Capillary Pressure

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Capillary Pressure

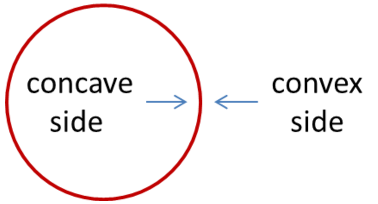
- When two immiscible fluids are in contact, there is a pressure discontinuity between the two fluids which depends upon the curvature of the interface separating the two fluids. This pressure difference or excess pressure is known as the capillary pressure.
- The pressure on the concave side of the interface is higher than that on the convex side of the interface, and the capillary pressure is given by Young-Laplace equation:

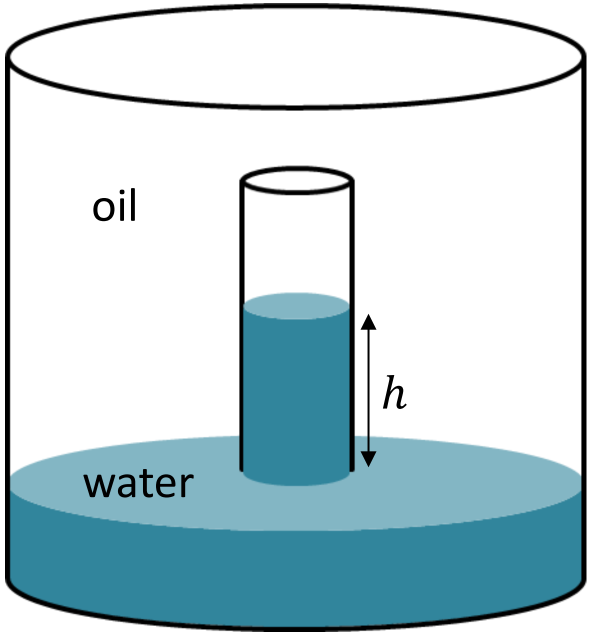
$$P_c = P_2 - P_1 = \sigma\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

- Where r_1 and r_2 are referred to as the principal radii of curvature of interface. If the two immiscible fluids are in contact with a solid surface, the interface will intersect the solid at an equilibrium contact angle θ:

$$P_c = \frac{2\sigma\cos\theta}{r}$$

- Laplace equation can be derived by considering the mechanical equilibrium of the interface or by energy considerations.

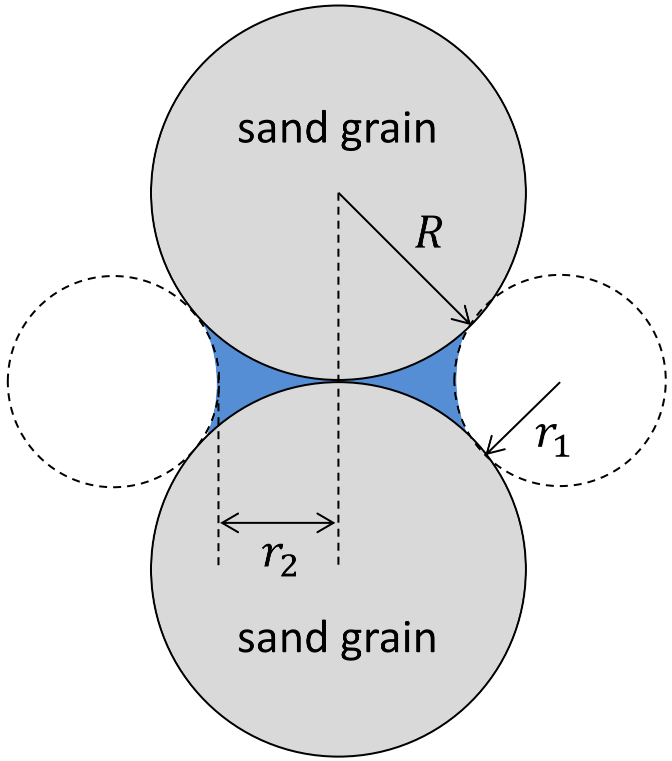




$$h = \frac{2\sigma_{wo}\cos\theta}{r(\rho_w - \rho_o)g}$$

Pendular ring of water between two spheres

- Consider two spherical grains each of radius R in contact with each other.
- A pendular ring of water of volume V that is small compared to the volume of each sphere is associated with the point of contact as shown in the figure to the right.
- Otherwise, spheres are surrounded by air. The contact angle of the water on the material of the spheres is zero.
- Let r_1 and r_2 be the principle radii of curvature of the air-water interface and σ be the interfacial tension between the water and the air.
- Show the relationship between r_1, r_2 and R.
- Given $r_2 = 10\ \mu\text{m}$, $R = 80\ \mu\text{m}$ and $\sigma = 72\ \text{dynes/cm}$, calculate the capillary pressure of the system in psi.
- Calculate the adhesive force holding the grains together.
- Graph the relationship between RP_c/σ vs. r_2/R

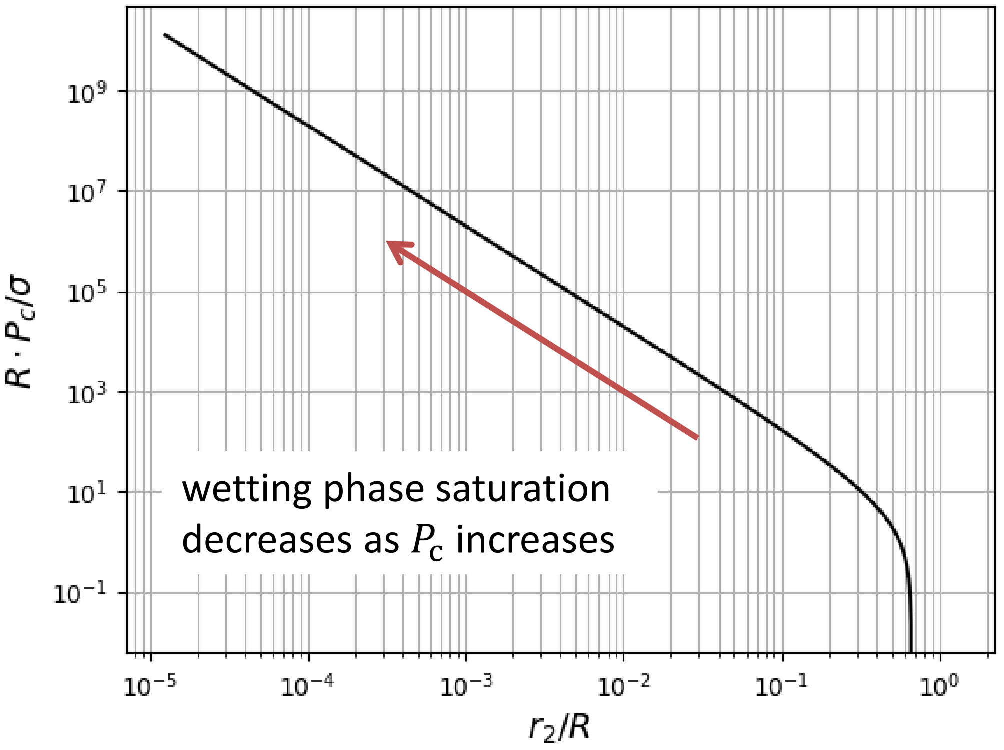


Pendular ring of water between two spheres

- In this example, the principal radii of curvature are on opposite sides of the interface. By sign convention, one radius will be positive and the other will be negative in the Laplace equation:

$$P_\mathrm{c} = \sigma \cos\theta \left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

- If the wetting fluid saturation in the pendular ring is reduced, r_1 and r_2 will be reduced. However, r_1 will be reduced more than r_2 as the wetting phase recedes into the corners of the contact of the grains.
- As a result, the capillary pressure will increase. And vice versa is true for the opposite scenario.
- Low wetting phase saturation corresponds to high capillary pressure and high wetting phase saturation corresponds to low capillary pressure.

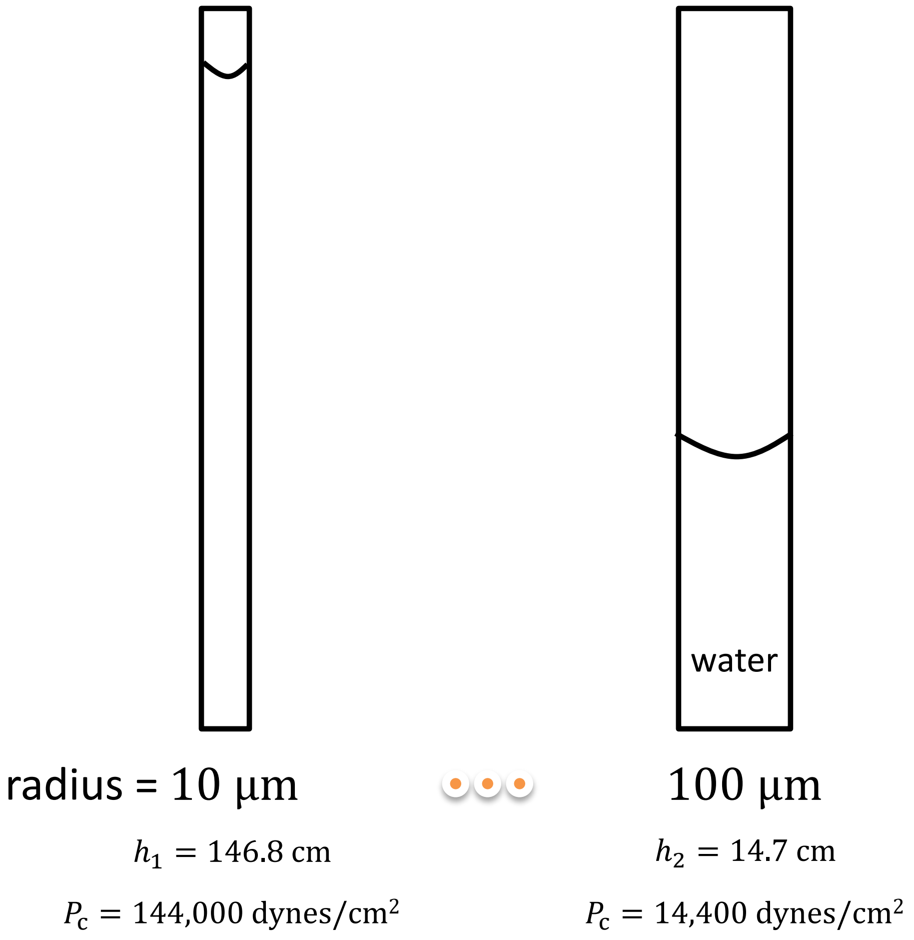


Outline

- Capillary pressure vs. saturation relationship
- Drainage capillary pressure curve
- Imbibition capillary pressure curve and hysteresis
- Effects of wettability and interfacial tension
- Capillary pressure lab-measurements
- Capillary end effect in a laboratory core
- Empirical capillary pressure models
- Capillary trapping in porous media
- Averaging capillary pressure data
- Determination of the initial static reservoir fluid saturations
- Pore size distribution
- Calculation of permeability

