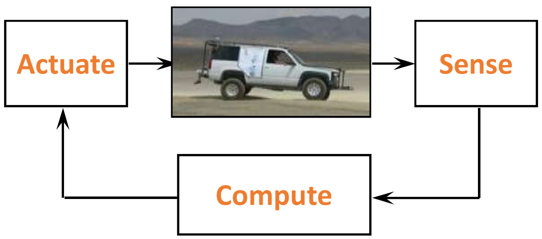
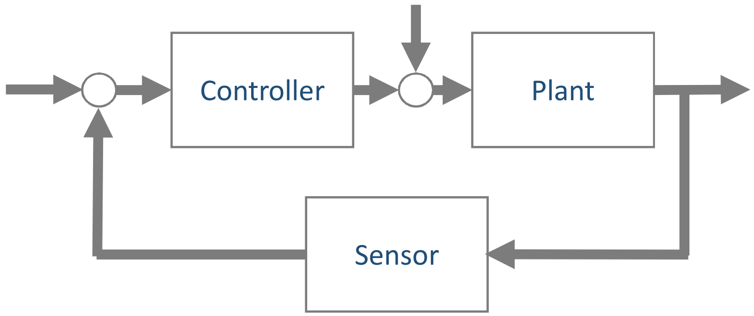
Block Diagrams

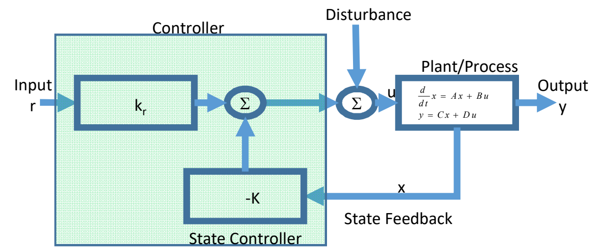
Dr. Mitch Pryor

Block diagrams

- We have been using block diagrams informally throughout the course.



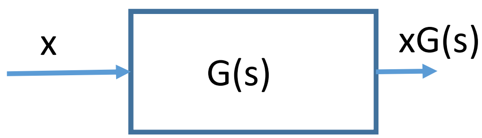




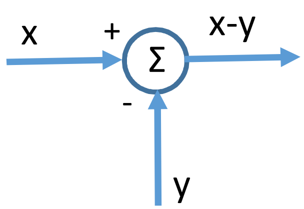
- Time to formalize the meaning of the components of the block diagram
  - Allows us to design controllers visually
    - Software packages exist to do this (will discuss later)
  - Allows us to then simplify complex block diagrams mathematically
- Block diagrams provide a simple mechanism to derive the transfer function for a closed loop system.

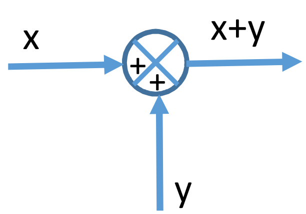
Components

The **paths** represent variable values which are passed within the system

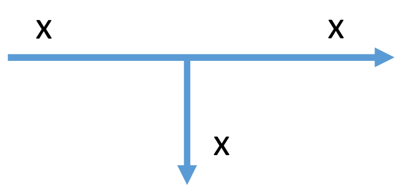


Blocks represent System components which are represented by transfer functions (***Laplace Functions***) and multiply their input signal to produce an output





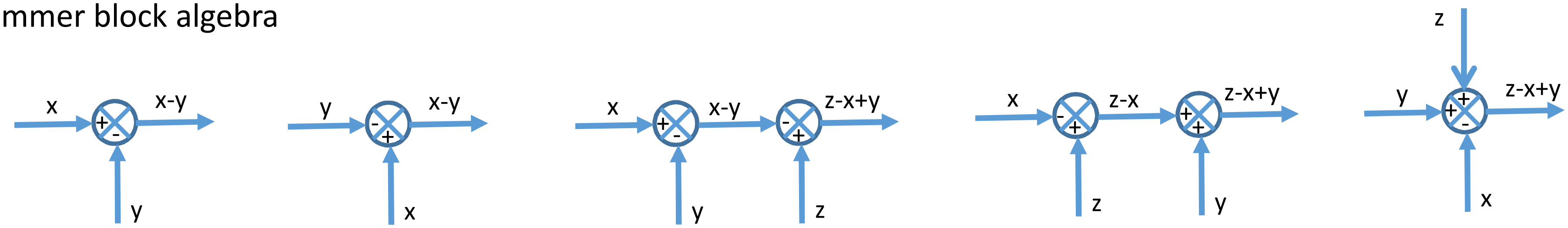
Addition and subtraction of signals are represented by a **summer block** with the operation indicated on the arrow. (Often, if there are no signs, the default is addition.)



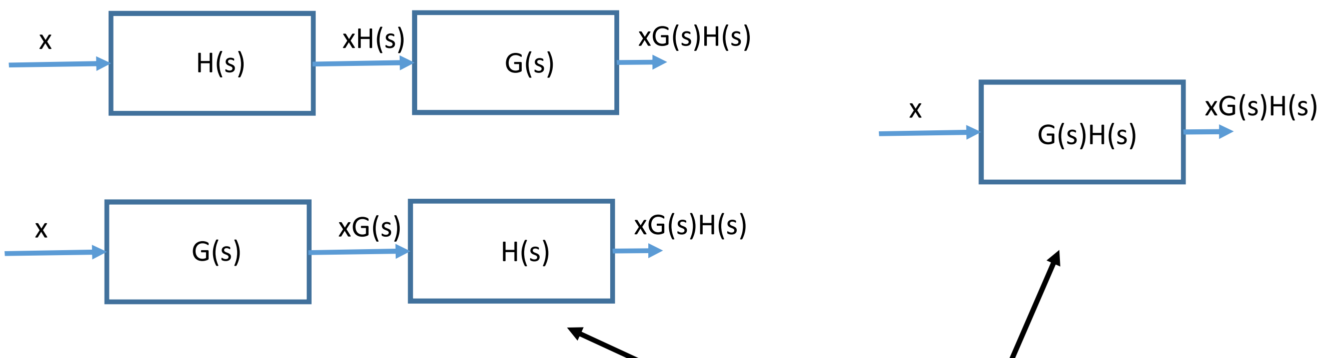
Branch points occur when a value is placed on two lines: no modification is made to the signal

Block Diagram Algebra

Summer block algebra



Transfer Block Multiplication



Note:

xG(s)H(s) = xH(s)G(s)

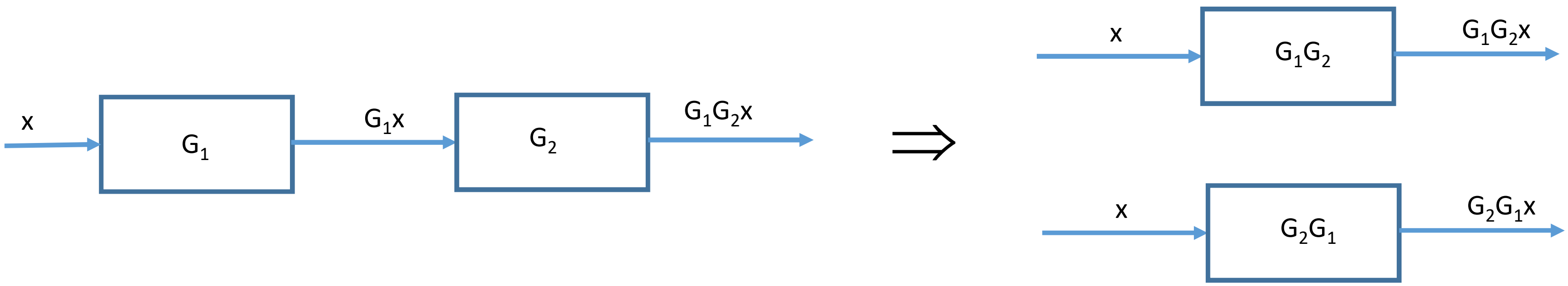
G(s)+H(s) = H(s)+G(s)

1st example of *Block Diagram Reduction*

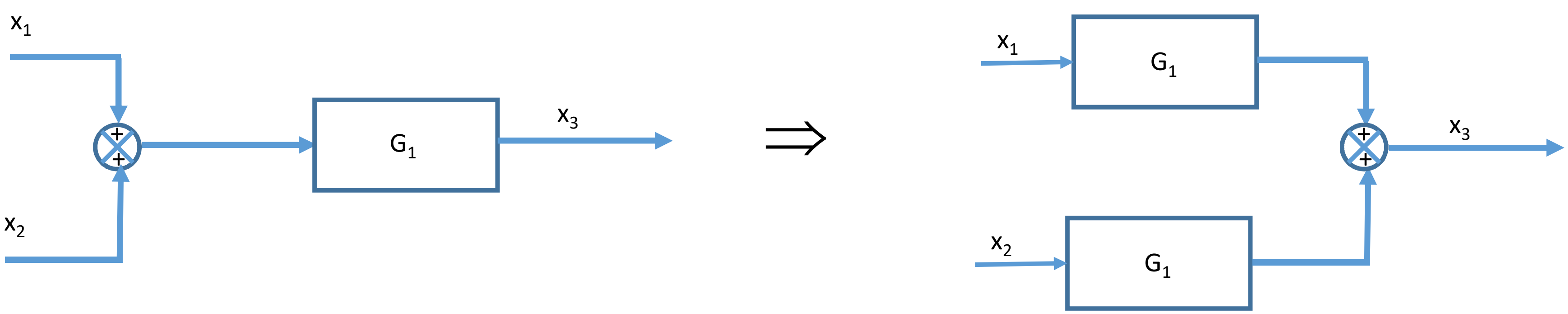
Block Diagram Algebra

From these simple rules, follows some logically valid algebraic operations.

