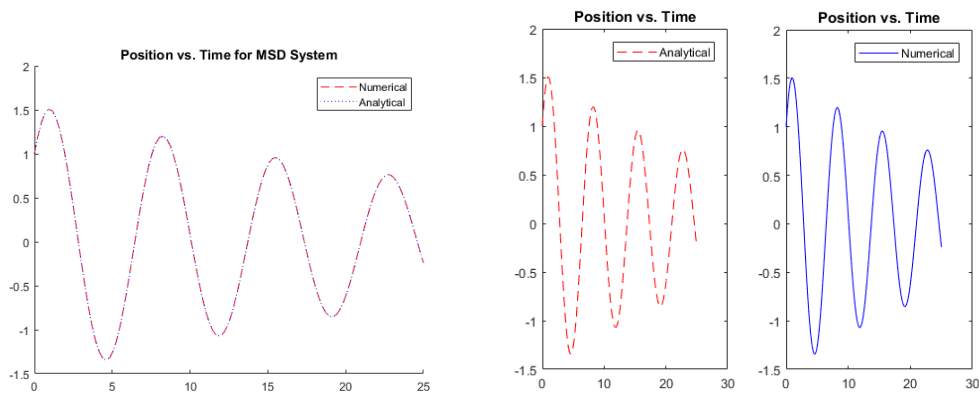


HW 3

1.)