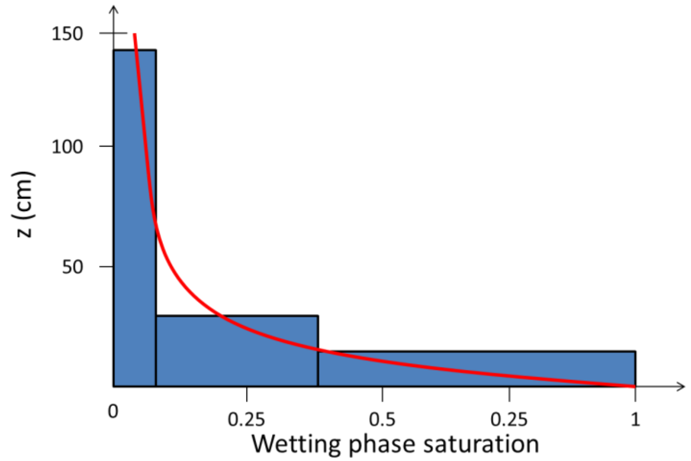
Capillary Pressure vs. Saturation Relationship

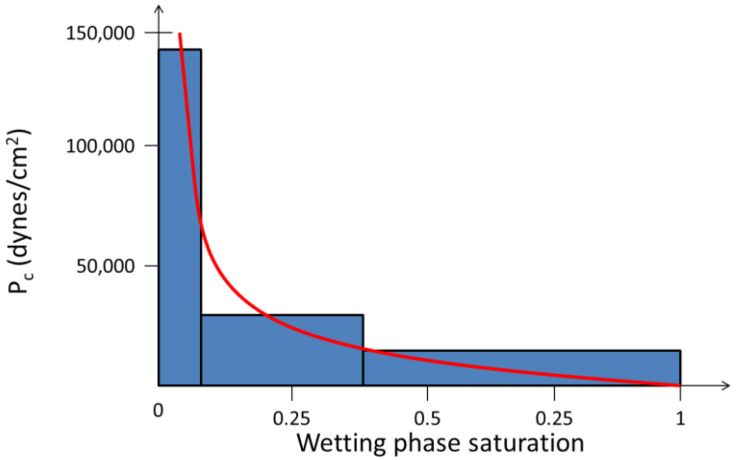
- Before considering the capillary pressure vs. saturation relationship for a porous medium, it is instructive to consider the relationship for an idealized medium consisting of a bundle of capillary tubes of varied radii.
- Let the bundle of capillary tubes medium be dipped into the wetting phase and allowed to attain capillary equilibrium as shown to the right:
  - Wetting phase is water, and air is a non-wetting phase
  - For water rising in glass tubes, we can assume $\theta = 0$
  - The surface tension of water is 72 dynes/cm at 25°C
  - For this scenario, we can calculate the heights as:
  $$h = \frac{0.1468}{r}\ \mathrm{cm}^2$$
  - And the capillary pressure as:
  $$P_\mathrm{c} = \frac{144}{r}\ \mathrm{dyne/cm}^2$$
  - The volume of water in the tube can be calculated from the area of tube and the height obtained to calculate the fraction of the total water volume in each tube.
- Let the model consist of ten capillary tubes with the radii changing in between 10 and 100 µm.



Capillary Pressure vs. Saturation Relationship

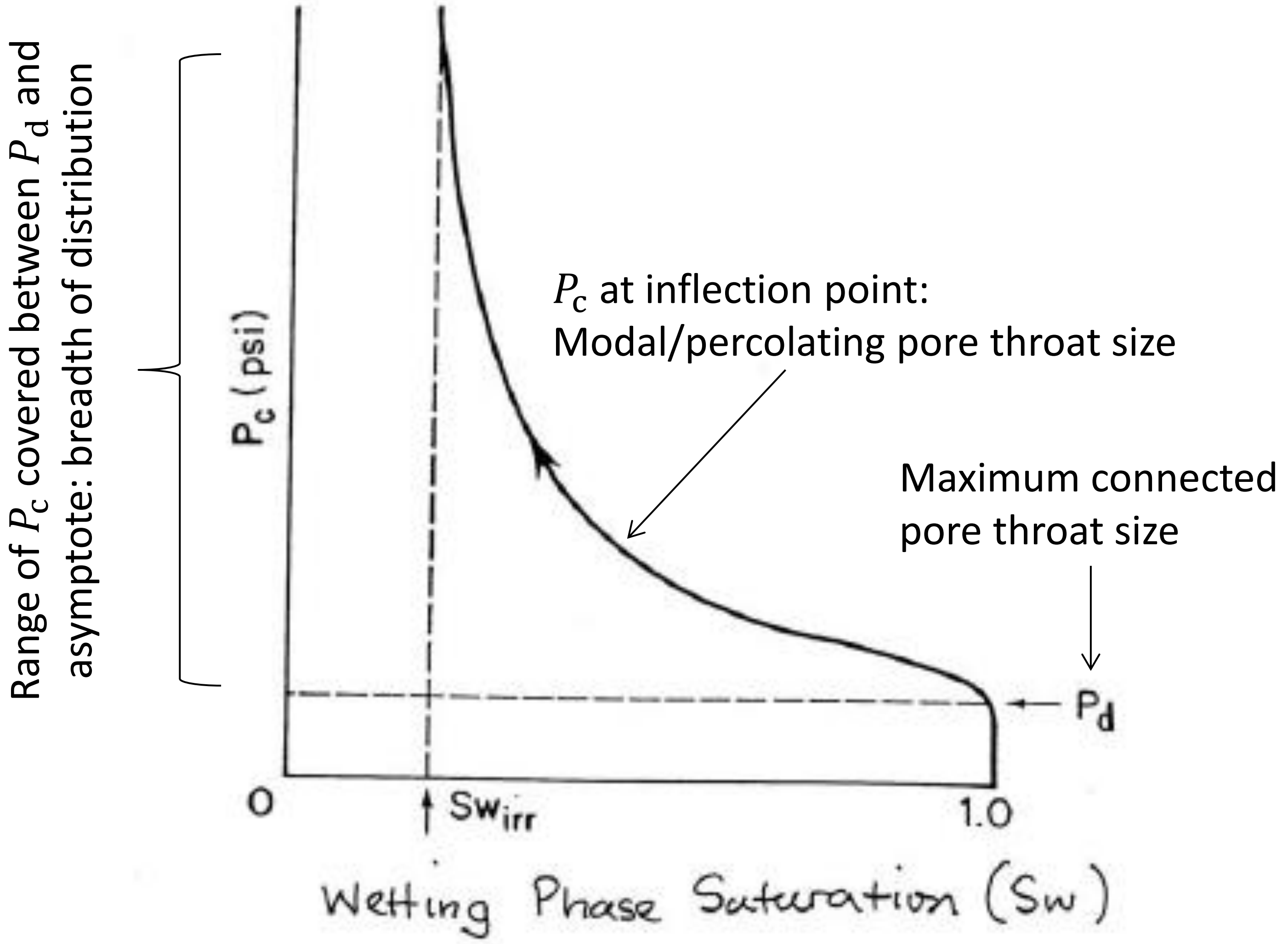
- Figures to the right show the capillary pressure vs. wetting phase saturation for the idealized model described above.
- In top figure, the capillary pressure is presented as height above the free water level. This presentation is can be used to determine the water saturation distribution in a petroleum reservoir.
- In bottom figure, the capillary pressure is given in psi. This presentation is useful for calculating pore size distribution.
- The curves have a staircase shape because of the limited number and size of capillary tubes, and it will approach a smooth curve if more tubes are used and the difference in the tube diameters are made small.
- The only limitation of this model is the absence of an irreducible wetting phase saturation. There is no possibility of trapping an irreducible saturation for a model consisting of straight and isolated capillary tubes.





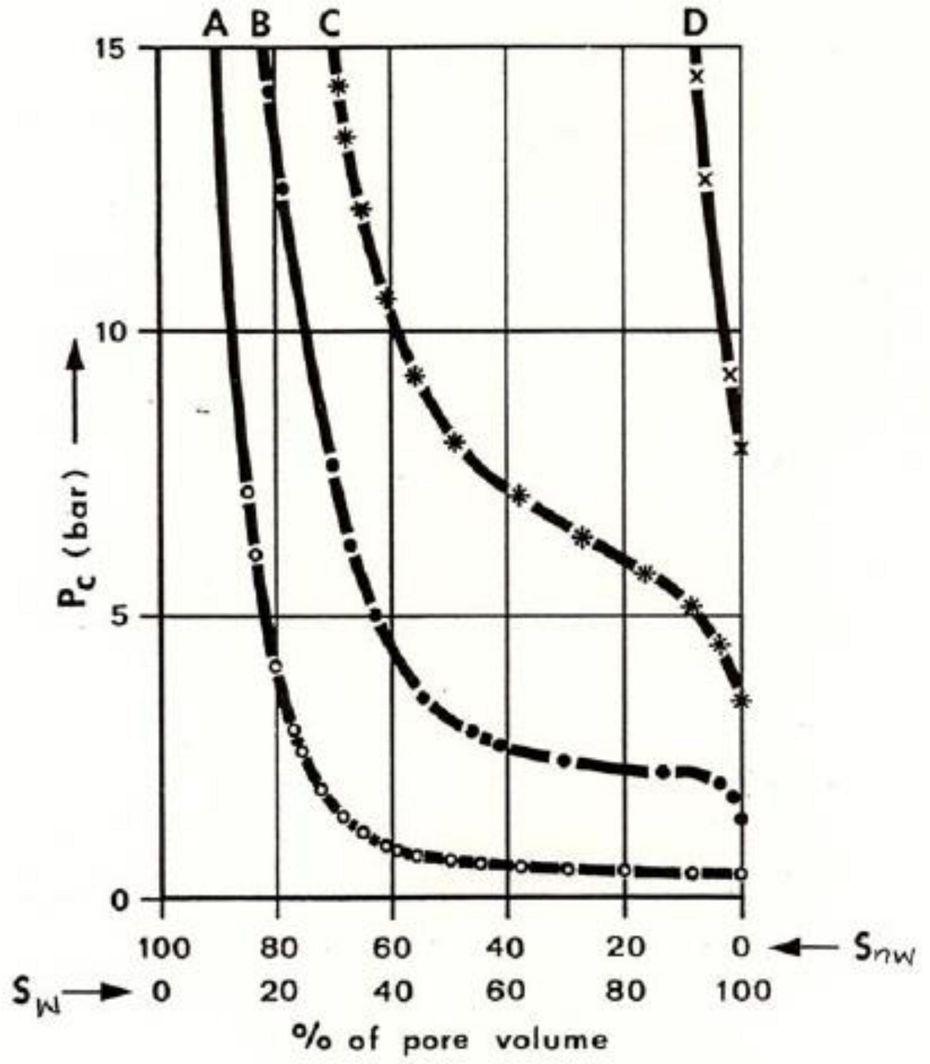
Drainage curves and pore size distributions

- If the idealized porous medium was replaced by an actual porous medium and the experiment repeated, the drainage capillary pressure curve would look like the one shown to the right.
- The minimum pressure, which is known as the displacement pressure, the threshold pressure or the entry pressure, is determined by the size of the largest pores connected to the surface of the medium. This is the minimum pressure difference required to initiate the drainage.
- At the irreducible wetting phase saturation, the capillary pressure curve becomes nearly vertical. The irreducible wetting phase saturation is a function of the grain-pore size, the wettability of the medium, and the interfacial tension between the wetting and non-wetting fluids.