1)

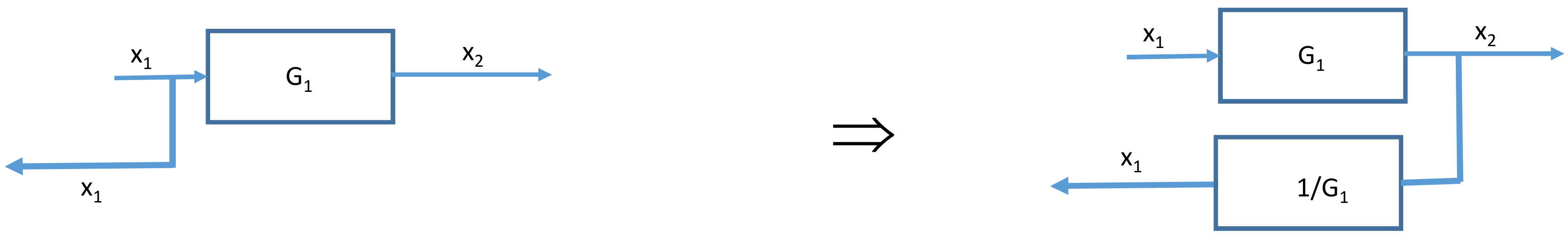


2)

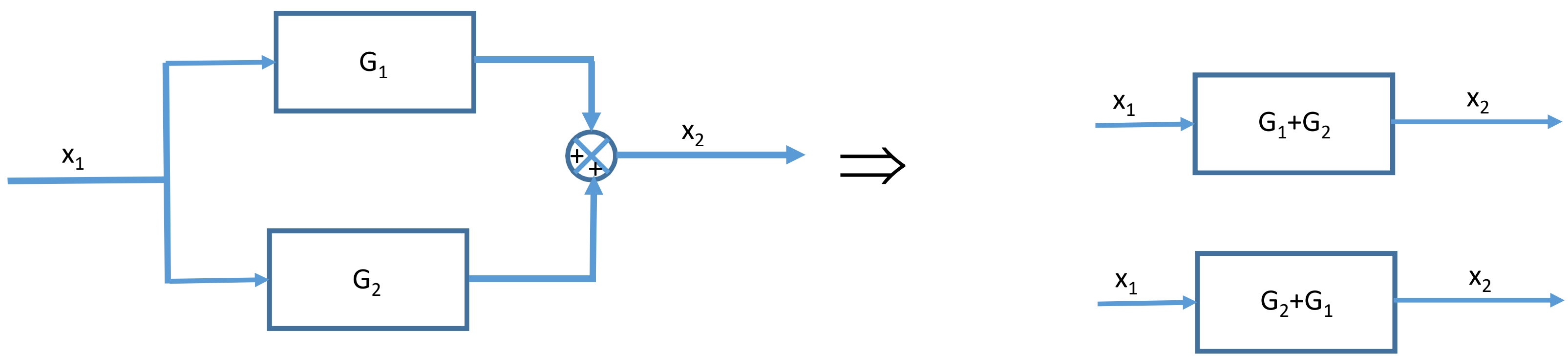


Block Diagram Algebra

3)

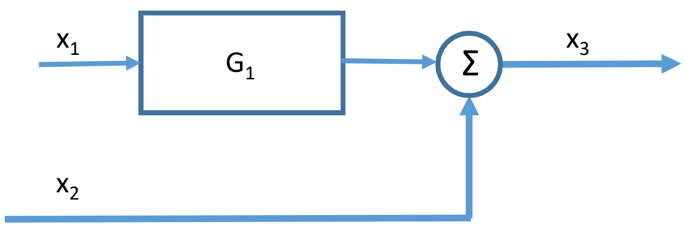


4)

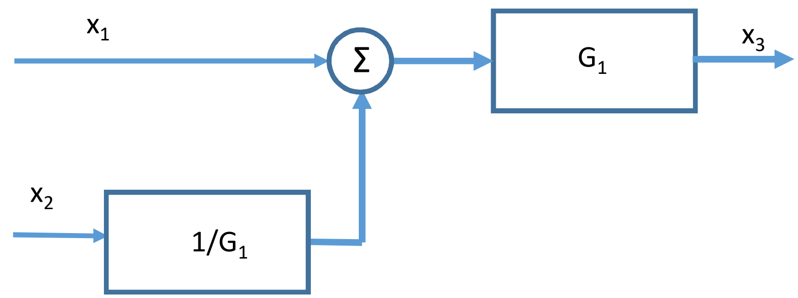


Block Diagram Algebra

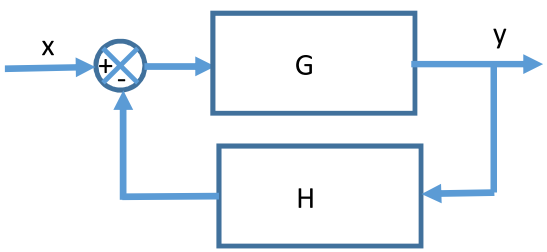
5)



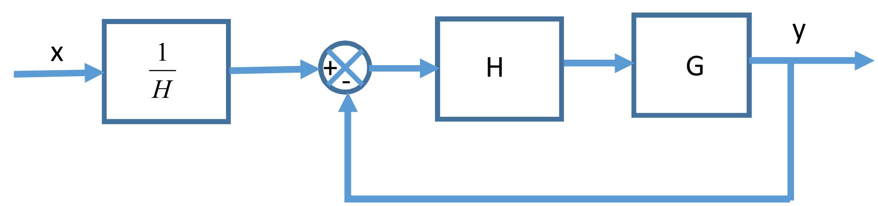
$\Rightarrow$



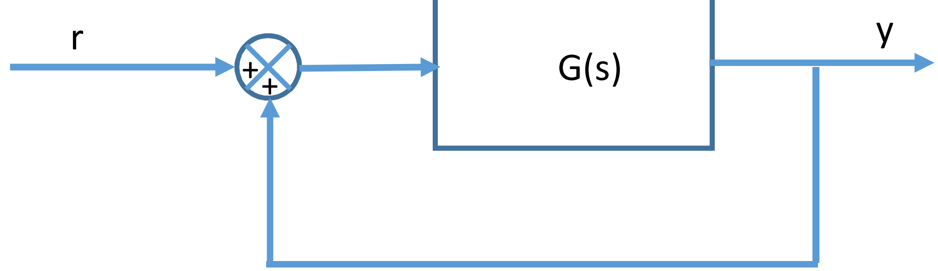
6)



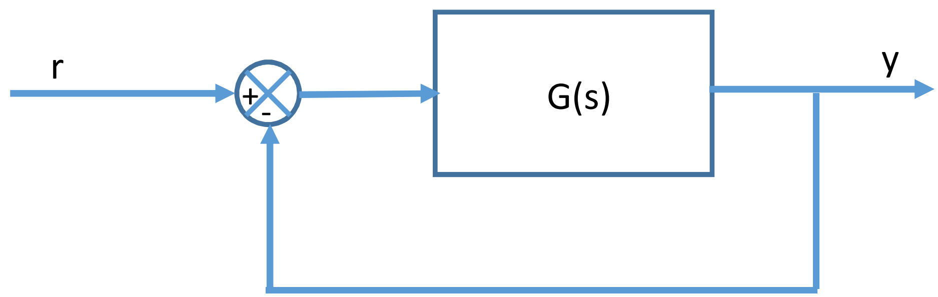
$\Rightarrow$



Closed Loops Systems

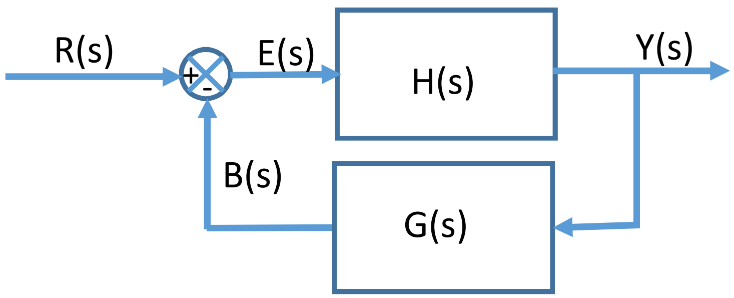


A positive feedback system



A negative feedback system

Simplification of a closed loop system



$$Y(s) = H(s)E(s)$$
$$Y(s) = H(s)(R(s) - B(s))$$
$$Y(s) = H(s)(R(s) - G(s)Y(s))$$
$$Y(s) = H(s)R(s) - H(s)G(s)Y(s)$$

$$Y(s) + H(s)G(s)Y(s) = H(s)R(s)$$

$$Y(s)(1 + H(s)G(s)) = H(s)R(s)$$

$$\boxed{T_{cl}(s) = \frac{Y(s)}{R(s)} = \frac{H(s)}{1 + H(s)G(s)}}$$

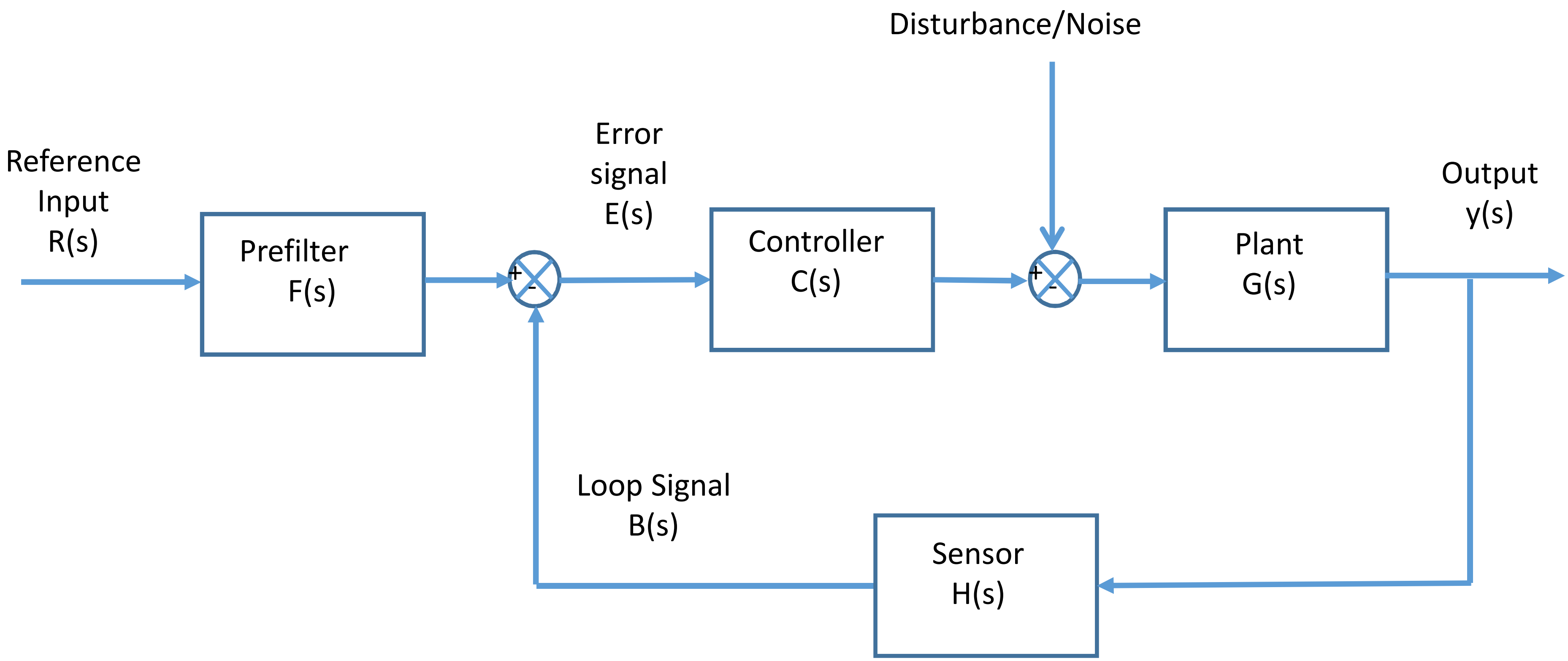
Note the sign switched from feedback sign!

<u>Typical Control Blocks and Signals</u>

R(s) – Reference signal or desired output
E(s) – Error signal
H(s) – Often C(s) P(s)
  C(s) – The control law that produces a u for the model or plant
  P(s) – The model of the system to be controlled or plant
Y(s) – The output of the system
G(s) – Sensor signal (if necessary)
B(s) – Loop transfer signal (last signal value prior to feedback)

*not standardized. Nomenclature varies from textbook to textbook

Closed System Nomenclature, cont'd



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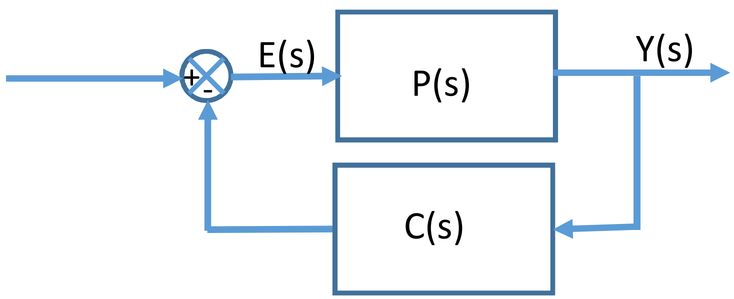
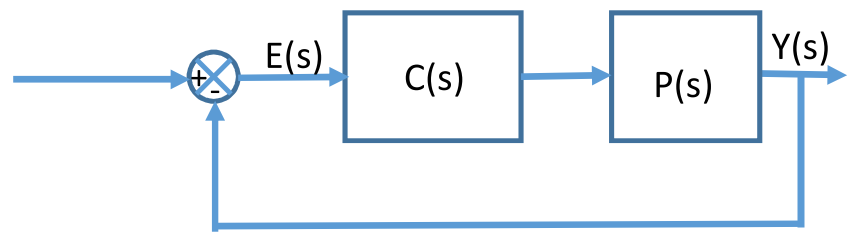
Two common control configurations



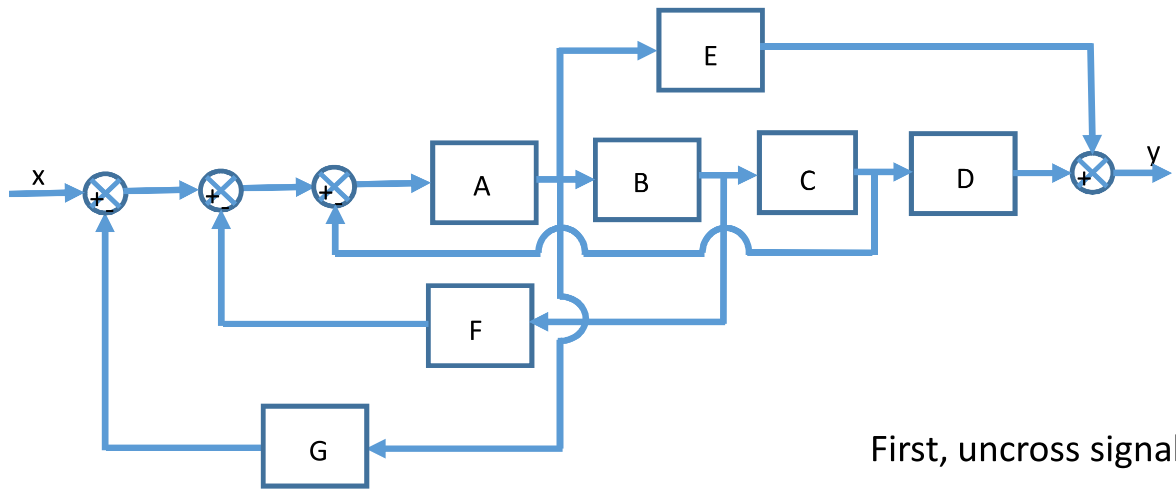


$$T_{cl}(s) = \frac{Y(s)}{R(s)} = \frac{\mathrm{P}(s)}{1 + C(s)\mathrm{P}(s)}$$

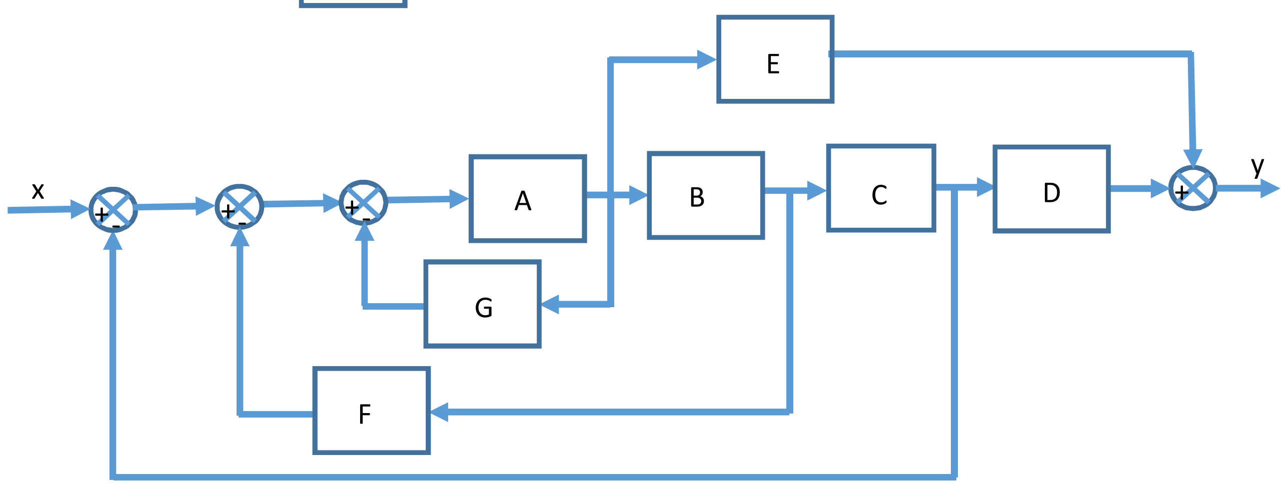
$$T_{cl}(s) = \frac{Y(s)}{R(s)} = \frac{\mathrm{C}(s)\mathrm{P}(s)}{1 + C(s)\mathrm{P}(s)}$$