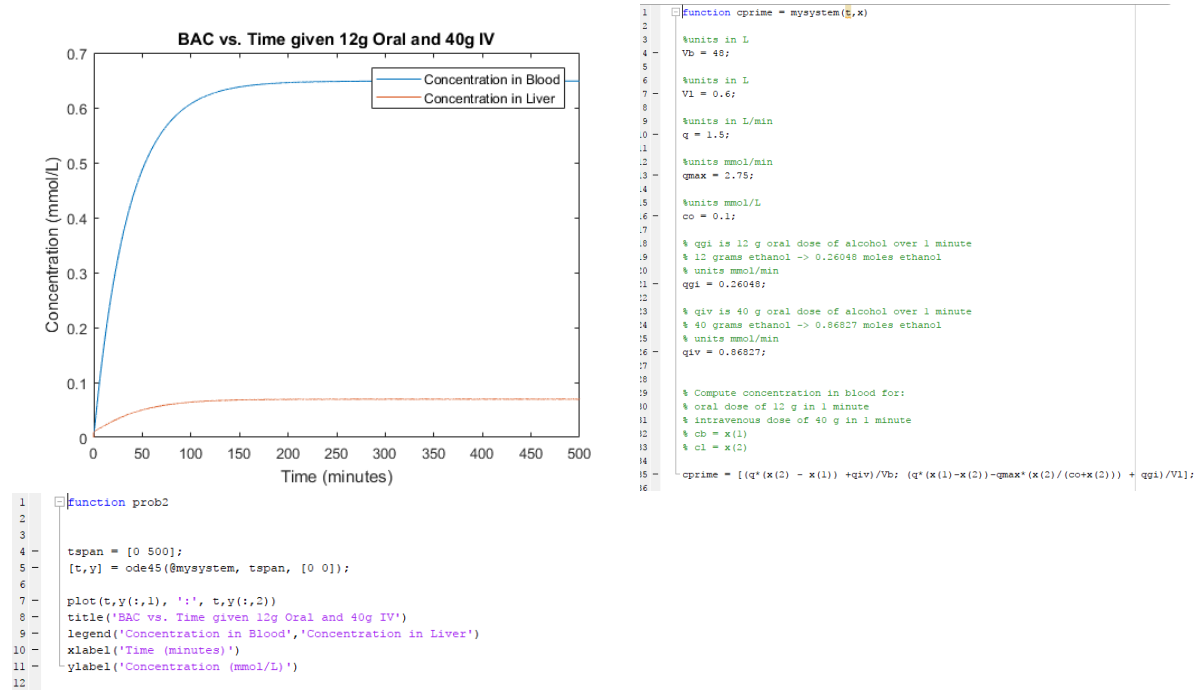


I included two plots to make sure you can see the different functions.

```

1 function problem1
2 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
3 % spring mass damper solver
4 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
5
6
7
8 to = 0;
9 tf = 25;
10
11
12
13 [t,x] = ode45('myode',[to tf], [1,1]);
14
15 subplot(1,2,1);
16 plot (t,x(:,1), 'r--')
17 title ('Position vs. Time')
18 legend('Analytical')
19 subplot(1,2,2);
20 plot(t, analytical(t), 'b')
21 %t = linspace(0,100);
22 %plot (t,analytical(t), 'x', 'blue')
23 title ('Position vs. Time')
24 legend('Numerical')
25
26 %
27 hold on;
28 plot (t,x(:,1), 'r--');
29 plot(t, analytical(t), 'b')
30 title('Position vs. Time for MSD System')
31 legend('Numerical', 'Analytical')
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```

a.) The doses were calculated and converted into mmol/min. The estimated time for these doses is taken to be one minute.



Determine equilibrium points for:

$$\dot{x} + 1.2\dot{x} - 4x + x^3 = 0$$

$$\begin{aligned} z_1 = x &\rightarrow \dot{z}_1 = z_2 \\ z_2 = \dot{x} &\rightarrow \dot{z}_2 = -1.2z_2 + 4z_1 - z_1^3 \end{aligned} \quad \frac{d\vec{z}}{dt} = \begin{bmatrix} 0 & 1 \\ 4 & -1.2 \end{bmatrix} \vec{z} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} z_1^3$$

Let z_e be an eq. point:

z_e is an eq. point for $\frac{d\vec{z}}{dt} = F(\vec{z})$ iff $F(\vec{z}_e) = 0$

So, for what z is this eq. true:

$$\begin{bmatrix} 0 & 1 \\ 4 & -1.2 \end{bmatrix} \vec{z} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} z_1^3 = 0$$

This is also the set of equations,

$$\begin{aligned} z_2 &= 0 \leftarrow \text{this gives points of form } (z_1, 0) \\ 4z_1 - 1.2z_2 &= z_1^3 \end{aligned}$$

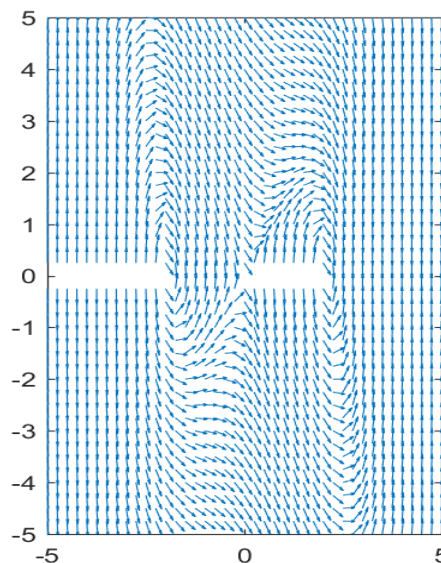
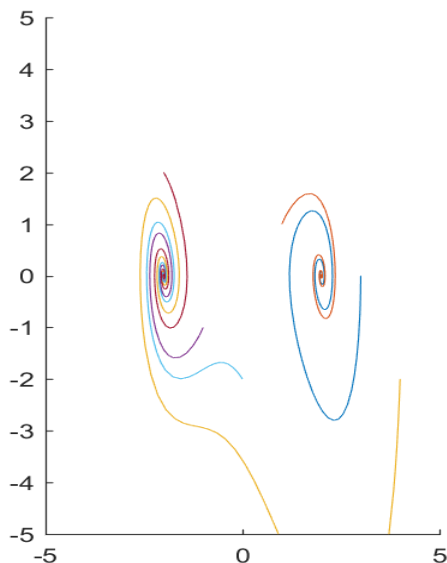
This amounts to finding solutions to:

$$z^3 - 4z = 0, \text{ where } z = z_1$$

$$z = 0 \quad z^2 - 4 = 0 \rightarrow z^2 = 4 \rightarrow z = \pm 2$$

Therefore, the system has the following equilibrium points:

