PID Tuning Example

(dated meme edition)

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Lesson Objective

- Quick example of analytically tuning PID controller for a more complex system.

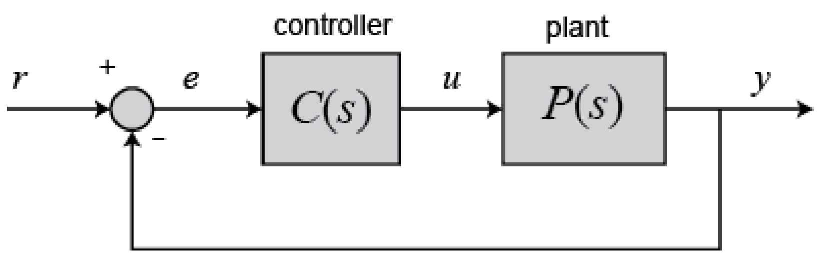
The Question

Consider a model of the dynamic response of a high capacity material handling system in which the the desired position is $r(t)$=1.0 m and the output signal $y(t)$ is the actual, measured position (m).

$$P(s) = \frac{2}{6s^3 + 11s^2 + 6s + 1}$$

Design a controller where $e_{ss}=0$. There should be **no overshoot** and the **response should be as rapid as possible** without violating the first two requirements. Note, the trial and error process for designing controllers discussed in class is unlikely to guarantee a provably optimal solution, but do briefly explain how you arrived at your answer.

Summary of differences



$$K_p + \frac{K_i}{s} + K_d s = \frac{K_d s^2 + K_p s + K_i}{s}$$

PID Gain	Rise time	Overshoot	Settling time	Steady-state error
Kp	Decrease	Increase	Small Change	Decrease
Ki	Decrease	Increase	Increase	Eliminate
Kd	Small Change	Decrease	Decrease	No Change

(**small print:** correlations may not always be accurate, your mileage may vary, non-refundable in Michigan, highly coupled interactions may be observed, prohibited from asserting in Oklahoma, changing one value can cause the effects of the other variables to change and/or exhibit erratic behavior, experimentation may lead to increased blood pressure, sleep deprivation, and general sense of internal rage.)

Option 1

$$P(s) = \frac{2}{6s^3 + 11s^2 + 6s + 1}$$

```
>>pidtuner
```

Tune by hand

$$P(s) = \frac{2}{6s^3 + 11s^2 + 6s + 1}$$

Start with our PID Controller…

$$\mathrm{u}(s) = \frac{k_d\left(s^2 + \frac{k_p}{k_d}s + \frac{k_i}{k_d}\right)}{s} = \frac{k_d\left(s^2 + as + b\right)}{s}$$

Tune by hand

$$P(s) = \frac{2}{6s^3 + 11s^2 + 6s + 1}$$

Plug in…

$$T_{cl}(s) = \frac{C(s)P(s)}{1 + C(s)P(s)}$$

$$= \frac{\left[\frac{k_d\left(s^2 + as + b\right)}{s}\right]\left[\frac{2}{6s^3 + 11s^2 + 6s + 1}\right]}{1 + \left[\frac{k_d\left(s^2 + as + b\right)}{s}\right]\left[\frac{2}{6s^3 + 11s^2 + 6s + 1}\right]}$$

$$\frac{2k_d\left(s^2 + as + b\right)}{6s^4 + 11s^3 + 6s^2 + s + 2k_d\left(s^2 + as + b\right)}$$

$$= \frac{2k_d s^2 + 2k_d as + 2k_d b}{6s^4 + 11s^3 + \left(6 + 2k_d\right)s^2 + \left(1 + 2k_d a\right)s + \left(2k_d b\right)}$$

Tune by hand

$$P(s) = \frac{2}{6s^3 + 11s^2 + 6s + 1}$$

Plug in…

$$T_{cl}(s) = \frac{2k_d s^2 + 2k_d a s + 2k_d b}{6s^4 + 11s^3 + \left(6 + 2k_d\right)s^2 + \left(1 + 2k_d a\right)s + \left(2k_d b\right)}$$

Now what? Let’s set a, b and k_D all to 1…cuz why not?

