

1

In how many different ways can the letters of the word 'STATISTICS' be arranged

S T A T I S T I C S, 10 letters. 3 Ss, 3 Ts, 2 Is

$10! / 3! 3! 2!$

2

$P(A) = .3$, $P(B) = .4$, $P(A \cap B) = .18$

Mut Exclusive? is $(A \cap B) = 0$? aka $(A \cap B) = 1$?

NO

Is $1 - P(A) = P(B)$?

NO

Independent? Does $P(A \cap B) = P(A) * P(B)$?

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$, whether its independent or not

$P(A \cup B) = P(A \cap B) + P(B)$ (All of A not also covered by B, so add all of B to get OR)

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$.3 + .4 - (.18 + .4) = .12$

$.3 * .4 = .12$

YES!

3

5/10 first, then if that happened 4/9 next, then if that happened 3/8 thrid!

4

$1 - \text{pbinom}(39, 50, .7)$

5

You buy one \$5 raffle ticket for a new car valued at \$10,000.

One hundred tickets are sold. What is the expected value of your gain?

$-5 * 99/100 + 9995 * 1/100$

6

A baseball player has a batting average of 0.320.

This is the general probability that he gets a hit each time he is at bat.

What is the probability that he gets his first hit in the third trip to bat?

`pnbinom(2, 1, .32)`

`pgeom(2, .32)`

7

Telephone calls enter a college switchboard on the average of three every two minutes. What is the probability of 5 or more calls arriving in a 6-minute period? (Please type the numerical answer and the R-code)

`1-ppois(4, (1.5*6))`

3 calls in two minutes = 1.5 calls every minute * 6 minutes

8

Suppose that X is a random variable and follows the Chi- squared distribution with ten degrees of freedom. Find the probability that X is at least 13.

`1-pchisq(13, 10)`

9

One sample of size 35 students grades was taken from a hybrid class and another sample of size 40 taken from a standard lecture format class. Both classes were for the same subject.

The population mean course grade in percent for the hybrid class is 74 with the population standard deviation of 16.

The population mean grades from the standard lecture class is 76 percent with the population standard deviation of 9.

Find the probability that the sample mean grade in the standard lecture course will be greater by more than 1% than in the hybrid class.

`1-pnorm(1, 76-74, sqrt(9^2/40 + 16^2/35))`

10

An insurance company wants to audit health insurance claims in its very large database of transactions. In a quick attempt to assess the level of overstatement of this database, the insurance company selects at random 100 items from the database (each item represents a dollar amount). Suppose that the population mean overstatement of the entire database is \$8, with the population standard deviation \$20. Find the probability that the sample mean of the 100 would be less than \$6.50.

```
pnorm(6.5, 8, 20/sqrt(100))
```

#11

Construct a 96% Confidence Interval for the population mean based on the following data: 45, 55, 67, 45, 68, 79, 98, 87, 84, 82. Assume that the measurements are selected from a normal population.

```
x11 <- c(45, 55, 67, 45, 68, 79, 98, 87, 84, 82)
```

```
mean(x11)+qt(.98, 9)*sd(x11)/sqrt(10)
mean(x11)-qt(.98, 9)*sd(x11)/sqrt(10)
```

#12

Suppose a sample of 25 UM students are given an IQ test. If the sample has a standard deviation of 13.2 points, find a 95% confidence interval for the population standard deviation.

```
sqrt((24*(13.2)^2)/qchisq(.975, 24))
sqrt((24*(13.2)^2)/qchisq(.025, 24))
```

15

In the population of Americans who drink coffee, the average daily consumption is 3 cups per day. UM wants to know if our students tend to drink more coffee than the national average. They ask a random sample of 10 students how many cups of coffee they drink each day and got the following results: 0,3,3,4,2,3,4,6,3,6 . Do they have evidence that their students drink more than the national average? Find the p-value of the test statistics.

```
x15 <- c(0,3,3,4,2,3,4,6,3,6)
```

```
ts15 <- (mean(x15)-3)/(sd(x15)/sqrt(10))
```

```
pnorm(ts15) = 0.761789
```

Since the p-value is within 2.5% and 97.5%, it cannot be concluded that UM students drink more coffee than the national average.