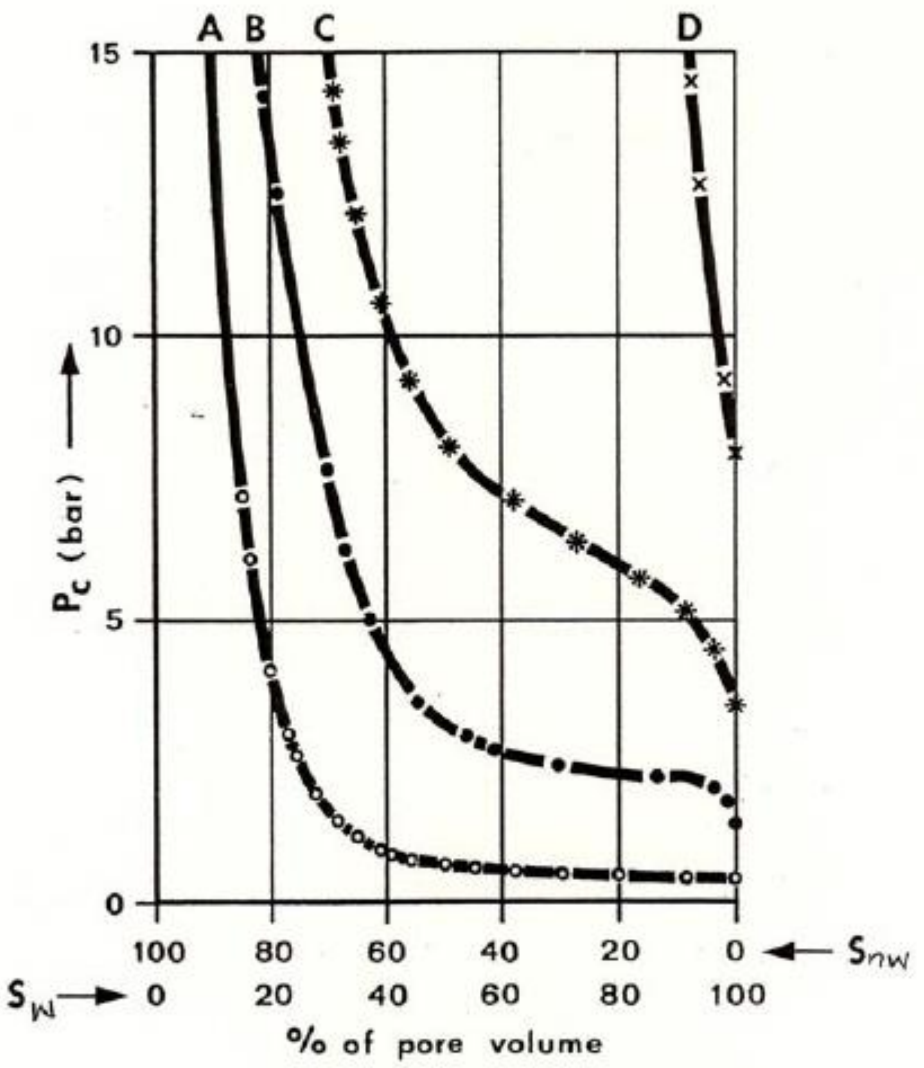
Drainage Capillary Pressure Curve

- What information does the capillary pressure curve for a reservoir rock provide about the rock? Assume that we have four different rock types with the P_c curves given to the right:
- Rock A
  - It has the least displacement pressure. Therefore, it has the largest pores connected to the surface.
  - Its capillary pressure curve remains essentially flat as the wetting phase saturation is decreased from 100% to 60% which means that many of the pores are invaded by the non-wetting fluid at essentially the same capillary pressure. Hence, it has uniform pores or is well sorted.
  - It has the least irreducible wetting phase saturation, indicating that it has relatively larger grains and pores than other rocks.
- Rock B
  - It has a higher displacement pressure than A. Therefore, it has smaller pores than A.
  - The capillary pressure curve at the high wetting phase saturation is relatively flat, indicating good sorting.
  - It has a higher irreducible wetting phase saturation than A, which is consistent with its fine grains and pores.



Drainage Capillary Pressure Curve

- Rock C
  - It is even more fine grained than B because of its higher displacement pressure.
  - The shape of its capillary pressure curve shows that a higher capillary pressure is required at each wetting phase saturation to desaturate the rock. This means that C has a wider pore size distribution than A and B. Therefore, C is poorly sorted.
  - It has a higher irreducible water saturation than B, which is consistent with its finer grains and pores.
- Rock D
  - It is extremely fine grained because of its very high displacement pressure.
  - It has very steep capillary pressure curve, it is extremely poorly sorted.
  - It has very high irreducible wetting phase saturation.
  - It would be a very poor reservoir rock. Without being told, one can easily infer that this rock is essentially made of clay.



Drainage Capillary Pressure Curve

Largest pore throats? **A**

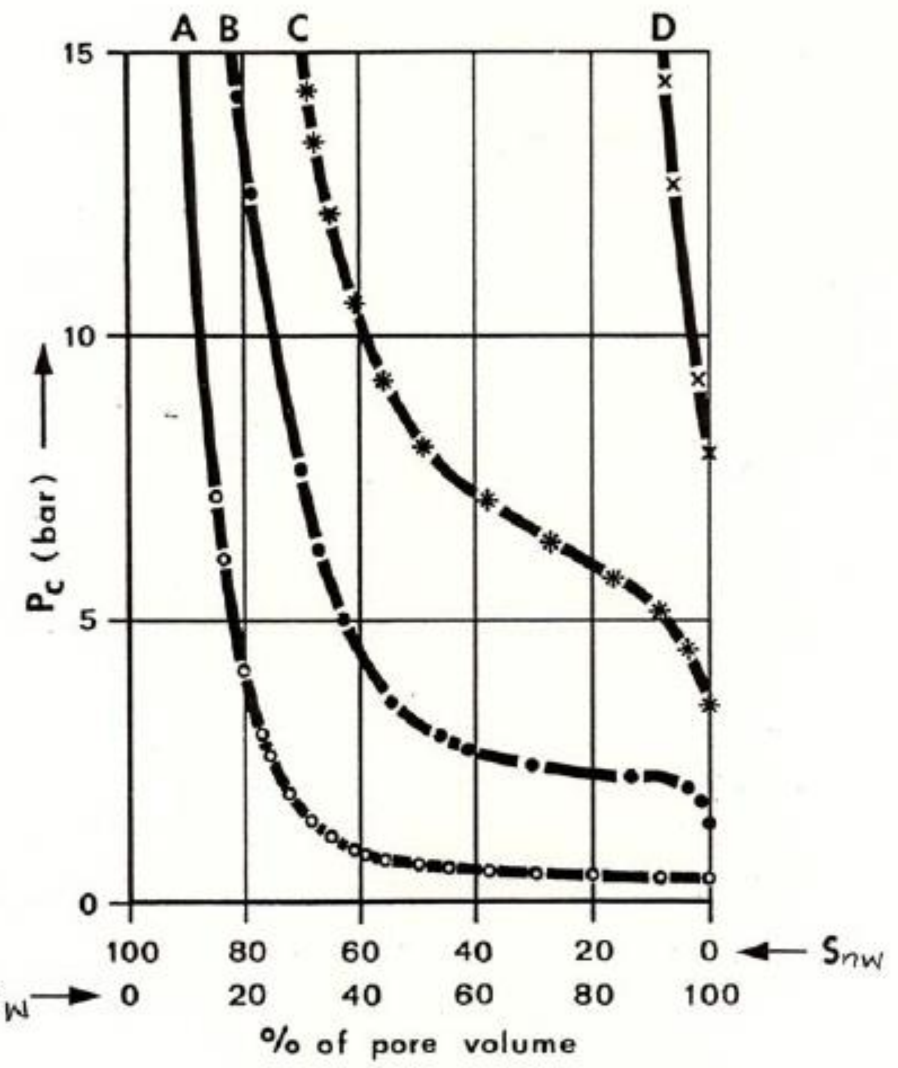
Smallest pore throats? **D**

Narrowest size distribution? **D**

Broadest size distribution? **C**

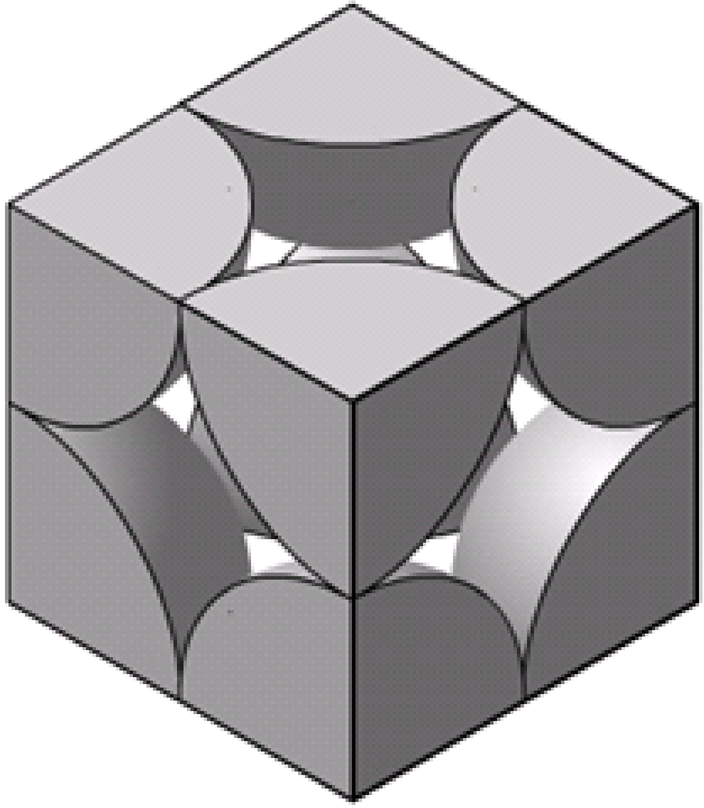
Largest modal size? **A**

Smallest modal size? **C? (does D have an inflection point?)**



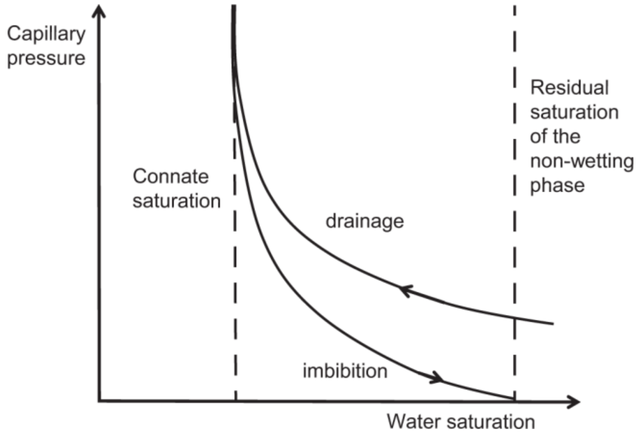
Drainage Capillary Pressure Curve

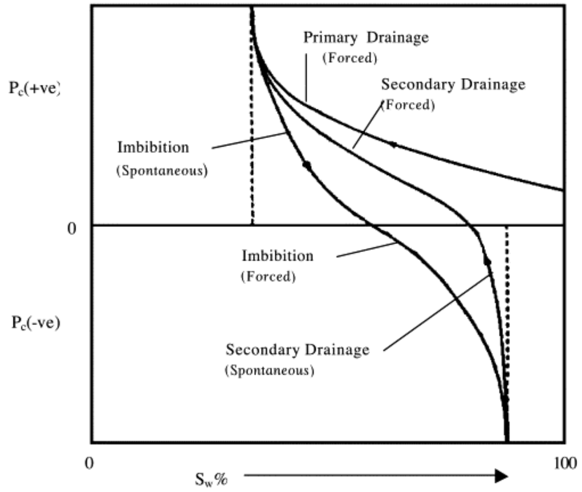
- Entrances to pore bodies are controlled by pore throats, but pore throats comprise very small fraction of pore volume.
- As an example, in simple cubic packing:
  - Radius of sphere inscribed in center (pore body):
    $$\left(2\sqrt{3}-2\right)r \approx 1.464r$$
  - Radius of sphere inscribed in face (pore throat):
    $$\left(2\sqrt{2}-2\right)r \approx 0.828r$$
- Drainage curves give us volume of pore bodies connected to pore throats of a certain size.



