EE 362K HW 8
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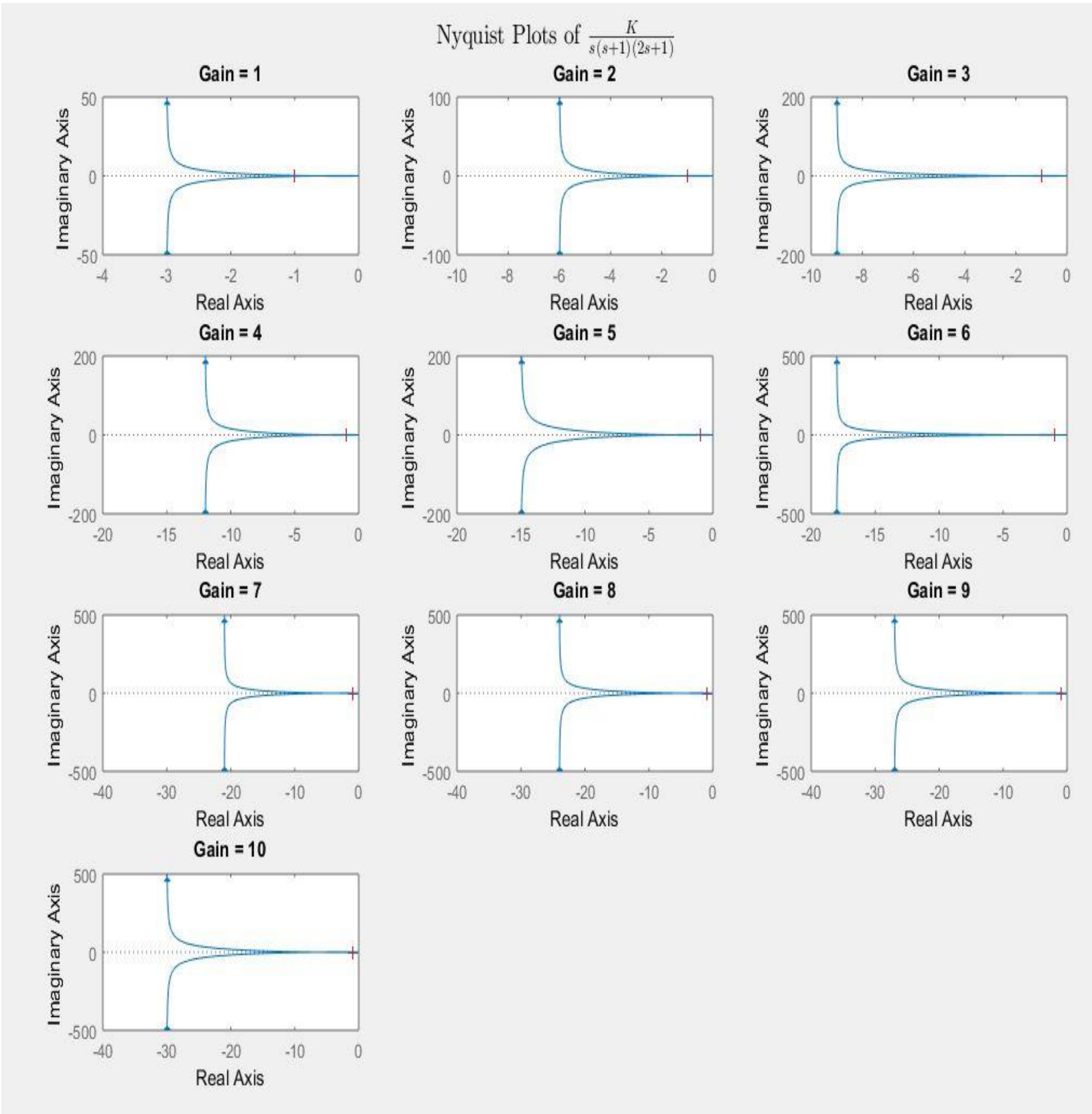
Question 1



The above Nyquist plots result in the following gain margins:

```
>> nplot
For gain K = 1 the gain margin is 0.50062
For gain K = 2 the gain margin is Inf
For gain K = 3 the gain margin is Inf
For gain K = 4 the gain margin is Inf
For gain K = 5 the gain margin is Inf
For gain K = 6 the gain margin is Inf
For gain K = 7 the gain margin is Inf
For gain K = 8 the gain margin is Inf
For gain K = 9 the gain margin is Inf
For gain K = 10 the gain margin is Inf
```