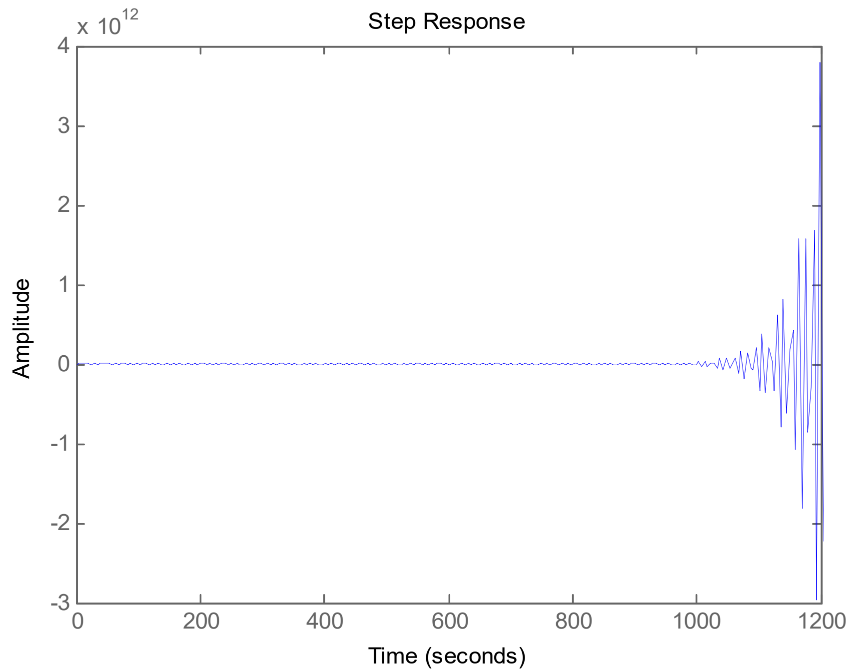
```
k = 1; a = 1; b = 1;
num = [ 2*k 2*k*a 2*k*b ]
den = [ 6 11 6+2*k 1+2*k*a 2*k*b ]
sys = tf( num, den );
step( sys );
```

Tune by hand

$$P(s) = \frac{2}{6s^3 + 11s^2 + 6s + 1}$$

What did de we get?

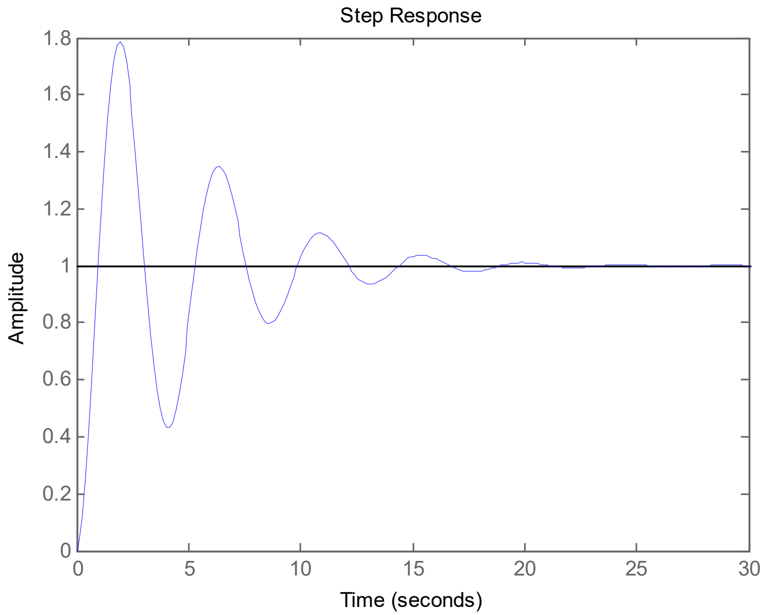
```
k = 1; a = 1; b = 1;
num = [ 2*k 2*k*a 2*k*b ]
den = [ 6 11 6+2*k 1+2*k*a 2*k*b ]
sys = tf( num, den );
step( sys );
```



