Single Phase Transient Flow

For the reservoir and fluid data given below, calculate pressure vs. radius for different times. In one graph, for the distances in between 0.25 and 1000 ft, show the pressures at 1, 10, and 100 days with three different lines (label them in the plot). You need to calculate exponential integral at many points to get a smooth plot especially close to wellbore. The initial reservoir pressure was 5000 psi. The stabilized flow rate is 300 STB/day.

reservoir info	oil properties	water properties	well info
$k = 80\ \mathrm{mD}$	$S_\mathrm{o} = 0.75$	$S_\mathrm{w} = 0.25$	$r_\mathrm{w} = 0.25\ \mathrm{ft}$
$c_\mathrm{f} = 3 \cdot 10^{-6} \dfrac{1}{\mathrm{psi}}$	$c_\mathrm{o} = 5 \cdot 10^{-6} \dfrac{1}{\mathrm{psi}}$	$c_\mathrm{w} = 3 \cdot 10^{-6} \dfrac{1}{\mathrm{psi}}$	
$\phi = 0.18$	$B_\mathrm{o} = 1.2\ \mathrm{bbl/STB}$		
$h = 50\ \mathrm{ft}$	$\mu_\mathrm{o} = 3.0\ \mathrm{cp}$		

Analytical Solution

The final form of the transient equation in SI units is:

$$P = P_\mathrm{i} - \frac{q\mu}{4\pi kh} \mathrm{Ei}\left(-\frac{\phi\mu c_t r^2}{4kt}\right)$$

First, we are doing the unit conversion. Given the equation above, what are the conversion factors to be used if all the parameters are given in oil field units?

- What should be the pressure difference in psi to flow a 1 cp viscosity fluid with 1 bbl/day flow rate in a core with 1 mD permeability and 1 ft length?

$$p = \frac{1}{4\pi} \frac{1\left[\frac{\mathrm{bbl}}{\mathrm{day}}\right] 1[\mathrm{cp}]}{1[\mathrm{mD}]1[\mathrm{ft}]} = \frac{1}{4\pi} \frac{\frac{0.1589}{24 \cdot 60 \cdot 60}\left[\frac{\mathrm{m}^3}{\mathrm{s}}\right] 0.001[\mathrm{Pa} \cdot \mathrm{s}]}{9.869 \cdot 10^{-16}[\mathrm{m}^2] 0.3048[\mathrm{m}]} = 973065[\mathrm{Pa}] \frac{1}{6894}\left[\frac{\mathrm{psi}}{\mathrm{Pa}}\right] = 70.6[\mathrm{psi}]$$

- What is the value of the constant part of exponential integral?

$$\frac{1[\mathrm{cp}] \cdot 1\left[\frac{1}{\mathrm{psi}}\right] \cdot 1[\mathrm{ft}^2]}{4 \cdot [\mathrm{mD}] \cdot 1[\mathrm{day}]} = \frac{0.001[\mathrm{Pa} \cdot \mathrm{s}] \cdot \frac{1}{6894}\left[\frac{1}{\mathrm{Pa}}\right] \cdot 0.0929[\mathrm{m}^2]}{4 \cdot 9.869 \cdot 10^{-16}[\mathrm{m}^2] \cdot 86400[\mathrm{s}]} = 39.5$$

- So for the oil field units, the final form of the transient equation is:

$$P = P_\mathrm{i} - 70.6\frac{q\mu}{kh} \mathrm{Ei}\left(-39.5\frac{\phi\mu c_t r^2}{kt}\right)$$

Second, we calculate the total compressibility:

$$c_t = S_\mathrm{w} c_\mathrm{w} + S_\mathrm{o} c_\mathrm{o} + c_\mathrm{f} = 0.25 \cdot 3 \cdot 10^{-6}\left[\frac{1}{\mathrm{psi}}\right] + 0.75 \cdot 5 \cdot 10^{-6}\left[\frac{1}{\mathrm{psi}}\right] + 3 \cdot 10^{-6}\left[\frac{1}{\mathrm{psi}}\right] = 7.5 \cdot 10^{-6}\left[\frac{1}{\mathrm{psi}}\right]$$

Third, we find the time interval where the solution is valid:

$$3.79 \cdot 10^5 \frac{\phi\mu c_t r_\mathrm{w}^2}{k} < t < 948\frac{\phi\mu c_t r_\mathrm{e}^2}{k}$$

In the equation above, parameters should be in oil field units except time in hours. Replacing the numbers:

$$\frac{3.79 \cdot 10^5 \cdot 0.18 \cdot 3 \cdot 7.5 \cdot 10^{-6} \cdot 0.25^2}{80} < t \quad \Rightarrow \quad t > 0.001199 \text{ hours}$$

Where all the given time values satisfy this condition. Since the exterior reservoir size has not been provided, we calculate the minimum size reservoir which can still act as an infinite at the maximum time questioned, at 100 days:

$$100 \cdot 24 < \frac{948 \cdot 0.18 \cdot 3 \cdot 7.5 \cdot 10^{-6} \cdot r_e^2}{80} \quad \Rightarrow \quad r_e > 7071.6 \text{ ft}$$

Final step, we calculate the coefficients inside and outside of exponential integral as:

$$-39.5\frac{\phi \mu c r^2}{kt} = -39.5\frac{0.18 \cdot 3.0[\text{cp}] \cdot 7.5 \cdot 10^{-6}\left[\frac{1}{\text{psi}}\right] \cdot r^2}{80[\text{mD}] \cdot t} = -2 \cdot 10^{-6}\frac{r^2}{t}$$

$$70.6\frac{q\mu}{kh} = 70.6\frac{-300[\text{STB/day}] \cdot 1.2[\text{bbl/STB}] \cdot 3.0[\text{cp}]}{80[\text{mD}] \cdot 50[\text{ft}]} = -19.062$$

And for this question, the final form of equation becomes:

$$P(r,t) = 5000 + 19.062 \cdot \text{Ei}\left(-2 \cdot 10^{-6}\frac{r^2}{t}\right)$$

Calculating pressure at certain points and time:

$$P(100 \text{ ft}, 1 \text{ day}) = 5000 + 19.062 \cdot (-3.3547077833097094) = 4936 \text{ psi}$$

$$P(328 \text{ ft}, 1 \text{ day}) = 5000 + 19.062 \cdot (-1.1632460247506078) = 4978 \text{ psi}$$

Plotting Results

Below is the plot of the data generated with python. The script is attached separately: