Lossless T-Line Extension Model
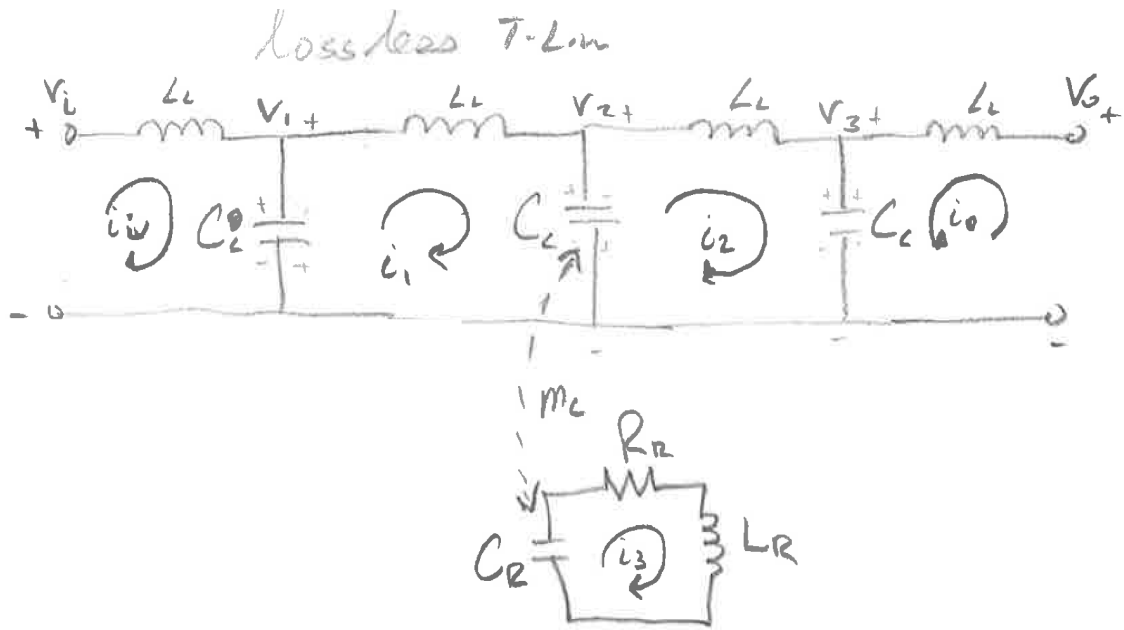
At port 1		At port 2	
Open-circuit input impedance	$z_{11} = \left.\dfrac{V_1}{I_1}\right|_{I_2=0}$	Open-circuit forward transimpedance	$z_{21} = \left.\dfrac{V_2}{I_1}\right|_{I_2=0}$
Open-circuit reverse transimpedance	$z_{12} = \left.\dfrac{V_1}{I_2}\right|_{I_1=0}$	Open-circuit output impedance	$z_{22} = \left.\dfrac{V_2}{I_2}\right|_{I_1=0}$

EqnMatrix =

$$\left\{\left\{\left(\mathbb{i}\,\omega\,\frac{LL}{2} + \frac{1}{\mathbb{i}\,\omega\,CL}\right), \frac{-1}{\mathbb{i}\,\omega\,CL}, 0, 0, 0\right\}, \left\{\frac{-1}{\mathbb{i}\,\omega\,CL}, \left(\frac{2}{\mathbb{i}\,\omega\,CL} + \mathbb{i}\,\omega\,LL\right), \frac{-1}{\mathbb{i}\,\omega\,CL}, 0, \mathbb{i}\,\omega\,Mc\right\},\right.$$

$$\left\{0, \frac{-1}{\mathbb{i}\,\omega\,CL}, \left(\frac{2}{\mathbb{i}\,\omega\,CL} + \mathbb{i}\,\omega\,LL\right), \frac{1}{\mathbb{i}\,\omega\,CL}, \mathbb{i}\,\omega\,Mc - \mathbb{i}\,\omega\,ML\right\}, \left\{0, 0, \frac{1}{\mathbb{i}\,\omega\,CL}, \left(\mathbb{i}\,\omega\,\frac{LL}{2} + \frac{1}{\mathbb{i}\,\omega\,CL}\right), 0\right\},$$