$$(0, 0), (2, 0), (-2, 0)$$



4.)

By hand, find eigenvalues and eigenvectors.

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \text{ Solve } \det(A - \lambda I) = 0$$

$$(1-\lambda)(1-\lambda) - 1 = 0$$

$$\lambda^2 - 2\lambda = 0$$

$$\lambda(\lambda-2) = 0$$

$$\lambda = 0 \text{ and } \lambda - 2 = 0$$

$$\lambda_1 = 0$$

$$\lambda_2 = 2$$

$$(A - 0I) = 0 \quad \lambda_1$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \vec{x} = 0 \rightarrow \vec{x}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$(A - 2I) = 0 \quad \lambda_2$$

$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \vec{x} = 0 \rightarrow \vec{x}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

For the first matrix, $\lambda_1 = 0, \vec{x}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$
 $\lambda_2 = 2, \vec{x}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 3 & 0 \\ 2 & -4 & 2 \end{bmatrix}$$

$$\det(A - \lambda I) = 0$$

$$\det \begin{bmatrix} 1-\lambda & 2 & 0 \\ 0 & 3-\lambda & 0 \\ 2 & -4 & 2-\lambda \end{bmatrix} = 0$$

$$(1-\lambda)(3-\lambda)(2-\lambda) - 2[0] + 0[\dots] = 0$$

$$(1-\lambda)(3-\lambda)(2-\lambda) = 0$$

$$\lambda_1 = 1; \lambda_2 = 2; \lambda_3 = 3$$

$$(A - \lambda_1 I) \vec{x}_1 = 0 \quad \lambda_1$$

$$\begin{bmatrix} 0 & 2 & 0 \\ 0 & 2 & 0 \\ 2 & -4 & 1 \end{bmatrix} \vec{x}_1 = 0 \quad \vec{x}_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$$

$$(A - \lambda_2 I) \vec{x}_2 = 0 \quad \lambda_2$$

$$\begin{bmatrix} -1 & 2 & 0 \\ 0 & 1 & 0 \\ 2 & -4 & 0 \end{bmatrix} \vec{x}_2 = 0 \quad \vec{x}_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$(A - \lambda_3 I) = 0 \quad \lambda_3$$

$$\begin{bmatrix} -2 & 2 & 0 \\ 0 & 0 & 0 \\ 2 & -4 & -1 \end{bmatrix} \vec{x}_3 = 0 \quad \vec{x}_3 = \begin{bmatrix} 0 \\ -2 \\ 0 \end{bmatrix}$$

For the second matrix:

$$\lambda_1 = 1$$

$$\lambda_2 = 2$$

$$\lambda_3 = 3$$

$$\vec{x}_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} \quad \vec{x}_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad \vec{x}_3 = \begin{bmatrix} 0 \\ -2 \\ 0 \end{bmatrix}$$

