

Hash Tables

Victor Milenkovic

Department of Computer Science
University of Miami

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- ▶ Skip List (SkipMap): add, find, and remove in $O(\log n)$ expected time.



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- ▶ When you look up a name, it is nice to be able to go forward or back a few names in case you misspelled it.
- ▶ If you want Milenkovic in a hash table, you better not look for Milenkovich because it will be far away.



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- ▶ We will look at the one for String:

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public int hashCode()
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Returns a hash code for this string. The hash code for a String object is computed as

$$s[0]*31^{(n-1)} + s[1]*31^{(n-2)} + \dots + s[n-1]$$

using int arithmetic, where $s[i]$ is the i th character of the string, n is the length of the string, and $^$ indicates exponentiation. (The hash value of the empty string is zero.)

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WHAT???

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String name = "Milenkovic";  
for (int i = 0; i < name.length(); i++) {  
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An int can only hold integers in the range from -2147483648 to 2147483647.



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- ▶ Using powers of 31 makes these all different.



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```
int hashIndex (String name, int m) {  
    int code = name.hashCode();  
    int index = code % m;  
    if (index < 0)  
        index += m;  
    return index;  
}
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- ▶ But it is also -3 remainder 2 . Check $-3*7+2=-19$.



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- ▶ Open Addressing



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- ▶ Entries in the same list in the first table will be in different lists in the second table.



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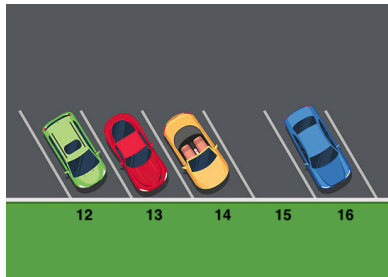
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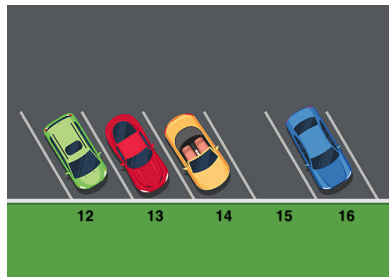
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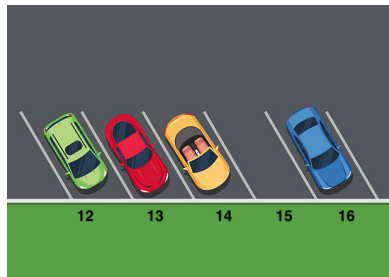


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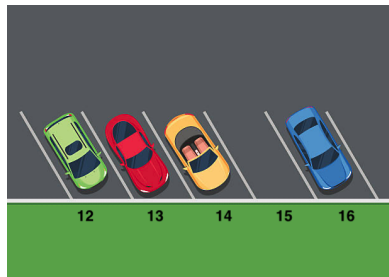
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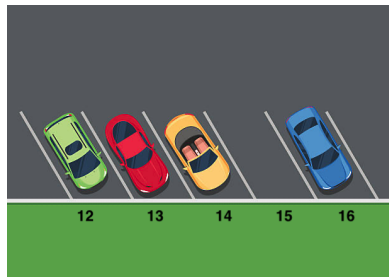
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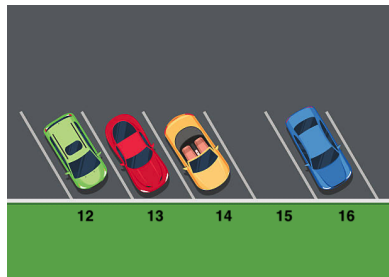
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Here is how to add.

- ▶ Suppose my assigned number is 12.
- ▶ But 12 is full.
- ▶ So are 13 and 14.

