

## Group Assignment 1

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### I. Presudo-Code

#### 1. Enumeration

```
ENUMERATION(a[1,...,n])  
  define a result variable  
  for i = 1:n  
    for j = i:n  
      define a current variable  
      for k = j:n  
        add two numbers together and store it in current  
        take the max of current and result and store it in result  
      return max sum found so far
```

#### 2. Better Enumeration

```
BETTER_ENUMERATION(a[1, ..., n])  
  for i = 1:n  
    sum <- 0  
    for j = i:n  
      current += a[j]  
      if sum < current OR i <- 0  
        sum <- current  
      keep max sum found so far  
    return max sum found so far
```

#### 3. Dynamic Programming

```
DYNAMIC(a[1,...,n])  
  define a variable named current  
  define a variable named result  
  for i = 1:n  
    current += a[i]  
    if current less than 0 then set current to 0
```

take the max of current and result and store it in result  
return max sum found so far

## II. Run-time Analysis

### 1. Enumeration

The run-time of Enumeration Algorithm is  $O(n^3)$ , and the sum of it is  $\sum_{i=1}^n \sum_{j=i}^n \sum_{k=j}^n 3k + 2$ , which means that running every size of input array is much slower, the output of runtime is below. As the picture shows that this algorithm is not an optimized one, it increases faster when it comes to 400 array size, which is nearly three times of 300 array size.

```
Array size: 100
Run time: 0.021281435s
****
Array size: 200
Run time: 0.172305706s
****
Array size: 300
Run time: 0.588032311s
****
Array size: 400
Run time: 1.4199510650000002s
****
Array size: 500
Run time: 2.8589634409999998s
****
Array size: 600
Run time: 4.790457604s
****
Array size: 700
Run time: 7.439985529000001s
****
Array size: 800
Run time: 11.216020258s
****
Array size: 900
Run time: 16.102422386s
****
```

### 2. Better Enumeration

The run-time of Better Enumeration Algorithm is  $O(n^2)$ . It means that running every size of input array is slower than  $O(n)$ , which is the best run-time of maximum subarray, and the sum of it is  $\sum_{i=1}^n \sum_{j=i}^n 2j + 3$ , the output of runtime is below. As the picture shows, run time increases rapid when it comes to 4000 array size, which is nearly twice of 3000 array size.

