

John Sigmon

Js85773

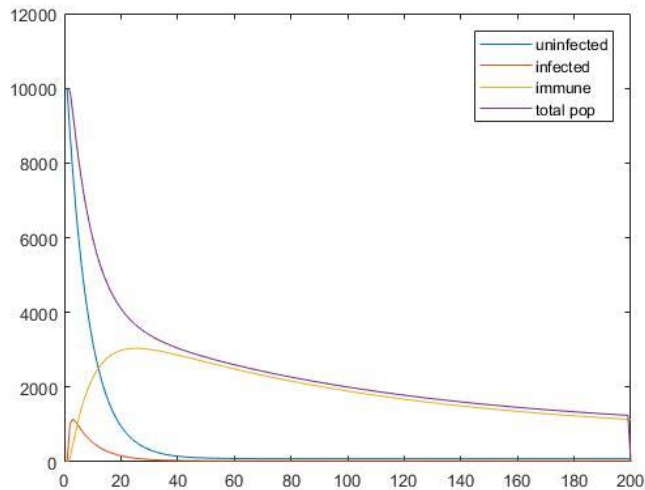
EE 362K

HW 2

1.) I completed the exercises

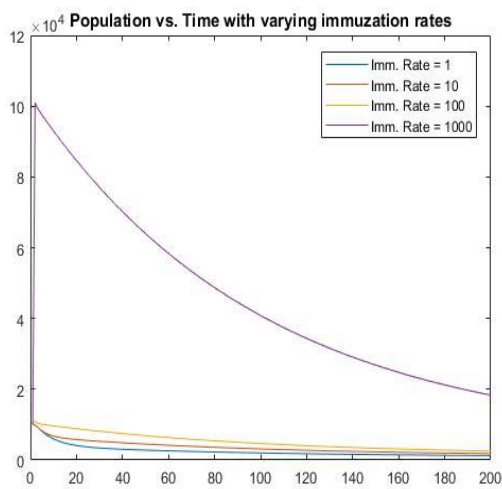
2.)

Code and Graph for part 1 (modification of code)



```
Editor - C:\Users\John\Google Drive\Classwork\EE362K\HW2\Problem2\HW_2_2.m
hw5.m x mysystem.m x rk4.m x epidemic.m x HW_2_2.m x +
1 clear all;
2 z = [10000,10,0,0]';
3 %
4 z(1,1) = 10000; %initial uninfected pop
5 z(2,1) = 10; %initial infected pop
6 z(3,1) = 0; %initial immunized/cured
7 z(4,1) = 0; %dead
8 %
9 a=1; b=10; c=1; %%/100/day die, infected, immunized
10 d=50; e=25; %%/100/day of infected who die or are cured
11 f=1; %%/100/day of immune who die
12 u(1) = 10; u(2) = 10; %%/uninfected and infected added per day.
13 A = [-a-b-c 0 0 0; b -d-e 0 0; c e -f 0; a d f 0]./100;
14 B = [1 0; 0 1; 0 0; 0 0];
15 C = [eye(3) zeros(3,1)];
16 day=1:200;
17 for i=1:length(day)-1
18 z(:,i+1)= z(:,i) + A*z(:,i)+B*u';
19 for j=1:4
20 if z(j,i+1) < 0
21 z(j,i+1) = 0;
22 end
23 end
24 end
25
26 pop= zeros(1,length(day));
27 for i=1:length(day)-1
28 pop(i) = z(1,i)+z(2,i)+z(3,i);
29 end
30 plot(day,C*z, day, pop)
31
32 legend('uninfected','infected','immune','total pop');
```

