Mechanical Properties

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Rock Mechanical Properties

- Rock mechanical properties, such as Poisson's ratio, shear modulus, Young's modulus, bulk modulus and compressibility can be obtained from two different sources:
  - Laboratory measurements, which allow for direct measurements of strength parameters and static elastic behavior with recovered core material from discrete depths;
  - Downhole measurements through wireline logging, which allow the determination of dynamic elastic constants from the continuous measurement of compressional and shear velocities.
- The mechanical properties obtained from laboratory core tests may be slightly to considerably different from those existing in-situ. Core alteration during and after drilling also may influence the results.

Outline

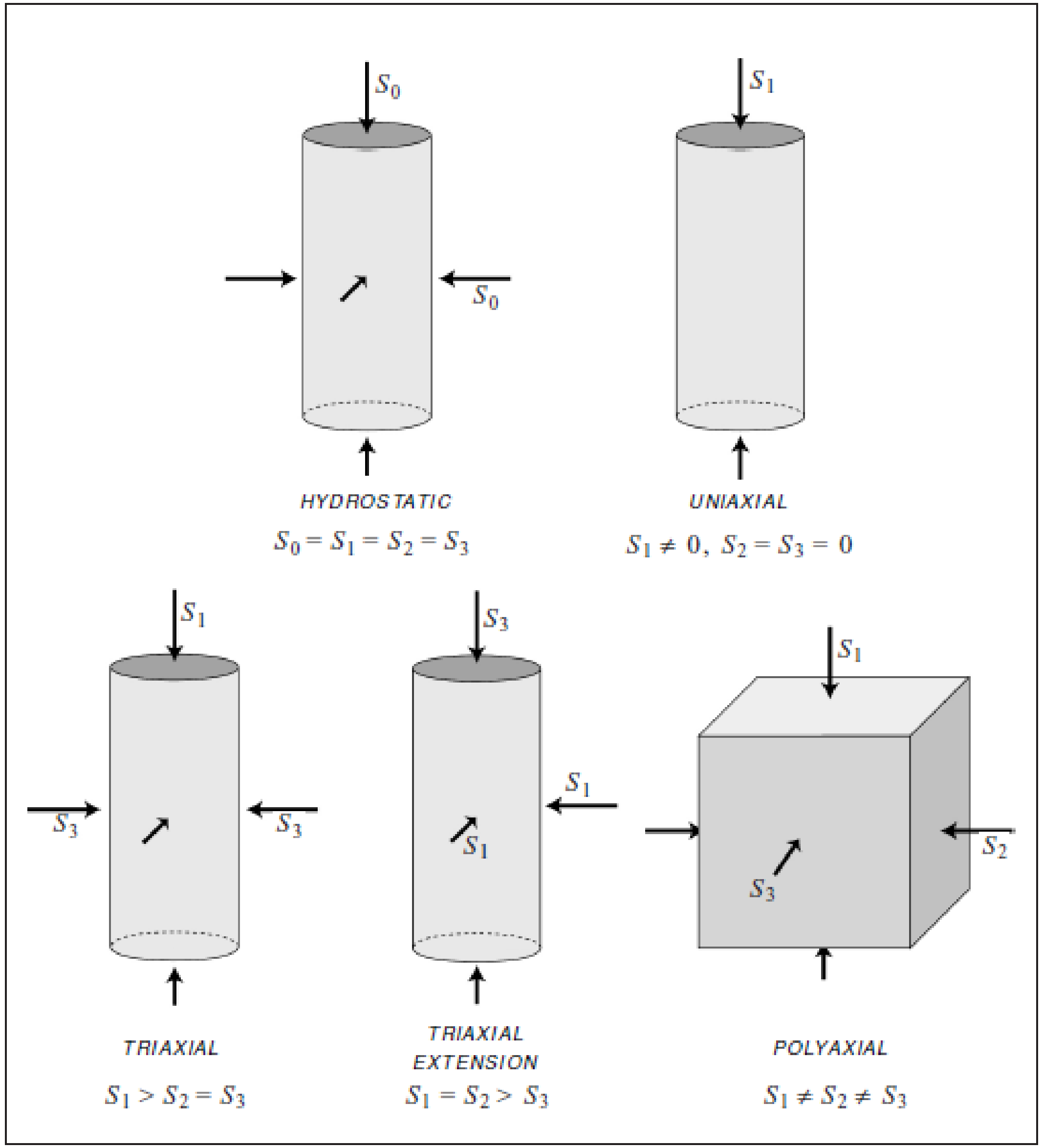
- Rock Deformation
- Linear Elasticity
- Porous Media Representation
- Wave Propagation in Elastic Media
- Rock Embedment Strength