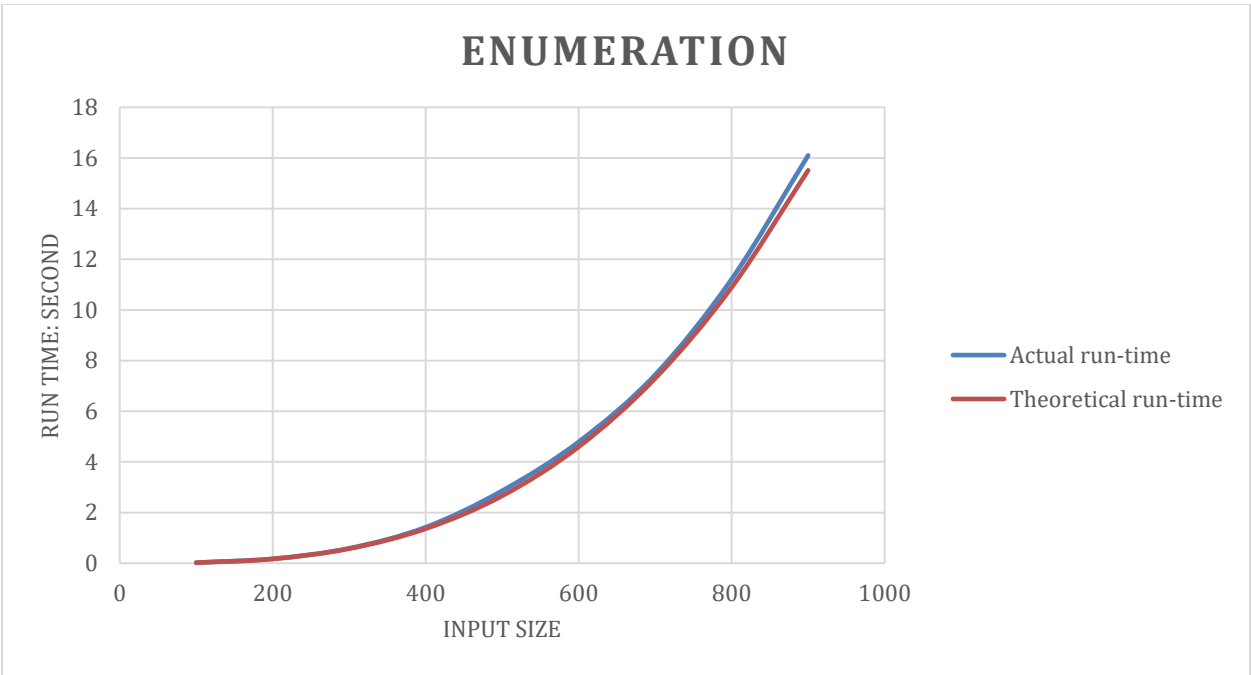
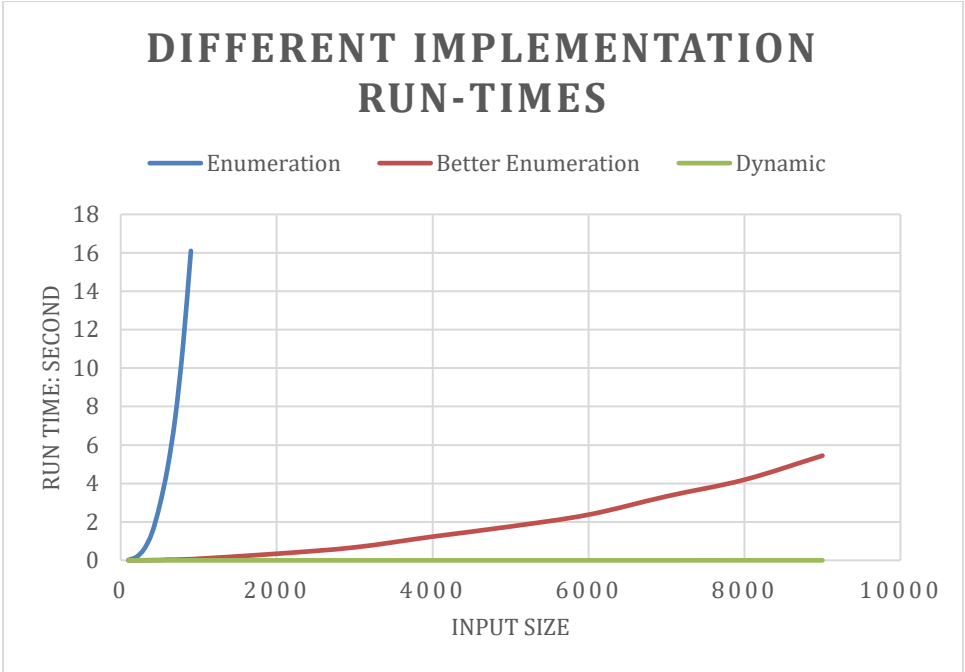
better_enumeration	
Array size: 100	Array size: 1000
Run time: 0.000632379000000024 s	Run time: 0.08355299100000002 s
****	****
Array size: 200	Array size: 2000
Run time: 0.004530359999999997 s	Run time: 0.34362644900000006 s
****	****
Array size: 300	Array size: 3000
Run time: 0.005592859999999998 s	Run time: 0.6754097010000001 s
****	****
Array size: 400	Array size: 4000
Run time: 0.011794885999999997 s	Run time: 1.234526448 s
****	****
Array size: 500	Array size: 5000
Run time: 0.016973205000000005 s	Run time: 1.7622367620000001 s
****	****
Array size: 600	Array size: 6000
Run time: 0.034041493000000006 s	Run time: 2.3761689949999996 s
****	****
Array size: 700	Array size: 7000
Run time: 0.036710887999999998 s	Run time: 3.3299819150000003 s
****	****
Array size: 800	Array size: 8000
Run time: 0.048138226000000006 s	Run time: 4.195108458 s
****	****
Array size: 900	Array size: 9000
Run time: 0.06147872400000001 s	Run time: 5.4466013019999998 s
****	****