Let's up the gain to 10

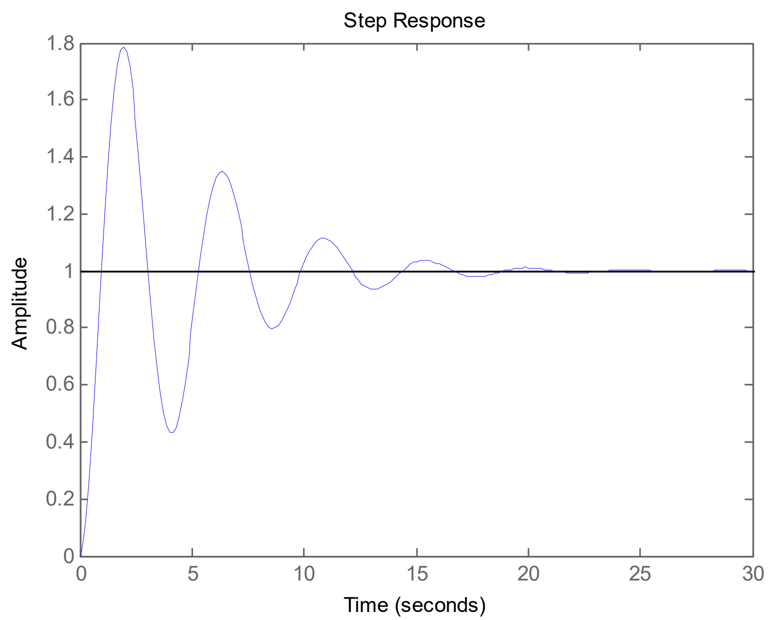
$$P(s) = \frac{2}{6s^3 + 11s^2 + 6s + 1}$$

```
k = 10; a = 1; b = 1;
num = [ 2*k 2*k*a 2*k*b ]
den = [ 6 11 6+2*k 1+2*k*a 2*k*b ]
sys = tf( num, den );
step( sys );
```



Hmmm.

$$P(s) = \frac{2}{6s^3 + 11s^2 + 6s + 1}$$



We have overshoot, but no steady state error, so this seems like a good place to focus.