Code and Graph for part 2 (pop vs imm. rates)



```
Editor - C:\Users\John\Google Drive\Classwork\EE362K\HW2\Problem2\epidemic.m
hw5.m x mysystem.m x rk4.m x epidemic.m x HW_2_2.m x hw22.m x
1 function [pop]=epidemic(c)
2 %input is rate of immunization/day
3 clear z;
4 clear pop;
5 z = [10000,10,0,0]';
6 %
7 % z1 = initial uninfected pop
8 % z2 = initial infected pop
9 % z3 = initial immunized/cured
10 % z4 = dead
11 %
12 a=1; b=10; %%/100/day die, infected, immunized
13 d=50; e=25; %%/100/day of infected who die or are cure
14 f=1; %%/100/day of immune who die
15 u(1) = 10; u(2) = 10; %%/uninfected and infected added per day
16 A = [-a-b-c 0 0 0; b -d-e 0 0; c e -f 0; a d f 0]./100;
17 B = [1 0; 0 1; 0 0; 0 0];
18 C = [eye(3) zeros(3,1)];
19 day=1:200;
20 for i=1:length(day)-1
21 z(:,i+1)= z(:,i) + A*z(:,i)+B*u';
22 for j=1:4
23 if z(j,i+1) < 0
24 z(j,i+1) = 0;
25 end
26 end
27 end
28 %
29 pop= zeros(1,length(day));
30 for i=1:length(day)-1
31 pop(i) = z(1,i)+z(2,i)+z(3,i);
32 end
33 %
34 pop = [1,1,1,0]*z;
```

John Sigmon

Js85773

```
Editor - C:\Users\John\Google Drive\Classwork\EE362K\HW2\Problem2\hw22.m
hw5.m  x  mysystem.m  x  rk4.m  x  epidemic.m  x  HW_2_2.m  x  hw22.m  x  +
1      %hw 2 problem 2
2
3      t = 1:200;
4      plot(t,epidemic(1),t,epidemic(10),t,epidemic(100),t,epidemic(1000));
5      title('Population vs. Time with varying immuzation rates')
6      legend('Imm. Rate = 1', 'Imm. Rate = 10', 'Imm. Rate = 100', 'Imm. Rate = 1000');
7
```

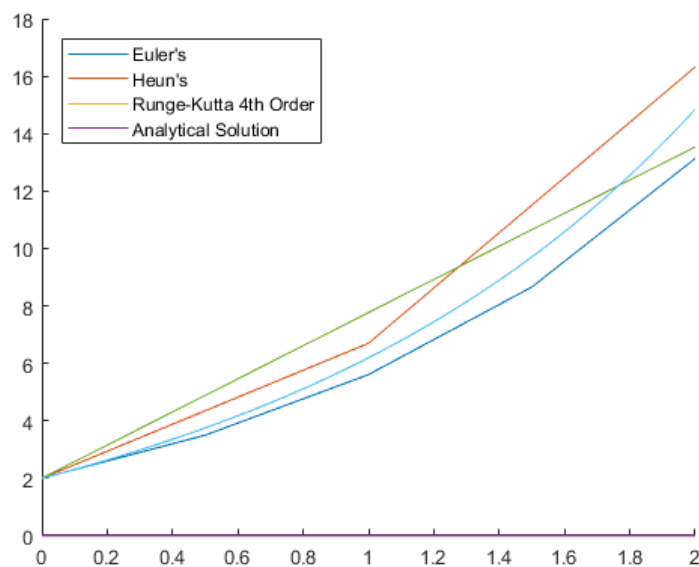
3.)

