1 Stability Re-visited

In your group, use the line graph of the net flow rate for this stock to answer the following questions.

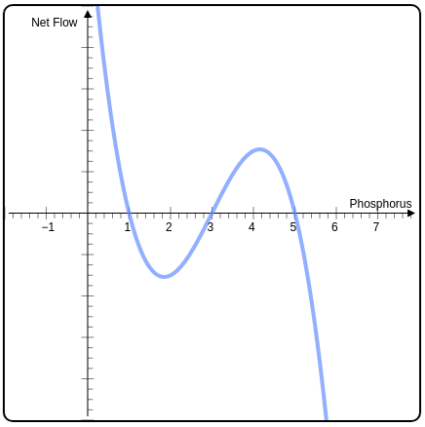


Figure: The line graph for the net flow rate of phosphorus. There net flow rate is zero when phosphorus values are 1, 3, and 5. The net flow rate decreases for phosphorus amount less than about 2, increases for phosphorus amounts between about 2 and 4, and decreases for phosphorus amounts greater than about 4.

1. What are the equilibrium value(s) of this system? How do you know?

Solution (click to open)

2. Classify each equilibrium value as stable or unstable.

Solution (click to open)

2 Sensitivity

Work in your group to fill out the following table to compute the sensitivities to pollution at various equilibrium values.

Use the first graph to set your initial pollution value, and find the associated initial phosphorus equilibrium value by either moving the point on the horizontal axis, or setting the value manually so that the net flow rate is 0. Similarly, use the second graph to set your next pollution value, and find the associated next phosphorus equilibrium value.

The first row is completed for you to use as a guide.

Initial Pollution Value:

$P_0 = 0$

0 3 6 9 12 15 18

Initial Phosphorus Value, $S_0 =$ 1

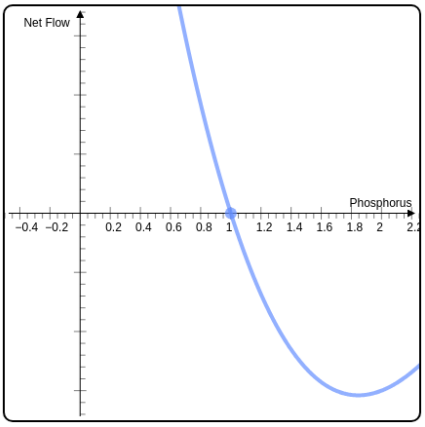


Figure: The initial phosphorus value is currently 1. It is an equilibrium value because the net flow rate is 0.

Next Pollution Value:

$P_1 = 0$

0 3 6 9 12 15 18

Next Phosphorus Value, $S_1 =$ 1

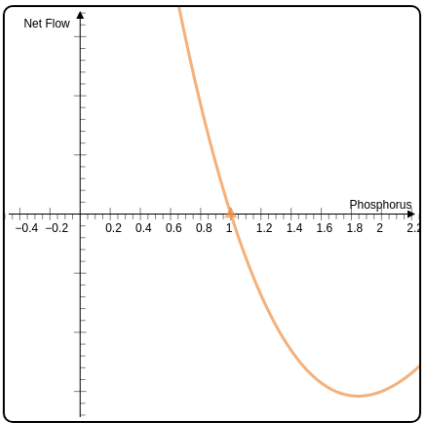


Figure: The next phosphorus value is currently 1. It is an equilibrium value because the net flow rate is 0.

P_0	S_0	P_1	S_1	**Sensitivity**
0	1	0.25	1.006	$\frac{1.006-1}{0.25-0} = 0.0238$
6	Check Work		Check Work	Check Work
12	Check Work		Check Work	Check Work
15	Check Work		Check Work	Check Work

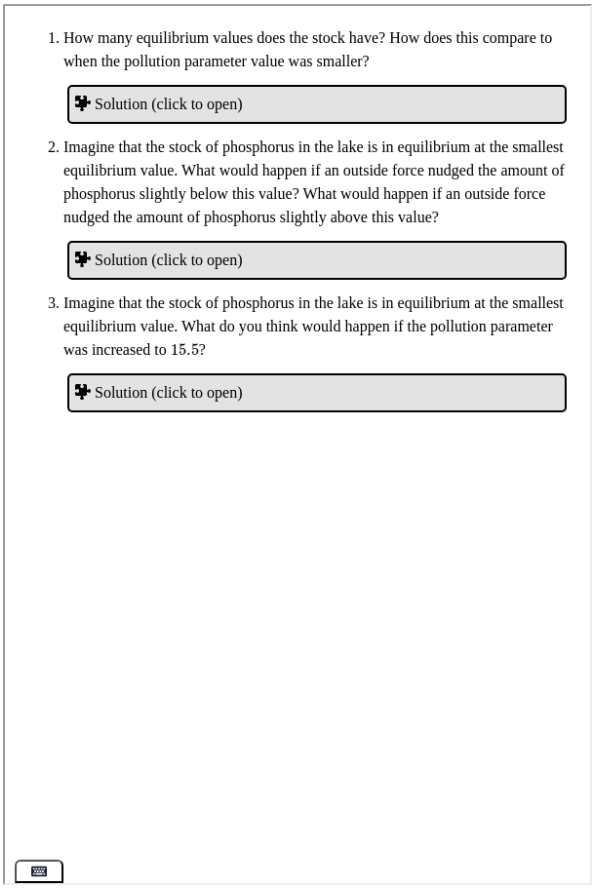
1. Once your table is completed correctly, discuss the following questions in your group.