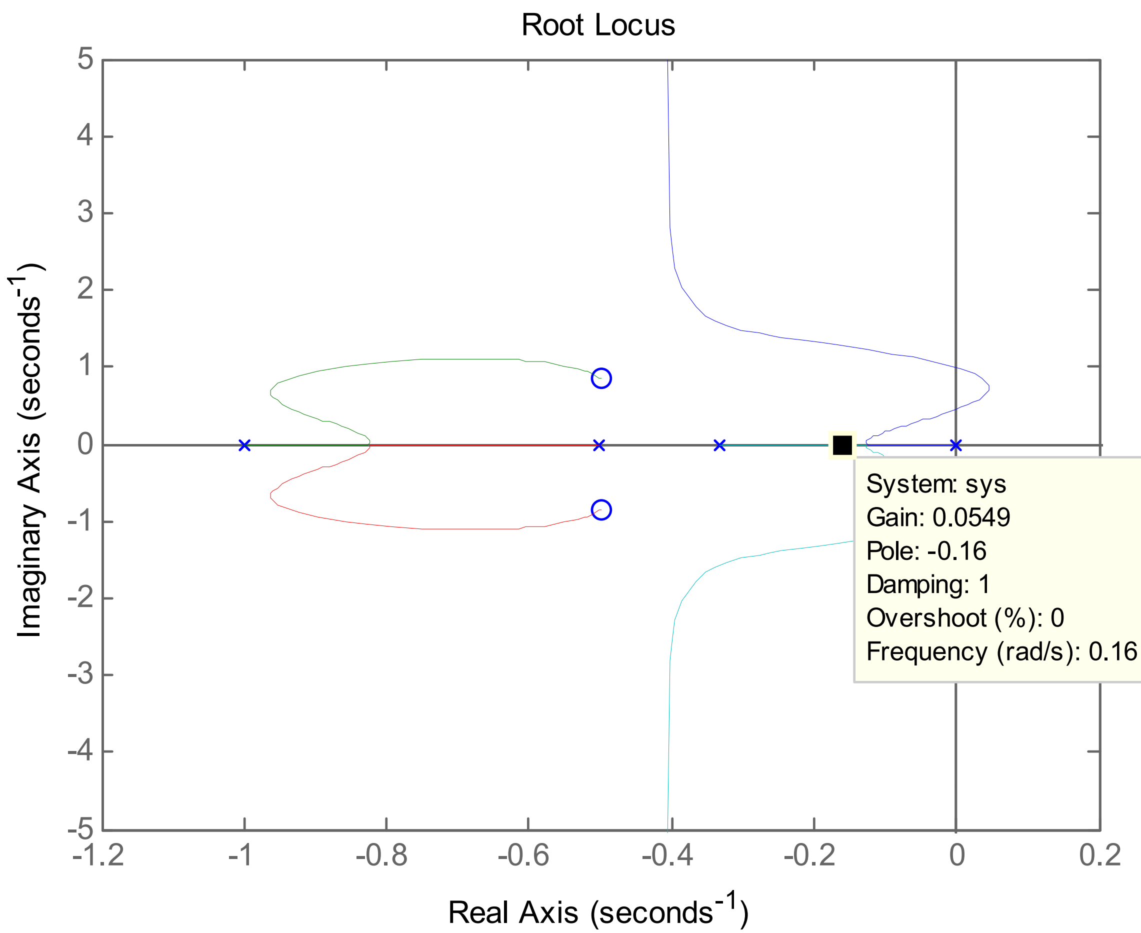
But I can't get rid of the overshoot no matter what k_D I pick.

Let's check the Root Locus

$$P(s)=\frac{2}{6s^3+11s^2+6s+1}$$

$$\begin{aligned}CE &= 1+k_d C'(s)P(s)\\ &= 1+k_d\left[\frac{\left(s^2+as+b\right)}{s}\right]\left[\frac{2}{6s^3+11s^2+6s+1}\right]\\ &= 1+2k_d\frac{\left(s^2+as+b\right)}{6s^4+11s^3+6s^2+s}\\ &= 1+k_d\frac{\left(2s^2+2as+2b\right)}{6s^4+11s^3+6s^2+s}\end{aligned}$$

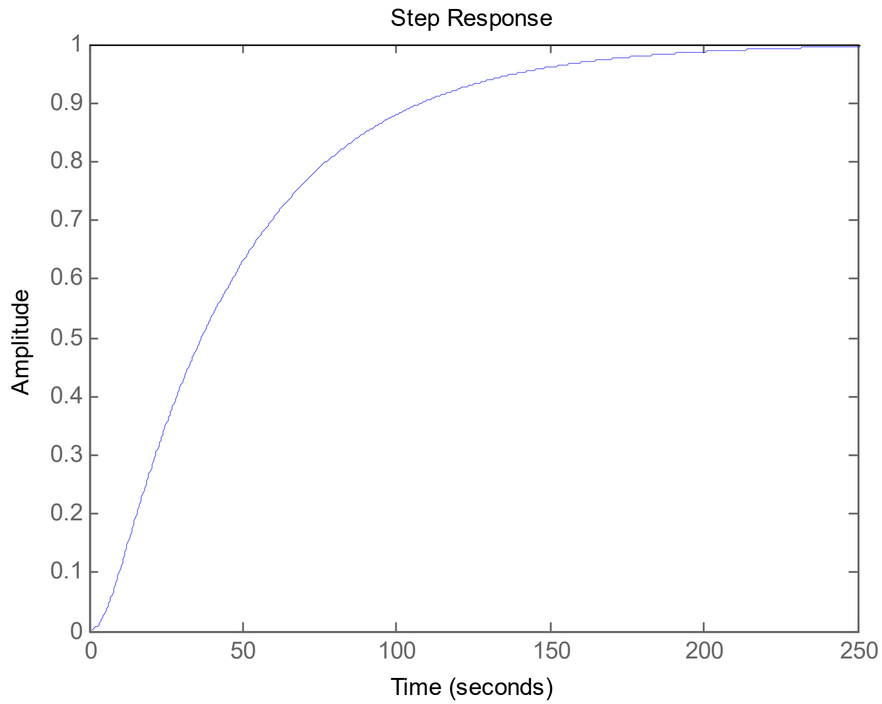
```
clear all
a = 1; b = 1;
num = [ 2 2*a 2*b ];
den = [ 6 11 6 1 ];
sys = tf( num, den );
rlocus( sys );
```

