The University of Texas at Austin
Department of Electrical and Computer Engineering

EE362K: Introduction to Automatic Control – Fall, 2017

Problem Set 5

Reading Suggestion: Chapters 6.1-6.4 and 7.1 – 7.3 of Åström & Murray

1. Determine if the following system is controllable/reachable.

$$\frac{d\mathbf{z}}{dt} = \begin{bmatrix} -4 & -12 & -7 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \mathbf{z} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u$$
$$y = \begin{bmatrix} 0 & 1 & 3 \end{bmatrix} \mathbf{z}$$

2. Determine if the system in problem 1 is observable.

3. Given the duality of the canonical forms for systems that are both reachable and observable, what does it mean for a system to be one and not the other? In other words, if a system is – for example – observable, why can't the determined coordinate transformation not be used to formulate the system in controllable canonical form?

4. Consider the following system

$$\frac{dx}{dt} = \begin{bmatrix} 3 & 0 & 1 \\ -3 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u$$
$$y = \mathbf{C}x + [0]u$$

(a) Determine if the system is observable for the following values of **C**.

$$\mathbf{C}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}^T, \mathbf{C}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}^T, \mathbf{C}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}^T$$

(b) For the system with $\mathbf{C}_2$, rewrite the system in observable canonical form. Use this result to find the transformation **T**, necessary to put the system in *observable* canonical form.
(c) For the system with $\mathbf{C}_2$, consider the linear observer (**A-LC**) where **L=0**. Does **L=0** produce or not produce a functional (even if inefficient) observer for the system? Explain your answer.
(d) Design a linear observer that asymptotically converges to the true trajectory, i.e. so that the error ($e = x - \hat{x}$) goes to zero. If you found that the suggested observer in part (c) did converge, find an estimator that converges approximately 5-10 times faster. If it did not converge, simple find one that does.

5. Design a state feedback controller for the two vessel system discussed in multiple lectures including as an example for developing an observer in the example given on October 19th. Identify a controller that regulates the level of concentration in vessel to at 10 mL/L within 30 seconds with little or no overshoot. Assume you can only measure the concentration in Vessel 1. The observer values should converge on the correct values 5-10 times faster than the controller converges on the desired value. The observer should work no matter what the actual initial conditions are. So let your observer initially assume the concentrations are zero, but the patient actual begins treatment with 5 mL/L in each vessel. Turn in the graphs and code showing your controller works with the prescribed observer. (+10 points if you also turn in a graph showing your observer converging with the actual concentration values)

6. Complete Problem 7.10 from the text as shown below. *(note: much of the necessary work to set up this problem was given as an example in class.)*

7.10 (Observer design for motor drive) Consider the normalized model of the motor drive in Exercise 2.10 where the open loop system has the eigenvalues $0, 0, -0.05 \pm i$. A state feedback that gave a closed loop system with eigenvalues in -2, -1 and $-1 \pm i$ was designed in Exercise 6.11. Design an observer for the system that has eigenvalues -4, -2 and $-2 \pm 2i$. Combine the observer with the state feedback from Exercise 6.11 to obtain an output feedback and simulate the complete system.