*not standardized. Nomenclature varies from textbook to textbook

Block Diagram Reduction, Example 1

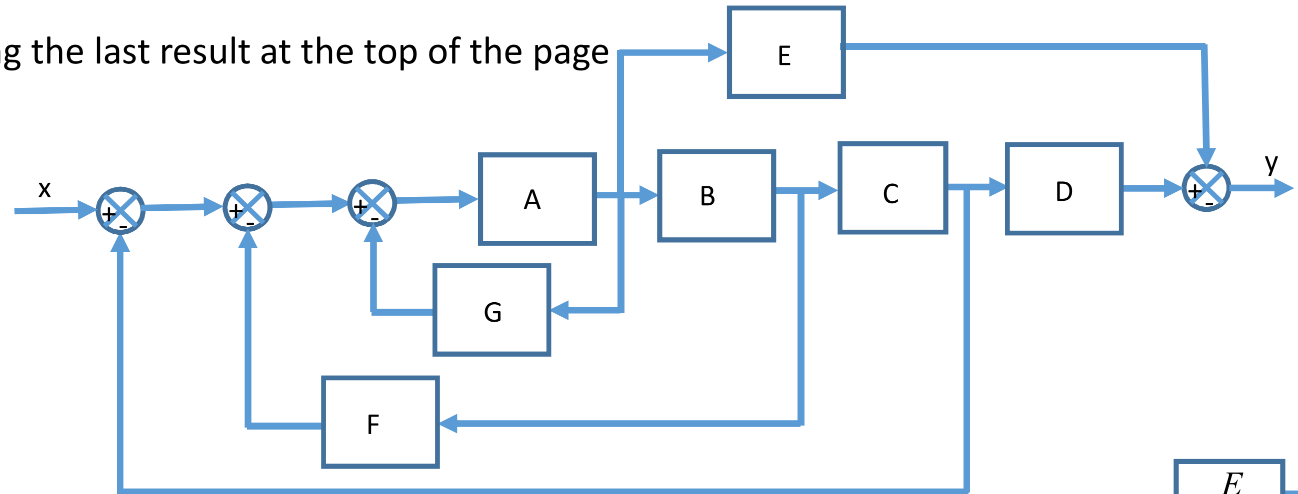


First, uncross signals where possible

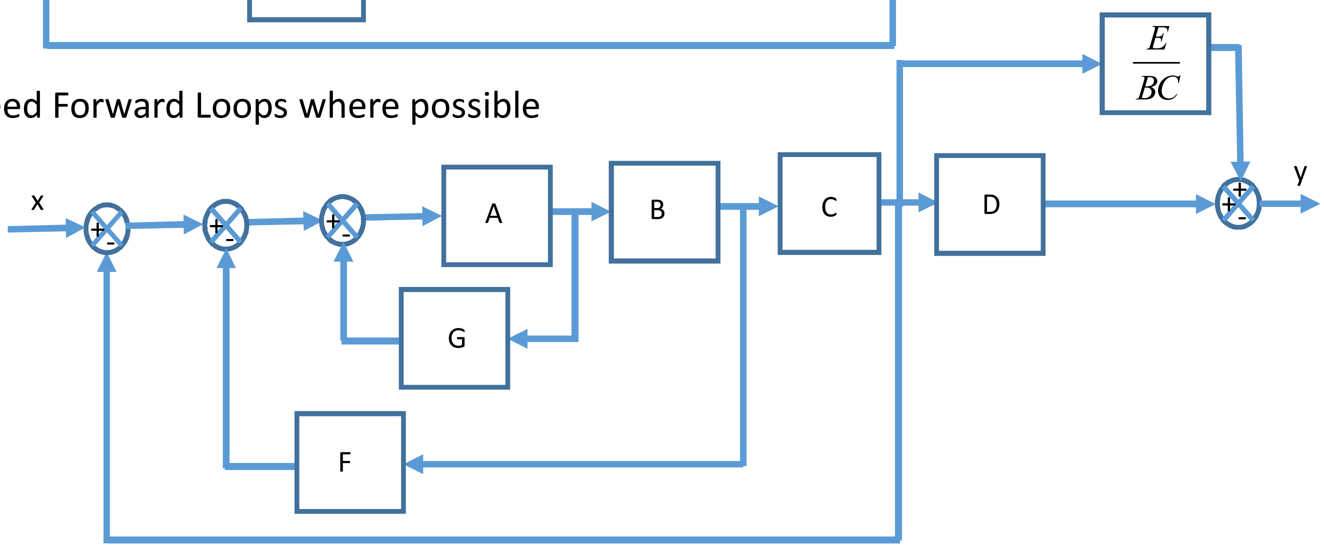


Block Diagram Reduction, Example 1

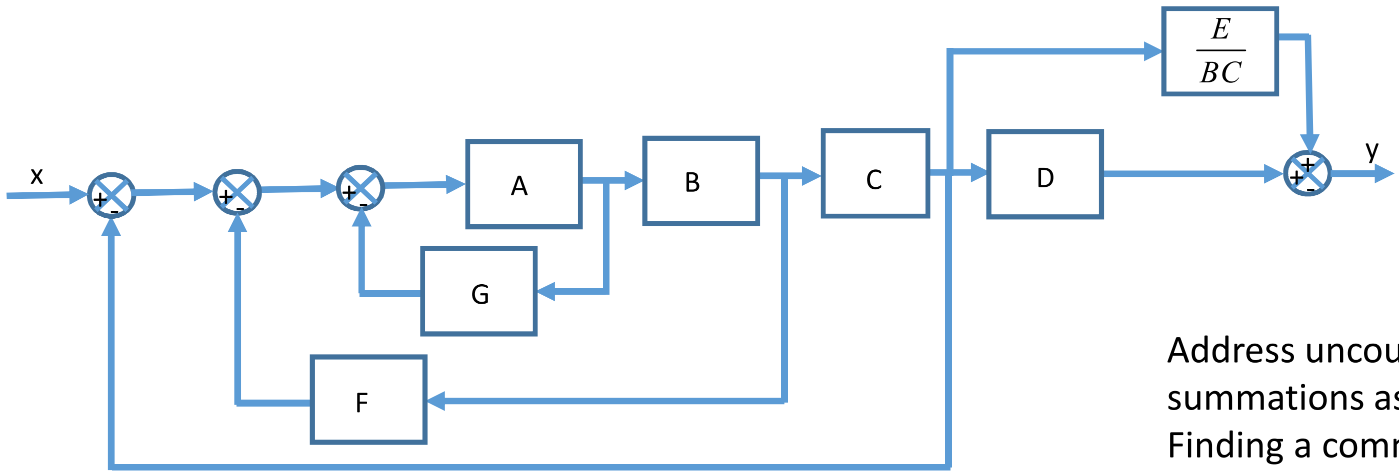
Always putting the last result at the top of the page



Next: Reduce Feed Forward Loops where possible

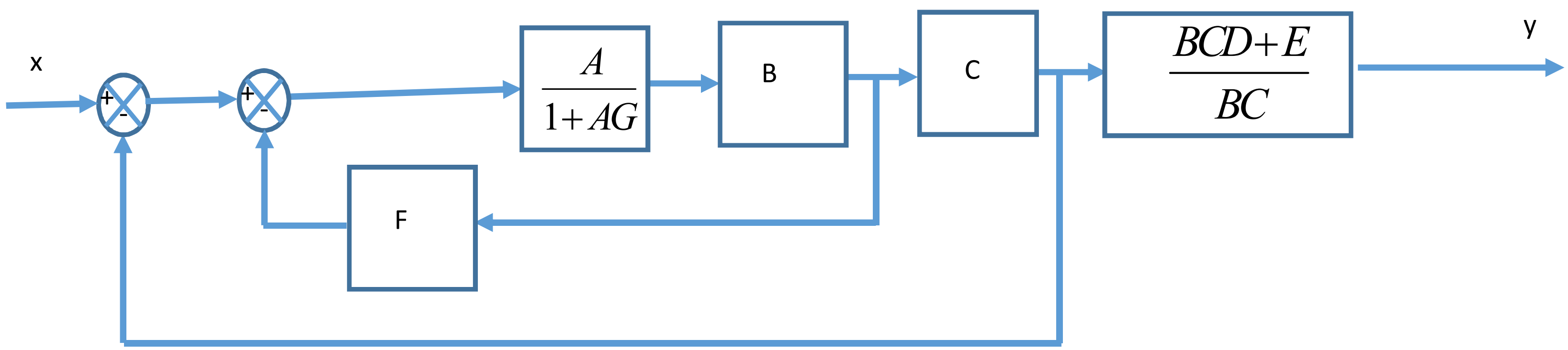


Block Diagram Reduction, Example 1

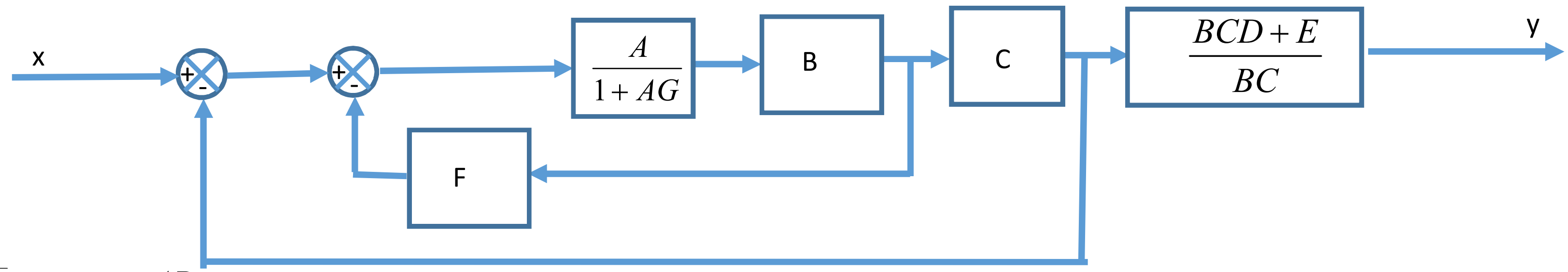


Address uncoupled summations as they occur. Finding a common denominator may simplify things later.