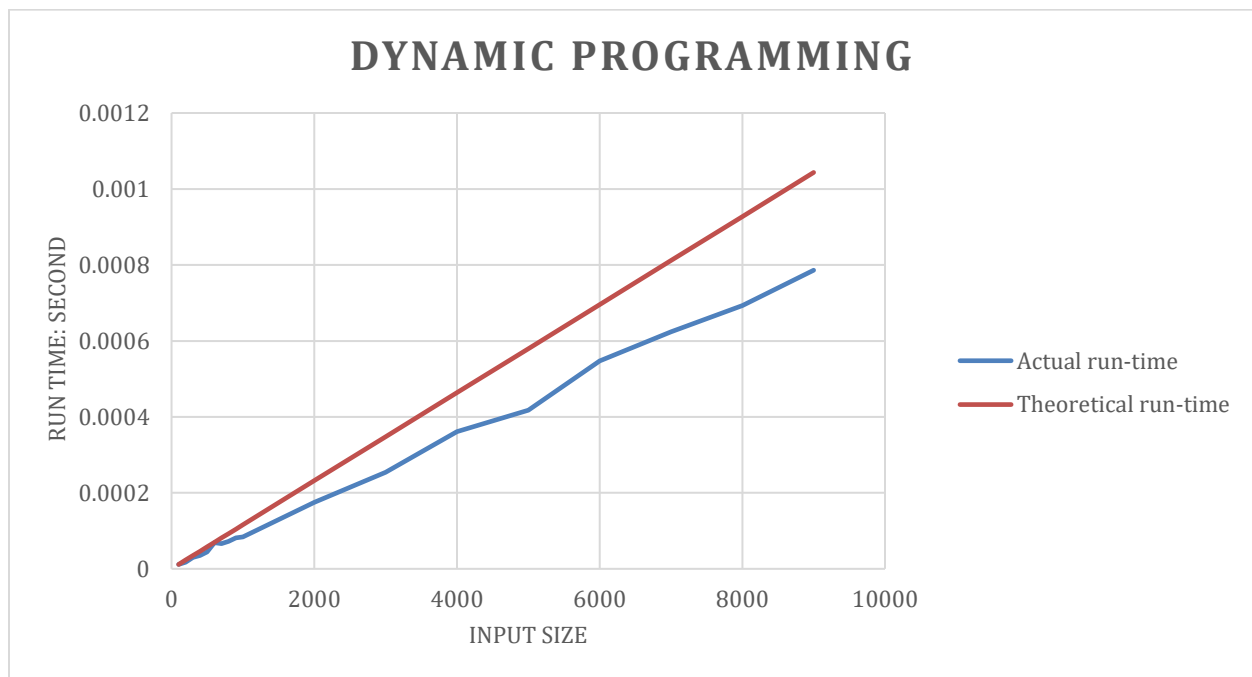
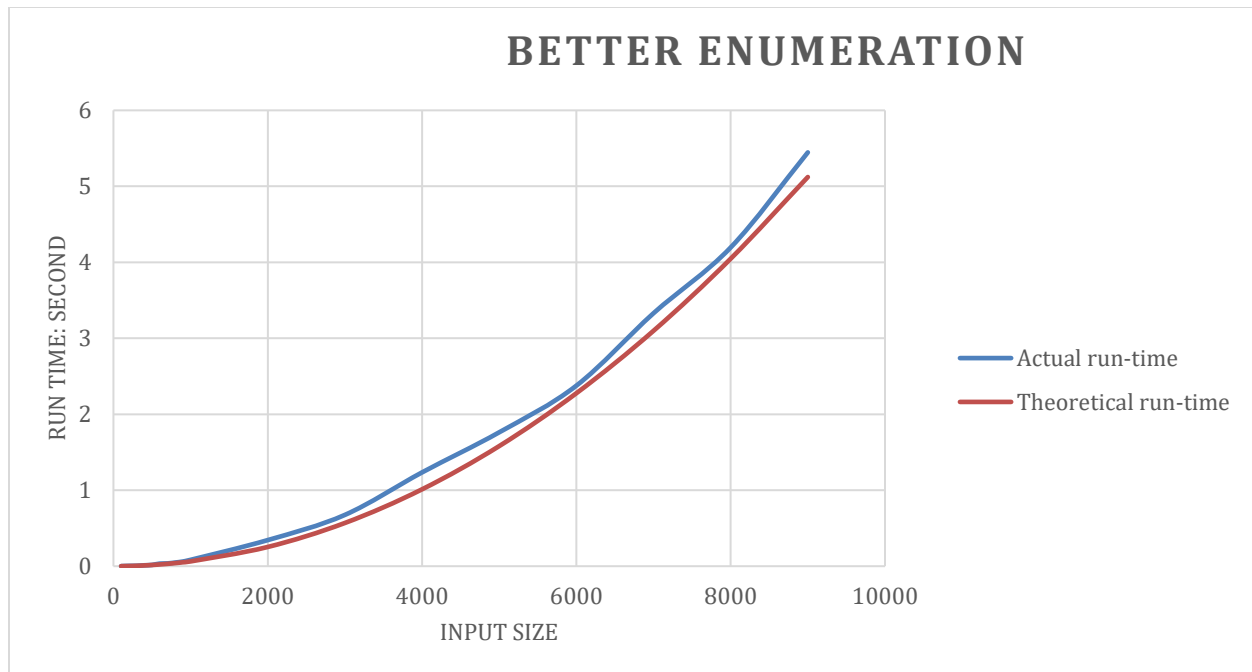
### 3. Dynamic Programming