$$\left.\left\{0, \mathbb{i}\,\omega\,Mc, -\left(\mathbb{i}\,\omega\,Mc - \mathbb{i}\,\omega\,ML\right), 0, \left(RR + \mathbb{i}\,\omega\,LR + \frac{1}{\mathbb{i}\,\omega\,CR}\right)\right\}\right\};$$

VVector = {vi, 0, 0, vo, 0};

Ans = FullSimplify[

  LinearSolve[EqnMatrix, VVector] /. $\left\{Mc \to kc\,(CR\,CL)^{1/2}, ML \to kL\,(LR\,LL)^{1/2}\right\}$];

FullSimplify$\left[\text{FullSimplify}\left[\dfrac{\frac{i3}{\mathbb{i}\,\omega\,CL} + i4\left(\mathbb{i}\,\omega\,\frac{LL}{2} + \frac{1}{\mathbb{i}\,\omega\,CL}\right)}{i1}\right.\right.$ /.

   {i1 → Ans[[2]], i2 → 0, i3 → Ans[[3]], i4 → Ans[[4]]} /. {vo → 1, vi → 1}$\Big]\Big]$

$\left(-8\,\mathbb{i}\,CR\,\sqrt{CL\,CR}\,kc\,kL\,\sqrt{LL\,LR}\,\omega^2 - \mathbb{i}\,CL^3\,LL^4\,\omega^6\left(-1 + CR\,\omega\left(-\mathbb{i}\,RR + \left(1 + kL^2\right)LR\,\omega\right)\right) + \right.$

$\quad 4\,LL\left(-3\,\mathbb{i} + CR\,\omega\left(3\,RR + \mathbb{i}\left(3 + kL^2\right)LR\,\omega + 5\,\mathbb{i}\,CL\,\sqrt{CL\,CR}\,kc\,kL\,\sqrt{LL\,LR}\,\omega^3\right)\right) -$

$\quad \mathbb{i}\,CL\,LL^2\,\omega^2\left(-19 + CR\,\omega\left(-19\,\mathbb{i}\,RR + \left(19 + 10\,kL^2\right)LR\,\omega + 12\,CL\,\sqrt{CL\,CR}\,kc\,kL\,\sqrt{LL\,LR}\,\omega^3\right)\right) +$

$\quad \left.2\,CL^2\,LL^3\,\omega^4\left(-4\,\mathbb{i} + CR\,\omega\left(4\,RR + \mathbb{i}\left(\left(4 + 3\,kL^2\right)LR\,\omega + CL\,\sqrt{CL\,CR}\,kc\,kL\,\sqrt{LL\,LR}\,\omega^3\right)\right)\right)\right)\Big/$