Capillary pressure hysteresis and capillary imbibition

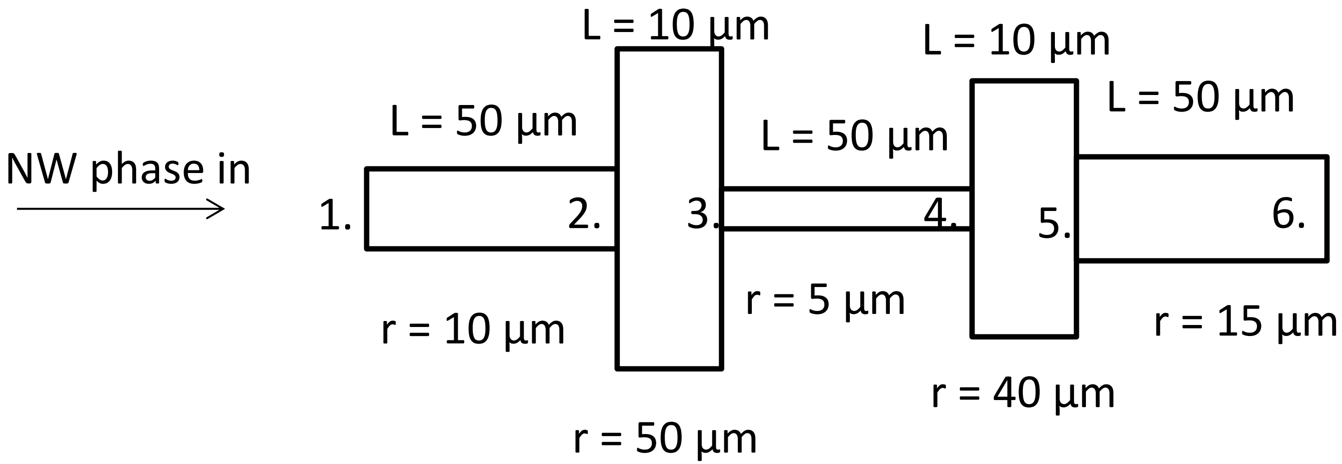
- Capillary pressure curves show a marked hysteresis depending on whether the curve is determined under a drainage process or an imbibition process. In other words, at any wetting phase saturation, the drainage capillary pressure is higher than the imbibition capillary pressure.
- At a capillary pressure of zero, the spontaneous imbibition curve terminates at a wetting phase saturation that may or may not correspond to the true residual non-wetting phase saturation depending on the wettability of the rock.
- Capillary pressure hysteresis can be explained in a variety of ways. The most intuitive one is the explanation based on pore structure.
- The drainage curve should be used for estimating the initial fluid saturation distribution in the reservoir whereas the imbibition curve should be used for analyzing a water flood performance in a water-wet reservoir.

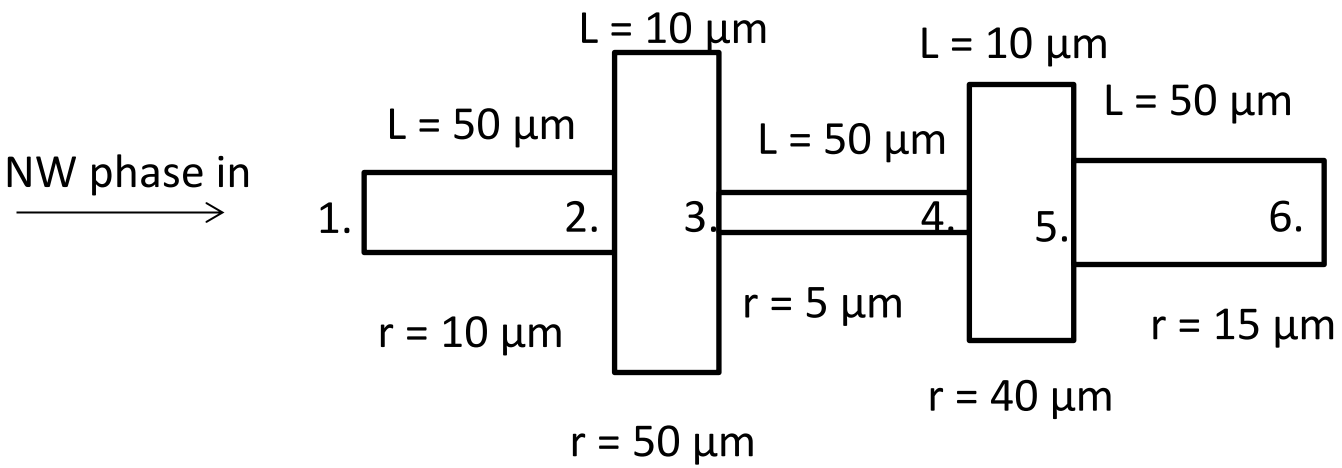




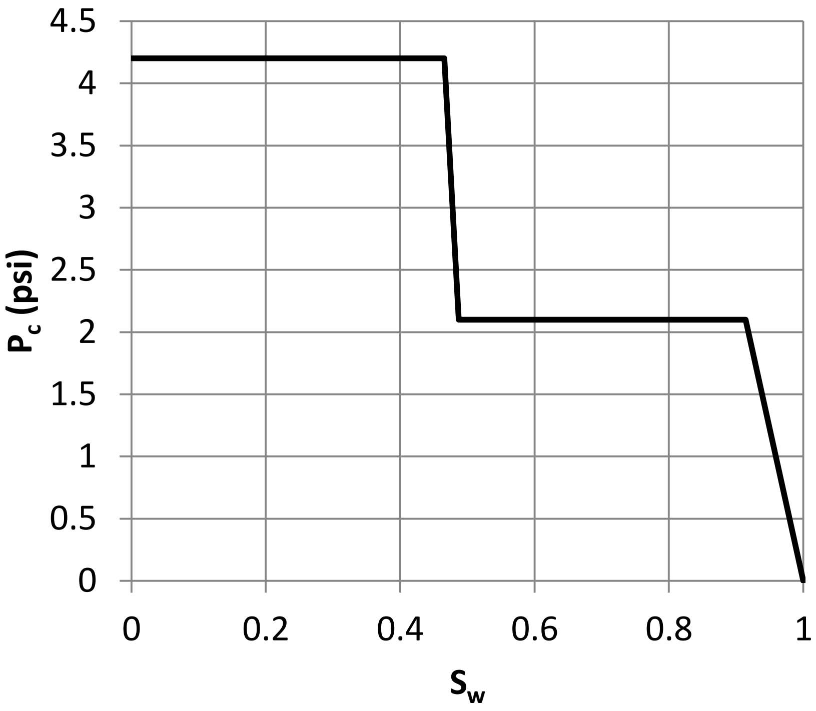
Capillary pressure hysteresis and capillary imbibition

- Why is this? Consider a very simple pore network of bodies and throats to the right.
- Let's assume bodies and throats are cylindrical, and that the wetting fluid is water and the non-wetting fluid is air so σ = 72 $\mathrm{dynes/cm}$ and θ = $0°$.

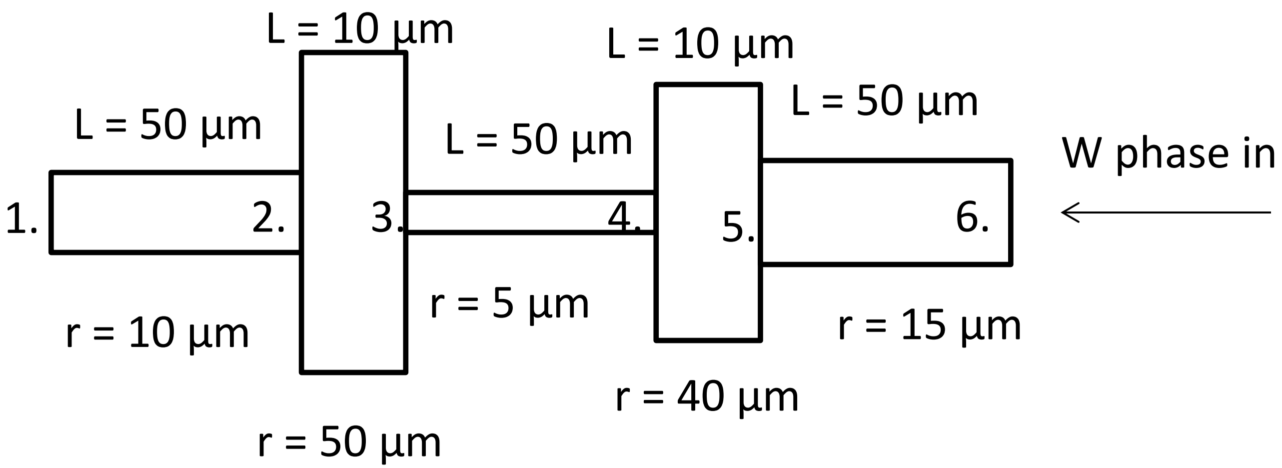




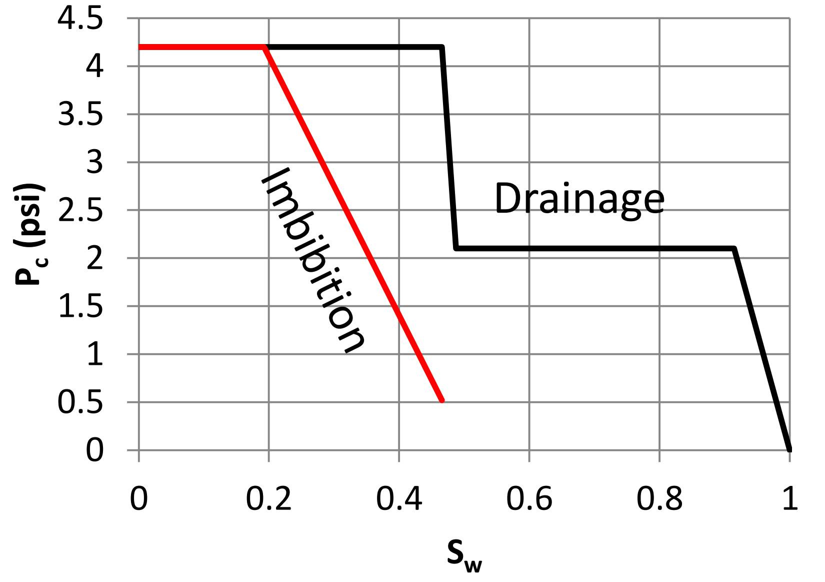
Drain to point…	Pc (psi)	Sw
1	0	1
2	2.1	0.91
3	2.1	0.49
4	4.2	0.47
5	4.2	0.19
6	4.2	0



Spontaneous imbibition: Wetting phase and Non-wetting phase pressures held constant