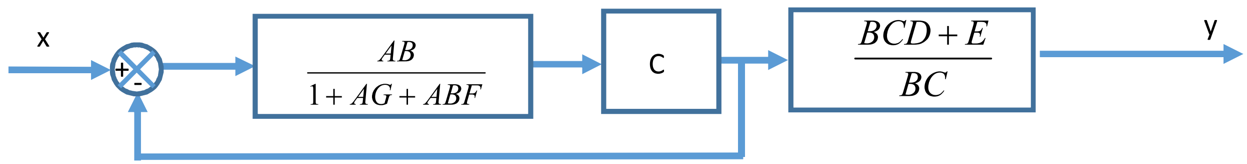
Reduce Feedback Loops starting with the inner most First



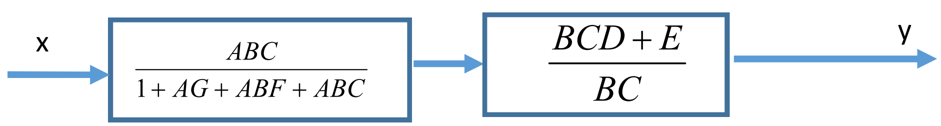
Block Diagram Reduction, Example 1

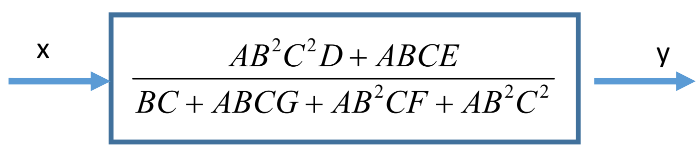


$$\frac{\dfrac{AB}{1+AG}}{1+\dfrac{FAB}{1+AG}}=\frac{AB}{1+AG+ABF}$$

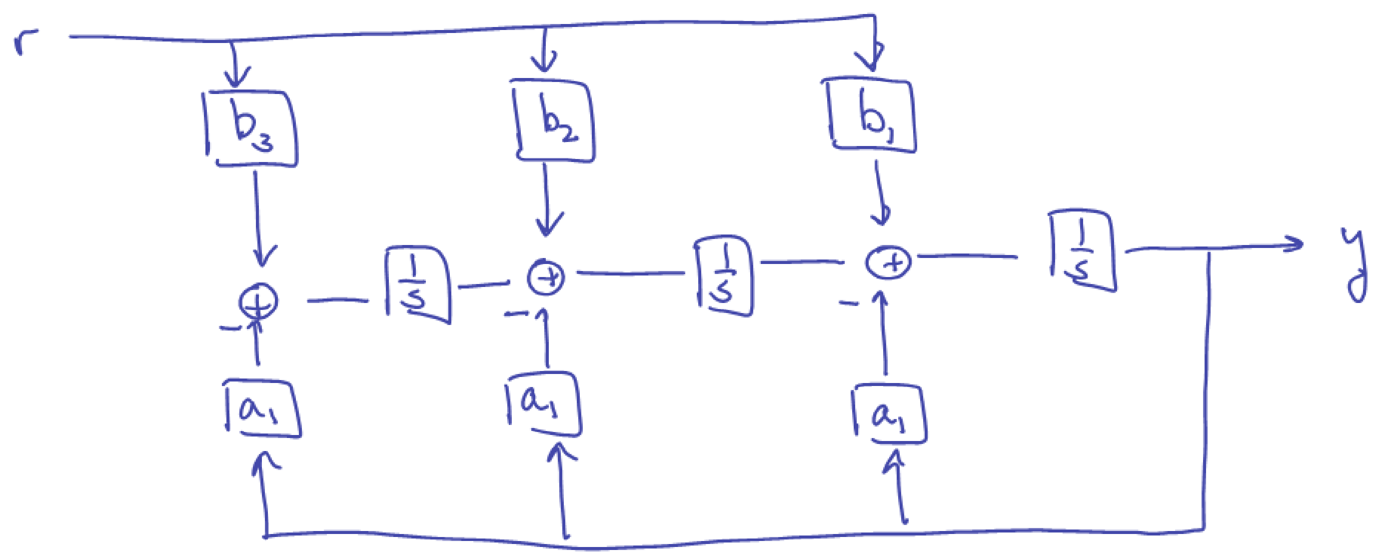


$$\frac{\dfrac{ABC}{1+AG+ABF}}{1+\dfrac{ABC}{1+AG+ABF}}=\frac{ABC}{1+AG+ABF+ABC}$$