$\left(2\,CL\,\omega\left(2\,CR\,kc\left(-2\,CL\,CR\,kc + 3\,\sqrt{CL\,CR}\,kL\,\sqrt{LL\,LR}\right)\omega^2 + \right.\right.$

```mathematica
          CL LL² ω² (-1 + CR ω (-i RR + (1 + kL²) LR ω)) +
          LL (3 + CR ω (3 i RR - (3 + 2 kL²) LR ω + CL kc (2 CL CR kc - 3 √(CL CR) kL √(LL LR)) ω³)))))
Z21a[CL_, LL_, RR_, LR_, CR_, kc_, kL_, ω_] :=
    (-8 i CR √(CL CR) kc kL √(LL LR) ω² - i CL³ LL⁴ ω⁶ (-1 + CR ω (-i RR + (1 + kL²) LR ω)) +
       4 LL (-3 i + CR ω (3 RR + i (3 + kL²) LR ω + 5 i CL √(CL CR) kc kL √(LL LR) ω³)) -
       i CL LL² ω² (-19 + CR ω (-19 i RR + (19 + 10 kL²) LR ω + 12 CL √(CL CR) kc kL √(LL LR) ω³)) +
       2 CL² LL³ ω⁴ (-4 i + CR ω (4 RR + i ((4 + 3 kL²) LR ω + CL √(CL CR) kc kL √(LL LR) ω³)))) /
     (2 CL ω (2 CR kc (-2 CL CR kc + 3 √(CL CR) kL √(LL LR)) ω² +
          CL LL² ω² (-1 + CR ω (-i RR + (1 + kL²) LR ω)) +
          LL (3 + CR ω (3 i RR - (3 + 2 kL²) LR ω + CL kc (2 CL CR kc - 3 √(CL CR) kL √(LL LR)) ω³))))
χ[T1_, T2_, ωs0_, ω_, γ_] := (     T1 (ωs0 - ω)                    +  i      T1                    )
                               ─────────────────────────────       ──────────────────────────────
                               1 + T2² (ωs0 - ω)² + γ² 1 T1 T2       1 + T2² (ωs0 - ω)² + γ² 1 T1 T2

gL = .000025 ;
gR = .007 ;
kkc = 0.01 × 0;
kkL = 0.005 × 0;
Rv = .001;
T1 = 4 × 10⁻⁹;
T2 = .1 × 10⁻⁹;
γ = 2.8 ;
Show[Plot[Im[
    χ[T1, T2, 1.4 × 250 × 10⁹, f, γ]
   ], {f, 100 × 10⁹, 750 × 10⁹}, PlotRange → All, AspectRatio → 1/4, PlotStyle → Automatic]
]
LR[T1_, T2_, ωs0_, ω_, γ_, Dr_, dr_, gr_] :=
   4 π 10⁻⁷ (1 + gr χ[T1, T2, ωs0, ω, γ]) Dr/2 (Log[8 Dr/dr] - 2)

lineStyle = {Black, Dashed};
line1 = Line[{{398 × 10⁹, -6 × 10¹⁹}, {398 × 10⁹, 6 × 10⁹}}];
line2 = Line[{{440 × 10⁹, -6 × 10¹⁹}, {440 × 10⁹, 6 × 10⁹}}];
```

Coupled Only

```mathematica
gL =  0 × 1000 × 10⁻⁸ ;
gR =  4700 × 10⁻⁸ ;
kkc = 0.255 ;
kkL = 0.065 ;
Rv = .0008;
output = Table[2 Re[
      1 - (Z21a[8.854 × 10⁻¹², 4 π 10⁻⁷ (1 + gL χ[T1, T2, 300 × 10⁹ i, f, γ]), Rv, LR[T1, T2,
              300 × 10⁹ i, f, γ, 10 × 10⁻⁹, 0.095 × 10⁻⁹, gR], 1.93 × 10⁻¹⁰, kkc, kkL, f] /
           Z21a[8.854 × 10⁻¹², 4 π 10⁻⁷ (1 + gL χ[T1, T2, 295 × 10⁹ i, f, γ]), Rv, LR[T1,
              T2, 295 × 10⁹ i, f, γ, 10 × 10⁻⁹, 0.095 × 10⁻⁹, gR], 1.93 × 10⁻¹⁰, kkc, kkL, f])
      ], {f, 255 × 10⁹, 700 × 10⁹, .5 × 10⁹}, {i, 1.7, 1.2, -.025}];
Export["C:/Users/sidabras/Desktop/CoupledOnly.CSV", output, "CSV"]
Clear[output]
```