Imbibe to point…	P_c (psi)	S_w
6	4.2	0
5	4.2	0.19
4	0.52	0.47
3	-	-
2	-	-
1	-	-



*To imbibe past point 4, either P_w needs to be increased or P_{nw} decreased

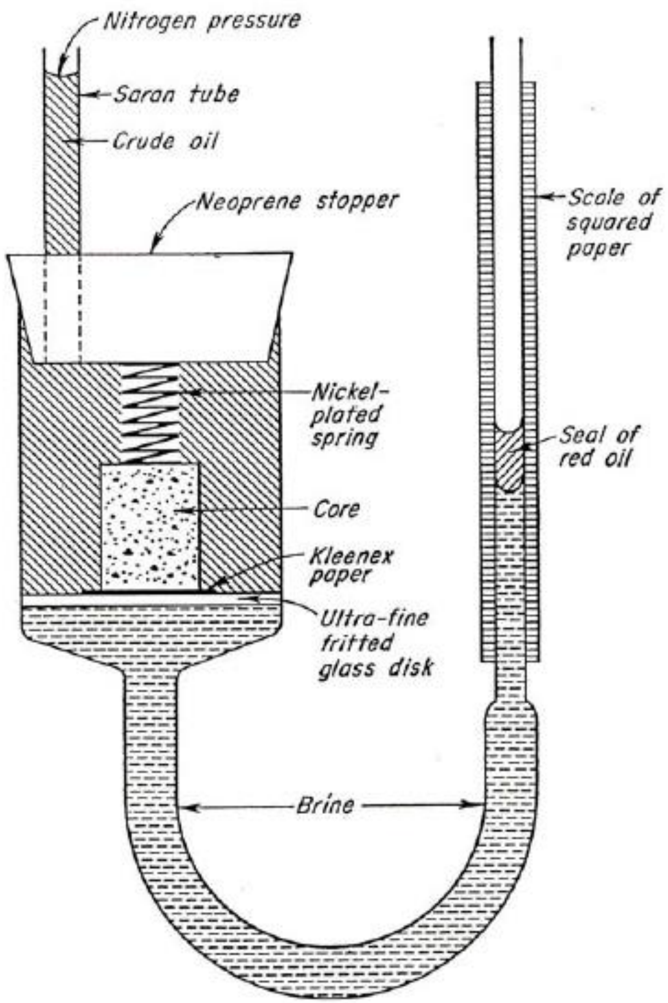
Capillary Pressure Lab-Measurements

- Porous Plate Method
- Mercury Injection Method
- Centrifuge Method

Capillary Pressure Lab-Measurements

Porous Plate Apparatus

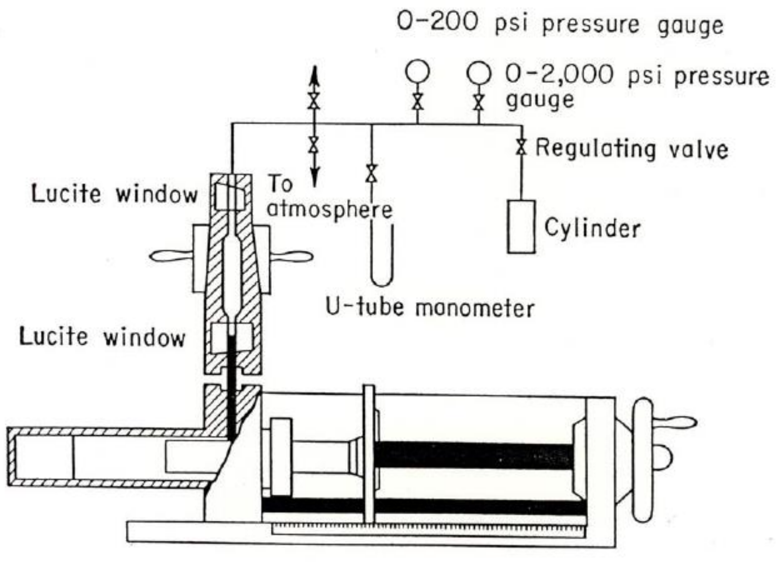
- Capillary pressure is measured by placing the sample, initially saturated with a wetting fluid, in a vessel filled with the non-wetting fluid.
- The bottom of the vessel consists of a semi-permeable plate, which allows the wetting phase displaced from the sample to pass through while blocking the passage of the non-wetting phase.
- The plate is typically made of porcelain or fritted glass, and it must have a displacement pressure that is higher than the largest capillary pressure to be measured. Typically, the maximum capillary pressure that can be measured is 200 psi.
- Pressure of the non-wetting fluid is increased in steps and the system is allowed to achieve equilibrium after each pressure change, and the volume of wetting phase displaced at each pressure step is measured.
- The porous plate apparatus can be used to measure the imbibition capillary pressure as well as the drainage curve.
- It takes too long to obtain the entire capillary pressure curve, it is not unusual for the experiment to take several weeks to complete



Capillary Pressure Lab-Measurements

Mercury Injection Method

- Mercury is a non-wetting phase.
- Nitrogen pressure is applied in successive increments and at each step, mercury is injected to maintain the mercury level with the graduation on the capillary.
- The mercury-air capillary pressure versus saturation relationship is calculated from the volume of mercury forced into the sample pore space as a function of applied nitrogen pressure.
- The mercury injection method is very fast. The curves can be obtained in a matter of hours.
- The major disadvantage is that the core can no longer be used for other tests after mercury injection.
- The method also cannot be used to determine the irreducible wetting phase saturation.



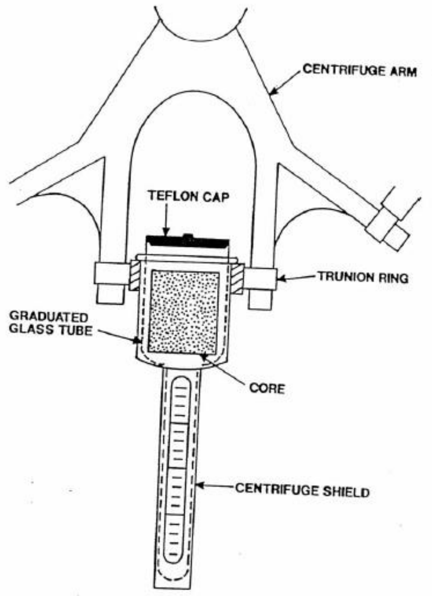
Mercury injection capillary pressure (MICP) apparatus

Capillary Pressure Lab-Measurements

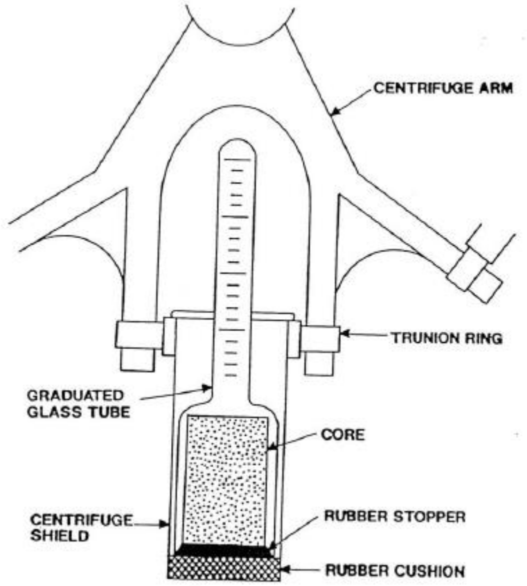
Centrifuge Method

- The sample saturated with a wetting fluid is placed in a centrifuge cup containing the non-wetting fluid.
- It is rotated at a series of constant angular velocities
- The amount of wetting fluid displaced at equilibrium at each velocity is measured with the aid of a stroboscopic light.
- Capillary pressure and saturation is calculated from rotational speed and the volume of wetting fluid displaced, respectively.
- This experiment is fast and allows the capillary pressure measurement to be completed in a day or less.

Oil displacing water



Water displacing oil



Ex: Centrifuge Method

- Table to the right shows the data obtained in a centrifuge experiment for determining the air-water capillary pressure curve of a core sample. Other data from the experiment are as follows:
  - Core length = 2.0 cm
  - Core diameter = 2.53 cm
  - Core pore volume = 1.73 cm^3
  - Core permeability = 144 mD
  - Water-Air density difference = 0.9988 g/cm^3
  - Distance to core inlet face r_1 = 6.6 cm
  - Distance to core outlet face r_2 = 8.6 cm
- Calculate the capillary pressure curves for the sample using both methods for the inlet water saturation.
- Compare the capillary pressure curves obtained from two methods.
- Calculate the acceleration imposed on the inlet of the core at the centrifuge speed of 5690 RPM and compare to the acceleration due to the gravity.

Centrifuge Speed (RPM)	Volume of Water Displaced (cc)
1300	0.30
1410	0.40
1550	0.50
1700	0.60
1840	0.70
2010	0.75
2200	0.80
2500	0.90
2740	1.00
3120	1.05
3810	1.10
4510	1.20
5690	1.25

Capillary end effect in a laboratory core

- This phenomenon has several undesirable consequences:
  - The observed breakthrough recovery of the non-wetting phase will be falsely high and will give a false sense of displacement efficiency.
  - The wetting phase saturation distribution in the core will be opposite what would normally be expected, with the wetting phase saturation being higher toward the core outlet than in the rest of the core.
  - In the unsteady state method for relative permeability measurement, if there is capillary end effect in the experiment, the calculated relative permeabilities will be wrong.

Capillary Pressure Lab-Measurements

- Typically, capillary pressure curves are measured in the laboratory using fluids other than reservoir fluids.
- The conversion of laboratory capillary pressure data to reservoir conditions is done using Laplace Equation.
- This ability to scale the laboratory capillary pressure data to reservoir conditions provides the flexibility for making laboratory capillary measurements with more convenient fluids than reservoir fluids.

- From the following two equations:

$$(P_\mathrm{c})_\mathrm{lab} = \frac{2(\sigma \cos\theta)_\mathrm{lab}}{r_\mathrm{m}}$$

$$(P_\mathrm{c})_\mathrm{reservoir} = \frac{2(\sigma \cos\theta)_\mathrm{reservoir}}{r_\mathrm{m}}$$

- We get the following relation by assuming the same mean radius:

$$(P_\mathrm{c})_\mathrm{reservoir} = (P_\mathrm{c})_\mathrm{lab} \frac{(\sigma \cos\theta)_\mathrm{reservoir}}{(\sigma \cos\theta)_\mathrm{lab}}$$

- The equation above can be used to convert lab results to reservoir conditions.

Empirical Capillary Pressure Models

- Often, it is desirable to fit analytical models to capillary pressure curves to simplify reservoir performance calculations, especially in numerical simulations.
- In the models, capillary pressure appears as a derivative. Therefore, an analytical model will allow the derivatives to be calculated without numerical noise.
- Brooks-Corey and Van Genuchten Models are two examples for such models.

Brooks-Corey Model

- Based on this model, all the drainage capillary pressure curves can be represented by linear function to the right.
- P_d is a constant determined from fitting capillary pressure value at $S = 1$ on a log-log plot of P_c and S.
- λ is the pore size distribution index obtained from the slopes of the straight lines. A large value of λ corresponds to a narrow pore size distribution whereas a small value of λ indicate a wide pore size distribution. A uniform pore size corresponds to $\lambda \to \infty$.
- The model cannot adequately fit a capillary pressure curve for poorly sorted porous media because of the inflection (S-shape part of a curve) as typically observed in such rocks.

Brooks-Corey Model

Drainage capillary pressure model

$$\ln S = -\lambda \ln P_\mathrm{c} + \lambda \ln P_\mathrm{d}$$

$$P_\mathrm{c} = P_\mathrm{d} \cdot S^{-\frac{1}{\lambda}}$$

$$S = \frac{S_\mathrm{w} - S_\mathrm{wirr}}{1 - S_\mathrm{wirr}}$$

Imbibition capillary pressure model

$$P_\mathrm{c} = P_\mathrm{d}\left[S^{-\frac{1}{\lambda}} - 1\right]$$

$$S = \frac{S_\mathrm{w} - S_\mathrm{wirr}}{1 - S_\mathrm{wirr} - S_\mathrm{nwr}}$$

Ex: Brooks-Corey Model

- Fit the Brooks-Corey model to the air-water drainage capillary pressure data given to the right.
- Derive the spontaneous imbibition capillary pressure curve for the sample using the Brooks-Corey model.