a.)  $x(2) = 13.1334$

b.)  $x(2) = 16.3198$

c.)  $x(2) = 13.5388$

d.) graph and code



```

function [y] = eulers

% This function solves equations of the type dy/dt = f(y, t)
% Method: y(new) = y(old) + slope * step size

% First define variables
clear t
clear y

h = 0.5;          % Step size
t = 0:h:2;

y = zeros(1, length(t)); % y = a vector
y(1) = 2;         % y(IC) = y0

for i = 1: length(t)-1
    phi = 4*exp(0.8*t(i)) - 0.5 * y(i);
    y(1, i+1) = y(1, i) + phi * h;
end

plot(t,y,'b:'); hold on;

function [y] = heuns

% This function solves equations of the type dy/dt = f(y, t)
% Method: y(new) = y(old) + slope * step size

% First define variables
clear t; clear y;

h = 1;          % Step size
t = 0:h:2;
y = zeros(1, length(t)); % y = a vector
y(1,1) = 2;         % y(IC) = y0

for i = 1: length(t)-1
    si = 4*exp(0.8*t(i)) - 0.5*y(1,i); % si is the slope at beg of interval
    y(1, i+1) = y(1,i) + si*h; % find y(i+1) w/ si
    sf = 4*exp(0.8*(t(i+1))) - 0.5*y(1,i+1); % estimate final slope
    phi = (si+sf)/2; % let slope be avg between i/f
    y(i+1) = y(i) + phi*h;
end

plot(t,y, 'r--'); hold on;

1 function [y] = rk4
2
3 clear y;
4 h=2;
5 t = 0:h:2;
6 y = zeros(1,length(t));
7 y(1)=2; %Initial Condition
8 F_ty = @(t,y) (4)*exp(0.8*t) - 0.5*y;
9
0 for i=1:length(t)-1
1 k1 = F_ty(t(i), y(i));
2 k2 = 2 * F_ty(t(i) + 0.5*h, y(i) + 0.5*h*k1);
3 k3 = 2 * F_ty(t(i) + 0.5*h, y(i) + 0.5*h*k2);
4 k4 = F_ty(t(i) + h, y(i) + h*k3);
5 y(i+1) = y(i) + (k1 + k2 + k3 + k4)/6;
6 end
7
8 plot(t,y, 'g-.'); hold on;

```

John Sigmon

Js85773

```
function [yt]=yt
-
-   t = linspace(0,2);
-   yt = (4/1.3)*(exp(0.8*t)-exp(-0.5*t)) + 2*exp(-0.5*t);
-
-   plot(t, yt, 'c');
```

```
1 function hw3
2
3     eulers;
4     heuns;
5     rk4;
6     yt;
7     legend('show','Location','northwest');
8     legend('Euler's','Heun's','Runge-Kutta 4th Order','Analytical Solution');
9     hold off;
10
11
```

4.)

Something is wrong with these numbers... I went back over code. Ran out of time.

$y(2.5) = 18.657$  by runge kutta

$y(2.5) = 7.9102$  by analysis

absolute error = 1.36

```
1 function [error] = hw4
2
3 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
4 % HW question 4 - Error calculation of RK4 analysis of ODE (f_ty) and
5 % analytical solution (f_t = y)
6 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
7
8 f_ty = @(t,y) (1+t)*sqrt(y);          %%ODE
9 tf = 2.5;
10 h = 2.5;
11 ic = 1;
12
13
14 [t1, y1] = rk4(f_ty,h,tf,ic);          %%RK4 approximation of solution
15 n = length(t1);
16 rkans = y1(n);
17 f_t = @(t) ((t+(t.^2)/2)/2).^2;
18 ans = f_t(2.5);
19 error = abs((f_t(2.5) - rkans))/f_t(2.5);
20
21
```

```

1 function [t,y]=rk4(F_ty,h,tf,ic)
2 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
3 % Pass in an ODE F_ty and a step size h, this method returns two
4 % vectors, t and y which you can plot as classical fourth order Runge Kutta
5 % solutions to your ODE F_ty
6 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
7
8
9 clear y;
10 t = 0:h:tf;           %Time span of interest
11 y = zeros(1,length(t));
12 y(1)=ic;              %Initial Condition
13
14 for i=1:length(t)-1
15     k1 = F_ty(t(i), y(i));
16     k2 = 2 * F_ty(t(i) + 0.5*h, y(i) + 0.5*h*k1);
17     k3 = 2 * F_ty(t(i) + 0.5*h, y(i) + 0.5*h*k2);
18     k4 = F_ty(t(i) + h, y(i) + h*k3);
19     y(i+1) = y(i) + (k1 + k2 + k3 + k4)/6;
20 end

```

5.)

