Section III, in which the convergence and boundedness of the states are demonstrated based on Lyapunov function-based stability analysis. In Section IV, a series of numerical simulations are performed to verify the theoretical proof for the designed control. Last, Section V gives the conclusion.

II. Modeling and Problem Formulation

This crane system, shown in Fig. 1, is comprised of a hoisting cable and a payload in the bottom of the cable. The cable is lifted by the upper motor with an axial speed $v(t)$, namely, $v(t) < 0$, while it is simultaneously subject to both boundary longitudinal and transverse disturbances denoted by $d_z(t)$ and $d_w(t)$. For the constraint problems, the boundary elastic deflections $w(l(t), t)$ and $z(l(t), t)$ are required to satisfy the limitations $|z(l(t), t)| \le C_z$ and $|w(l(t), t)| \le C_w$, where C_z and C_w are positive constants. Longitudinal controller $u_{z1}(t)$ and transverse controller $u_{w1}(t)$ are applied at the lower boundary of the cable, and the longitudinal controller $u_{z2}(t)$ and transverse controller $u_{w2}(t)$ are implemented at the upper boundary of the cable.

Definition 1: In order to make this paper more concise, throughout this paper the notations $* = *(x, t)$, $(\cdot)_x = \frac{\partial(\cdot)}{\partial x}$, $(\cdot)_{xx} = \frac{\partial^2(\cdot)}{\partial x^2}$, $(\cdot)_t = \frac{\partial(\cdot)}{\partial t}$, $(\cdot)_{tt} = \frac{\partial^2(\cdot)}{\partial t^2}$, $*^{L[l(t)]} = *(l(t), t)$ and $*^{L[0]} = *(0, t)$ are used.

A. Modeling for the Nonlinear Crane System

The coupling of longitudinal and transverse motions of the cable is considered in this paper, and the crane cable is moving upward. The dynamic model of the crane system can be deduced by using Hamilton's principle.

The parameters of the crane system are listed as follows: m_p describes the lump mass of the payload and ρ_c is given as the cable's unit distributed mass. In addition, the material derivative $\frac{D(\cdot)}{Dt}$ is taken into consideration in the dynamical analysis and $\frac{D(\cdot)}{Dt} = \frac{\partial(\cdot)}{\partial t} + v(t)\frac{\partial(\cdot)}{\partial x}$, where, $\dot{x} = v(t)$. EA is the axial stiffness of the cable. At time t, the elastic deflection $w(x, t)$ represents displacement of the flexible cable in X direction for position x, and $z(x, t)$ describes the Y

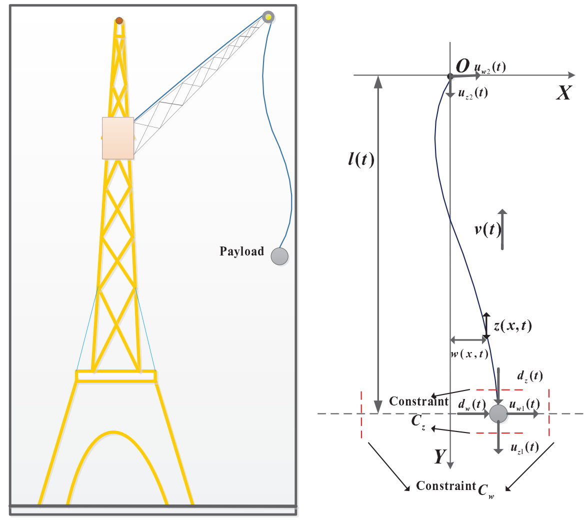

Fig. 1. A crane system in two dimensional space

direction. In addition, $l(t)$ is time-varying which is given as $l(t) = L + \int_0^t v(\varsigma)d\varsigma$.

Refereing the previous works [25], [32], obtain the kinetic energy function of the crane system as

$$\begin{aligned} E_k(t) &= \frac{m_p}{2}[v(t) + \frac{Dz^{L[l(t)]}}{Dt}]^2 + \frac{1}{2}\rho_c\int_0^{l(t)}[v(t) + \tfrac{Dz}{Dt}]^2 dx \\ &+\frac{m_p}{2}[\frac{Dw^{L[l(t)]}}{Dt}]^2 + \frac{1}{2}\rho_c\int_0^{l(t)}(\tfrac{Dw}{Dt})^2 dx \\ &+\frac{1}{2}\rho_c[L - l(t)][v(t)]^2, \end{aligned} \quad (1)$$

where $\frac{D(\cdot)}{Dt}$ denotes material derivative. We obtain the potential energy function as

$$\begin{aligned} E_p(t) &= \tfrac{EA}{2}\int_0^{l(t)}[z_x + \tfrac{1}{2}(w_x)^2]^2 dx \\ &+\tfrac{1}{2}\int_0^{l(t)}P(x, t)(w_x)^2 dx, \end{aligned}$$

where the tension $P(x, t)$ [46] is time-varying and distributed and given as

$$P(x, t) = \{m_p + \rho_c[l(t) - x]\}[g - \dot{v}(t)], \quad (2)$$

where g is gravitational acceleration. We obtain the virtual work function of the external boundary disturbances as

$$\delta W_d(t) = d_w(t)\delta w^{L[l(t)]} + d_z(t)\delta z^{L[l(t)]}. \quad (3)$$

The virtual work function of the proposed boundary control forces $u_{z1}(t)$, $u_{w1}(t)$, $u_{z2}(t)$ and $u_{w2}(t)$ is given as

$$\begin{aligned} W_b(t) &= u_{z1}(t)\delta z^{L[l(t)]} + u_{w1}(t)\delta w^{L[l(t)]} + u_{z2}(t)\delta z^{L[0]} \\ &+u_{w2}(t)\delta w^{L[0]}. \end{aligned} \quad (4)$$