S_w	Drainage P_c psi
1.000	1.973
0.950	2.377
0.900	2.840
0.850	3.377
0.800	4.008
0.750	4.757
0.700	5.663
0.650	6.781
0.600	8.195
0.550	10.039
0.500	12.547
0.450	16.154
0.400	21.787
0.350	31.817
0.300	54.691
0.278	78.408

Van Genuchten Model

- An empirical capillary pressure model that preserves the shape of the capillary pressure curve at high wetting phase saturations.
- This model is widely used in soil physics and in hydrology.
- The model for drainage process is given by:

$$S = \left[\frac{1}{1+(\alpha P_{\mathrm{c}})^{n}}\right]^{m} \quad \text{where} \quad S = \frac{S_{\mathrm{w}} - S_{\mathrm{wirr}}}{1 - S_{\mathrm{wirr}}}$$

- where α, n and m are the fitting parameters.

Ex: Van Genuchten Model

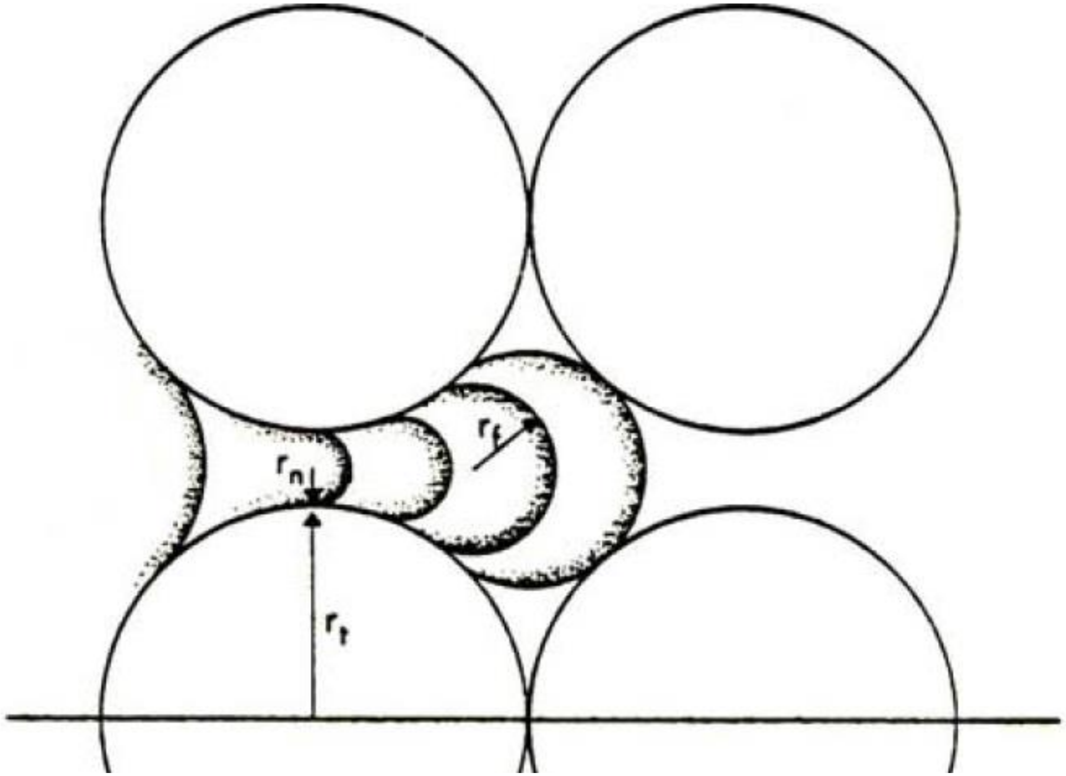
- Fit the van Genuchten model to the air-water drainage capillary pressure data given in the table and compare the result of the model to the original data.

S_w	P_c (psi)
0.20	50.0
0.24	39.0
0.25	38.0
0.28	35.0
0.30	34.0
0.35	32.0
0.40	30.0
0.50	28.5
0.56	28.0
0.60	27.8
0.68	27.0
0.70	26.8
0.70	26.8
0.75	26.0
0.80	24.8
0.85	23.0
0.90	21.0
0.95	18.0
1.00	13.5

Capillary trapping in porous media

- Capillary trapping ensures that an immiscible displacement at normal interfacial tensions and rates is never complete. There is always a residual phase that is trapped.
- Several models, such as the pore doublet model and the snap-off model, have been proposed to explain capillary trapping.

Snap-Off Model of Capillary Trapping



Advancing non-wetting phase will break in the pore throat when P_c at the throat exceeds that at the front of the drop:

$$\sigma\left(\frac{1}{r_n}-\frac{1}{r_t}\right) > \frac{2\sigma}{r_f}$$

(for a perfectly wetting fluid)

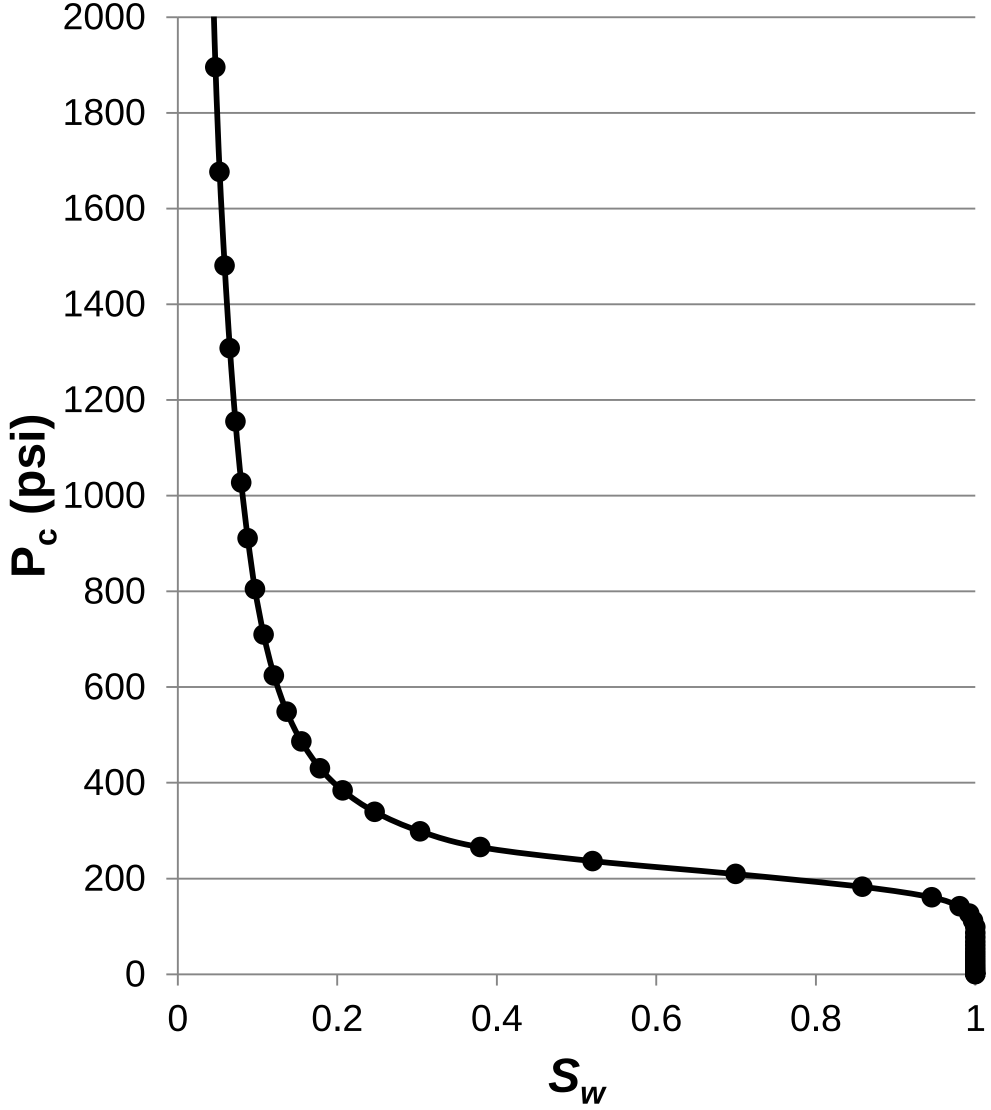
Capillary snap-off, residual non-wetting phase saturation, and hysteresis

Averaging capillary pressure data

- The capillary pressure curves for rock samples from the same reservoir having different permeabilities will be different.
- It is often necessary to average the capillary pressure data for cores from the same reservoir believed to have the same pore structure in order to obtain one capillary pressure curve that can be used for reservoir performance analysis.
- This averaging can be done using Leverett J-function, which is a dimensionless capillary pressure function:

$$J(S_{\mathrm{w}}, \Gamma) = \frac{P_{\mathrm{c}}\sqrt{k/\phi}}{\sigma \cos\theta}$$

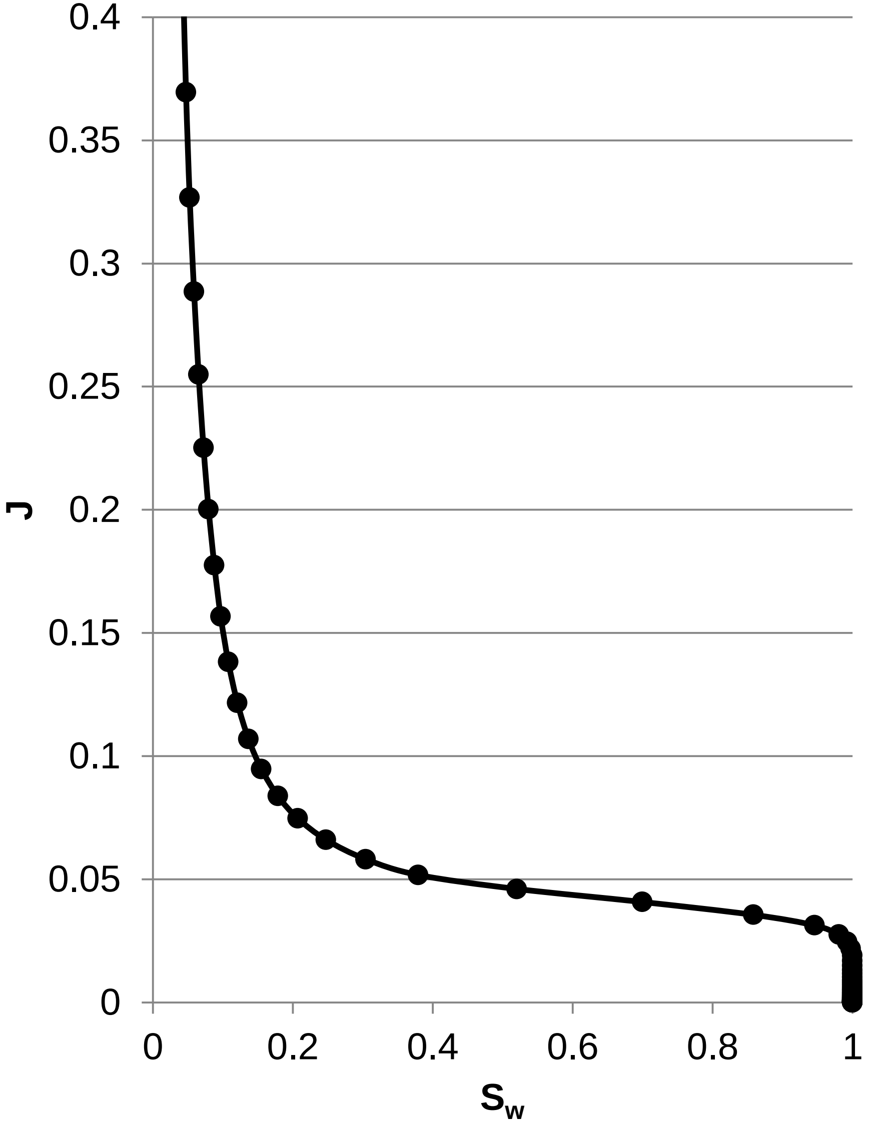
- where Γ is a dimensionless pore structure function that accounts for such things as pore size distribution, tortuosity, cementation and dead-end pores. The equation above suggests that porous media that have the same pore structure but different permeability and porosity will have the same Leverett J-function.