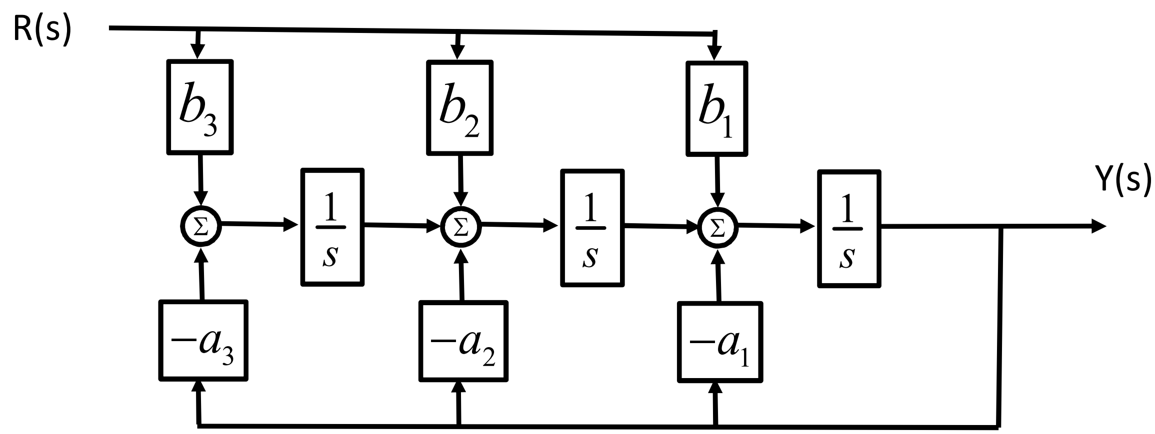


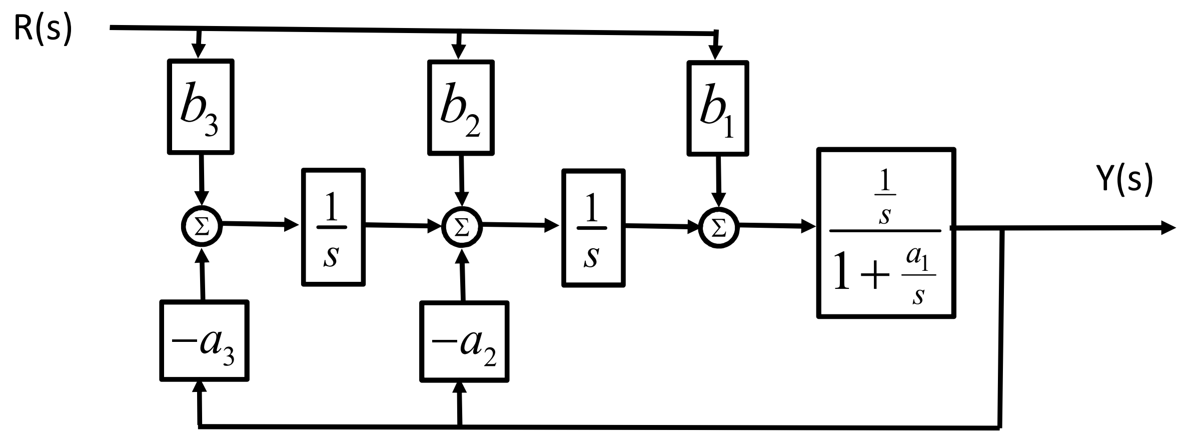


Block Diagram Reduction, Example 2

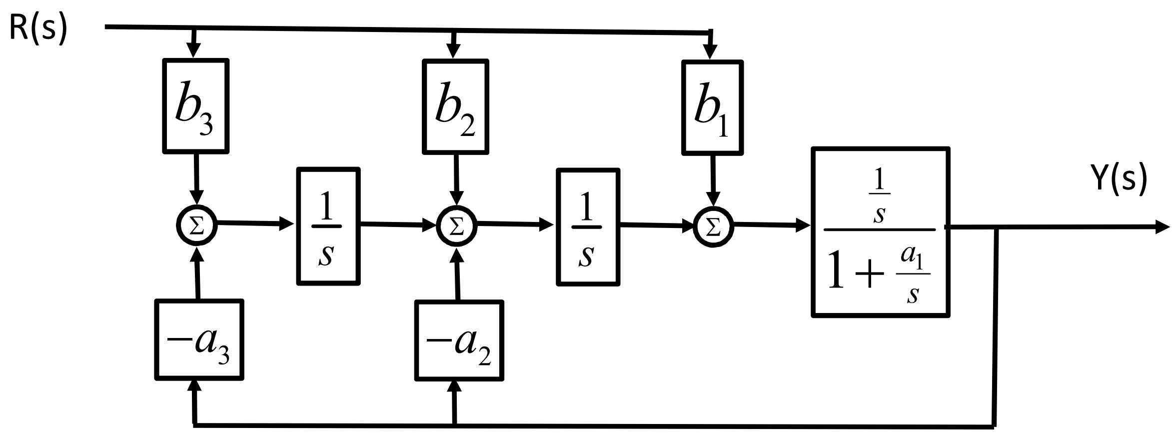


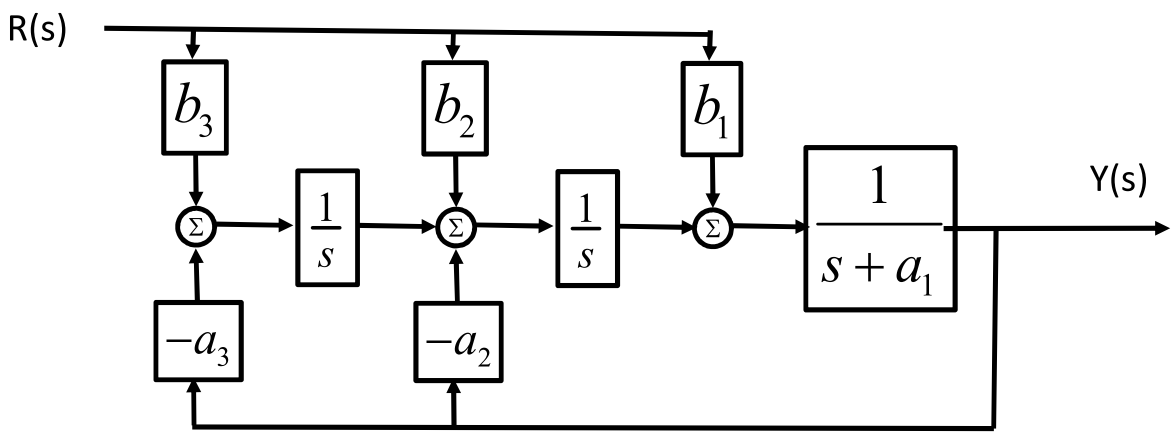
Solution by reduction



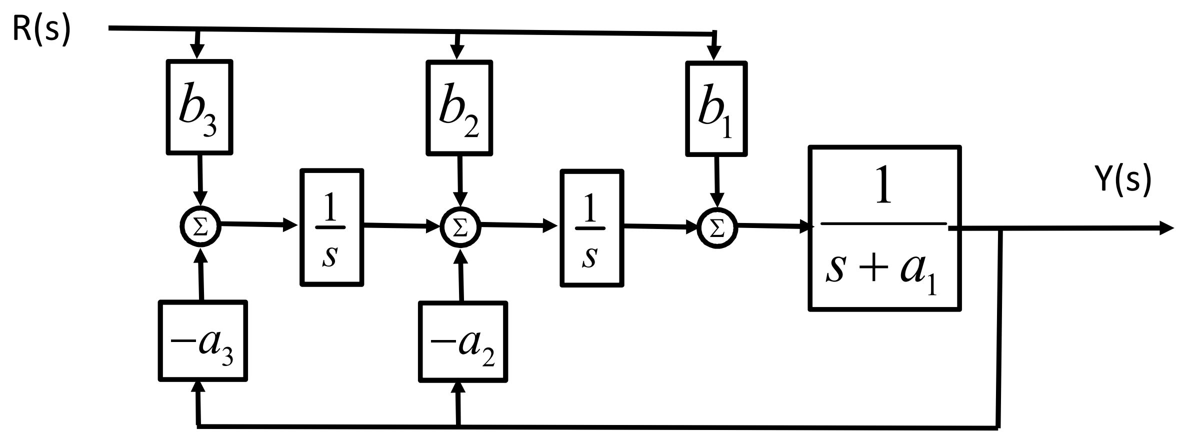


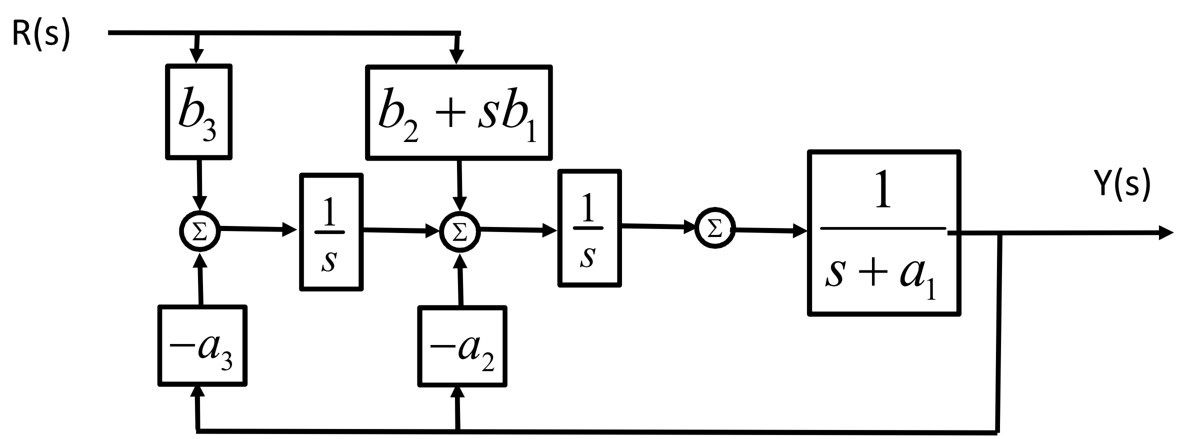
Solution by reduction



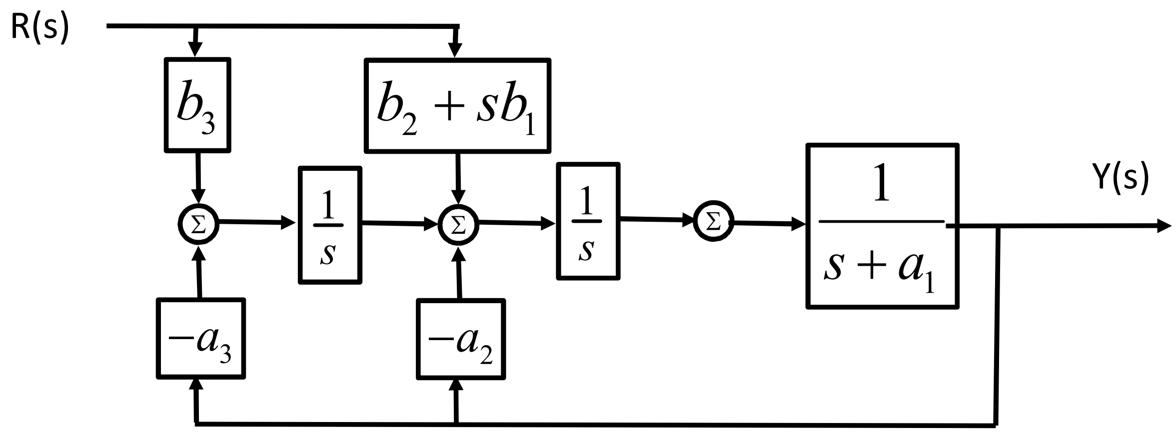


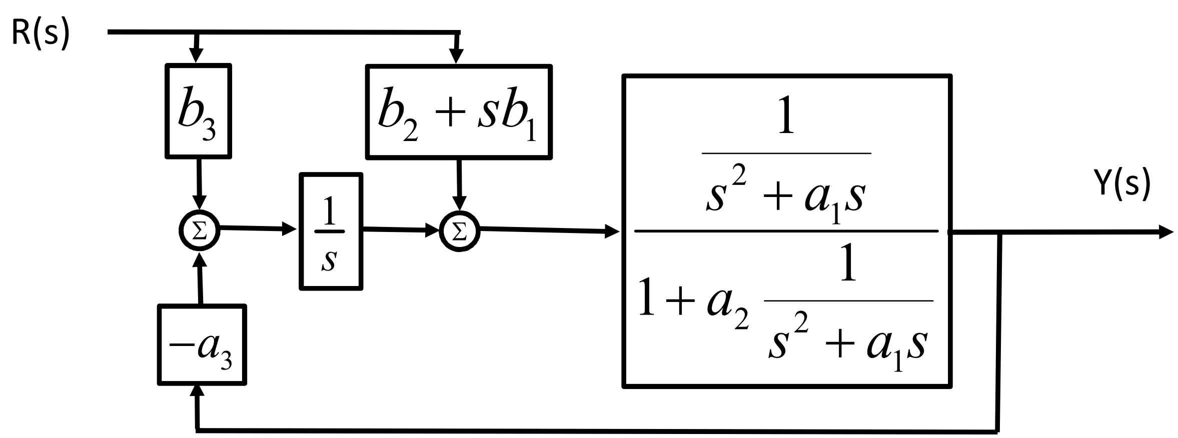
Solution by reduction





Solution by reduction





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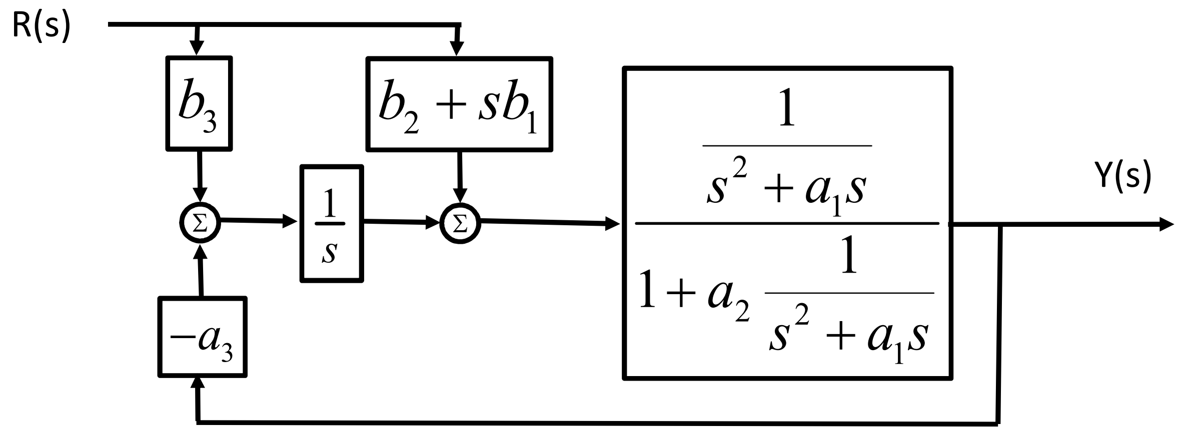