(a) How do the sensitivities change as pollution values change?

(b) Are there pollution values that can change the *number* of equilibrium values in the system? How does this seem to be connected to sensitivities?

3 Tipping Points

Place the pollution parameter slider in `[cross-reference to target(s) "fig-net-flow-parameter" missing or not unique]` to 15.25, and answer the following questions in your group.



1. How many equilibrium values does the stock have? How does this compare to when the pollution parameter value was smaller?

   Solution (click to open)

2. Imagine that the stock of phosphorus in the lake is in equilibrium at the smallest equilibrium value. What would happen if an outside force nudged the amount of phosphorus slightly below this value? What would happen if an outside force nudged the amount of phosphorus slightly above this value?

   Solution (click to open)

3. Imagine that the stock of phosphorus in the lake is in equilibrium at the smallest equilibrium value. What do you think would happen if the pollution parameter was increased to 15.5?

   Solution (click to open)

4 Tipping Point Behaviors

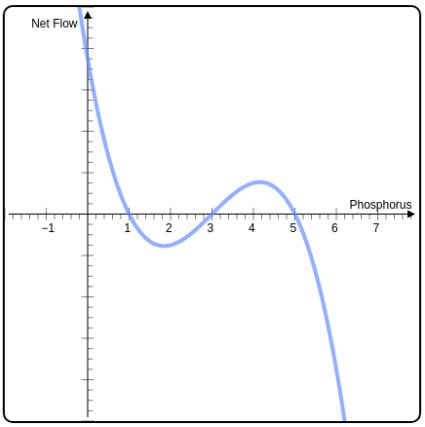


Figure: A line graph for the net flow rate of phosphorus, which has the same general shape as our previous net flow graphs. When the pollution parameter is between 0 and 7.75, there are 3 equlibrium values. When the pollution parameter is about 7.75, there are 2 equilibrium values. When the polution parameter is greater than 7.75, there is one equilibrium value.

$P = 0$

0 3 6 9 12 15 18

1. What are the tipping point values for the parameter P?

2. Imagine the phosphrus stock is in equilibrium when $P = 0$ at a phosphorus value of 1. If the parameter value exceeds the tipping point value closest to $P = 0$, how will the phosphorus value change?

3. If the parameter value exceeds the tipping point value closest to $P = 0$, is it possible for the phosphorus value to return to an equilibrium value of 1?

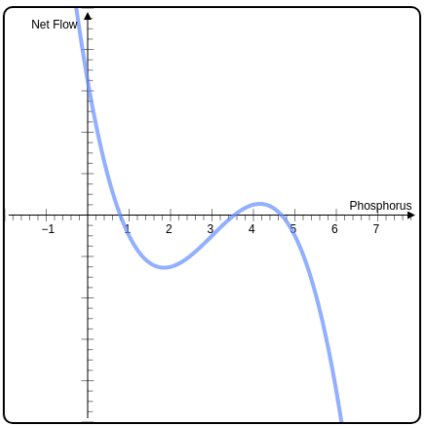


Figure: A line graph for the net flow rate of phosphorus, which has the same general shape as our previous net flow graphs. When the pollution parameter is between 0 and 17.5, there is 1 equlibrium value. When the pollution parameter is about 17.5, there are 2 equilibrium values. When the polution parameter is between 17.5 and 32.5, there are 3 equilibrium values. When the pollution parameter is about 32.5, there are 2 equilibrium values. When the pollution parameter is greater than 32.5, there is one equilibrium value.

$P = 20$

0 10 20 30

4. What are the tipping point values for the parameter P?

5. Imagine the phosphrus stock is in equilibrium when $P = 20$ at a phosphorus value of 0.8. If the parameter value exceeds the tipping point value which is larger than $P = 20$, how will the phosphorus value change?

6. If the parameter value exceeds the tipping point value which is larger than $P = 20$, is it possible for the phosporus value to return to an equilibrium value of less than 1?