1 Stability Introduction

The images below represent three different systems in equilibrium. The stock value being measured is the red ball's position in space, and since all three balls are balanced and not moving on their respective surfaces, each ball is in equilibrium. Now imagine that a gust of wind blows over each system.

- If the wind blows so that each ball moves just slightly to the right, describe how you think the ball will behave afterwards.
- If the wind blows so that each ball moves just slightly to the left, describe how you think the ball will behave afterwards.

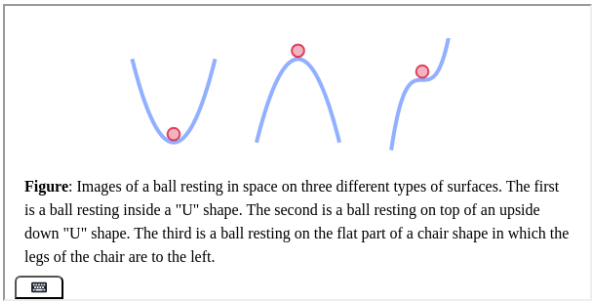

Figure: Images of a ball resting in space on three different types of surfaces. The first is a ball resting inside a "U" shape. The second is a ball resting on top of an upside down "U" shape. The third is a ball resting on the flat part of a chair shape in which the legs of the chair are to the left.

2 Stability In Continuous Dynamical Systems

In your group, use the graphs below to answer the following questions.

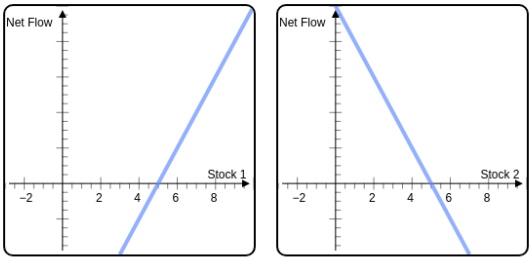

Figure: On the left, a line graph that shows the net flow rate increasing as stock 1 values increase. The net flow rate is zero when the stock 1 value is 5. On the right, a line graph that shows the net flow rate decreasing as stock 2 values increase. The net flow rate is zero when the stock 2 value is 5.

1. What are the equilibrium values of Stock 1 and Stock 2? How do you know?

   Solution (click to open)

2. Imagine that Stock 1 is in equilibrium. If the stock's value is nudged to a slightly lower value, how will the stock behave? If the stock's value is nudged to a slightly higher value, how will the stock behave? Sketch a line graph of the stock's value over time in each case.

   Solution (click to open)

3. Imagine that Stock 2 is in equilibrium. If the stock's value is nudged to a slightly lower value, how will the stock behave? If the stock's value is nudged to a slightly higher value, how will the stock behave? Sketch a line graph of the stock's value over time in each case.

   Solution (click to open)

3 Half-Stable Equilibria

In your group, do the following:

1. Construct a line graph for the Net Flow with respect to Stock value that you think would result in behavior around the equilibrium value that is reminiscent of the third system in Worksheet 1.[1]

2. Make a conjecture (this means an educated prediction) about how you could tell whether an equilibrium value was stable or unstable in a continuous dynamical system based off of its net flow rate line graph.

[1] We will call equilibrium values with this behavior **half-stable**.

4 Stability in Discrete Dynamical Systems

In the following examples, work in your group to make similar computations. You will keep track of your results in a table, and also see the visual representation as a scatter plot to help with your analysis.

Initializing....

1. In your group, write down a conjecture (this means an educated prediction) about how you could tell what the behavior of an equilibrium value will be in a discrete dynamical system based off of its net flow rate line graph.