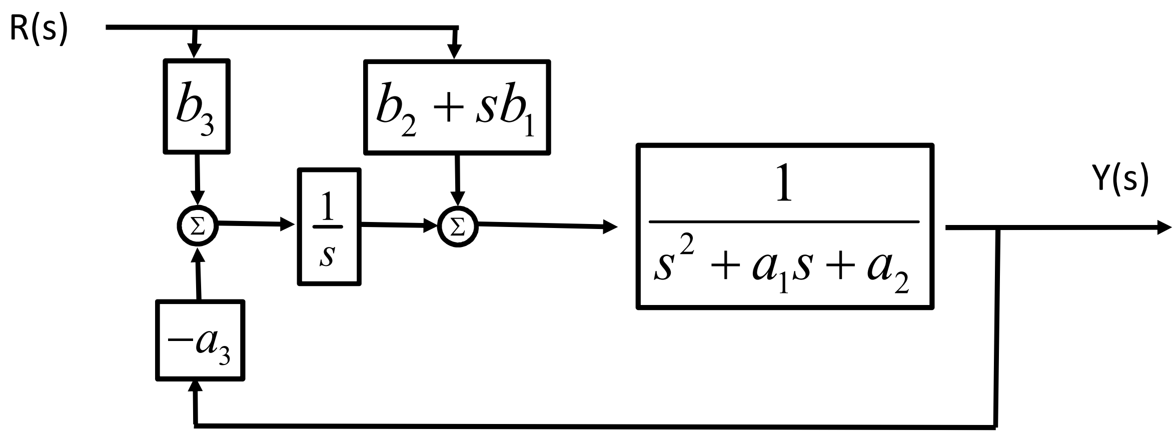
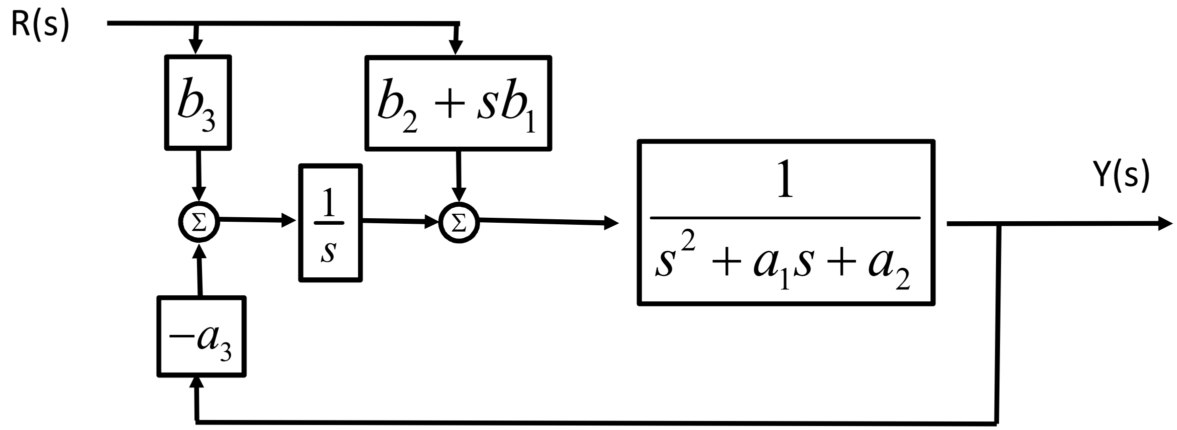
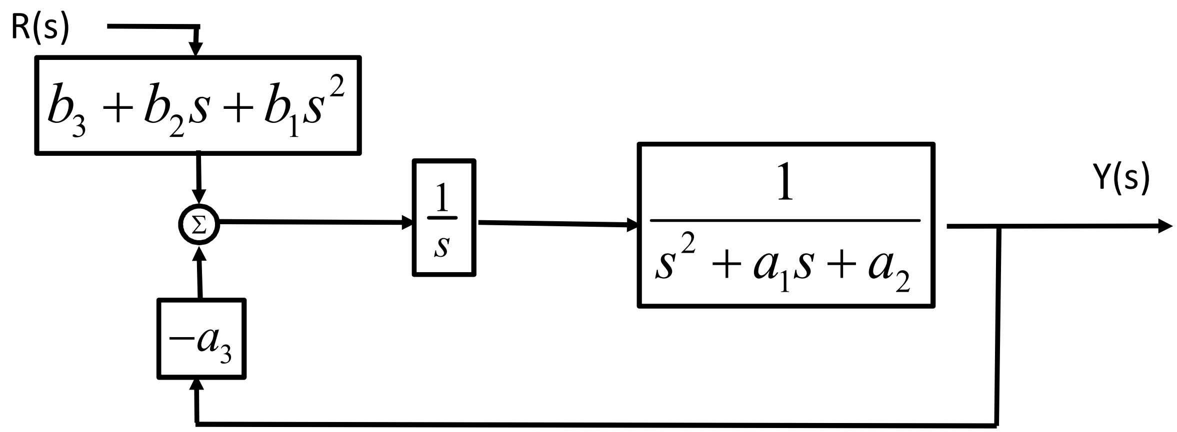


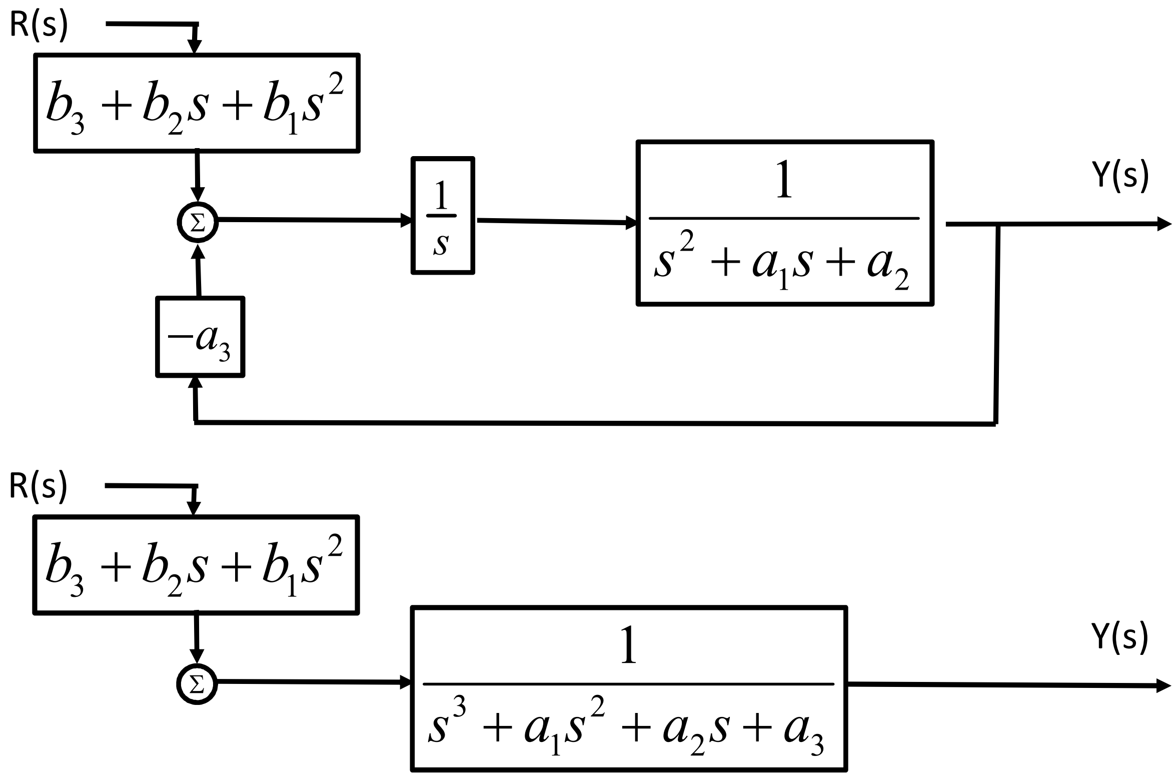


Solution by reduction

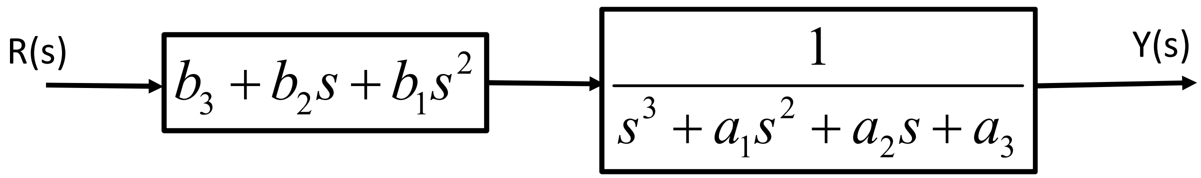


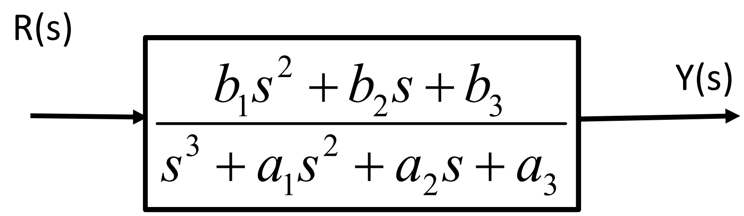


Solution by reduction



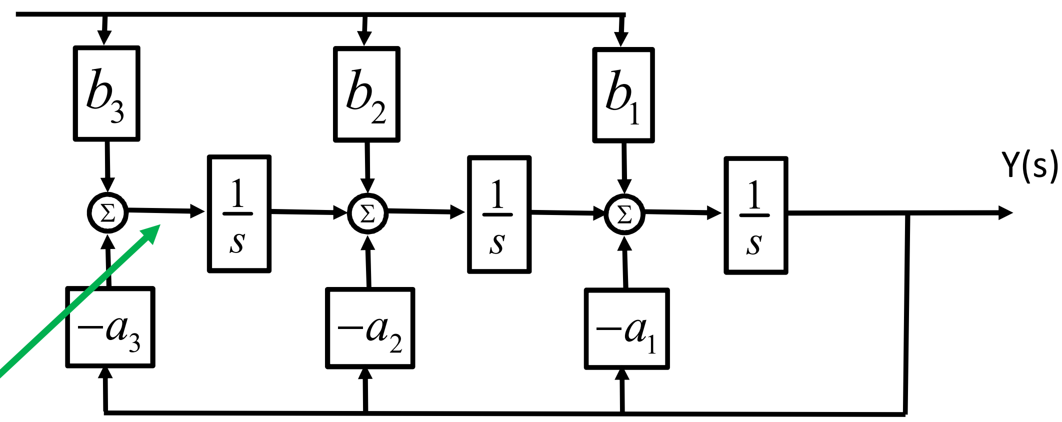
Solution by reduction





Solution by inspection

Back to the original diagram…



$$Y(s)s^3 = b_3R(s) - a_3Y(s) + b_2sR(s) - a_2sY(s) + b_1s^2R(s) - a_1s^2Y(s)$$

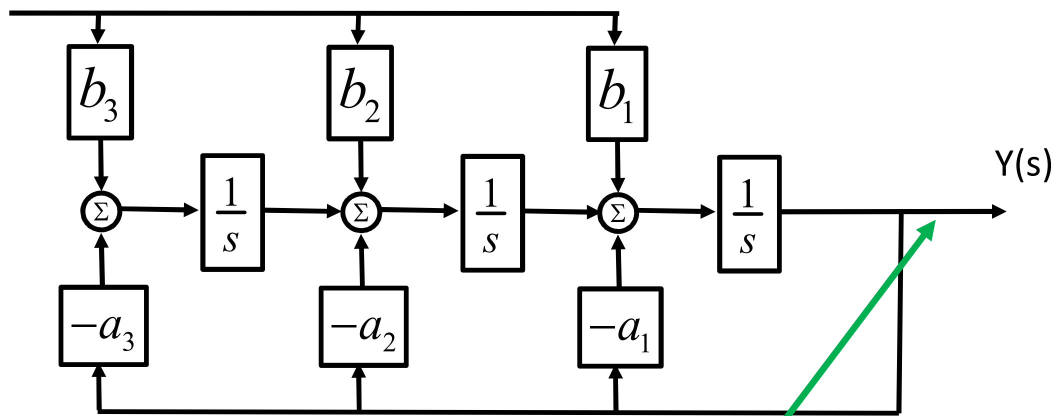
Rearrange (Y's to the left R's to the right)……