Then, we get the total virtual work function as

$$\delta W(t) = \delta W_d(t) + \delta W_b(t). \quad (5)$$

Combining Leibniz's integral rule [47], the system's governing equations are deduced using Hamilton's principle, which is expressed as $\int_0^{l(t)}\delta[E_k(t) - E_p(t) + W(t)]dx = 0$.

$$\rho_c[\dot{v}(t) + \frac{D^2 z}{Dt^2}] - EA\frac{\partial}{\partial x}[z_x + \frac{1}{2}(w_x)^2] = 0, \quad (6)$$

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$$\rho_c \frac{D^2 w}{Dt^2} - \frac{\partial}{\partial x}[P(x,t)w_x] - EA\frac{\partial}{\partial x}[z_x w_x + \frac{1}{2}(w_x)^3] = 0, \quad (7)$$

$\forall(x,t) \in (0, l(t)) \times [0, \infty)$, meanwhile, the boundary conditions are deduced as

$$u_{z1}(t) = m_p[\dot{v}(t) + \frac{D^2 z^{L[l(t)]}}{Dt^2}] - d_z(t) + \frac{EA}{2}[2z_x^{L[l(t)]} + (w_x^{L[l(t)]})^2], \quad (8)$$

$$u_{z2}(t) = -EA[z_x^{L[0]} + \frac{1}{2}(w_x^{L[0]})^2], \quad (9)$$

$$u_{w1}(t) = m_p\frac{D^2 w^{L[l(t)]}}{Dt^2} - d_w(t) + P(l(t),t)w_x^{L[l(t)]} + \frac{EAw_x^{L[l(t)]}}{2}[2z_x^{L[l(t)]} + (w_x^{L[l(t)]})^2], \quad (10)$$

$$u_{w2}(t) = -P(0,t)w_x^{L[0]} - \frac{EAw_x^{L[0]}}{2}[2z_x^{L[0]} + (w_x^{L[0]})^2]. \quad (11)$$

B. Preliminaries

Some related concepts are given in this section in order to explain the following research of this topic.

Assumption 1: Considering the finite energy of the disturbances $d_z(t)$ and $d_w(t)$, we assume that will can guarantee $|d_z(t)| \leq \bar{d}_z(t)$ and $|d_w(t)| \leq \bar{d}_w(t)$, where the constants $\bar{d}_z(t)$ and $\bar{d}_w(t) \in \mathbb{R}^+$.

Assumption 2: Combining (2), the distributed tension $P(x,t)$ is assumed to be nonnegative, namely, the acceleration of the cable is confined by $|\dot{v}(t)| < g$. Further, it can be deduced that $P'(x,t)$, and $\dot{P}(x,t)$ are also bounded. Therefore, there are a set of positive constants to satisfy $P(x,t) \in [P_a, P_b]$, $P'(x,t) \in [P'_a, P'_b]$ and $\dot{P}(x,t) \in [\dot{P}_a, \dot{P}_b]$.

Assumption 3: The initial state values of the crane system are assumed to be bounded in L^2-norm and a nonnegative constant F_0 is existing to satisfy $\int_0^L \{[w(x,0)]^2 + [z(x,0)]^2 + [\dot{w}(x,0)]^2 + [\dot{z}(x,0)]^2\}dx \leq F_0$.

III. Control Design

An active boundary control strategy are designed to reduce the longitudinal-transverse coupled vibrations of the flexible cable for the crane system in this paper and to keep the boundary displacements of the tip payload in the given limitations, namely, to keep $|z^{L[l(t)]}| < C_z$ and $|w^{L[l(t)]}| < C_w$. To achieve the control objectives, the boundary control design and the stability analysis will be completed cooperatively, where, the controllers $u_{z1}(t)$ and $u_{w1}(t)$ act at the lower boundary of the cable and $u_{z2}(t)$ and $u_{w2}(t)$ at the upper boundary of the cable, and barrier Lyapunov function is constructed to realize the output constraint. The design process of the control laws is illustrated in Fig. 2.

The challenge for the topic is how to cope with the coupling of the two-dimensional elastic deflections for the flexible longitudinally moving cable. The Lyapunov candidate function $F(t)$ is constructed as

$$F(t) = F_e(t) + F_a(t) + F_c(t), \quad (12)$$

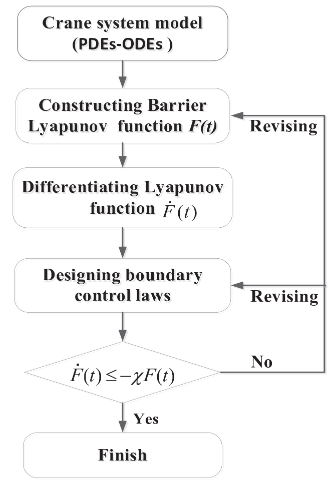

Fig. 2. The design process of the boundary control for the crane system

where

$$F_e(t) = \frac{a\rho_c}{2}\int_0^{l(t)}[v(t) + \frac{Dz}{Dt}]^2 dx + \frac{a\rho_c}{2}\int_0^{l(t)}[\frac{Dw}{Dt}]^2 dx + \frac{aEA}{2}\int_0^{l(t)}[z_x + \frac{1}{2}(w_x)^2]^2 dx + \frac{a}{2}\int_0^{l(t)} P(x,t)(w_x)^2 dx, \quad (13)$$

$$F_a(t) = \frac{am_p}{2}[u_{aw}(t)]^2 \ln\frac{2C_w^2}{C_w^2 - (w^{L[l(t)]})^2} + \frac{am_p}{2}[u_{az}(t)]^2 \ln\frac{2C_z^2}{C_z^2 - (z^{L[l(t)]})^2}, \quad (14)$$

$$F_c(t) = b\rho_c\int_0^{l(t)} x[v(t) + \frac{Dz}{Dt}]z_x dx + b\rho_c\int_0^{l(t)} x\frac{Dz}{Dt}w_x dx, \quad (15)$$

