Lab 0

Evan, Jane, Manu

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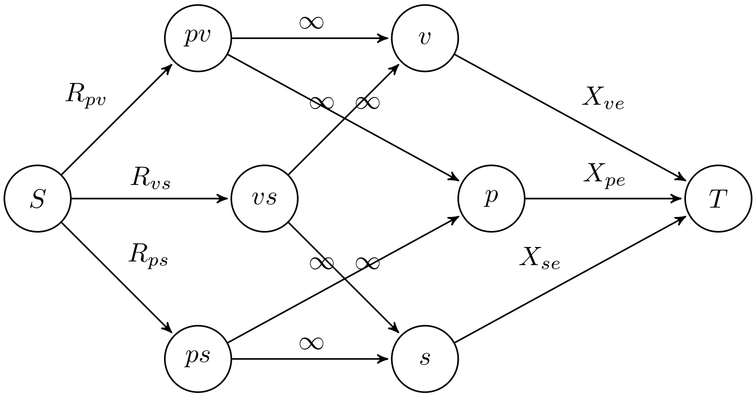
1 Introduction

1.1 Part 1 - Setting up the problem

1. If the sum of Team A's current wins and remaining games is less than or equal to the win count of the team with the most wins currently, then Team A is eliminated from receiving the Dunkins Donuts gift card.
   Following this logic, teams Vicky and Prava should be eliminated. Vicky could not possibly win more than 77+3 = 80 games, so she will be beat by Emily no matter what. Prava cannot win more than 80+3 = 83 games, and since Emily already has that many wins, Prava can no longer get the most wins. According to her defeatist attitude, she should quit now.



2. (a)

   This graph shows the capacities of the flow on each edge.

   R_ij: Games remaining between team i and team j

   X_ij: w_j + r_j - w_i, where team j is the team who is the focus of the graph. The number of games team i would need to win to catch up to team j (the focus team).

   The edges connecting a node ij to a node i represent the number of games played between team i and team j and won by i. The capacity need not be defined because the capacity is only limited to the incoming flow, Rij.

(b) In this specific problem, to create the graph specific to Emily's team, the capacities would be as follows:

$R_{pv} = 2$
$R_{vs} = 0$
$R_{ps} = 0$
$X_{ve} = 14$
$X_{pe} = 11$
$X_{se} = 13$

(c) To solve the problem, build a flow network using the method explained in Question 2 for each team in the system. For each team, if the maximal flow of that team's network is less than the capacity of the edges on the source node (the number of remaining games between the other teams), then the team in question should be eliminated.

(d) As for why this makes sense, consider what each set of edges represents:

- The initial edges from the source node are the remaining games that each team has to play, *excluding those against the team under consideration*.
- The second set of edges, between the matches and the teams, represent the hypothetical results of the matches (but have the same or greater capacity as the first set of edges).
- The capacities of the final set of edges, connected to the source node, represent the number of wins necessary to beat the team under consideration, that the team under consideration cannot control (for matches that the team under consideration is a part of, assume they will win - we give them the best chance to win in this analysis so we know if they are definitely eliminated.).

With these representations, the condition described in part 2c) can be rephrased as "If any other team has more remaining games than number necessary to beat the considered team, then the considered team should be eliminated."

3. (a) The following expressions represent the flow as a Linear Programming problem:

(f_{ij} represents the flow on the edge connecting node i to node j.)

Maximize $f_{vT} + f_{pT} + f_{sT}$

Subject to:

$f_{Spv} \leq R_{pv}$
$f_{Svs} \leq R_{vs}$
$f_{Sps} \leq R_{ps}$
$f_{pvv}, f_{pvp}, f_{vsv}, f_{vss}, f_{psp}, f_{pss} \leq \infty$

$f_{vT} \leq X_{ve}$
$f_{pT} \leq X_{pe}$
$f_{sT} \leq X_{se}$
$f_{Spv} = f_{pvv} + f_{pvp}$
$f_{Svs} = f_{vsv} + f_{vss}$
$f_{Sps} = f_{psp} + f_{pss}$
$f_{pvv} + f_{vsv} = X_{ve}$
$f_{pvp} + f_{psp} = X_{pe}$
$f_{vss} + f_{pss} = X_{se}$
$f_{Spv}, f_{Svs}, f_{Sps}, f_{pvv}, f_{pvp}, f_{vsv}, f_{vss}, f_{psp}, f_{pss}, f_{vT}, f_{pT}, f_{sT} \geq 0$

(b) This formulation tries to maximize the wins of the other teams. When we compare the other teams potential to Emily's max potential, we can determine if Emily is eliminated yet.

1.2 Part 2 - Implementation

We think the test cases are sufficient - they check all expected results of the various example divisions. One issue we ran into is that we weren't resetting our graphs in between evaluations - so the internal graph was eventually an amalgamation of all of the previous team graphs, which unsurprisingly led to some incorrect results. A test case that ran each team calculation with a fresh division object would have caught that explicitly, but since the current tests still failed we don't think it is necessary.