LETTER

On the Distribution of MISO Channel Capacity in the Interference-Limited System*

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SUMMARY In this paper, the exact distribution of the channel capacity of MISO (multiple-input single-output) systems subject to co-channel interference is derived from an information theoretic viewpoint. It is found that the MISO channel capacity in the interference-limited channel follows the F-distribution. By using these capacity distributions, the outage capacity in Rayleigh fading channels can be accurately computed. We confirm the accuracy of our analysis by performing simulations. Our results exactly match those of the empirical simulations of interference-limited systems.

key words: *MISO, channel capacity distribution, interference*

1. Introduction

A formula for the capacity of noise-limited MIMO (multiple-input multiple-output) systems is exactly derived, and it is proved that the MIMO system can help achieve high spectral efficiency [1], [2]. However, if interfering users are present, the co-channel interference results in significant channel impairment and the MIMO system capacity is greatly reduced. In [3], the effect of interference is quantified based on the simulation results. According to [3], the capacity of MIMO channels subject to co-channel interference is significantly smaller than that of noise-limited MIMO systems. The capacity degradation due to interference is also reported in [4]. In [5] and [6], an analytical approach to calculating the MIMO capacity in the presence of interference is provided. However, the exact distribution of MIMO capacity cannot be derived since MIMO calculation is not easily tractable because matrix operations need to be performed. In contrast, the exact distribution of the capacity of MISO (multiple-input single-output) systems, which is a subset of MIMO, can be provided.

In this paper, the exact distribution of the channel capacity of MISO (multiple-input single-output) systems subject to co-channel interference is derived from an information theoretic viewpoint, which can offer new insights into the behavior of channel capacity; a new method for MISO system analysis is also presented. It is found that the MISO channel capacity in the noise-limited channel follows the chi-square distribution with $2M_T$ degrees of freedom (also shown by Example 7 in [1]), and the capacity in the interference-limited channel follows the F-distribution with $2M_T$ numerator degrees of freedom and $2M_TN_I$ denominator degrees of freedom, where M_T is the number of transmit antennas and N_I is the number of interferers. Using these capacity distributions, we can accurately compute the outage capacity for Rayleigh fading channels-this is the main contribution of this paper. We confirm the accuracy of our analysis by performing simulations. Our results exactly match the results of empirical simulations for noise-limited and interference-limited systems.

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2. MISO Channel Capacity in the Presence of Interference

2.1 MISO Channel Capacity with Interference

Assume that an $M_T \times 1$ MISO system is considered over a frequency flat-fading channel where M_T is the number of transmit antennas. Denote $\mathbf{x}$ as the $M_T \times 1$ input signal and $\mathbf{h}$ as $1 \times M_T$ channel vectors for $\mathbf{x}$. Also, denote $\mathbf{x}_i$ as the $M_i \times 1$ interference signal vectors from the ith interferer and $\mathbf{h}_i$ as $1 \times M_i$ channel vectors for $\mathbf{x}_i$ where M_i is the number of transmit antennas at the interferer. The complex entries of the channel vector are independent with uniformly distributed phase and normalized Rayleigh distributed magnitude modeled by the complex Gaussian random variable $\mathcal{CN}(0, 1)$. Therefore, the output signal vector can be expressed by

$$y = \sqrt{\frac{E_s}{M_T}}\mathbf{h}\mathbf{x} + \sum_{i=1}^{N_I}\sqrt{\frac{E_i}{M_i}}\mathbf{h}_i\mathbf{x}_i + n \tag{1}$$

where N_I is the total number of interferers, n is the additive Gaussian noise and E_s and E_i are the powers allocated to the desired user and the ith interferer, respectively.

According to [4], it can be assumed that both channel vectors $\mathbf{h}$ and $\mathbf{h}_i$ are independent with independent and identically distributed (i.i.d) elements and $\mathcal{E}\{\mathbf{x}\mathbf{x}^H\} = \frac{E_s}{M_T}\mathbf{I}_{M_T}$ and $\mathcal{E}\{\mathbf{x}_i\mathbf{x}_i^H\} = \frac{E_i}{M_i}\mathbf{I}_{M_i}$ for the transmit covariance matrices of the desired signal and interference, respectively. From [6], the MISO capacity with interference can be defined as

$$C = \log_2\left(1 + \frac{E_s}{M_T}\mathbf{h}\mathbf{h}^H\left(\sum_{i=1}^{N_I}\frac{E_i}{M_i}\mathbf{h}_i\mathbf{h}_i^H + N_o\right)^{-1}\right)$$
$$= \log_2\left(1 + \frac{E_s}{M_T}\lambda\left(\sum_{i=1}^{N_I}\frac{E_i}{M_i}\lambda_i + N_o\right)^{-1}\right). \quad (2)$$

Note that the channel gain λ in (2) is can be obtained as $\lambda = \mathbf{h}\mathbf{h}^H = \sum_{k=1}^{2M_T} h_k^2$ and λ_i is $\lambda = \mathbf{h}_i\mathbf{h}_i^H = \sum_{k=1}^{2M_i} h_k^2$ where h_k is a normally distributed random variable with mean zero and variance $\sigma^2 = 1/2$.

