The University of Texas at Austin
Department of Electrical and Computer Engineering

EE362K: Introduction to Automatic Control – Fall, 2017

Problem Set 4

Reading Suggestion: Section 4.4 (except for section on K-R Invariance Principle), 5.1-5.4 and 6.1-6.4 (except for the section on LQRs) of Åström & Murray

1. Complete exercise 4.4 from the text.

4.4 (Lyapunov functions) Consider the second-order system

$$\frac{dx_1}{dt} = -ax_1, \qquad \frac{dx_2}{dt} = -bx_1 - cx_2,$$

where $a, b, c > 0$. Investigate whether the functions

$$V_1(x) = \frac{1}{2}x_1^2 + \frac{1}{2}x_2^2, \qquad V_2(x) = \frac{1}{2}x_1^2 + \frac{1}{2}(x_2 + \frac{b}{c-a}x_1)^2$$

are Lyapunov functions for the system and give any conditions that must hold.

2. Complete exercise 5.3 from the text by applying the convolution equation.

5.3 (Pulse response for a compartment model) Consider the compartment model given in Example 5.7. Compute the step response for the system and compare it with Figure 5.10b. Use the principle of superposition to compute the response to the 5 s pulse input shown in Figure 5.10c. Use the parameter values $k_0 = 0.1$, $k_1 = 0.1$, $k_2 = 0.5$ and $b_0 = 1.5$.

3. Verify the following system is reachable (i.e. controllable). If so, determine its reachable canonical form.

$$\frac{d\mathbf{z}}{dt} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & 3 & 5 \end{bmatrix}\mathbf{z} + \begin{bmatrix} 0 \\ 0 \\ 1/2 \end{bmatrix}u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}\mathbf{z}$$

4. For the system given in Problem 3, use the convolution equation to plot all three states for 30 seconds. Assume the initial conditions are **z**=[1 0 1] and that there is a step input. Repeat the problem for an impulse input with a value of 3.

5. For the system given in problem 3, determine the coordinate system $\tilde{x}$ for the canonical system.

6. For the system in problem 3, use the general equation to determine the gain matrix for a controller producing the system dynamics associated with eigenvalues of -1, -3, and -6 as well as -2, -1±5j. For each system, graph the input value u for the first 30 seconds if the desired output is a y=1.0.

7. Is it possible to use the generalized (Acker's) solution to determine state feedback gains if the desired eigenvalues contain imaginary components?

8. For what range of values for alpha and beta is the system below not controllable?

$$\frac{d\mathbf{z}}{dt} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \alpha & 1 & 1 \end{bmatrix} \mathbf{z} + \begin{bmatrix} \beta \\ 0 \\ 0 \end{bmatrix} u$$

(Bonus +20 pts) Complete Problem 5.4 from the text.

5.4 (Matrix exponential for second-order system) Assume that $\zeta < 1$ and let $\omega_d = \omega_0\sqrt{1-\zeta^2}$. Show that

$$\exp\begin{pmatrix} -\zeta\omega_0 & \omega_d \\ -\omega_d & -\zeta\omega_0 \end{pmatrix} t = \begin{pmatrix} e^{-\zeta\omega_0 t}\cos\omega_d t & e^{-\zeta\omega_0 t}\sin\omega_d t \\ -e^{-\zeta\omega_0 t}\sin\omega_d t & e^{-\zeta\omega_0 t}\cos\omega_d t \end{pmatrix}.$$