J function example

Air-water capillary pressure for a particular sample is shown. The porosity of this sample was 53% and the permeability was 0.0022 mD.

What P_c would correspond to S_w = 50% in a similar rock with 43% porosity and 0.0012 mD permeability?



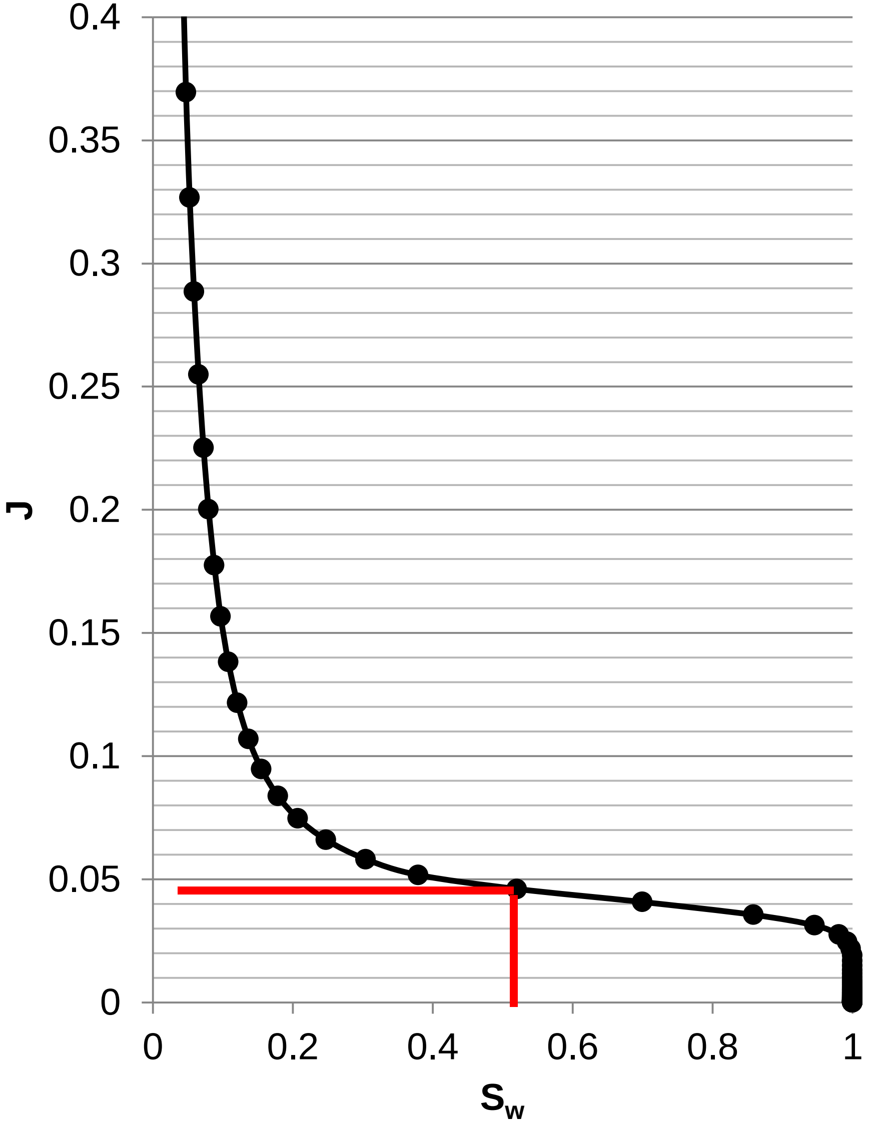
First compute J as a function of Sw. Since the measured curve was for air-water, σ = 72 dynes/cm. Let's assume the rock is water-wet (θ = 0°).

J at each P_c is computed as

$$J = \frac{P_c\sqrt{k/\phi}}{\sigma\cos\theta}$$

$$= P_c\sqrt{\frac{0.0022}{0.53}\times 9.869\times 10^{-12}\,\frac{\text{cm}^2}{\text{mD}}}\,\frac{1}{72}\times 68948\,\frac{\text{baryes}}{\text{psi}}$$

(doing the unit conversion ensures that J is truly non-dimensional and can be applied to any unit system)



Now find J at $S_w = 0.5$.

J = 0.045.

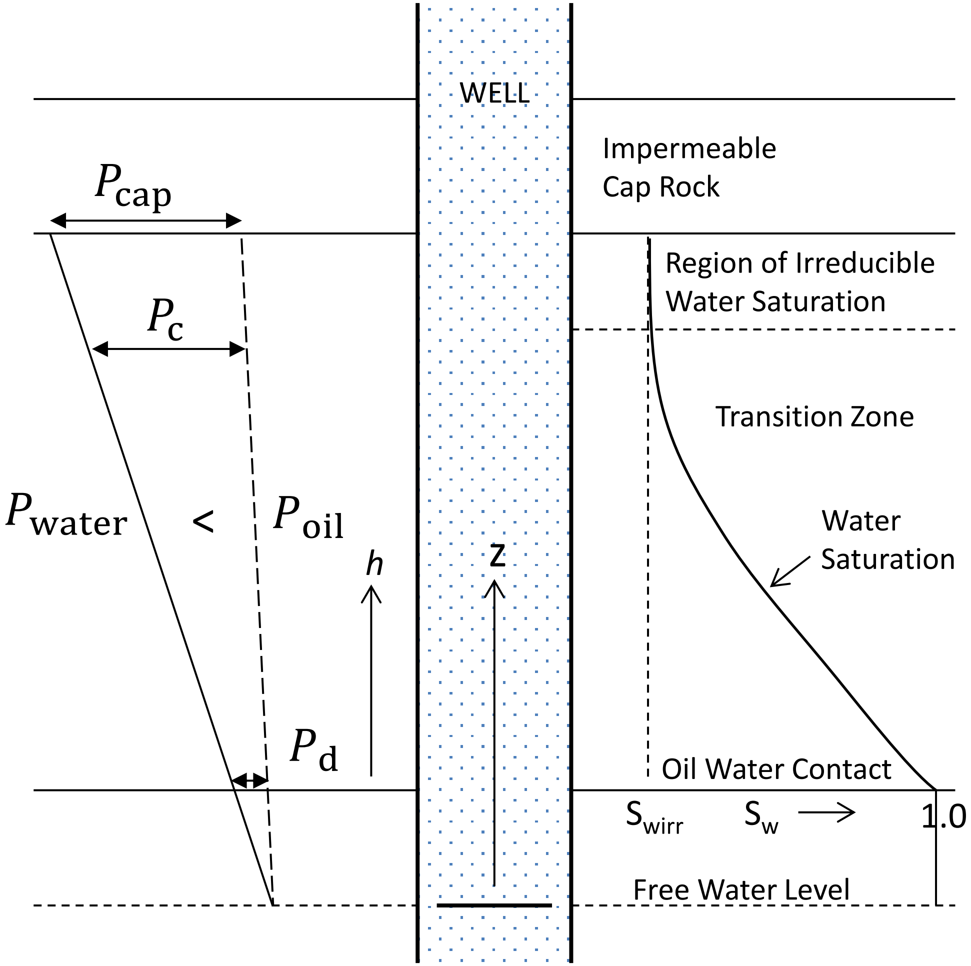
Now convert to our new rock:

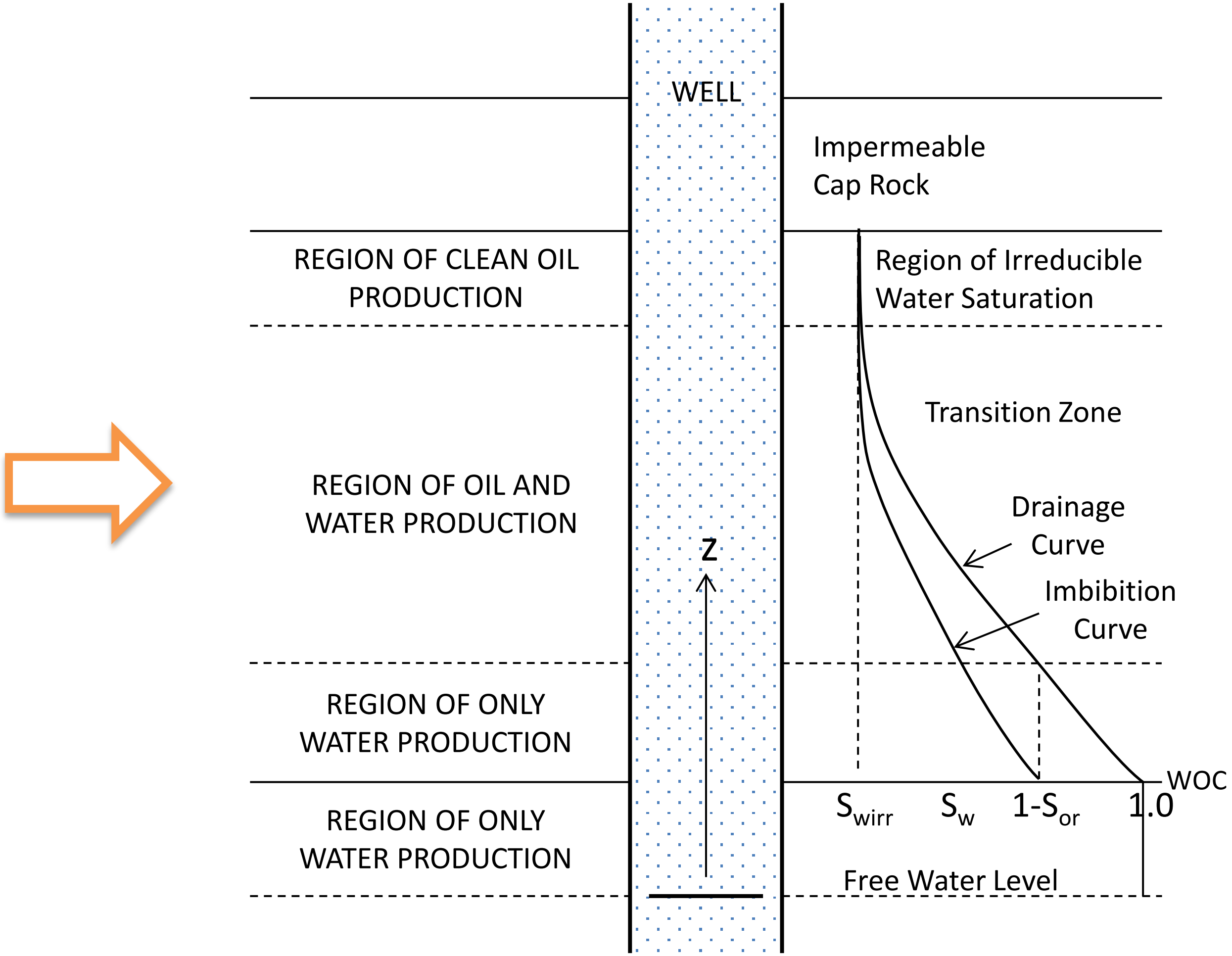
k = 0.0012 mD = 1.2×10^{-14} cm^2
porosity = 43%