This code gives the preceding output and plots.

```
clear all;
close all;
K = 1:1:10;
warning('off', 'all')

for i = 1:10
    n = K(i);
    d = [2 3 1 K(i)];
    sys = tf(n, d);
    subplot(4,3,i);
    nyquist(sys);
    title(['Gain = ', num2str(i)]);
    [gm] = margin(sys);
    disp(['For gain K = ', num2str(i), ' the gain margin is ', num2str(gm)]);
end
for i = 1:10

end
set(0, 'defaultTextInterpreter','latex');
suptitle('Nyquist Plots of $\frac{K}{s(s+1)(2s+1)+K}$')
```

The value of K for which the system is neutrally stable can be found from the first gain value and its associated gain margin.

$$for\ K = 1, GM = 0.5 \quad and \quad K^* = K \times GM \quad \rightarrow \quad K^* = 0.5$$

Then K* is the desired gain for which the system in neutrally stable. This calculation can be verified by multiplying any of the other K values by their associated gain margins.
Thus, the range of K where the system is asymptotically stable is:

$$0 < K < 0.5$$

<u>Question 2</u>

<u>Part a</u>

I attempted to draw my Nyquist plot for this system by hand by separating the closed loop transfer function into real and imaginary parts. Then, evaluating the real and imaginary parts for different values of omega would allow the plot to be sketched by hand. I must have been making some calculation as no matter what I kept ending up with the same real and imaginary functions, and the points I plotted did not match the Nyquist turned up by MATLAB. Clearly the real part of the closed loop transfer function evaluated at jw should be negative, but I could not make it so. I provided both.

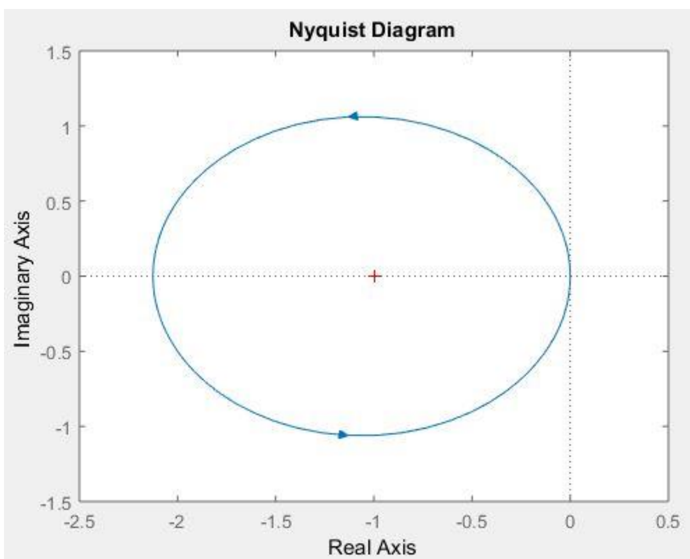
$$T_{OL}(s) = \frac{K}{s-2} \rightarrow T_{CL}(s) = \frac{K}{s-2+K} \text{ and } K = 4.25 \rightarrow T_{CL}(s) = \frac{4.25}{s+2.25}$$

We separate $Re\{T_{CL}\}$ and $Im\{T_{CL}\}$ and evaluate $s = jw$

$$T_{CL}(jw) = \frac{4.25}{jw+2.25} \cdot \frac{2.25-jw}{2.25-jw} = \frac{9.5625 - 4.25jw}{2.25^2 + w^2} = \frac{9.5625}{5.0625+w^2} - j\frac{4.25w}{5.0625+w^2}$$

$$Re\{T_{CL}\} = R(w) = \frac{9.5625}{5.0625+w^2}$$

$$Im\{T_{CL}\} = I(w) = -\frac{4.25w}{5.0625+w^2}$$




```
>> prob2
Gain margin is: 0.47059
Phase margin is: 61.927
```

```
clear all;
close all;
K = 4.25;

sys = tf(K, [1 -2]);
nyquist(sys);
[gm pm wgm wpm] = margin(sys);

disp(['Gain margin is: ', num2str(gm)]);
disp(['Phase margin is: ', num2str(pm)]);
```

Part b

To calculate the gain margin, first we let $G(s) = \frac{N(s)}{D(s)}$ and then evaluate $Im\{N(j\omega)D^*(j\omega)\} = 0$

$$G(j\omega) = \frac{4.25}{j\omega - 2}$$
$$\Rightarrow Im\{N(j\omega)D^*(j\omega)\} = -4.25j\omega$$

$$-4.25j\omega = 0 \; only\ when\ \omega = 0$$

$$\omega = \omega_{pc} = 0 \quad \text{and} \quad GM = \frac{-1}{G(j\omega_{pc})} \qquad \Rightarrow GM = \frac{-1}{\frac{4.25}{-2}} = 0.47$$