where, a, b are two positive weighting constants. C_z and C_w are used to ascertain $|z^{L[l(t)]}| \leq C_z$ and $|w^{L[l(t)]}| \leq C_w$. $u_{az}(t)$ and $u_{aw}(t)$ are auxiliary functions constructed as

$$u_{az}(t) = z_x^{L[l(t)]} + v(t) + \frac{Dz^{L[l(t)]}}{Dt}, \quad (16)$$

$$u_{aw}(t) = \frac{Dw^{L[l(t)]}}{Dt} + w_x^{L[l(t)]}. \quad (17)$$

The boundary control laws which are illustrated in Fig. 3 are designed as follow:

$$\begin{aligned} u_{z1}(t) = & -k_3 u_{az}(t) + EA[z_x^{L[l(t)]} + \frac{1}{2}(w_x^{L[l(t)]})^2] \\ & -m_p z_{xt}^{L[l(t)]} - m_p v(t) z_{xx}^{L[l(t)]} + \{-k_4 u_{az}(t) \\ & -\frac{m_p u_{az}(t) z^{L[l(t)]}(Dz^{L[l(t)]}/Dt)}{C_z^2 - [z^{L[l(t)]}]^2} \\ & -\frac{EAw_x^{L[l(t)]}}{2}[2z_x^{L[l(t)]} + (w_x^{L[l(t)]})^2] \\ & -P(l(t),t)w_x^{L[l(t)]}\}/\ln\frac{2C_z^2}{C_z^2 - [z^{L[l(t)]}]^2} \\ & -\mathrm{sgn}[u_{az}(t)]\bar{d}_z, \end{aligned} \quad (18)$$

$$u_{z2}(t) = -k_6[v(t) + \frac{Dz^{L[0]}}{Dt}], \quad (19)$$

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$$
\begin{aligned}
u_{w1}(t) &= -k_1 u_{aw}(t) + w_x^{L[l(t)]}\{P(l(t),t) + EA[z_x^{L[l(t)]} \\
&\quad + \frac{1}{2}(w_x^{L[l(t)]})^2]\} - m_p w_{xt}^{L[l(t)]} - m_p v(t) w_{xx}^{L[l(t)]} \\
&\quad + \{-k_2 u_{aw}(t) - EA[z_x^{L[l(t)]} + \frac{1}{2}(w_x^{L[l(t)]})^2] \\
&\quad - m_p u_{aw}(t) \frac{w^{L[l(t)]} D w^{L[l(t)]}/Dt}{C_w^2 - [w^{L[l(t)]}]^2}\} \\
&\quad / \ln \frac{2C_w^2}{C_w^2 - [w^{L[l(t)]}]^2} - \mathrm{sgn}[u_{aw}(t)]\bar{d}_w, \qquad (20) \\
u_{w2}(t) &= -k_5 \frac{D w^{L[0]}}{Dt}, \qquad (21)
\end{aligned}
$$

where k_1, k_2, k_3, k_4, k_5 and k_6 are positive control gains.

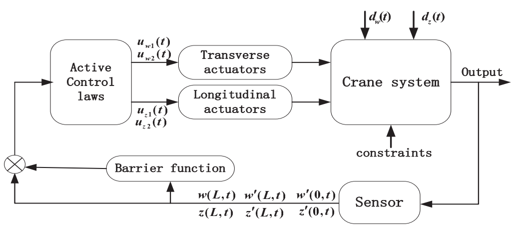

Fig. 3. The procedure of the control design for the crane system

Lemma 1: By choosing appropriate parameters, the Lyapunov candidate function (12) can be proven to be positive definite and obtained as

$$0 \le \chi_1[A(t) + F_a(t)] \le F(t) \le \chi_2[A(t) + F_a(t)], \qquad (22)$$

where $\chi_1 = \min\{[\frac{a}{2}\min\{\rho_c, P_a, EA\} - \frac{1}{2}b\rho_c L], 1\}$ and $\chi_2 = \max\{[\frac{a}{2}\max\{\rho_c, P_b, EA\} - \frac{1}{2}b\rho_c L], 1\}$ are two positive constants, and the auxiliary function $A(t)$ is defined as

$$
\begin{aligned}
A(t) &= \int_0^{l(t)} \{[\frac{Dw}{Dt}]^2 + [z_x + \frac{1}{2}(w_x)^2]^2 + [v(t) + \frac{Dz}{Dt}]^2 \\
&\quad + [w_x]^2\} dx. \qquad (23)
\end{aligned}
$$

Proof: See Appendix A. ∎

Lemma 2: The time derivation of the Lyapunov candidate function (12) is negative definite.

$$\dot{F}(t) \le -\chi F(t), \qquad (24)$$

where χ is a positive constant.
Proof: See Appendix B. ∎

Combing (24), we can analyze the stabilization of the system with the control.

Theorem 1: Based on the Assumption 3, the closed-loop system with the novel boundary controllers can be proven to be convergence and the desired boundary limitations will not be violated.
Proof: Transform (24) as

$$
\begin{aligned}
&\frac{d}{dt}[F(t)\exp(\chi t)] \le \exp(\chi t), \\
&F(t) \le F(0)\exp(-\chi t). \qquad (25)
\end{aligned}
$$

From the form of (25), it is known $F(t)$ is exponential convergence. Further, it can be yielded as

$$w^2 \le 2L \int_0^{l(t)} (w_x)^2 dx + 2L[w^{L(l(t))}]^2 \le \frac{2L}{\chi_1} F(t) + 2LC_w^2.$$

Further, we can deduce that $w(x,t)$ is convergence,

$$|w(x,t)| \le \sqrt{\frac{2L}{\chi_1} F(0)\exp(-\chi t) + LC_w^2}, \qquad (26)$$

Using inequality condition $2[z_x]^2 \le [w_x]^2$ [48], the state $|z(x,t)|$ can be proven to be also convergence.

$$|z(x,t)| \le \frac{1}{\sqrt{2}}|w(x,t)| \le \sqrt{\frac{L}{\chi_1} F(0) + LC_w^2}. \qquad (27)$$

∎

Remark 1: (25) shows that $F(t)$ is a convergence and bounded function $\forall t \in [0, \infty)$, therefore, from (35) and (33), we can know that $A(t)$, $F_e(t)$ and $F_a(t)$ are also bounded. Then, we can deduce that the energy equations $E_k(t)$ and $E_p(t)$ are proven to be bounded. Using the cited Properties 1 and 2, we can further guarantee that the states z_{xx}, z_{xt}, w_{xx} and w_{xt} are also convergence and bounded.