16

```
qnorm(.985)
```

```
qnorm(.015)
```

17

Suppose that X is a continuous random variable with the probability density function $f(x)$ equals $(3/8)x^2$, $0 \leq x \leq 2$.

a) What is the probability that X is exactly 1?

0%, it's a continuous variable so the odds of exactly one specific number is 0.

b) Find the variance of X .

$E[X^2] - E[X]^2$

```
v1_17 <- integrate(function(x) (3/8)*x^4, 0, 2)
```

```
v2_17 <- integrate(function(x) x*(3/8)*x^2, 0, 2)
```

```
v1_17$value - (v2_17$value)^2
```

18

At a police station in a large city, calls come in at an average rate of three calls per minute. We can assume that the number of calls received during a time period follows a Poisson distribution.

a) Find the average time between two successive calls (in seconds).

3 calls every 60 seconds

=

1 call every 20 seconds, so the average is 20 seconds

b) Find the probability that after a call is received, the next call occurs in less than ten seconds.

```
1 - ppois(0, 1/2)
```

This is 'the opposite of' getting 0 calls in the next 10 seconds, which is 1/2 of

one call every 20 seconds

19

Suppose that X and Y are independent random variables with $E[X] = 3$, $E[X^2] = 25$, $E[Y] = 1$, $E[Y^2] = 37$. Find $\text{Var}[X-2Y]$.

$$\begin{aligned}\text{Var}[X-2Y] &= \\&= \text{Var}[X] + 4\text{Var}[Y] \\&= E[X^2] - E[X]^2 + 4(E[Y^2] - E[Y]^2) \\&= (25-9) + 4(37-1) = 160\end{aligned}$$

20

Consider body mass index (BMI) in a population of 60 year old males in whom BMI is normally distributed and has a mean value of 30 and a standard deviation of 5.

a) what is the 90th percentile for BMI in a population of 60 year old males?
`qnorm(.9, 30, 5)`

b) What is the probability that a randomly selected 60 year old man has BMI more than 33?
`1-pnorm(33, 30, 5)`

c) What is the probability that a randomly selected 60 year old man has BMI exactly 30?
0, because it is a continuous variable, so the odds of exactly a certain value is always 0%.

21

$n=8$ for all the parts of this problem
Select a random sample of size n from a normal population with the mean 20 and the standard deviation 2.

a) Use `rnorm(n,20,2)` to generate the observations and store them as vector `x`. Please list the elements in your solutions.

```
rnorm(8, 20, 2)
```

```
x21 <- c(19.77084, 16.54689, 18.63999, 19.02081, 14.76077, 17.27738, 23.18267, 17.39614)
```

b) Find the sample mean.

```
mean(x21) = 18.32444
```

c) Generate the boxplot. (Upload your plot as the file response to this question)

```
boxplot(x21)
```

d) Use the sample to construct a 90% Confidence Interval for the population mean. Identify the point estimate, critical value, and margin of error. Assume that the population standard deviation is known ($\sigma=2$).

```
point estimate = xbar = mean(x21) = 18.32444
```

```
critical value = qnorm(.95) = 1.644854
```

```
margin of error = (qnorm(.95)*2)/sqrt(8) = 1.163087
```

```
mean(x21) + 1.163087
```

```
mean(x21) - 1.163087
```

```
confidence interval = [17.16135, 19.48752]
```

e) Suppose that your research objective is to show that the population average is different than 19. State the null and the alternative hypotheses. What is the conclusion based on the Confidence Interval from part d) ?

```
H0 >> mu0 = 19
```

```
H1 >> mu1 != 19
```

Since 19 is in the confidence interval, we **fail to reject the null hypothesis**