Rock Deformation

- Some the factors affecting the relationship between stress and strain for reservoir rocks:
  - Composition and lithology of rocks
  - Degree of cementation and alteration
  - Type of cementing material
  - Amount and type of fluids in the porous space
  - Compressibility of the rock matrix and fluids
  - Porosity and permeability
  - Reservoir pressure and temperature
- Laboratory techniques used to measure the separate and combined effects of these factors on the stress-strain relationship”
  - Hydrostatic Testing
  - Uniaxial Testing
  - Tri-axial Testing

Rock Failure

- Hydrostatic compression tests
- Uniaxial compressive tests
- Uniaxial tension tests
- Triaxial compression tests
- Triaxial extension tests
- True Triaxial Tests



Linear Elasticity

- For an ideal elastic body characterized by its Young’s modulus E and its Poisson’s ratio ν the compressive stresses σ_x, σ_y, σ_z required to yield strains ϵ_x, ϵ_y, ϵ_z are given as follows:

$$\epsilon_i = \frac{1+\nu}{E}\sigma_i - \frac{\nu}{E}\left(\sigma_x + \sigma_y + \sigma_z\right)$$

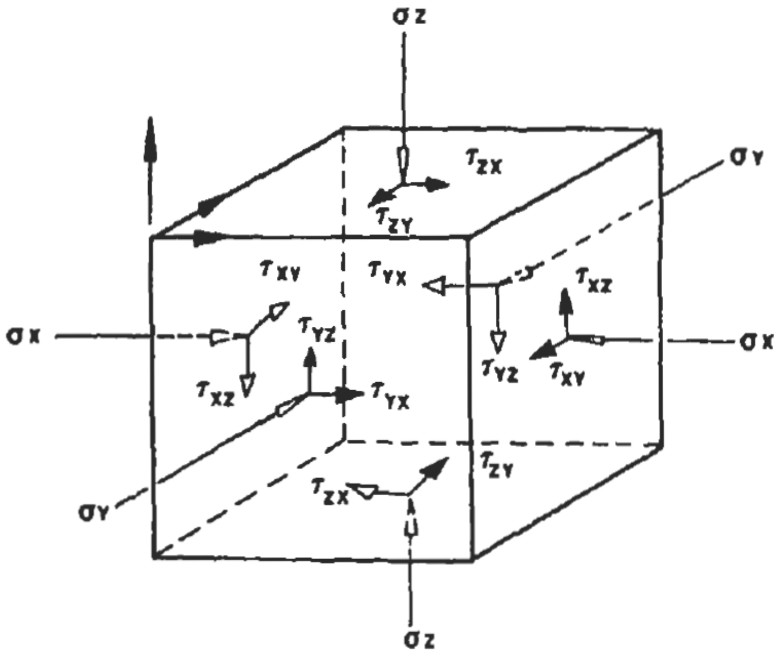
- In other form the same equation can be written as:

$$\sigma_i = 2G\epsilon_i + \frac{2G\nu}{1-2\nu}\left(\epsilon_x + \epsilon_y + \epsilon_z\right)$$

- The modulus of rigidity is related to Young’s modulus by the equation:

$$G = \frac{E}{2(1+\nu)}$$

- The equations relating stress and strain given here represent the most general linear constitutive relationship for an isotropic and homogeneous material. Whether or not such a relationship actually applies to real systems is a matter requiring experimental justification.



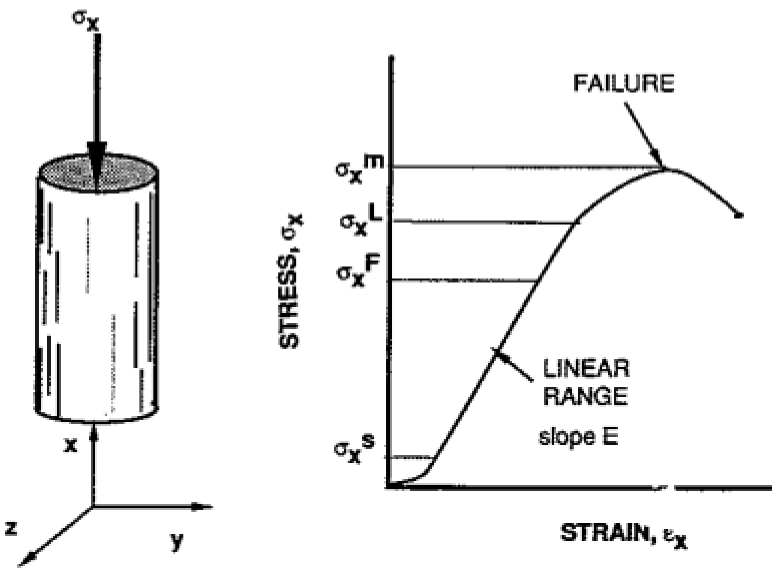
Three-dimensional stress field of a cubic element

