The University of Texas at Austin
Department of Electrical and Computer Engineering

EE362K: Introduction to Automatic Control – Fall, 2017
Problem Set 3

Suggested Reading: Chapters 4 (4.1-4.4) and 5 of Åström & Murray

1. Consider Slide 10 (titled: 2nd order solution, is it correct?) from the lecture on 2nd order systems. For a mass spring damper system with m=3, c=.25, and k=4, determine the damping coefficient, natural frequency, and damped frequency for the system. On a single graph, compare the analytical solution to numerical solution given by ode45( ). Can the damping coefficient, natural frequency, and damped frequency be determined from the graph? If so, how? And do the results match those found analytically? From inspection of the analytical solution, how would the system response change if the mass were doubled?

2. Complete problem 3.8 from the Astrom and Murray Book.

3.8 (Drug administration) The metabolism of alcohol in the body can be modeled by the nonlinear compartment model

$$V_b\frac{dc_b}{dt} = q(c_l - c_b) + q_{iv}, \qquad V_l\frac{dc_l}{dt} = q(c_b - c_l) - q_{\max}\frac{c_l}{c_0 + c_l} + q_{gi},$$

where $V_b = 48$ L and $V_l = 0.6$ L are the apparent volumes of distribution of body water and liver water, c_b and c_l are the concentrations of alcohol in the compartments, q_{iv} and q_{gi} are the injection rates for intravenous and gastrointestinal intake, $q = 1.5$ L/min is the total hepatic blood flow, $q_{\max} = 2.75$ mmol/min and $c_0 = 0.1$ mmol/L. Simulate the system and compute the concentration in the blood for oral and intravenous doses of 12 g and 40 g of alcohol.

2. Complete Exercise 4.3 from the book.

4.3 (Cruise control) Consider the cruise control system described in Section 3.1. Generate a phase portrait for the closed loop system on flat ground ($\theta = 0$), in third gear, using a PI controller (with $k_p = 0.5$ and $k_i = 0.1$), $m = 1000$ kg and desired speed 20 m/s. Your system model should include the effects of saturating the input between 0 and 1.

3. Mathematically (if possible) determine what the equilibrium points are for the system below and which (if any) are stable. Then, using MATLAB, produce a single phase-plane portrait for the following complex system for the initial conditions: [3, 0], [1,1],[4, -2] and two of your choosing where (if possible) one converges to an equilibrium point and one does not.

$$\ddot{x} + 1.2\dot{x} - 4x + x^3 = 0$$

4. By hand, find the eigenvalues and eigenvectors for the following matrices. Verify your answers using MATLAB and explain any discrepancies.

$$\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \qquad \mathbf{A} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 3 & 0 \\ 2 & -4 & 2 \end{bmatrix}$$

5. Linearize the system above about the operating point [1, 2], and then, by hand, determine the characteristic equation for **A**. By hand, determine the eigenvalues of **A**. You are certainly allowed to get hints from MATLAB but show your work. Then, use MATLAB to compute the following:

(a) The eigenvalues of the system at the point of interest.
(b) What does this condition number tell us about the system this matrix represents?
(c) The determinant of the A at the point of interest.
(d) The determinant of A^T at the point of interest.
(e) What is the product of the eigenvalues at the point of interest?

6. Find value of both states as a function of time by solving the following system by finding its eigenvalues and eigenvectors.

$$\dot{\mathbf{z}} = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \mathbf{z} \qquad\qquad \mathbf{z}(0) = \begin{bmatrix} 0 \\ -4 \end{bmatrix}$$

a) What is the analytical solution?
b) Create a phase plot for the system using ODE 45 for the set of initial conditions $z_1(0)$= [-4:1:4] and $z_2(0)$= [-4:1:4]. From this graph (or a similar one with more illustrative initial conditions), what is the relationship between the eigenvectors and the phase portrait?

7. Validate the equations for the performance metrics given for rise time, settling time, and %overshoot for Problem 1 above by comparing the calculated answers to values estimated from a graph showing the systems response to a step input.