With Transmission

```mathematica
gL = 1000 × 10⁻⁸;
 gR = 4000 × 10⁻⁸ ;
```

```
kkc = 0.255;
kkL = 0.065;
Rv = .0008;
output = Table[ 2 Re[
      1 - (Z21a[8.854 × 10⁻¹², 4 π 10⁻⁷ (1 + gL χ[T1, T2, 300 × 10⁹ i, f, γ]), Rv, LR[T1, T2,
           300 × 10⁹ i, f, γ, 10 × 10⁻⁹, 0.095 × 10⁻⁹, gR],  1.93 × 10⁻¹⁰, kkc, kkL, f] /
         Z21a[8.854 × 10⁻¹², 4 π 10⁻⁷ (1 + gL χ[T1, T2, 295 × 10⁹ i, f, γ]), Rv,  LR[T1,
             T2, 295 × 10⁹ i, f, γ, 10 × 10⁻⁹, 0.095 × 10⁻⁹, gR],  1.93 × 10⁻¹⁰, kkc, kkL, f])
      ], {f, 255 × 10⁹, 700 × 10⁹, 0.5 × 10⁹}, {i, 1.7, 1.2, -.025}];
Export["~/Desktop/WithTransmission.CSV", output, "CSV"]
Clear[output]
```

Only Transmission

```
gL = 1000 × 10⁻⁸ ;
gR = 4000 × 10⁻⁸;
kkc = 0.255 × 0;
kkL = 0.065 × 0;
Rv = .0008;
output = Table[2 Re[
      1 - (Z21a[8.854 × 10⁻¹², 4 π 10⁻⁷ (1 + gL χ[T1, T2, 300 × 10⁹ i, f, γ]), Rv,  LR[T1, T2,
           300 × 10⁹ i, f, γ, 10 × 10⁻⁹, 0.095 × 10⁻⁹, gR],  1.93 × 10⁻¹⁰, kkc, kkL, f] /
         Z21a[8.854 × 10⁻¹², 4 π 10⁻⁷ (1 + gL χ[T1, T2, 295 × 10⁹ i, f, γ]), Rv,  LR[T1,
             T2, 295 × 10⁹ i, f, γ, 10 × 10⁻⁹, 0.095 × 10⁻⁹, gR],  1.93 × 10⁻¹⁰, kkc, kkL, f])
      ], {f, 255 × 10⁹, 700 × 10⁹, 0.5 × 10⁹}, {i, 1.7, 1.2, -.025}]; *)
Export["~/Desktop/OnlyTransmission.CSV", output, "CSV"]
Clear[output]
```