2.2 Capacity Distribution in Interference-Limited Channel

Let us consider a case where the noise power is negligible in comparison with the interference power. In this case, the achievable channel capacity is a function of SIR (Signal to Interference power Ratio). Therefore, from (2), the capacity in the interference-limited channel is represented as

$$C^i = \log_2\left(1 + \frac{E_s}{M_T}\frac{\lambda}{\sum_{i=1}^{N_I}\eta_i\lambda_i}\right) \quad (3)$$

where $\eta_i = E_i/M_i$.

First, we will derive the distribution of $Z = \sum_{i=1}^{N_I}\eta_i\lambda_i$ from the definition of the characteristic function. The characteristic function of Z, denoted by $\Phi_Z(t)$, is

$$\Phi_Z(t) = E\left[e^{jtZ}\right] = \prod_{i=1}^{N_I}\Phi_{\eta_i\lambda_i}(t) \quad (4)$$

where $j = \sqrt{-1}$ and $\Phi_{\eta_i\lambda_i}(t)$ is the characteristic function of $\eta_i\lambda_i$. Since $\lambda_i = \sum_{k=1}^{2M_i} h_k^2$ is a chi-square distributed with $2M_i$ degree of freedom, its characteristic function $\Phi_{\lambda_i}(t) = \prod_{k=1}^{2M_i}(1 - j2t)^{-1/2}$; hence

$$\Phi_{\eta_i\lambda_i}(t) = E\left[e^{jt\eta_i\lambda_i}\right] = \Phi_{\lambda_i}(\eta_i t) = \prod_{k=1}^{2M_i}(1 - j2\eta_i t)^{-1/2}. \quad (5)$$

Therefore

$$\Phi_Z(t) = \prod_{i=1}^{N_I}\prod_{k=1}^{2M_i}(1 - j2\eta_i t)^{-1/2}. \quad (6)$$

The probability distribution function (PDF) of Z, to be denoted by $f_Z(x)$, is obviously zero for $x \le 0$; hence, it can be obtained by the Fourier transform inversion formula [7], provided that $\Phi_Z(t)$ is integrable over the real line.

$$f_Z(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-jxt}\Phi_Z(t)dt. \quad (7)$$

To make $\Phi_Z(t)$ continuous, hence integrable, for all real t, $(1 - 2j\eta_i t)^{-1/2}$ can be defined using polar notation as

$$(1 - 2j\eta_i t)^{-1/2}$$
$$= \left(1 + 4\eta_i^2 t^2\right)^{-1/4}\exp\left[\frac{j}{2}\tan^{-1}(2\eta_i t)\right] \quad (8)$$

where $\frac{1}{2}\tan^{-1}(2\eta_i t)$ is continuous ranging from $-\pi/2$ to $\pi/2$ since $-\pi < \tan^{-1}(2\eta_i t) \le \pi$. Therefore

$$\Phi_Z(t) = \exp\left[j\left(-tx + \frac{1}{2}\sum_{i=1}^{N_I}\sum_{k=1}^{2M_i}\tan^{-1}(2\eta_i t)\right)\right]$$
$$\times\prod_{i=1}^{N_I}\prod_{k=1}^{2M_i}\left(1 + 4\eta_i^2 t^2\right)^{-1/4}. \quad (9)$$

Using (9), (7) can be derived as

$$f_z(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\exp\left[j\left(-tx + \frac{1}{2}\sum_{i=1}^{N_I}\sum_{k=1}^{2M_i}\tan^{-1}(2\eta_i t)\right)\right]$$
$$\times\prod_{i=1}^{N_I}\prod_{k=1}^{2M_i}\left(1 + 4\eta_i^2 t^2\right)^{-1/4} dt. \quad (10)$$

Since the imaginary part of the integrand is an odd function due to sine function, its integral over the real line is zero; thus

$$f_Z(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\cos\left(-tx + \frac{1}{2}\sum_{i=1}^{N_I}\sum_{k=1}^{2M_i}\tan^{-1}(2\eta_i t)\right)$$
$$\times\prod_{i=1}^{N_I}\prod_{k=1}^{2M_i}\left(1 + 4\eta_i^2 t^2\right)^{-1/4} dt$$
$$= \frac{1}{\pi}\int_{0}^{\infty}\cos\left(-tx + \frac{1}{2}\sum_{i=1}^{N_I}\sum_{k=1}^{2M_i}\tan^{-1}(2\eta_i t)\right)$$
$$\times\prod_{i=1}^{N_I}\left(1 + 4\eta_i^2 t^2\right)^{-M_i/2} dt. \quad (11)$$

Using (11), the cumulative distribution function (CDF) F_{C^i} in the interference-limited channel is obtained as (12) where $F_{\chi^2}(x; 2M_T)$ is the CDF of chi-square random variables with $2M_T$ degree of freedom.