Remark 2: Based on the definition of the barrier term in $F_a(t)$, we know that $F_a(t) \to \infty$ when the state $|z^{L[l(t)]}|$ approaches to the boundary limitation C_z or $|w^{L[l(t)]}|$ to C_w. Since $F_a(t)$ is clarified to be bounded, this function will not approach to infinite. Consequently $|z^{L[l(t)]}|$ and $|w^{L[l(t)]}|$ cannot violate the constraints C_z and C_w. Given that the initial conditions of the states $z(L,0)$ and $w(L,0)$ do not violate the limitations C_z and C_w, we can deduce that the states $z^{L[l(t)]}$ and $w^{L[l(t)]}$ will never change beyond the given limitations, namely, the output constraints can be guaranteed.

Remark 3: Combining the proven deductions, it can be used to analyze the boundedness of the designed control (18)-(21). For all consisted signals, including z, z_x, z_t, z_{xt}, w, w_x, w_t and w_{xt} are reduced to be bounded, we can conclude that the designed boundary controllers (18)-(21) in this paper are also bounded. Through the various sensors and backward difference algorithm, we can measure the position signals, inclination angle signals and velocity signals. These available boundary signals can guarantee the designed boundary control strategy implement.

IV. Numerical simulation

The crane system parameters are $L = 10$ m, $m_p = 10$ kg, $\rho_c = 1$ kg/m, $EA = 10$ N, $g = 9.8$ m/s^2, and we set the boundary output constraint $C_z = 1$ m and $C_w = 1$ m on the longitudinal and transverse direction, respectively. The corresponding initial conditions are $w(x,0) = 0$ m, $\dot{w}(x,0) = \frac{5}{3}x$ m/s, $z(x,0) = 0$ m, $\dot{z}(x,0) = \frac{1}{3}x$ m/s. The boundary disturbances $d_z(t)$ and $d_w(t)$ are taken as $d_z(t) = d_w(t) = 0.2\sin(0.2t) + 0.3\sin(0.3t) + 0.5\sin(0.5t)$.

From above equation, it can be seen that $d_z(t) \le 1$ and $d_w(t) \le 1$. Therefore we set $\bar{d}_z = \bar{d}_w = 1$.

In order to highlight the effectiveness of the proposed control method, PD controllers are taken into account for comparison. The PD controllers at $l(t)$ point are described as

$$
\begin{aligned}
u_{z3}(t) &= -k_{pz3} z^{L[l(t)]} - k_{dz3} z_x^{L[l(t)]}, \qquad (28) \\
u_{w3}(t) &= -k_{pw3} w^{L[l(t)]} - k_{dw3} w_x^{L[l(t)]}, \qquad (29)
\end{aligned}
$$

and the PD controllers at 0 point are expressed as:

$$u_{z4}(t) = -k_{pz4}z^{L[0]} - k_{dz4}z_x^{L[0]}, \tag{30}$$

$$u_{w4}(t) = -k_{pw4}w^{L[0]} - k_{dw4}w_x^{L[0]}, \tag{31}$$

where PD control parameters are given as $k_{pz3} = 10$, $k_{dz3} = 7$, $k_{pw3} = 20$, $k_{dw3} = 7$, $k_{pz4} = 10$, $k_{dz4} = 10$, $k_{pw4} = 10$ and $k_{dw4} = 10$. The parameters of our proposed control (18-21) are chosen as $k_1 = 10$, $k_2 = 1000$, $k_3 = 10$, $k_4 = 1000$, $k_5 = 15$ and $k_6 = 3$. In order to compare the performance in different moving velocity of cable, a fast hoisting mode where $v(t) = -0.1 - 0.01t$ and a slow hoisting mode where $v(t) = -0.05 - 0.005t$ are discussed. All control parameters remain the same for both modes.

In the fast hoisting mode, the length of the cable is $l(t) = 10 - 0.1t - 0.5t^2$. During the hoising process, the length of the cable decreases. Such three situations are compared at both boundaries of the cable, 0 point in Figs. 5, 7 and $l(t)$ point in Figs. 4, 6. From the simulation results, the vibration of the cable will gradually increase under the boundary disturbance and will shift toward the direction of initial velocity if neither boundary is applied with control force. While PD control and the proposed control can both alleviate the vibration, our proposed control exhibits better performance due to its smaller overshoot and less stable time compared with PD control. With the proposed control, the equilibrium position of the cable converges to a small neighborhood near 0, which accords with our theoretical results (26) and (27). And for sake of the output constraint, it can be seen that the vibration amplitude of the $z(l, t)$ and $w(l, t)$ does not violate the constraints C_z and C_w.

In the slow hoisting mode, where the length of the cable is $l(t) = 10 - 0.05t - 0.0025t^2$, Figs. 8-11 reveals that our proposed control also behaves well when the speed of the cable is small. The comparison with PD controllers demonstrates the effectiveness of our proposed control.