```
>> a
a =
     1     1
     1     1

>> [V,D] = eig(a)
V =
   -0.7071    0.7071
    0.7071    0.7071

D =
     0     0
     0     2

>> a = [1,2,0;0,3,0;2,-4,2]
a =
     1     2     0
     0     3     0
     2    -4     2

>> [V,D] = eig(a)
V =
     0    0.4472    0.4082
     0     0     0.4082
    1.0000   -0.8944   -0.8165

D =
     2     0     0
     0     1     0
     0     0     3
```

V returns the eigenvectors associated with the eigenvalues in D. In both cases, the answers are correct. The discrepancies are because eigenvectors are not unique. In particular, these eigenvectors that MATLAB returned *appear* to be normalized.

5.)

Linearize the system about the point (1,2)

System: $\ddot{x} + 1.2\dot{x} - 4x + x^3 = 0$

Convert to S.S. form

$$\begin{aligned} z_1 &= x & \dot{z}_1 &= z_2 \\ z_2 &= \dot{x} & \dot{z}_2 &= 4z_1 - 1.2z_2 - z_1^3 \end{aligned} \rightarrow \begin{aligned} f_1(z_1, z_2) &= z_2 \\ f_2(z_1, z_2) &= 4z_1 - 1.2z_2 - z_1^3 \end{aligned}$$

Find Jacobian

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial z_1} & \frac{\partial f_1}{\partial z_2} \\ \frac{\partial f_2}{\partial z_1} & \frac{\partial f_2}{\partial z_2} \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 1 \\ 4-3z_1^2 & -1.2 \end{bmatrix}$$

Evaluate J at (1,2)

$$J|_{(1,2)} = \begin{bmatrix} 0 & 1 \\ 4-3 & -1.2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -1.2 \end{bmatrix} = A$$

Then our system is:

$$\dot{z} = A\bar{z} \quad \begin{aligned} \text{trace } A &= \tau = a+d \\ \text{det } A &= \Delta = ad-bc \end{aligned}$$

$$\lambda_1 = \frac{\tau + \sqrt{\tau^2 - 4\Delta}}{2} \quad \lambda_2 = \frac{\tau - \sqrt{\tau^2 - 4\Delta}}{2}$$

$$\lambda_1 = \frac{-1.2 + \sqrt{1.2^2 - 4(-1)}}{2} = 0.566$$

$$\lambda_2 = \frac{-1.2 - \sqrt{1.2^2 - 4(-1)}}{2} = -1.766$$

The handwritten calculations to linearize the system are to the left, including eigenvalue calculations.

Below is the results of parts a through e, and the associated code.

From part b- the condition number is very small, therefore it is likely that the system is not "approaching a singularity", or, is relatively stable in the neighborhood of (1,2).