3. On the Distribution of MISO Channel Capacity with Homogeneous Interferers

In [8], the impact of the homogeneity of the interferers distribution on the outage capacity of a SISO system is analyzed. In order to investigate more statistical characteristics of MISO channel capacity from (12), let us assume that each interferer is homogeneous. That is, each interferer has the same number of transmit antennas and amount of allocated power per each antenna.

From (2), the capacity with homogenous interferers in the interference-limited channel is represented as

$$C^h = \log_2\left(1 + \frac{E_s}{M_T}\frac{M_T}{E_I}\frac{\lambda}{\sum_{i=1}^{N_I}\lambda_i}\right)$$
$$= \log_2\left(1 + \beta\frac{\lambda}{\sum_{i=1}^{N_I}\lambda_i}\right) \quad (13)$$

where $\beta = E_s/E_I$ is the SIR.

Since λ_i is chi-square random variable with $2M_T$ degree of freedom, the sum of chi-square distributed random

$$F_{C^i}(R) = \int_0^\infty P\left[\log_2\left(1+\frac{E_s}{M_T}\frac{\lambda}{z}\right) \le R | z = \sum_{i=1}^{N_I}\eta_i\lambda_i\right] P[Z=z]\,dz$$
$$= \int_0^\infty P\left[\lambda \le \frac{M_T\cdot z}{E_s}\left(2^R-1\right)|z = \sum_{i=1}^{N_I}\eta_i\lambda_i\right] P[Z=z]\,dz$$
$$= \int_0^\infty F_{\chi^2}\left(\frac{M_T\cdot z}{E_s\sigma^2}\cdot\left(2^R-1\right); 2M_T\right) f_Z(z)\,dz. \quad (12)$$

variables $\sum_{j=1}^{N_I}\lambda_i$ is also chi-square distributed. However, the degree of freedom is changed into $2M_TN_I$ because N_I chi-square random variables are added. A random variable of the F-distribution [9] arises as the ratio of two chi-square distributed variables, that is,

$$s = \frac{\lambda/2M_T}{\sum_{i=1}^{N_I}\lambda_i/2M_TN_I} \sim F(2M_T, 2M_TN_I) \quad (14)$$

and the CDF is obtained as

$$F_F(x;m,n) = I\left(\frac{mx}{mx+n};\frac{m}{2},\frac{n}{2}\right) \quad (15)$$

where m and n are positive integers which represent the degree of freedom and $I(\cdot)$ is the regularized incomplete beta function.

The CDF F_{C^h} in the interference-limited channel is defined as

$$F_{C^h}(R) = P\left[C^h \le R\right]$$
$$= P\left[s \le \frac{N_I}{\beta}\left(2^R-1\right)\right]$$
$$= F_F\left(\frac{N_I}{\beta}\left(2^R-1\right); 2M_T, 2M_TN_I\right). \quad (16)$$

The PDF f_{C^h} in the interference-limited channel obtained as

$$f_{C^h}(R) = \frac{d}{dR}F_{C^h}(R)$$
$$= \frac{N_I}{\beta}\cdot 2^R\ln 2\cdot f_F\left(\frac{N_I}{\beta}\left(2^R-1\right); 2M_T, 2M_TN_I\right) \quad (17)$$

where $f_F(x;m,n)$ is the PDF of F-distribution with with numerator degrees of freedom m and denominator degrees of freedom n.

Using (17), ergodic capacity in the interference-limited channel obtained as

$$\bar{C}^h = \mathcal{E}\{C^i\}$$
$$= \int_0^\infty r f_R(r)dr$$
$$= \int_0^\infty \log_2\left(1+\frac{\beta}{N_I}s\right) f_F(s;2M_T,2M_TN_I)ds \quad (18)$$

where $f_R(r)$ is probability density function of the achievable capacity.

From (16), the p-outage capacity is

$$C^h(p) = \log_2\left(1+\frac{\beta}{N_I}F_F^{-1}(p\ |2M_T, 2M_TN_I)\right) \quad (19)$$

where $F_F^{-1}(p\ |2M_T, 2M_TN_I)$ is the inverse CDF of F-distribution with numerator degrees of freedom $2M_T$ and denominator degrees of freedom $2M_TN_I$ for the corresponding probabilities in p.