**Strong and Weak Coupling Studies**

```
gL =  0 × 1000 × 10⁻⁸ ;
gR =  100 × 4700 × 10⁻⁸ ;
kkc = 0.255 ;
kkL = 0.065 ;
Rv = .0008 / 5;
Show[Plot[Re[
      Z21a[8.854 × 10⁻¹², 4 π 10⁻⁷ (1 + gL χ[T1, T2, 300 × 10⁹ #, f, γ]), Rv,
        LR[T1, T2, 300 × 10⁹ #, f, γ, 10 × 10⁻⁹, 0.095 × 10⁻⁹, gR],  1.93 × 10⁻¹⁰, kkc, kkL, f]
      ], {f, 255 × 10⁹, 700 × 10⁹}, PlotRange → All, PlotPoints → 500, MaxRecursion → 0,
     AspectRatio → 1 / 4, PlotStyle → Red] & /@ {1.7`, 1.45`, 1.2`, 1}, PlotRange → All]
tab1 = Table[Re[
      Z21a[8.854 × 10⁻¹², 4 π 10⁻⁷ (1 + gL χ[T1, T2, 300 × 10⁹ #, f, γ]), Rv,
        LR[T1, T2, 300 × 10⁹ #, f, γ, 10 × 10⁻⁹, 0.095 × 10⁻⁹, gR],  1.93 × 10⁻¹⁰, kkc, kkL, f]
      ], {f, 255 × 10⁹, 700 × 10⁹, .1 × 10⁹}] & /@ Table[i, {i, 0.9, 1.9, .05}];
freq1 = {};
freq2 = {};
freq3 = {};
posout = {};
For[i = 1, i ≤ Length[tab1], i++,
 tlist = Table[i, {i, 0.9, 1.9, .05}];
 pos1 = FindPeaks[-tab1[[i]]];
 freq = Table[f, {f, 255 × 10⁹, 700 × 10⁹, .1 × 10⁹}][[pos1[[1, 1]]]];
 AppendTo[freq1, {tlist[[i]], freq}];
 If[Length[pos1] == 2,
   freq = Table[f, {f, 255 × 10⁹, 700 × 10⁹, .1 × 10⁹}][[pos1[[2, 1]]]];
```

```
     AppendTo[freq2, {tlist[[i]], freq}];
    ];
   If[Length[pos1] == 3,
    freq = Table[f, {f, 255 × 10^9, 700 × 10^9, .1 × 10^9}][[pos1[[2, 1]]]];
    AppendTo[freq2, {tlist[[i]], freq}];
    freq = Table[f, {f, 255 × 10^9, 700 × 10^9, .1 × 10^9}][[pos1[[3, 1]]]];
    AppendTo[freq3, {tlist[[i]], freq}];
   ];
   If[Length[pos1] == 4,