Meanwhile, the control force of PD control and the proposed control is shown in Figs. 12, 13, 14, 15, where our proposed control at 0 point is better than PD control in respect of amplitude and working time.

In general, the longitudinal-transverse coupled vibrations of the variable length crane system are proven to be suppressed with the active boundary control and the system to be stable by numerical simulation experiment which is consistent with the theoretical proof.

V. Conclusion

This paper has investigated a varying length cable under vibrations from the transverse and longitudinal with the external disturbances. The dynamic characteristics of crane system described with PDEs-ODEs have been derived by the Hamilton's principle, where both the changes in the tension and length of the cable are considered. As an additional achievement of the former step, the uniform ultimate boundedness of the cable system's states has been achieved with the proposed control laws generated from the the Lyapunov's direct method. Output constraints are applied using LBF. In the simulation study, by appropriate choices of the control parameters, the simulation examples have demonstrated the effectiveness and performance of the proposed control schemes. In this paper, although the stability of the crane system can be ensured by choosing the appropriate parameters, a guide rule for selecting the parameters is given. The intelligent algorithm is used to automatically tune the control parameters and the solvability of the inequalities for a flexible Timoshenko beam is studied in the one-dimensional space in [49]. They provide a idea and it is good topic to study the solvability of the inequalities and to tune the control parameters automatically for the crane system in future work.

Appendix A. Proof of Lemma 1

Using the inequality $2[z_x]^2 \leq [w_x]^2$ [48] and Young's inequality, $F_c(t)$ can be yielded as

$$\begin{aligned}|F_c(t)| \leq\ & \frac{1}{2}b\rho_c L\int_0^{l(t)}[v(t)+\frac{Dz}{Dt}]^2dx + \frac{1}{2}b\rho_c L\int_0^{l(t)}[z_x]^2xdx \\ & +\frac{1}{2}b\rho_c L\int_0^{l(t)}[\frac{Dw}{Dt}]^2dx + \frac{1}{2}b\rho_c L\int_0^{l(t)}[w_x]^2dx \\ \leq\ & \frac{3}{4}b\rho_c LA(t).\end{aligned} \tag{32}$$

Combining the equation $F_e(t)$ and (32), we can deduce that

$$\gamma_1 A(t) \leq F_e(t) + F_c(t) \leq \gamma_2 A(t), \tag{33}$$

where, $\gamma_1 = \frac{a}{2}\min\{\rho_c, P_a, EA\}$ and $\gamma_2 = \frac{a}{2}\max\{\rho_c, P_b, EA\}$. Notice that $F_a(t) \geq 0$ and $A(t) \geq 0$, if b is set as $0 < b < \frac{2\gamma_1}{\rho L}$ the following inequalities will hold

$$\begin{aligned}F(t) \geq\ & (\gamma_1 - \frac{1}{2}b\rho_c L)A(t) + F_a(t) \geq 0, \\ F(t) \leq\ & (\gamma_2 - \frac{1}{2}b\rho_c L)A(t) + F_a(t).\end{aligned} \tag{34}$$

When the parameters χ_1 and χ_2 are chosen as $\chi_1 = \min\{(\gamma_1 - \frac{1}{2}b\rho_c L), 1\}$ and $\chi_2 = \max\{(\gamma_2 - \frac{1}{2}b\rho_c L), 1\}$, we obtain

$$0 \leq \chi_1[A(t) + F_a(t)] \leq F(t) \leq \chi_2[A(t) + F_a(t)]. \tag{35}$$

Appendix B. Proof of Lemma 2

The time derivation of (12) is deduced as

$$\dot{F}(t) = \dot{F}_e(t) + \dot{F}_a(t) + \dot{F}_c(t). \tag{36}$$

Differentiating (13) with respect to time and substituting governing equations (6) and (7), we obtain

$$\begin{aligned}\dot{F}_e(t) =\ & \frac{a\rho_c v(t)}{2}[v(t) + \frac{Dz^{L[l(t)]}}{Dt}]^2 + \frac{a\rho_c v(t)}{2}[\frac{Dw^{L[l(t)]}}{Dt}]^2 \\ & +\frac{av(t)}{2}P(l(t),t)[w_x^{L[l(t)]}]^2 + \frac{aEAv(t)}{2}[z_x^{L[l(t)]} \\ & +\frac{1}{2}(w_x^{L[l(t)]})^2]^2 + aP(l(t),t)w_x^{L[l(t)]}\frac{Dw^{L[l(t)]}}{Dt} \\ & +au_{w2}(t)\frac{Dw^{L[0]}}{Dt} + aEA[z_x^{L[l(t)]} + \frac{1}{2}(w_x^{L[l(t)]})^2] \\ & \times[v(t) + \frac{Dz^{L[l(t)]}}{Dt}] + \frac{a}{2}\int_0^l[\frac{DP(x,t)}{Dt}][w'(x,t)]^2dx \\ & +au_{z2}(t)[v(t) + \frac{Dz^{L[0]}}{Dt}] + aEA[z_x^{L[l(t)]}w_x^{L[l(t)]} \\ & +\frac{1}{2}(w_x^{L[l(t)]})^3]\frac{Dw^{L[l(t)]}}{Dt}.\end{aligned} \tag{37}$$