4. Simulation Results

To confirm the accuracy of the derived formula, a Monte-Carlo simulation is performed. Simulation results are compared with the probability density functions in the interference-limited channel obtained by (17). As shown in Fig. 1, the derived formula exactly coincides with the empirical simulation results.

Figure 2 shows the outage probability according to target capacity R in the interference-limited channel obtained by using (16). Multiple transmit antennas increase the outage capacity below 0.5 as compared to SISO case. Unlike the noise-limited channel, however, there exists a gap between the ergodic capacity and Shannon capacity even if the large number of transmit antenna is used. This reflects that

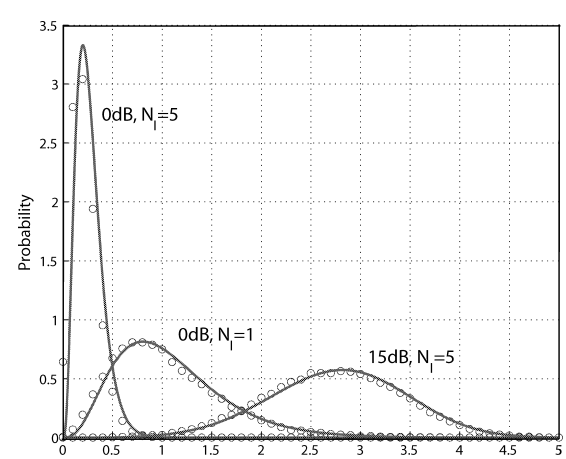

Fig. 1 Comparison between the empirical simulation (circle) and derived formula (line) results of 4×1 MISO system in the interference-limited channel.

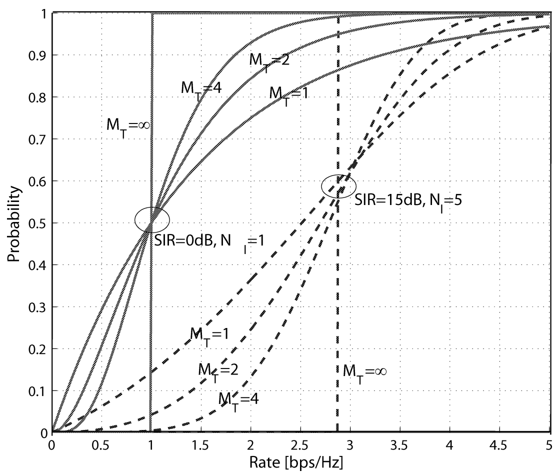

Fig. 2 Outage probability according to target capacity R in the interference-limited channel.

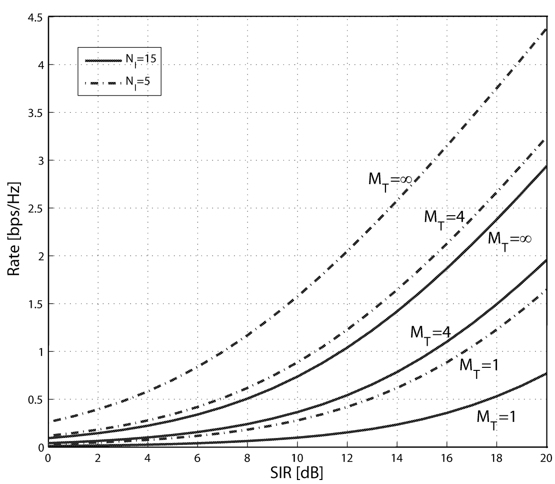

Fig. 3 10% outage capacity according to a given SIR and the number of interferers N_I in the interference-limited channel (solid line: $N_I = 15$, dot line: $N_I = 5$).

the performance of MISO system is limited by the interference.

Figure 3 shows the 10% outage capacity according to a given SNR and the number of interferers N_I in the interference-limited channel obtained by using (19). The 10% outage capacity increases with SIR and the number of transmit antennas, but the 10% outage capacity decreases by increasing the number of interferers. It is also found that the derived formula based on F-distribution clearly demonstrates the effect on capacity due to interference, which means the numerical results give a reasonable coverage of the analytical results.

5. Conclusion

In this paper, we develop a theoretical approach for deriving the ergodic capacity of MISO systems subject to co-channel interference. It is also found that the MISO channel capacity in the presence of homogenous interferers in the interference-limited channel follows the F-distribution with $2M_T$ numerator degrees of freedom and $2M_T N_I$ denominator degrees of freedom, where M_T is the number of transmit antennas and N_I is the number of interferers.

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