    Print["you missed!"];
   ];
  ]
Show[ListPlot[freq1],
 ListPlot[freq2, PlotStyle → Red],
 ListPlot[freq3, PlotStyle → Green], PlotRange → All]
```
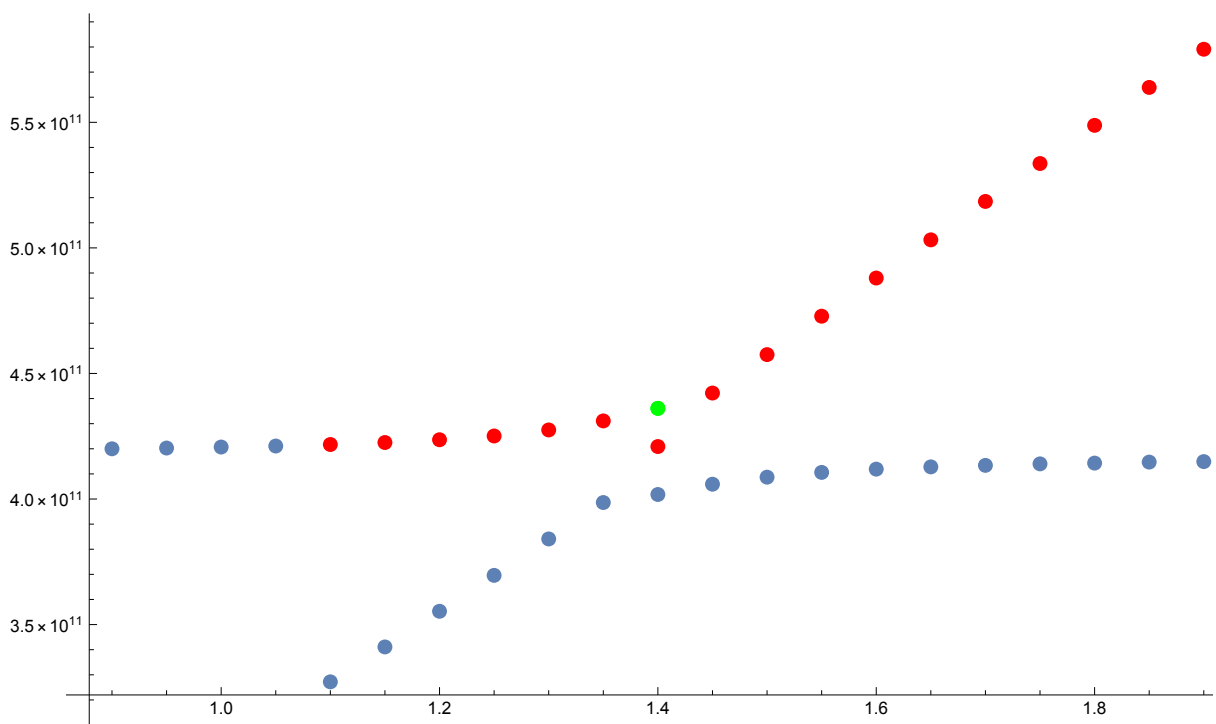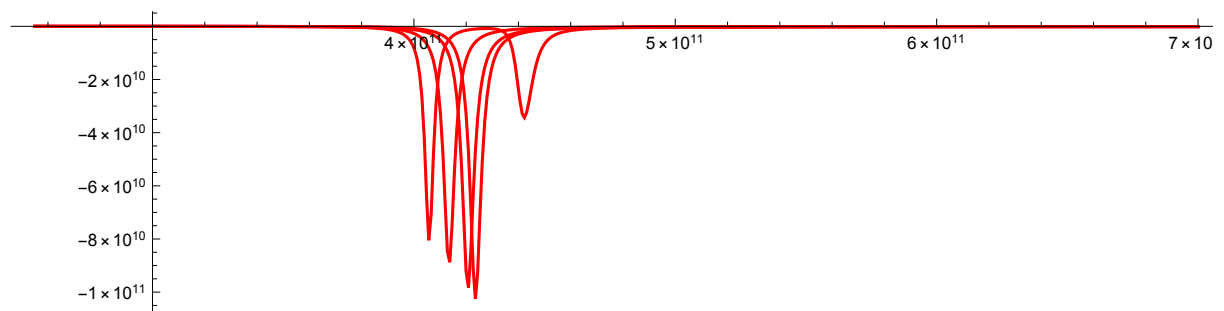


```
gL = 1 × 10^-8;
gR = 4.7 × 10^-8;
kkc = 0.255;
```

```mathematica
kkL = 0.065 ;
Rv = .0008;
Show[Plot[Re[
      Z21a[8.854 × 10⁻¹², 4 π 10⁻⁷ (1 + gL χ[T1, T2, 300 × 10⁹ #, f, γ]), Rv,
       LR[T1, T2, 300 × 10⁹ #, f, γ, 10 × 10⁻⁹, 0.095 × 10⁻⁹, gR],  1.93 × 10⁻¹⁰, kkc, kkL, f]
     ], {f, 255 × 10⁹, 700 × 10⁹}, PlotRange → All, PlotPoints → 500, MaxRecursion → 0,
    AspectRatio → 1 / 4, PlotStyle → Red] & /@ {1.7`, 1.45`, 1.2`, 1}, PlotRange → All]
tab1 = Table[Re[
      Z21a[8.854 × 10⁻¹², 4 π 10⁻⁷ (1 + gL χ[T1, T2, 300 × 10⁹ #, f, γ]), Rv,
       LR[T1, T2, 300 × 10⁹ #, f, γ, 10 × 10⁻⁹, 0.095 × 10⁻⁹, gR],  1.93 × 10⁻¹⁰, kkc, kkL, f]
     ], {f, 255 × 10⁹, 700 × 10⁹, .1 × 10⁹}] & /@ Table[i, {i, 0.9, 1.9, .005}];
freq5 = {};
freq6 = {};
For[i = 1, i ≤ Length[tab1], i++,
 tlist = Table[i, {i, 0.9, 1.9, .005}];
 pos1 = FindPeaks[-tab1[[i]]];
 freq = Table[f, {f, 255 × 10⁹, 700 × 10⁹, .1 × 10⁹}][[pos1[[1, 1]]]];
 AppendTo[freq5, {tlist[[i]], freq}];
 If[Length[pos1] == 2,
  freq = Table[f, {f, 255 × 10⁹, 700 × 10⁹, .1 × 10⁹}][[pos1[[2, 1]]]];
  AppendTo[freq6, {tlist[[i]], freq}];
 ];
]
Show[ListPlot[freq1],
 ListPlot[freq2, PlotStyle → Red],
 ListPlot[freq3, PlotStyle → Red],
 ListLinePlot[freq5, PlotStyle → Green], PlotRange → All]
```