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Differentiating $F_c(t)$ described by (13) with respect to time and substituting governing equations (6) and (7), we get

$$\begin{aligned}
\dot{F}_c(t) &= b\rho_c v(t)l(t)[v(t)+\frac{Dz^{L[l(t)]}}{Dt}]z_x^{L[l(t)]} \\
&-bEA\int_0^{l(t)}(w_x)^2 z_x dx+\frac{b\rho_c l(t)}{2}[v(t)+\frac{Dz^{L[l(t)]}}{Dt}]^2 \\
&-\frac{b\rho_c}{2}\int_0^{l(t)}[v(t)+\frac{Dz}{Dt}]^2dx+\frac{bEAl(t)}{2}[z_x^{L[l(t)]}]^2 \\
&+b\rho_c v(t)l(t)w_x^{L[l(t)]}\frac{Dw^{L[l(t)]}}{Dt}-\frac{bEA}{2}\int_0^{l(t)}(z_x)^2dx \\
&+bEAl(t)[w_x^{L[l(t)]}]^2 z_x^{L[l(t)]}+\frac{bl(t)}{2}P(l(t),t)[w_x^{L[l(t)]}]^2 \\
&-\frac{3bEA}{8}\int_0^{l(t)}[w_x]^4dx-\frac{b}{2}\int_0^{l(t)}xP'(x,t)[w_x]^2dx \\
&+\frac{3bEAl(t)}{8}(w_x^{L[l(t)]})^4-\frac{b}{2}\int_0^{l(t)}P(x,t)[w_x]^2dx \\
&+\frac{b\rho_c l(t)}{2}[\frac{Dw^{L[l(t)]}}{Dt}]^2-\frac{b\rho_c}{2}\int_0^{l(t)}[\frac{Dw}{Dt}]^2dx \\
&+b\rho_c v(t)\int_0^{l(t)}[v(t)+\frac{Dz}{Dt}]z_x dx \\
&+b\rho_c v(t)\int_0^{l(t)}[\frac{Dw}{Dt}]w_x dx. \qquad (38)
\end{aligned}$$

Combining boundary conditions (9), (10), and control laws (18), (21), the time derivative of $F_a(t)$ is

$$\begin{aligned}
\dot{F}_a(t) &\le -ak_1[u_{aw}(t)]^2\ln\frac{2C_w^2}{C_w^2-[w^{L[l(t)]}]^2}-ak_2[u_{aw}(t)]^2 \\
&-ak_3[u_{az}(t)]^2\ln\frac{2C_z^2}{C_z^2-[z^{L[l(t)]}]^2}-ak_4[u_{az}(t)]^2 \\
&-\frac{aEA}{2}[2z_x^{L[l(t)]}+(w_x^{L[l(t)]})^2][v(t)+\frac{Dz^{L[l(t)]}}{Dt}] \\
&-aEA[z_x^{L[l(t)]}]^2-\frac{3}{2}aEA[w_x^{L[l(t)]}]^2 z_x^{L[l(t)]} \\
&-\frac{aEA}{2}[w_x^{L[l(t)]}]^4-aEA\frac{Dw^{L[l(t)]}}{Dt}[z_x^{L[l(t)]}w_x^{L[l(t)]} \\
&+\frac{1}{2}(w_x^{L[l(t)]})^3]-aP(l(t),t)[w_x^{L[l(t)]}]^2 \\
&-aP(l(t),t)w_x^{L[l(t)]}\frac{Dw^{L[l(t)]}}{Dt}. \qquad (39)
\end{aligned}$$

Combining all above equalities and inequalities with the assumption of $v(t)<0$ and using the inequality $2[z_x]^2\le[w_x]^2$, we can express $\dot{F}(t)$ as

$$\begin{aligned}
\dot{F}(t) &\le -ak_1[u_{aw}(t)]^2\ln\frac{2C_w^2}{C_w^2-[w^{L[l(t)]}]^2}-c_1[u_{aw}(t)]^2 \\
&-ak_3[u_{az}(t)]^2\ln\frac{2C_z^2}{C_z^2-[z^{L[l(t)]}]^2}-c_2[u_{az}(t)]^2 \\
&-c_3\int_0^{l(t)}[v(t)+\frac{Dz}{Dt}]^2dx-c_4\int_0^{l(t)}[\frac{Dw}{Dt}]^2dx \\
&-\frac{1}{2}\int_0^{l(t)}c_5[w_x]^2dx-bEA\int_0^{l(t)}\{z_x+\frac{1}{2}[w_x]^2\}^2dx \\
&-\frac{bEA}{8}\int_0^{l(t)}[w_x]^4dx-c_6[v(t)+\frac{Dz^{L[l(t)]}}{Dt}]^2 \\
&-c_6[\frac{Dw^{L[l(t)]}}{Dt}]^2-k_5a[\frac{Dw^{L[0]}}{Dt}]^2-c_7[w_x^{L[l(t)]}]^2 \\
&-c_8[z_x^{L[l(t)]}]^2-k_6a[v(t)+\frac{Dz^{L[0]}}{Dt}]^2 \\
&-\frac{EAc_9}{2}[z_x^{L[l(t)]}+\frac{1}{2}(w_x^{L[l(t)]})^2]^2-\frac{EAc_{10}}{8}[w_x^{L[l(t)]}]^4, \qquad (40)
\end{aligned}$$

where δ_1, δ_2 and ϱ are positive constants chosen to satisfy the following conditions:

$$\begin{aligned}
c_1 &= ak_2+\frac{b\rho_c v(t)l(t)}{2}>0, \\
c_2 &= ak_4+\frac{b\rho_c v(t)l(t)}{2}>0, \\
c_3 &= \frac{b\rho_c}{2}-\frac{b\rho_c v(t)}{2\delta_1}>0, \\
c_4 &= \frac{b\rho_c}{2}-\frac{b\rho_c v(t)}{2\delta_2}>0, \\
c_5 &= bP_a+bxP'_a-a\dot{P}_b-av(t)P'_a-\frac{bEA}{2} \\
&\quad -\frac{b\rho_c\delta_1 v(t)}{4}-\frac{b\rho_c\delta_2 v(t)}{2}\ge\varrho>0, \\
c_6 &= -\frac{\rho av(t)}{2}-\frac{b\rho_c v(t)l(t)}{2}-\frac{b\rho_c l(t)}{2}\ge 0, \\
c_7 &= -\frac{av(t)P_a}{2}+aP_a+\frac{bl(t)P_a}{2} \\
&\quad -\frac{b\rho_c v(t)l(t)}{2}\ge 0, \\
c_8 &= aEA-\frac{bEAl(t)}{2}-\frac{b\rho_c v(t)l(t)}{2}\ge 0, \\
c_9 &= -av(t)-2bl(t)+3a\ge 0, \\
c_{10} &= a-bl(t)\ge 0.
\end{aligned}$$

and χ_3 is set as

$$\begin{aligned}
\chi_3 = \min\Big\{&(\frac{b\rho_c}{a}-\frac{b\rho_c}{a\delta_1}),(\frac{b\rho_c}{a}-\frac{b\rho_c}{a\delta_2}),\frac{\varrho}{P_b},\frac{2b}{a}EA, \\
&\frac{2k_1}{m_p},\frac{2k_3}{m_p}\Big\}.
\end{aligned}$$