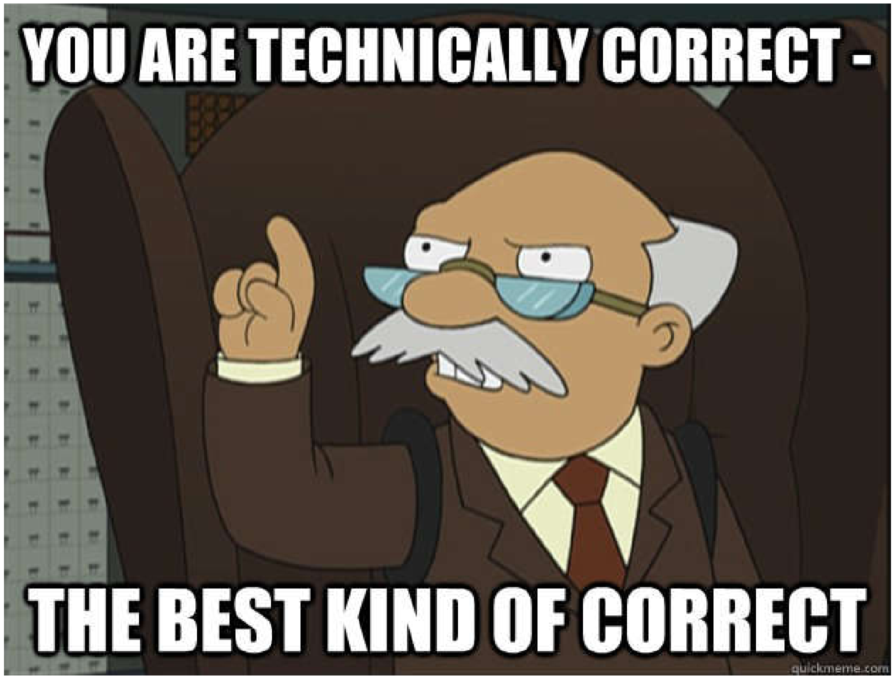


Cool. Let's use low gains!

$$P(s) = \frac{2}{6s^3 + 11s^2 + 6s + 1}$$

```
k = 0.01; a = 1; b = 1;
num = [ 2*k 2*k*a 2*k*b ]
den = [ 6 11 6+2*k 1+2*k*a 2*k*b ]
sys = tf( num, den );
step( sys );
```