$Pc = 72 * 0.045 * (0.43/1.2 \times 10^{-14})^{1/2}$
$= 1.94 \times 10^7$ baryes
= 281 psi

(compare to 233 psi for the first sample)

Determination of initial fluid saturation





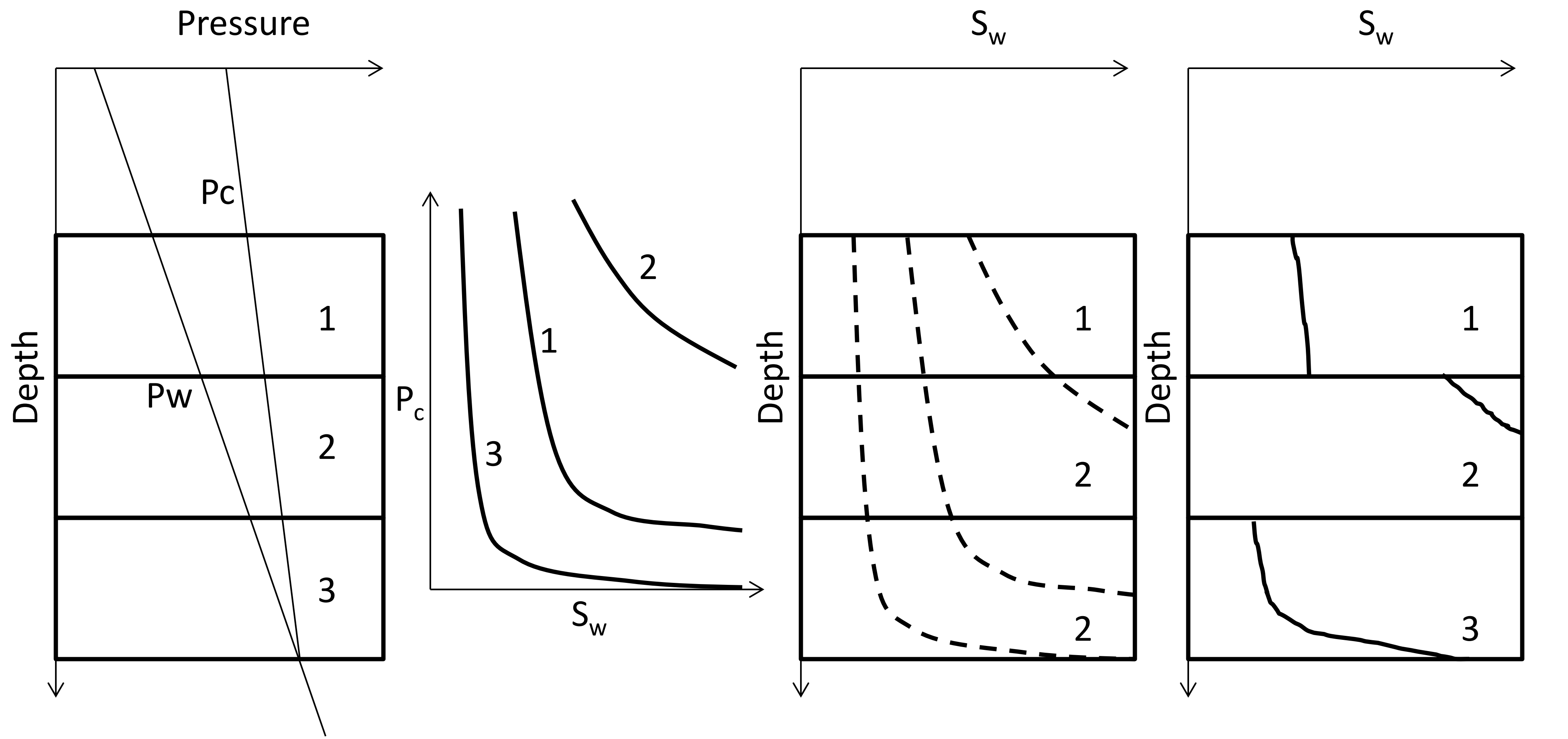
Note: to maintain the integrity of the cap rock, the capillary pressure at the cap rock should be less than the displacement pressure of the cap rock.

Ex 1: Determination of initial fluid saturation

- The company has discovered a massive reservoir sand. The oil column is 1000 ft thick from the top of the reservoir to the water-oil-contact. The reservoir is sufficiently clean to be treated as a homogenous reservoir.
- The drainage and imbibition capillary pressure curves for the reservoir (at reservoir conditions) are given by the Brooks-Corey model, and the data for this reservoir are as follows:
  - Water-oil density difference $\Delta\rho$ = 12.1 $\mathrm{lb/ft^3}$
  - Displacement pressure P_d = 4.2 psi
  - Residual oil saturation for the sand S_or = 30 %
  - Irreducible water saturation S_wirr = 22 %
  - Pore size distribution index λ = 2
- Where to perforate a well to ensure water-free production, at least initially:
  - Calculate the depth of the free water level
  - Calculate the maximum depth measured from the top of the reservoir that can be perforated and still produce clean oil initially
  - Calculate the depth measured from the top of the reservoir below which perforation will result in 100% water production
  - Calculate the minimum displacement pressure for the reservoir cap rock to maintain the seal integrity of this reservoir trap.

Determination of initial fluid saturation

In layered reservoirs with different drainage curves for each layer, the saturation can be computed by assuming that there is a single free water level.



Ex 2: Determination of initial fluid saturation

Table to the right bottom gives the properties of an idealized oil reservoir consisting of four layers with distinct petrophysical properties. The top of the reservoir is at 8000 ft below the surface and the oil water contact is at 8185 ft. Table on the next slide gives the drainage oil-water capillary pressure curve for Layer 1. All the layers have the same pore structure but different permeabilities and porosities.

- Calculate and plot the graph of the Leverett J-function for the reservoir.
- Calculate and plot the capillary pressure curves for Layers 2, 3 and 4, together with that of Layer 1.
- Calculate the depth of the free water level for the reservoir.
- Calculate and plot graphs of the initial water and oil saturations in the reservoir from 8000 ft to the free water level assuming the reservoir is in capillary equilibrium.
- Calculate and plot graphs of the water and oil pressures at the initial reservoir conditions.
- A well drilled into the reservoir has been perforated from 8090 to 8110 ft. Determine the type of reservoir fluid that will be produce initially.

$$\rho_{\mathrm{w}} = 1.036\ \mathrm{g/cm^3}$$

$$\rho_{\mathrm{o}} = 0.822\ \mathrm{g/cm^3}$$

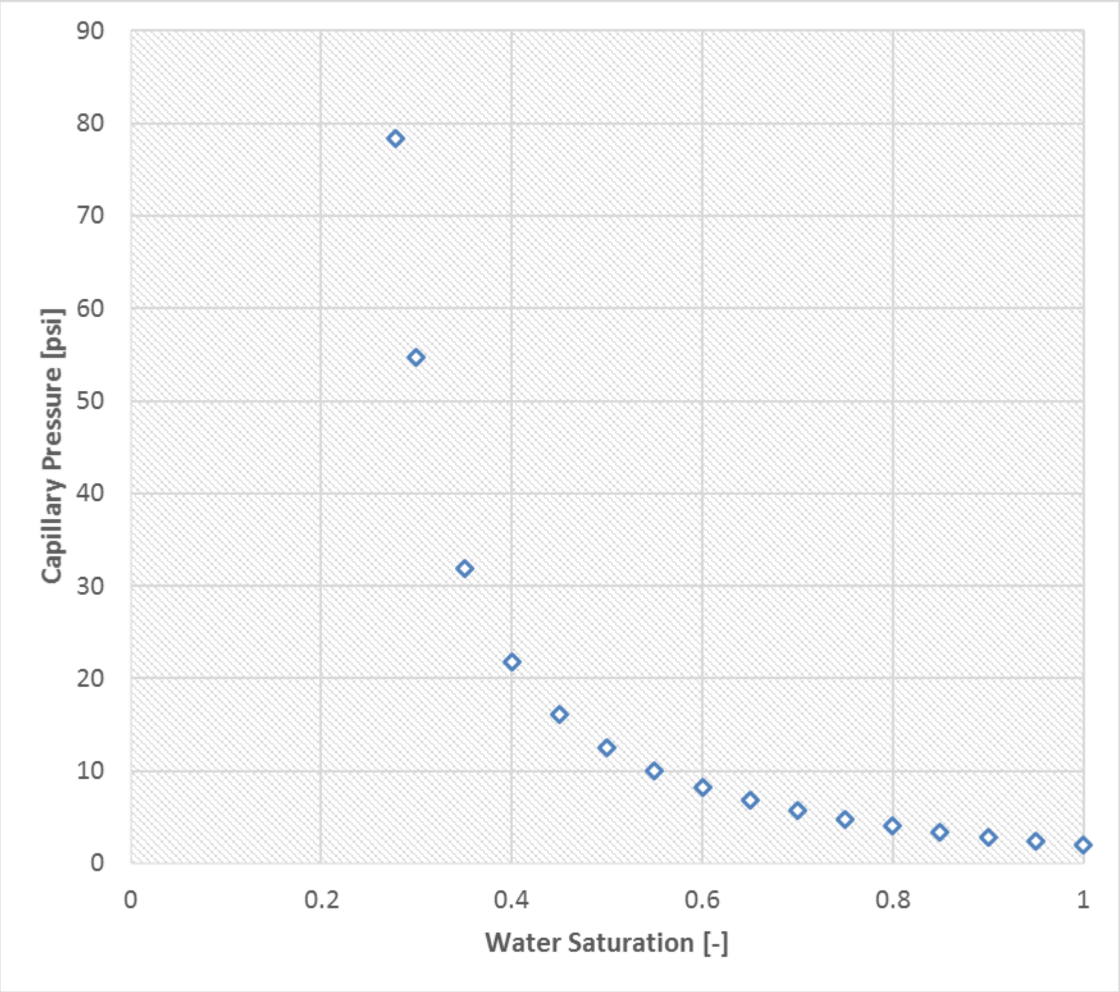
$$\sigma = 35\ \mathrm{dynes/cm}$$

$$\theta = 0°$$

	Layer 1	Layer 2	Layer 3	Layer 4
Depth (ft)	8000-8050	8050-8070	8070-8125	8125-8185
h (ft)	50	20	55	60
k (mD)	144	50	10	200
ϕ (%)	23.5	20	18	24

Ex 2: Determination of initial fluid saturation

S_w	P_c (psi)
1.000	1.973
0.950	2.377
0.900	2.840
0.850	3.377
0.800	4.008
0.750	4.757
0.700	5.663
0.650	6.781
0.600	8.195
0.550	10.039
0.500	12.547
0.450	16.154
0.400	21.787
0.350	31.817
0.300	54.691
0.278	78.408



Pore size distribution

- One of the principal applications of capillary pressure curve is for estimating the pore size distribution of porous media.
- Because mercury does not wet most solids, the capillary pressure curve derived from mercury injection is particularly well suited for probing the pore structure of porous media.
- As the capillary pressure is increased, pores with smaller pore throat sizes are invaded by mercury. If the mercury pressure is high enough, all the pores in the porous medium will be invaded by mercury.
- The cumulative volume of mercury injected vs. the capillary pressure can be used to determine the pore size distribution of the medium.