Combining (35) and (40), we have

$$\dot{F}(t)\le-\chi F(t), \qquad (41)$$

where $\chi=\frac{\chi_3}{\chi_2}>0$. ■

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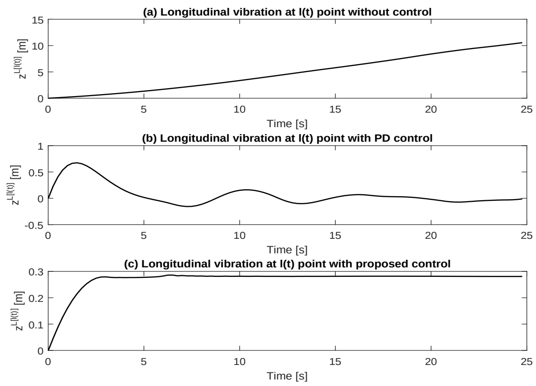

Fig. 4. Longitudinal vibration at the $l(t)$ point of the cable with (a) without control (b) PD control (c) our proposed control, with $v(t) = -0.1 - 0.01t$

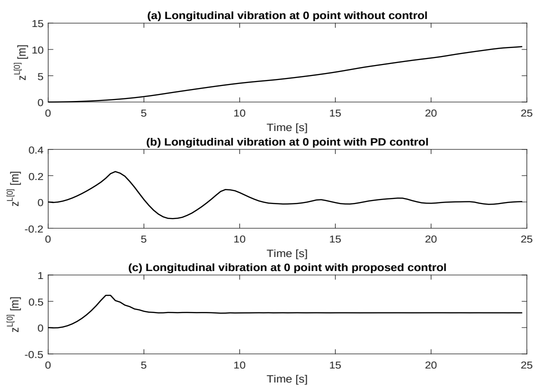

Fig. 5. Longitudinal vibration at the 0 point of the cable with (a) without control (b) PD control (c) our proposed control, with $v(t) = -0.1 - 0.01t$

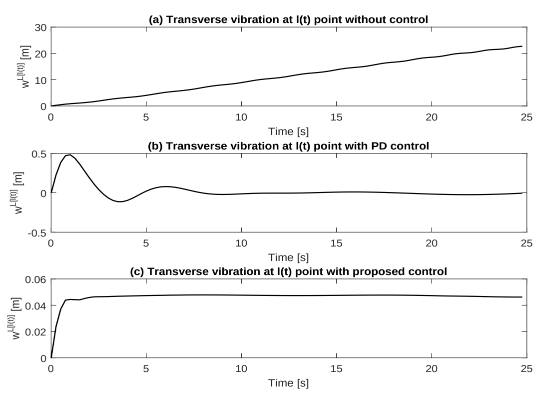

Fig. 6. Transverse vibration at the $l(t)$ point of the cable with (a) without control (b) PD control (c) our proposed control, with $v(t) = -0.1 - 0.01t$

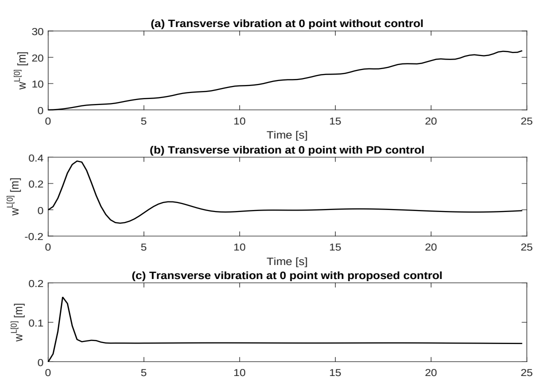

Fig. 7. Transverse vibration at the 0 point of the cable with (a) no control (b) PD control (c) our proposed control, with $v(t) = -0.1 - 0.01t$

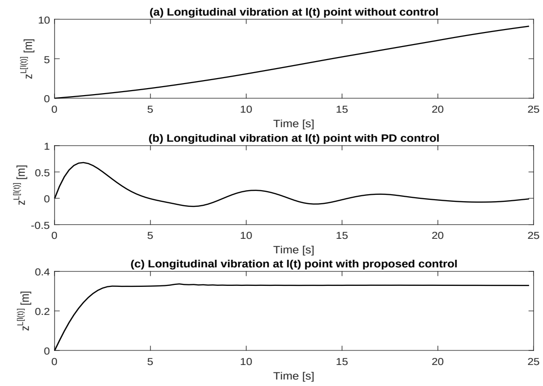

Fig. 8. Longitudinal vibration at the $l(t)$ point of the cable with (a) without control (b) PD control (c) our proposed control, with $v(t) = -0.05 - 0.005t$