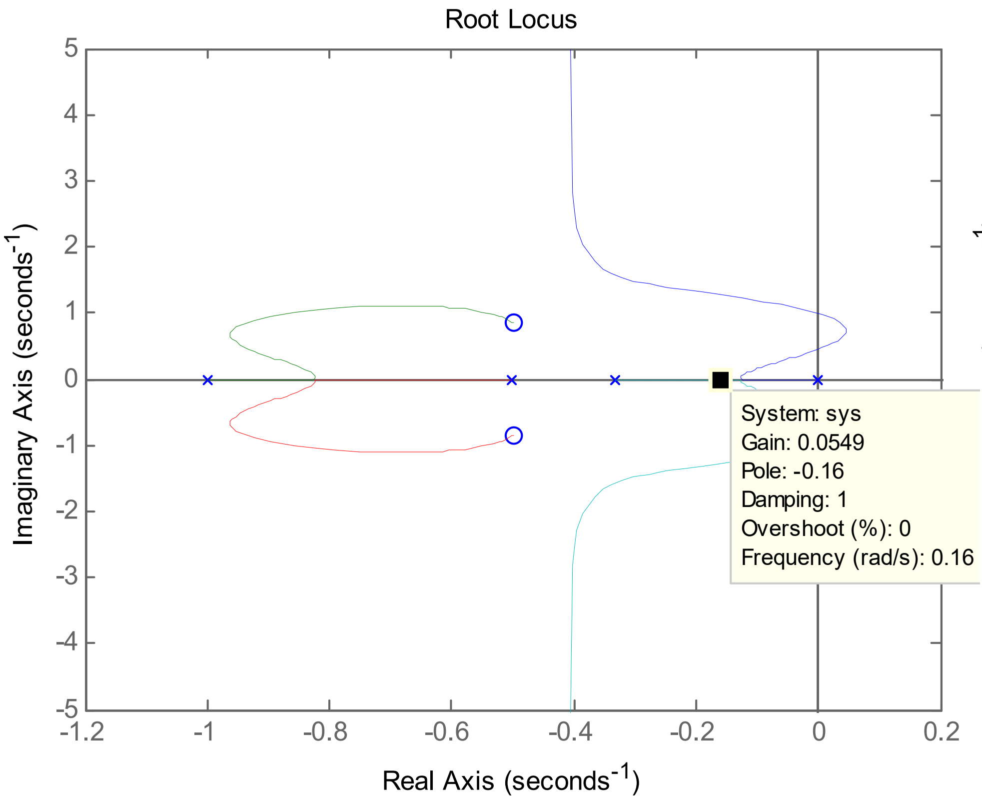


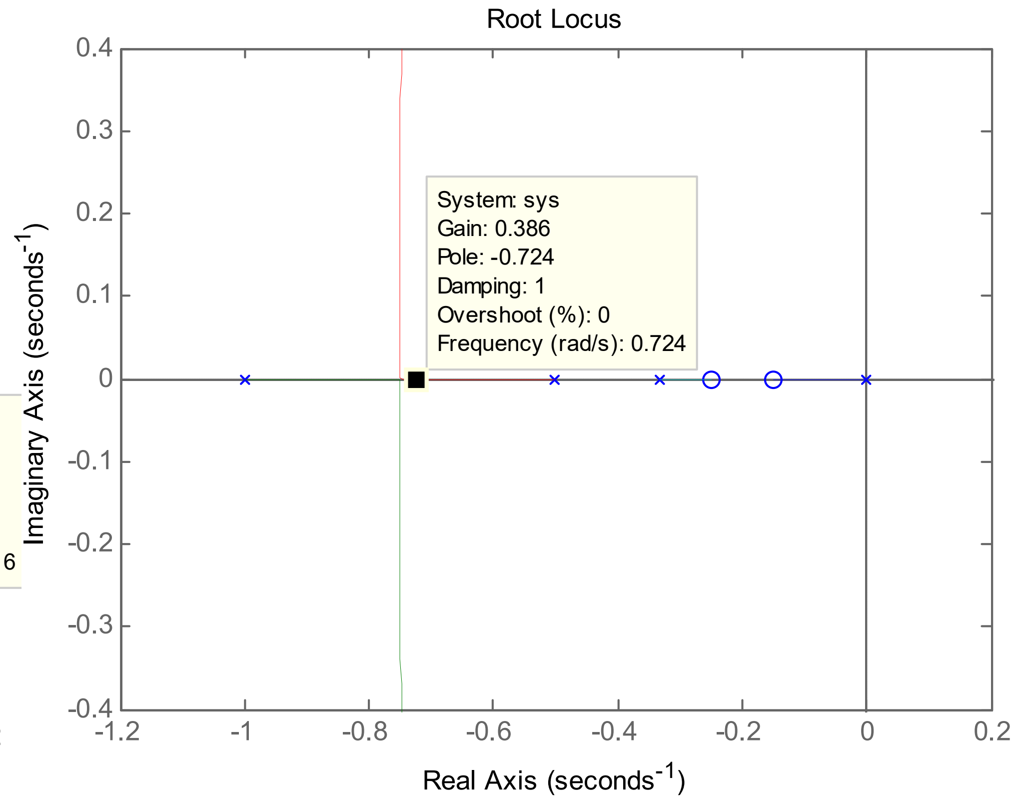


So, let's move the zeros!