```
>> prob6
part a.)
eigenvalues of A are
-1.7662      0
      0    0.5662
```

```
part b.)
the condition number of A is
3.1194
```

```
part c.)
the determinant of A at (1,2) is
-1
```

```
part d.)
the determinant of A^T at (1,2) is
-1
```

```
part e.)
the product of the eigenvalues is
-1
```

```
1 function prob6
2
3 A = [0 1; 1 -1.2];
4 deta = det(A);
5 detaI = det(A');
6 [V,D]= eig(A);
7 c = cond(A);
8 disp('part a. ');
9 disp('eigenvalues of A are');
10 disp(D);
11 disp('part b. ');
12 disp('the condition number of A is');
13 disp(c);
14 disp('part c. ');
15 disp('the determinant of A at (1,2) is');
16 disp(deta);
17 disp('part d. ');
18 disp('the determinant of A^T at (1,2) is');
19 disp(detaI);
20 disp('part e. ');
21 disp('the product of the eigenvalues is');
22 disp(det(D));
```

6.)

a)

Find the value of both states as a function of time
by finding eigenvalues and eigenvectors.
(Find $z_1 = z_1(t)$ and $z_2 = z_2(t)$)

$$\dot{\bar{z}} = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \bar{z} \quad \text{and} \quad \bar{z}(0) = \begin{bmatrix} 0 \\ -4 \end{bmatrix}$$

$$\downarrow A$$

$$\text{So, } (A - \lambda I)\bar{z} = 0 \rightarrow (1 - \lambda)(2 - \lambda) - 6 = 0$$

$$\text{or } \lambda^2 - 3\lambda - 4 = 0 \rightarrow (\lambda + 1)(\lambda - 4) = 0$$

$$\boxed{\begin{matrix} \lambda_1 = -1 \\ \lambda_2 = 4 \end{matrix}}$$

$$(A - (-1)I)\bar{\xi}_1 = 0 \rightarrow \begin{bmatrix} 2 & 2 \\ 3 & 3 \end{bmatrix} \bar{\xi}_1 = 0 \quad \bar{\xi}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$(A - (4)I)\bar{\xi}_2 = 0 \rightarrow \begin{bmatrix} -3 & 2 \\ 3 & -2 \end{bmatrix} \bar{\xi}_2 = 0 \quad \bar{\xi}_2 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$\boxed{\begin{matrix} \bar{\xi}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \\ \bar{\xi}_2 = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \end{matrix}} \quad \text{Then, the general solution is}$$

$$\bar{z}(t) = c_1 e^{-\lambda_1 t} \bar{\xi}_1 + c_2 e^{-\lambda_2 t} \bar{\xi}_2$$

From the initial conditions

$$\bar{z}(0) = \begin{bmatrix} 0 \\ -4 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} \bar{c} = \begin{bmatrix} 0 \\ -4 \end{bmatrix}$$

$$\boxed{\begin{matrix} c_1 = \frac{8}{5} \\ c_2 = -\frac{4}{5} \end{matrix}}$$

$$\left[\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & 5 & -4 \end{array} \right] \quad R_2 \leftrightarrow R_1$$

$$c_2 = -\frac{4}{5} \rightarrow c_1 - 2\left(-\frac{4}{5}\right) = 0$$

$$c_1 = \frac{8}{5}$$

Rewriting:

$$z_1(t) = \left(\frac{8}{5}\right)(1)e^{-t} - \frac{4}{5}(2)e^{4t} \rightarrow$$

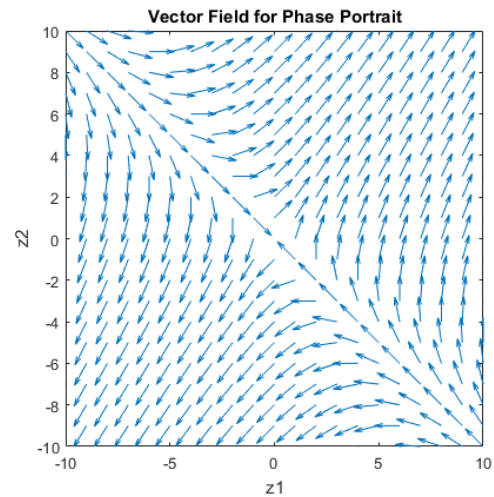
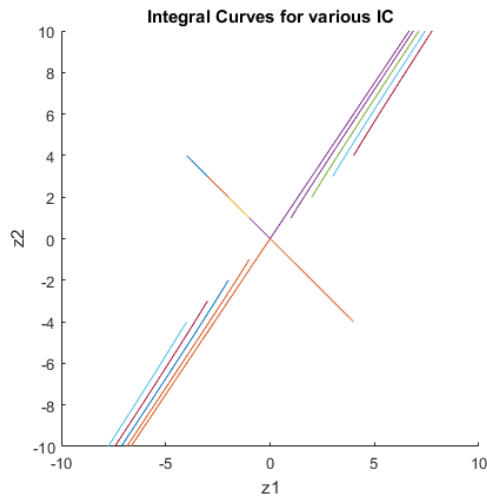
$$z_2(t) = \left(\frac{8}{5}\right)(-1)e^{-t} + \left(-\frac{4}{5}\right)(3)e^{4t} \rightarrow$$

$$\boxed{\begin{matrix} z_1(t) = \frac{8}{5}e^{-t} - \frac{8}{5}e^{4t} \\ z_2(t) = -\frac{8}{5}e^{-t} - \frac{12}{5}e^{4t} \end{matrix}}$$

We have $z_1 = z_1(t)$ and $z_2 = z_2(t)$ by way of finding eigenvalues and eigenvectors

6.)b.)

I created a phase plot, but did not find it very enlightening. I eventually created a few plots, including a vector field plot to better visualize the solutions. You can see that the system converges to the eigenspaces (linear combinations of eigenvectors) as $t \rightarrow \infty$. From the bottom plot you can clearly see the requested initial conditions plotted, as well as from $z_1(0) = [-4:1:4]$ and $z_2(0) = [4:1:-4]$. The eigenspace associated with the positive eigenvalue has solutions tending away from the origin (and equilibrium), while the eigenspace associated with the negative eigenvalue has solutions tending towards the origin (a stable equilibrium point.)