$$s^3Y(s) + a_1s^2Y(s) + a_2sY(s) + a_3Y(s) = b_1s^2R(s) + b_2sR(s) + b_3R(s)$$

$$\frac{Y(s)}{R(s)} = \frac{b_1s^2 + b_2s + b_3}{s^3 + a_1s^2 + a_2s + a_3}$$

Another, solution by inspection

Back to the original diagram…



$$Y(s) = \frac{b_3R(s) - a_3Y(s)}{s^3} + \frac{b_2R(s) - a_2Y(s)}{s^2} + \frac{b_1R(s) - a_1Y(s)}{s}$$

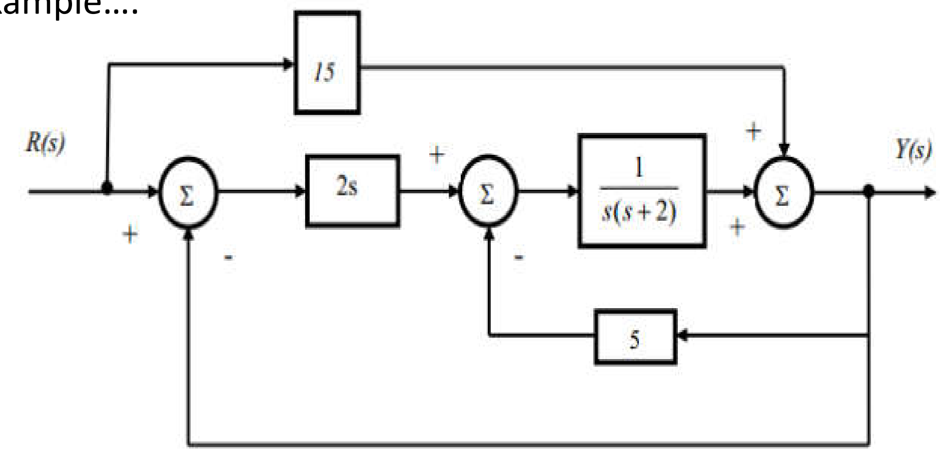
$$Y(s) + \frac{a_3Y(s)}{s^3} + \frac{a_2Y(s)}{s^2} + \frac{a_1Y(s)}{s} = \frac{b_3R(s)}{s^3} + \frac{b_2R(s)}{s^2} + \frac{b_1R(s)}{s}$$

$$Y(s)\left(1 + \frac{a_3}{s^3} + \frac{a_2}{s^2} + \frac{a_1}{s}\right)\frac{s^3}{s^3} = R(s)\left(\frac{b_3}{s^3} + \frac{b_2}{s^2} + \frac{b_1}{s}\right)$$

$$\boxed{\frac{Y(s)}{R(s)} = \frac{b_1s^2 + b_2s + b_3}{s^3 + a_1s^2 + a_2s + a_3}}$$

Block Diagram Reduction, Example 3

Let's do one final example….



While this can be done with standard reduction techniques, a good eye reveals it is a good candidate for solution by inspection.

$$Y(\mathrm{s}) = 15\mathrm{R(s)} + \left\{ \left(R(s) - Y(s) \right) \left(2s \right) - 5Y(s) \right\} \left(\frac{1}{s(s+2)} \right)$$

$$Y(\mathrm{s}) = 15\mathrm{R(s)} + \left\{ 2sR(s) - 2sY(s) - 5Y(s) \right\} \left(\frac{1}{s(s+2)} \right)$$

Block Diagram Reduction, Example 3

$$Y(\mathrm{s}) = 15\,\mathrm{R(s)} + \left\{2sR(s) - 2sY(s) - 5Y(s)\right\}\left(\frac{1}{s(s+2)}\right)$$

From the last slide…

$$Y(\mathrm{s}) = 15\,\mathrm{R(s)} + \left\{\frac{2s}{s(s+2)}R(s) - \frac{2s+5}{s(s+2)}Y(s)\right\}$$

Just algebra to isolate Y(s) and R(s)

$$Y(\mathrm{s})\left(1 + \frac{2s+5}{s(s+2)}\right) = \mathrm{R(s)}\left(15 + \frac{2s}{s(s+2)}\right)$$

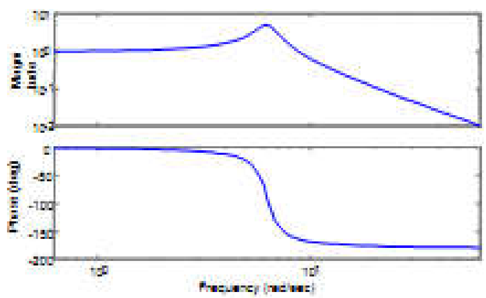
$$T(s) = \frac{Y(\mathrm{s})}{R(s)} = \frac{15 + \dfrac{2s}{s(s+2)}}{1 + \dfrac{2s+5}{s(s+2)}} = \frac{s(15s+32)}{s^2 + 4s + 5}$$

Summary

- We can use the algebraic tools to simplify block diagrams using reduction technique or inspection.
  - Soon, this will allow us to intuitively and visually discuss controller options AND have the tools to determine (and analyze) the resulting transfer function.

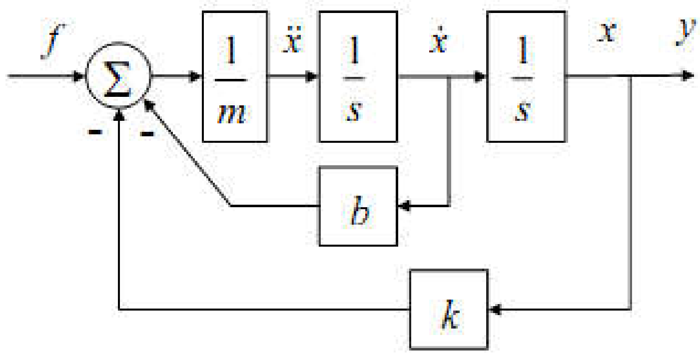
From on representation to another

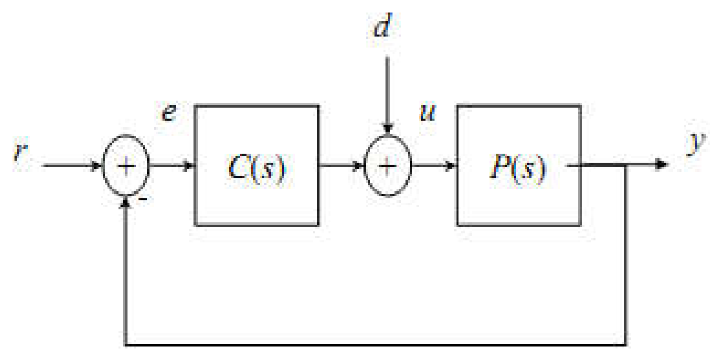
$$u = A\sin(\omega t) \longrightarrow \boxed{\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx + Du \\ x(0) &= 0\end{aligned}} \longrightarrow y_{ss} = |H(j\omega)|\, A \cdot \sin\left(\omega t + \angle H(j\omega)\right)$$



$$H(s) = C(sI - A)^{-1}B + D$$

$$H_{y_2 u_1} = H_{y_2 u_2} H_{y_1 u_1} = \frac{n_1 n_2}{d_1 d_2}$$





Summary of system models

- System representations used to design controllers.
  - Know which representation gives access to the needed information.
  - Understand advantages and disadvantages are asserted here.
  - Can convert from one representation to another
- ODEs
  - Commonly used for developing system models
  - Easy to solve for simple systems
  - Lots of trial and error when designing controllers,
  - Possible to design PID controllers for simple systems
  - Difficult to analytically solve complex systems
- State-space representations
  - Easy to solve numerically
  - Focus studies on canonical forms
  - Clear relationships for stability, reachability, and observability
  - Possible to design controller response as a state/output feedback controller.
  - Physical impact of modelled terms may not be clear
  - Difficult to evaluate performance over range of inputs
- Transfer functions
  - Excellent for examining system frequency response
  - Poor relation to physical parameters
  - Does not provide information on the transient response of the system.
    - No initial conditions information included.
  - Possible to design controller response including filters and PID controllers..
  - Clear relationship and design methodologies for PID controllers.
- Block diagrams
  - Useful to clarify system impacts in both time and frequency domain
  - Allows for possibility to design controllers with a GUI (Graphic User Interface)
  - Complex procedures for mapping from BDs to SS or TF.
- Other common representations are not covered in this class
  - (i.e. Signal flow graphs, Bond graphs, etc.)

Summary

- Formalized diagrams and their components
- Presented algebraic operations that allow for block diagrams to be modified and/or simplified
- Used this to derive the transfer function for a closed loop system.
- Summarize advantages and disadvantages of system representations