The phase margin (PM) is $PM = 180° + \angle G(j\omega_{gc})$ where ω_{gc} comes from evaluating $abs(G(j\omega)) = 1$.

$$abs(G(j\omega)) = 1 \Rightarrow 4.25^2 = 2^2 + \omega_c^2$$
$$\omega_c = 3.75$$

We then evaluate the angle of G at this frequency

$$tan^{-1}\left(\frac{Im\{G(\omega_c)\}}{Re\{G(\omega_c)\}}\right) = tan^{-1}\left(\frac{\frac{-15.9375}{18.0625}}{\frac{-8.5}{18.0625}}\right) = 61.88°$$

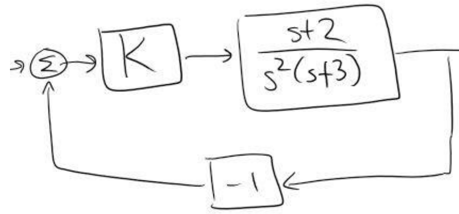
Then, $PM = 180° + 61.88° = 241.88°$

I suspect that MATLAB automatically shifts their phase margin by 180, and this answer is correct.

Question 3

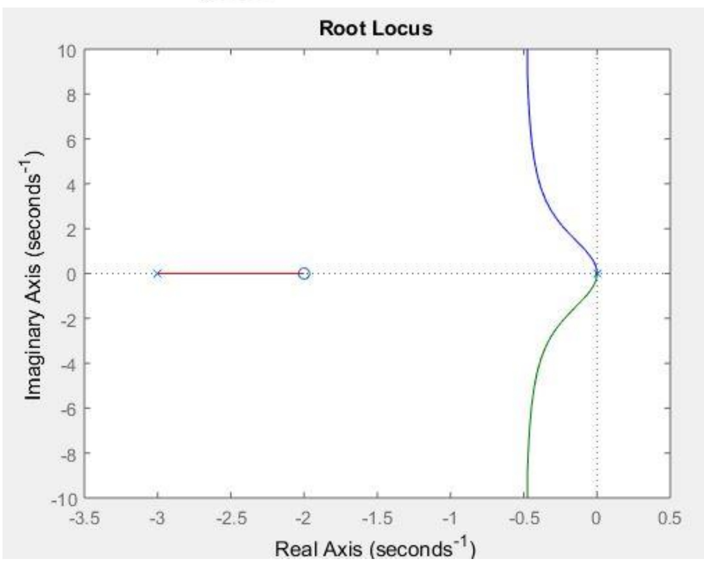
Part a

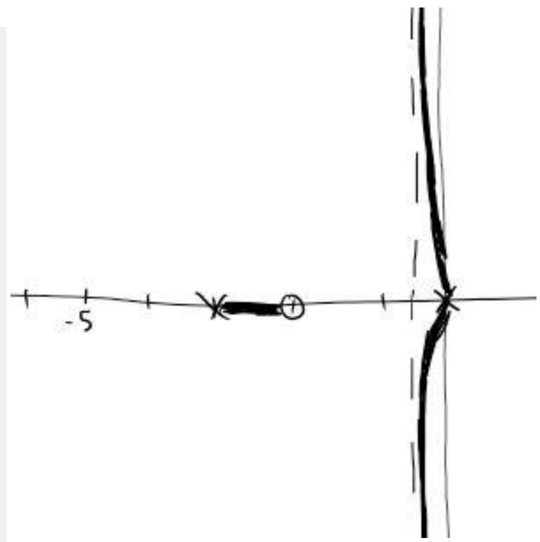
Hand sketch of root locus and calculations, with MATLAB confirmation.



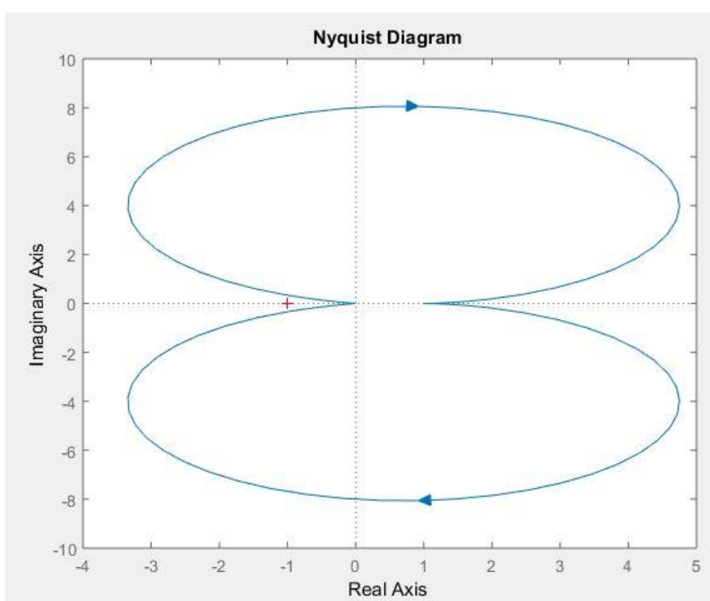
Open Loop Poles: 0 (multiplicity 2), -3
Zeros: -2