```

1 function prob7
2
3
4 IC = [-4, -4];
5 subplot(1,2,1);
6 hold on;
7 for i = 1:9
8     [t,z]=ode45(@mysystem,[0 20], IC);
9     plot(z(:,1),z(:,2));
10    IC(1) = IC(1) + 1;
11    IC(2) = IC(2) + 1;
12 end
13 IC = [-4, 4];
14 for i = 1:9
15     [t,z]=ode45(@mysystem,[0 20], IC);
16     plot(z(:,1),z(:,2));
17     IC(1) = IC(1) + 1;
18     IC(2) = IC(2) - 1;
19 end
20 xlabel('z1');
21 ylabel('z2');
22 title('Integral Curves for various IC')
23 hold off;
24
25 xlim([-10,10]);
26 ylim([-10,10]);
27 subplot(1,2,2);
28 [z1,z2] = meshgrid(-10:1:10,-10:1:10);
29 dz2 = (3*z1 + 2*z2);
30 dz1 = (z1 + 2*z2);
31 dz2u = 0.25*(dz2./sqrt(dz2.^2 + dz1.^2));
32 dz1u = 0.25*(dz1./sqrt(dz2.^2 + dz1.^2));
33 quiver(z1,z2,dz1u,dz2u);
34 xlim([-10 10]);
35 ylim([-10 10]);
36 xlabel('z1');
37 ylabel('z2');
38 title('Vector Field for Phase Portrait')
39 %title('Phase Portrait for Given System',
40 % 'Various Integral Curves shown with Vector Field')

```

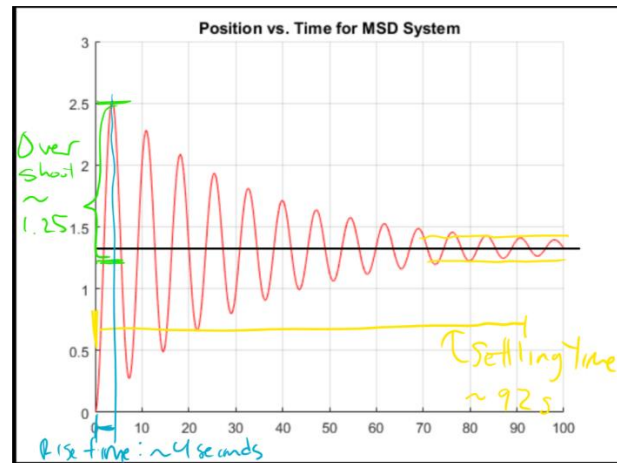
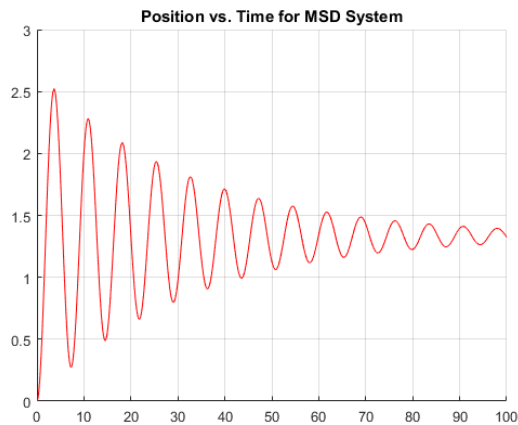
```

1 function zprime = mysystem (t,z)
2
3 zprime = [z(1) + 2*z(2); 3*z(1) + 2*z(2)];

```

7.)

Plotted below is a graph of the numerical (ode45) solution to the system given in problem 1 (mass spring damper.) Next to it are how I estimated the given metrics for this graph.



The estimates are in color, in case they are hard to read, they are as follows:

Overshoot = 1.25 meters

Rise time = 4 seconds

Settling time = 92 seconds

I used MATLAB to calculate the actual values.

The code and results are as follows.

```
>> clear all
>> prob8
risetime = 2.722848e+00
settling time = 9.600000e+01
overshoot = 8.927610e-01
```

Overshoot = 0.893 meters

Rise time = 2.72 seconds

Settling time = 96 seconds

Given the scale of the graph I think the estimates made from it are reasonable for a qualitative estimate of system response.


```

function prob8
% spring mass damper solver
% spring mass damper solver

to = 0;
tf = 100;

[t,x] = ode45(@mysystem,[to tf], [0,0]);
%{
subplot(1,2,1);
plot (t,x(:,1), 'r--')
xlim([0 100]);
title ('Position vs. Time')
legend('Numerical')
subplot(1,2,2);
plot(t, analytical(t), 'b')
xlim([0 100]);
%t = linspace(0,100);
%plot (t,analytical(t), 'x', 'blue')
title ('Position vs. Time')
legend('Analytical')
%}

%this code is for a graph on a single plot

hold on;
grid on;
plot (t,x(:,1), 'r--');
%plot(t, analytical(t), 'b:')
xlim([0 100])
title('Position vs. Time for MSD System')
%legend('Numerical', 'Analytical')

%}

% Calculating the performance metrics for MSD System
%}

% damping coefficient
c = 0.25;
% spring rate
k = 3;
% mass
m = 4;

syms wd wn zeta
wn = sqrt(k/m);
zeta = 0.5*sqrt(c.^2./(k*m));
wd = wn*sqrt(1-zeta.^2);
beta = atan(wd/wn);

risetime = (pi - beta)/wd;
settlingtime = 3/(zeta*wn);
overshoot = exp(-zeta*pi/sqrt(1-zeta.^2));

fprintf('risetime = %d\n', risetime)
fprintf('settling time = %d\n', settlingtime);
fprintf('overshoot = %d\n', overshoot);

```