Rock Deformation - **Exercise**

- A cubic-shaped sample of rock taken from a depth of 731 m is subjected to an axial compressive load of 1000 kPa. No confining pressure is applied to the lateral sides. What is the strain in the axial direction and the volume change of the sample?
- Young's modulus = $1.6 \times 10^{7}\ \mathrm{kPa}$
- Poisson's ratio = 0.276
- Permeability to air = $0.03 \times 10^{-15}\ \mathrm{m}^{2}$
- Porosity = 4.8 %

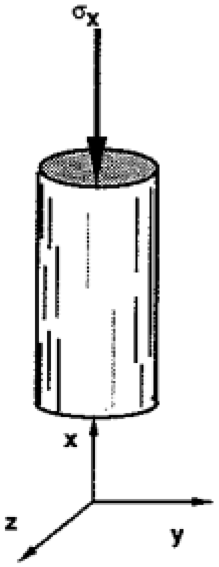
Linear Elasticity

- For stresses less than σ_x^S, the main impact of the compressive stress on the rock is to close any existing small cracks and fissures.
- Since few new cracks are created during this initial stage of compression, it is reversible; that is, if the stress is decreased, the stress-strain curve will be retraced.
- During this initial phase the stress-strain curve is not linear and the material tends to stiffen due to the closure of microscopic cracks.
- Once the applied stress exceeds σ_x^S the crack closure phase is complete and additional forces act to strain solids. Linear behavior is often observed for stresses exceeding σ_x^S.



Linear Elasticity

- Once the stresses are sufficient to initiate new cracks, as for example at σ_x^F, the process is both nonlinear and irreversible since relaxing the compressive force does not heal the newly created cracks.
- Finally, above σ_x^L the rapid growth of cracks is observed, and failure occurs at σ_x^M.
- The lithostatic pressure of in-situ rocks generally lies between σ_x^S and σ_x^F and thus over a limited range of conditions we would expect the equations of linear elasticity to be useful.



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Porous Media Representation

- Some of the pores are interconnected, thereby permitting the flow of fluid. When these pores are filled with a fluid, then the fluid pressure helps to support compressive stresses exerted on an element of rock volume.
- The representation of the mechanical properties of rock must therefore recognize the existence of a pore pressure p. One such representation is:

$$\sigma_i - \alpha p = 2G\epsilon_i + \frac{2G\nu}{1 - 2\nu}\left(\epsilon_x + \epsilon_y + \epsilon_z\right)$$

- where α is a material property and $\sigma_i - \alpha p$ can be interpreted as that part of the total stress supported by the rock skeleton. A reasonable approximation for α is 1.
- G and ν may be considered to be properties of the rock skeleton.