$\sigma = \frac{-3-(-2)}{|3-1|} = \frac{-1}{2}$





Nyquist plot from MATLAB



By inspection of the Nyquist plot, there is no value of K for which this closed loop system becomes unstable. Graphically, this can be seen by zooming in on the interval [-1, 0]. The contour does not cross the real axis, and this crossover distance to the imaginary axis must be non-zero for there to be a finite gain margin. The gain margin was calculated via MATLAB to confirm this is the case.

```
>> prob3
Gain margin is: Inf
```

I attempted to show this by hand with a goal of showing that the closed loop system evaluated at the phase crossover frequency would be equal to zero, thus forcing the gain margin to be infinite. I was not successful.

Calculate K for stability

$$H(s) = \frac{L(s)}{1+L(s)} \rightsquigarrow \frac{K(s+2)}{s^2(s+3)+K(s+2)}$$

$$H(s) = \frac{K(s+2)}{s^3+3s^2+Ks+2K}$$

First, we evaluate H at $j\omega$, then set $Im\{H(j\omega)\}$ to zero and solve for ω_{pc}. Finally, evaluate $H(j\omega_{pc})$ and solve $GM = -\frac{1}{H(j\omega_{pc})}$

$$H(j\omega) = \frac{j\omega K+2K}{(j\omega)^2(j\omega+3)+Kj\omega+2K} = \frac{2K+j\omega K}{-j\omega^3-3\omega^2+Kj\omega+2K} = \frac{2K+j\omega K}{2K-3\omega^2+j(K\omega-\omega^3)} \frac{(2K-3\omega^2)-j(K\omega-\omega^3)}{(2K-3\omega^2)-j(K\omega-\omega^3)} = \frac{-\omega^4(K)+\omega^2(K^2-6K)+4K^2}{(2K-3\omega^2)^2+(K\omega-\omega^3)^2} + j\frac{-\omega^3(5K)-\omega^2(K^2)+\omega(2K^2)}{(2K-3\omega^2)^2+(K\omega-\omega^3)^2}$$

Then, $Im\{H(j\omega)\} = \frac{-\omega^3 5K-\omega^2(K^2)+\omega(2K^2)}{D(j\omega)} = 0 \rightsquigarrow$

From here I could not see how to manipulate the equations to derive the result that I wanted.

```
close all;
L = tf([1 2], [1 3 1 2]);
rlocus(L);
figure;
nyquist(L);
figure;
step(L);
[gm, pm, wgm, wpm] = margin(L);
%disp([num2str(gm),' ', num2str(pm), ' ', num2s
disp(['Gain margin is: ', num2str(gm)]);
```

Question 4

First, realizing that G(s) = L(s), we can write that the condition for oscillation is when G(jw) = -1. This gives:

$$G(jw) = \frac{(ka_1 a_2)}{j\omega(j\omega + a_1)(j\omega + a_2)} = -1$$

$$\Rightarrow\ ka_1 a_2 = -j\omega(j\omega + a_1)(j\omega + a_2)$$

Then we separate into real and imaginary components.

$$(1) \qquad ka_1 a_2 = \omega^2(a_1 + a_2) + j(\omega^3 - \omega a_1 a_2)$$

$$k, a_1, a_2 \in \mathbb{R} \rightarrow \omega^3 - \omega a_1 a_2 = 0$$
$$\Rightarrow \omega^2 = a_1 a_2$$

Substituting our condition into (1).

$$k\omega^2 = \omega^2(a_1 + a_2) + j(0)$$
$$\Rightarrow k = a_1 + a_2$$

However, this is the condition for neutrally stable, i.e. undamped oscillation. To achieve any oscillation, we simply allow $k < a_1 + a_2$.

Question 5

I am not gonna lie, I tried to figure this out from the lecture notes, the textbook, and Wikipedia and was unsuccessful. I await your solutions.