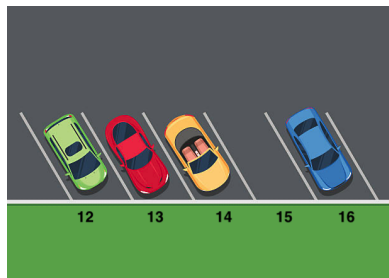
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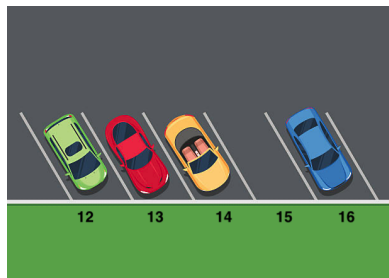
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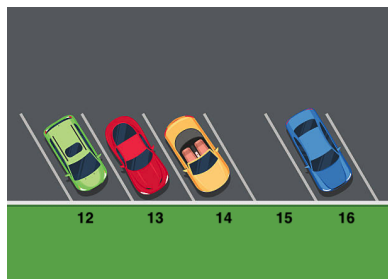


Find



Here is how to find Victor.

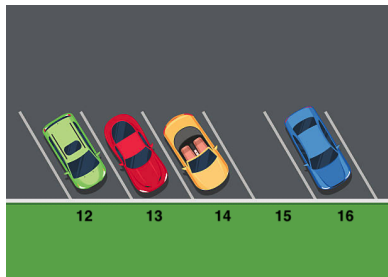
Find



Here is how to find Victor.

- ▶ You know my assigned number (hash index of Victor) is 12.

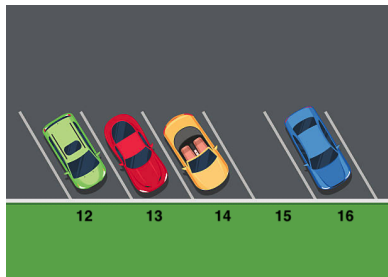
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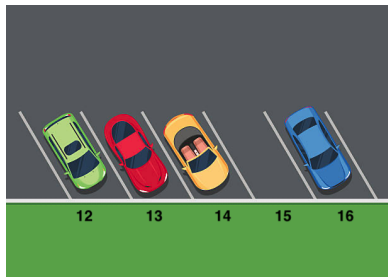
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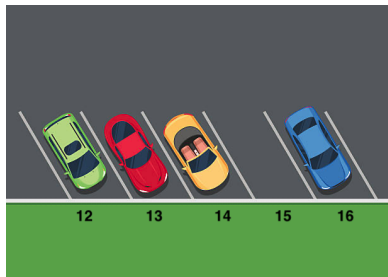
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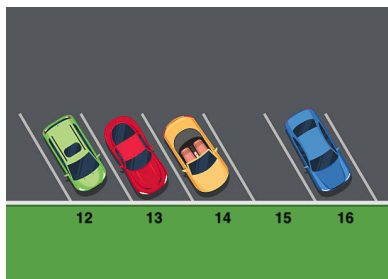
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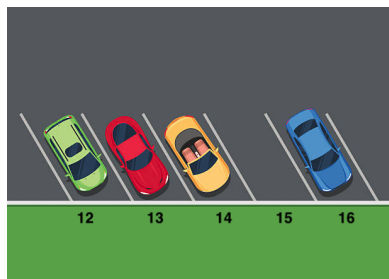
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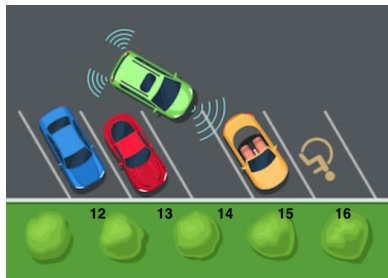


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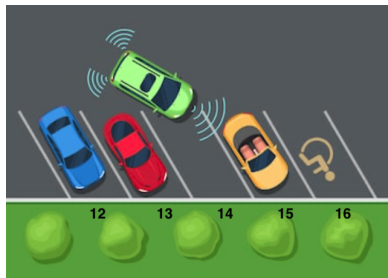
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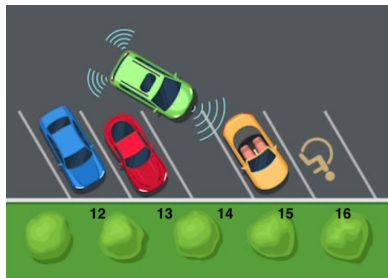


