1 Introduction to Density Histograms

A density histogram is a histogram which communicates the relative frequencies (e.g., 0.50) instead of the actual frequencies (e.g., 10). A density histogram for our example of 20 one liter samples is below.

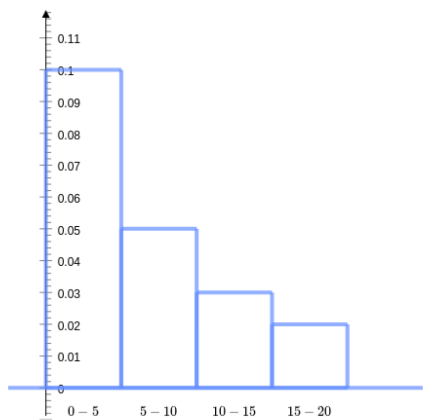


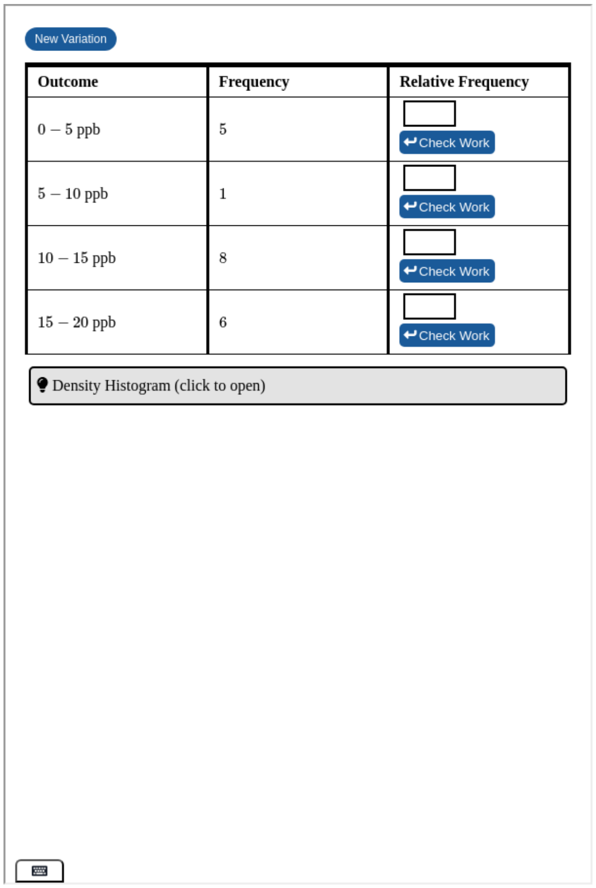
Figure: A density histogram communicating the relative frequencies from the previous table. The height of the 0 to 5 bin is 0.1, the height of the 5 to 10 bin is 0.05, the height of the 10 to 15 bin is 0.03, and the height of the 15 to 20 bin is 0.02.

1. In your group, answer the following question: how are the relative frequencies of each outcome being communicated in the histogram?

Hint. What is the area of each rectangle in the histogram? What is the sum of the areas of all of the rectangles in the histogram?

2 Using Relative Frequencies to Create Density Histograms

Suppose you and your group collect 20 new samples, given by the table below. Compute the relative frequencies of each outcome. Then create a frequency histogram and a density histogrm for the data.


New Variation

Outcome	Frequency	Relative Frequency
0 – 5 ppb	5	Check Work
5 – 10 ppb	1	Check Work
10 – 15 ppb	8	Check Work
15 – 20 ppb	6	Check Work

Density Histogram (click to open)

1. Choose new variations of the problem with the "New Variation" button. Before moving on, do at least 2 different variations, and write down an explanation for how to compute the relative frequency of an outcome and create a density histogram as if you were explaining it to someone new.

2. Do you think the shapes of the frequency histograms are related to the shapes of density histograms?

3 Law of Large Numbers Part 1

Let's first explore a random experiment that we are already familiar with: flipping a fair coin. Use the interactive provided to answer the questions below and discuss in your group.

1. In the random experiment of flipping a coin, what is the sample space? What is the probability of each outcome in the sample space?

   Solution (click to open)

2. Let's pretend for a moment that we wanted to verify that the coin was, in fact, a fair coin. One approach would be to approximate the probabilities of each outcome by flipping the coin several times and obtaining a density histogram, as we did above. Instead of having you actually flip the coin multiple times, we can simulate the process using the following interactive. A "0" means we flipped a heads, while a "1" means we flipped a tails. Use the program to flip your coin twice and verify the density histogram looks as you'd expect.

   How many times would you like to flip the coin? $N =$

   Results of your flips:

   Figure: A density histogram communicating the relative frequencies. The height of the Heads bin is NaN and the height of the Tails bin is NaN.

3. If you did not get one heads and one tails after flipping twice, do you think this means the coin is not fair? Why or why not?
4. Now flip your coin 10 times. Discuss how your results compare to the others in your group.
5. Repeat this process for flipping your coin 50, 100, 500, and $1,000$ times (be patient on this last one -- that's a lot of flips!). Do your observations change for how your results compare to others in your group as you increase the number of flips? Explain.
6. Enter your relative frequency of heads for $1,000$ flips in this shared spreadsheet.

4 Law of Large Numbers Part 2

Below is another interactive which allows you to (virtually) collect water samples from a reservoir, and returns the concentration of arsenic (in ppb) of your samples, as well as a density histogram of your samples. In your group, use the interactive along with the Law of Large Numbers to write a statement regarding your opinion about the arsenic levels in the reservoir. According to the Environmental Protection Agency, arsenic levels in drinking water should be less than 10 ppb[1].

How many times would you like to sample one liter of water? $N =$

Concentrations of your samples:

Figure: A density histogram communicating the relative frequencies. The height of the 0 to 5 bin is NaN, the height of the 5 to 10 bin is NaN, the height of the 10 to 15 bin is NaN, and the height of the 15 to 20 bin is NaN.

[1] www.epa.gov/dwreginfo/drinking-water-arsenic-rule-history