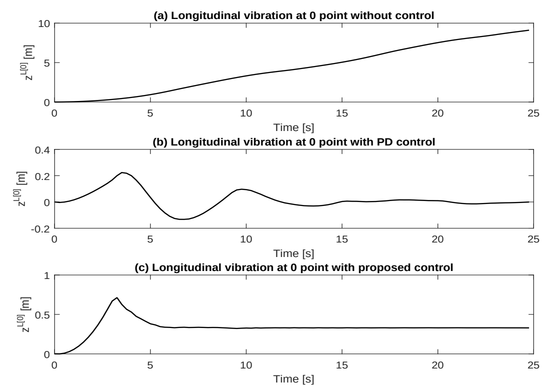

Fig. 9. Longitudinal vibration at the 0 point of the cable with (a) without control (b) PD control (c) our proposed control, with $v(t) = -0.05 - 0.005t$

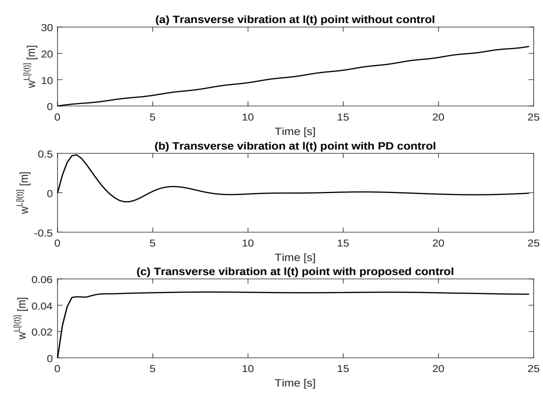

Fig. 10. Transverse vibration at the $l(t)$ point of the cable with (a) without control (b) PD control (c) our proposed control, with $v(t) = -0.05 - 0.005t$

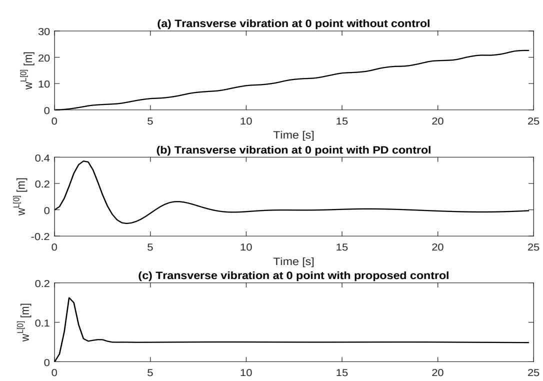

Fig. 11. Transverse vibration at the 0 point of the cable with (a) without control (b) PD control (c) our proposed control, with $v(t) = -0.05 - 0.005t$

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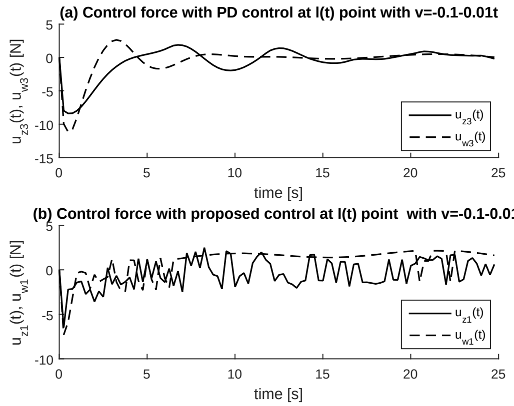


Fig. 12. The designed control forces at point $l(t)$ with $v(t) = -0.1 - 0.01t$

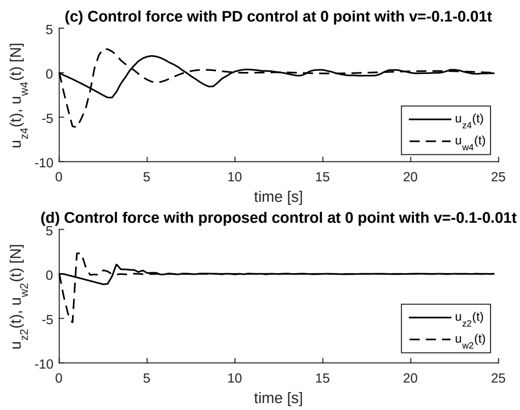


Fig. 13. The designed control forces at point 0 with $v(t) = -0.1 - 0.01t$

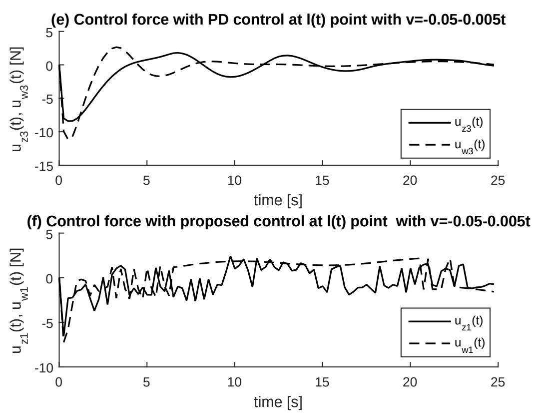
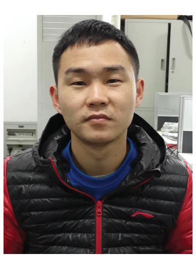


Fig. 14. The designed control forces at point $l(t)$ with $v(t) = -0.05 - 0.005t$



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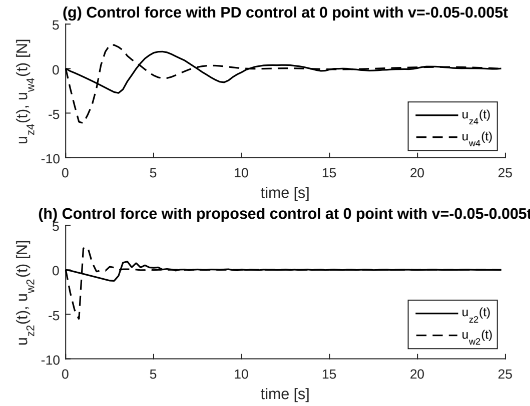

Fig. 15. The designed control forces at point 0 with $v(t) = -0.05 - 0.005t$

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