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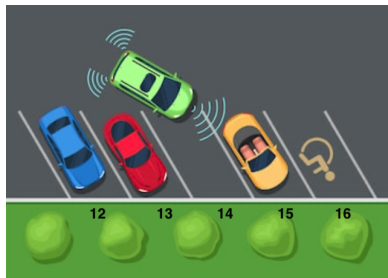
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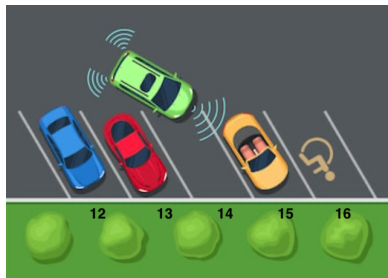
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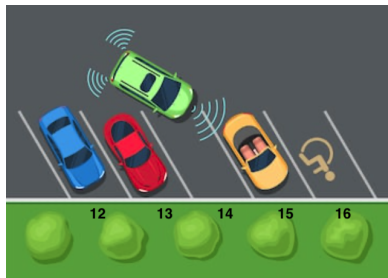
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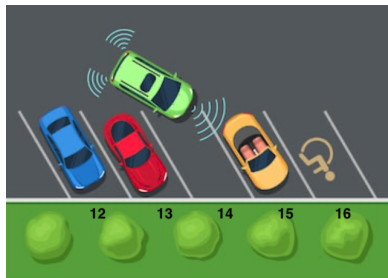
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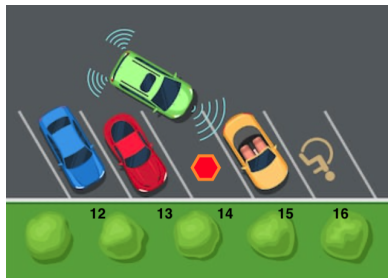
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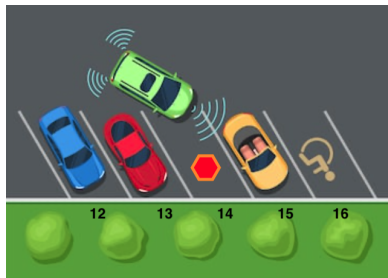
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- ▶ How do we fix this?

Marking DELETED spaces

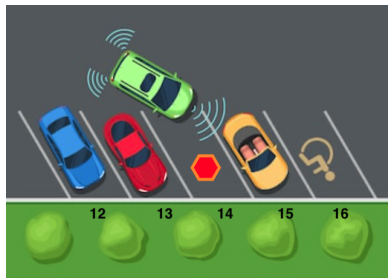


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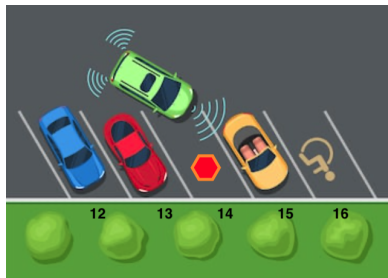
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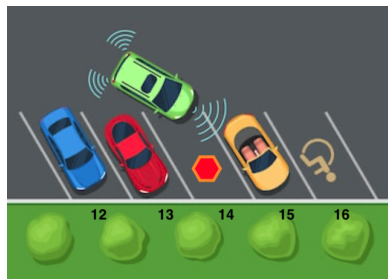
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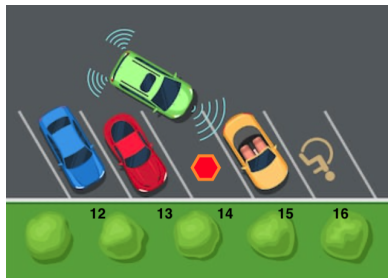
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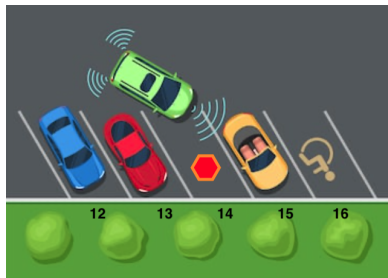
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- ▶ I park in the first space that is empty or has a traffic cone.

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- ▶ More suitable for real-time programming or operating systems.
- ▶ Item cycles through the array until it finds an empty spot.
- ▶ Removed items need to leave a “traffic cone”.
- ▶ Needs to rehash and possibly reallocate when less than $m/2$ spaces are empty.