The run-time of Dynamic Programming Algorithm is  $O(n)$ , which means that this is the best solution for maximum array problem among these three algorithms, and the sum of it is  $\sum_{i=1}^n 3i + 2$ . The output of runtime is below, as the picture shows that it increases gradually, and the time is much better than the Better Enumeration Algorithm.

dynamic	
Array size: 100	Array size: 1000
Run time: 1.1594000000003657e-05 s	Run time: 8.394200000000351e-05 s
****	****
Array size: 200	Array size: 2000
Run time: 1.762300000000827e-05 s	Run time: 0.000175305000000006 s
****	****
Array size: 300	Array size: 3000
Run time: 3.0145000000009192e-05 s	Run time: 0.000254144999999973 s
****	****
Array size: 400	Array size: 4000
Run time: 3.47819999999709e-05 s	Run time: 0.000360812000000002 s
****	****
Array size: 500	Array size: 5000
Run time: 4.4057999999996e-05 s	Run time: 0.0004178549999999476 s
****	****
Array size: 600	Array size: 6000
Run time: 6.86379999999587e-05 s	Run time: 0.000547247000000007 s
****	****
Array size: 700	Array size: 7000
Run time: 6.63189999999524e-05 s	Run time: 0.000623767999999968 s
****	****
Array size: 800	Array size: 8000
Run time: 7.23479999999986e-05 s	Run time: 0.0006933340000000038 s
****	****
Array size: 900	Array size: 9000
Run time: 8.11589999999732e-05 s	Run time: 0.0007860879999999904 s
****	****

III. Experimental Run-time Analysis





We can know from the plot that the actual run-time of our implementation is similar to the theoretical run-time. And the most efficient algorithm implementation is the dynamic one, the second is the better enumeration, and the last one is the enumeration. There are some deviations because the compiler needs extra cost for processing.