$$C(s) = \frac{k_d\left(s^2 + as + b\right)}{s}$$

If we place these on the real axis, we may allow for higher gains to be associated with no overshoot. So let's place the zeros at -.25 and -.15 which means that a=0.4 and b=0.0375.

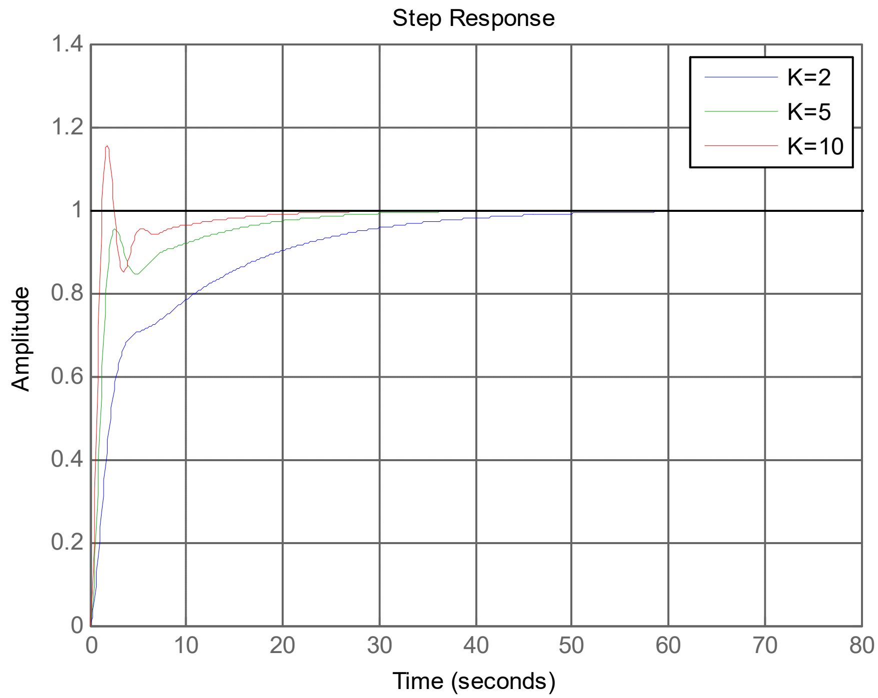




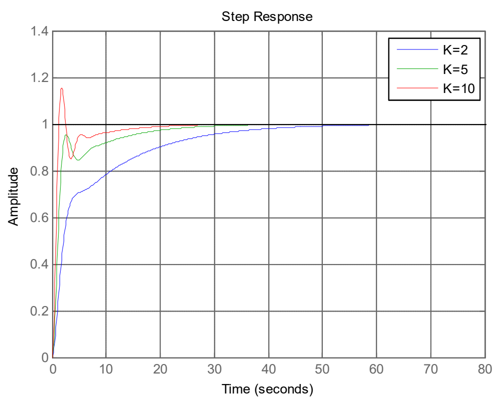
Now lets play with the gain.

$$P(s) = \frac{2}{6s^3 + 11s^2 + 6s + 1}$$

```
k = 2; %5 %10
a = 0.4; b = 0.0375;
num = [ 2*k 2*k*a 2*k*b ]
den = [ 6 11 6+2*k 1+2*k*a 2*k*b ]
sys = tf( num, den );
step( sys );
```



How would Ziegler-Nichols Do?



$$G_c(s) = K_P\left(1 + \frac{1}{T_I s} + T_D s\right)$$

Type	k_p	T_i	T_d
P	1/a		
PI	0.9/a	3τ	
PID	1.2/a	2τ	0.5τ