Hooke's Law

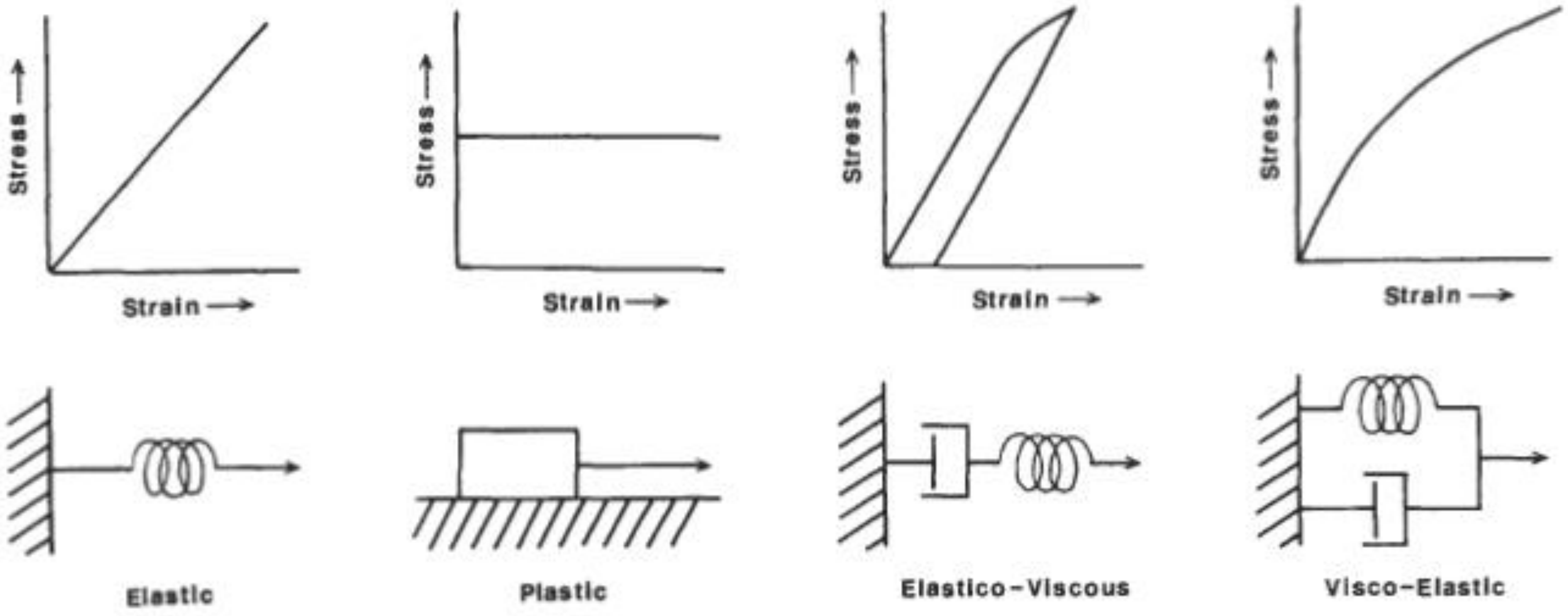


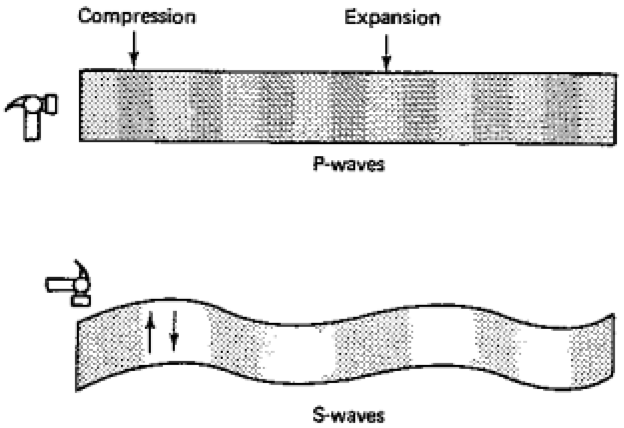
Figure 9.6. *Stress-strain relationships with related mechanical models [6].*

Wave Propagation in Elastic Media

- Consider a long rod that has been struck by a hammer at one of its ends. If the blow is delivered along the rod’s axis, P-wave will be induced. If, on the other hand, a blow is delivered perpendicular to the rod’s axis, it results in shear, S-wave; a change of shape without change of volume.
- Longitudinal waves (P-waves)

  Mechanical longitudinal waves are also called compressional waves, because they produce compression and rarefaction when traveling through a medium, and pressure waves, because they produce increases and decreases in pressure.
- Transverse waves (S-waves)

  Mechanical transverse waves will propagate along the axis with a certain velocity depending on how strongly the material resists a shearing force.
- If the bar is struck at a certain angle, then both waves are generated. The velocities of the two waves are in general different.



Wave Propagation in Elastic Media

- Dynamic mechanical properties:

  Quite often, while making static mechanical property measurements, the p and s-wave velocities are measured. This requires a pulse transmitter-receiver setup that typically operates at 1 MHz but can span a range of high frequencies. The measured transit times of the p and s-waves are used to calculate the velocities which can then be converted to dynamic elastic moduli as follows:

$$E = \frac{v_s^2 \rho \left[3\left(\frac{vp}{v_S}\right)^2 - 4 \right]}{\left(\frac{vp}{v_S}\right)^2 - 1} \qquad \nu = \frac{1}{2} \frac{\left(\frac{vp}{v_S}\right)^2 - 2}{\left(\frac{vp}{v_S}\right)^2 - 1}$$

  where E is Young's modulus, ν is Poisson's ratio, ρ is bulk density, v_p is the p-wave velocity and v_s is the s-wave velocity.