## HW2 Problem 5

Wednesday, September 20, 2017 6:40 PM

Convert ODE to S.S. Form:  $\ddot{x} + 3\dot{x} + 0.5x + 4x = u(t)$

Consider  $u(t) = 0$

Then;

$$\begin{aligned} z_1 &= x & \dot{z}_1 &= z_2 \\ z_2 &= \dot{x} & \dot{z}_2 &= z_3 \\ z_3 &= \ddot{x} & \dot{z}_3 &= -4z_1 - 0.5z_2 - 3z_3 \end{aligned}$$

So;

$$\frac{d\bar{z}}{dt} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & -0.5 & -3 \end{bmatrix} \bar{z}$$

Then, we are looking for  $z_1$  ( $z_1 = x$ ) at  $t=4$

Running the code below shows  $z_1 = 0.1373 \times 10^{-3}$  for 1 step  
and,  $z_1 = 0.5829 \times 10^{-3}$  for 4 steps

```

1 clear all
2
3 h = 1;
4 tf = 4;
5 clear y;
6 t = 0:h:tf;           %Time span of interest
7 y = zeros(3,1,length(t));
8 y(:,1,1) = [1,1,1]'; %Initial Condition
9
10 dy = @(y) [0,1,0;0,0,1;-4,-0.5,-3]*y(:,1);
11
12 %k's become vectors, so
13
14 for i=1:length(t)-1
15     k1 = dy(y(:,1,i));
16     k2 = 2 * dy(y(:,1,i) + 0.5*h*k1);
17     k3 = 2 * dy(y(:,1,i) + 0.5*h*k2);
18     k4 = dy(y(:,1,i) + h*k3);
19     y(:,1,i+1) = y(:,1,i) + (k1 + k2 + k3 + k4)/6;
20 end

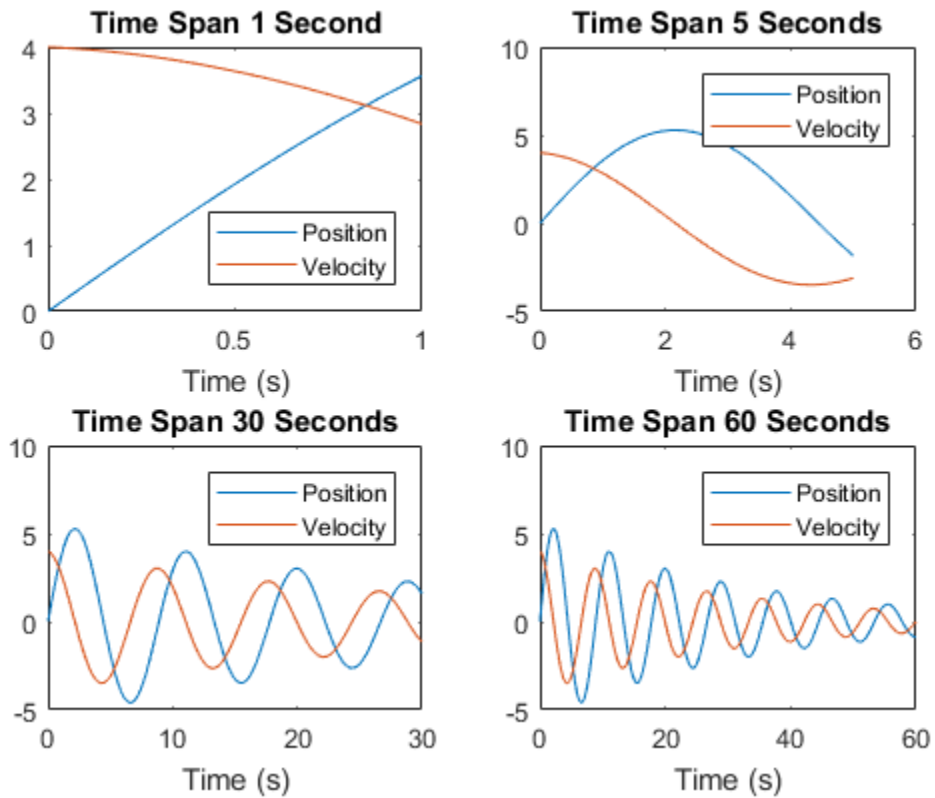
```

John Sigmon  
Js85773

```
1 function zprime=mysystem(t,z)
2
3 - zprime = [z(2);z(3);-4*z(1) - 0.5*z(2) - 3*z(3)];
4     %zprime is a column vector
5
```

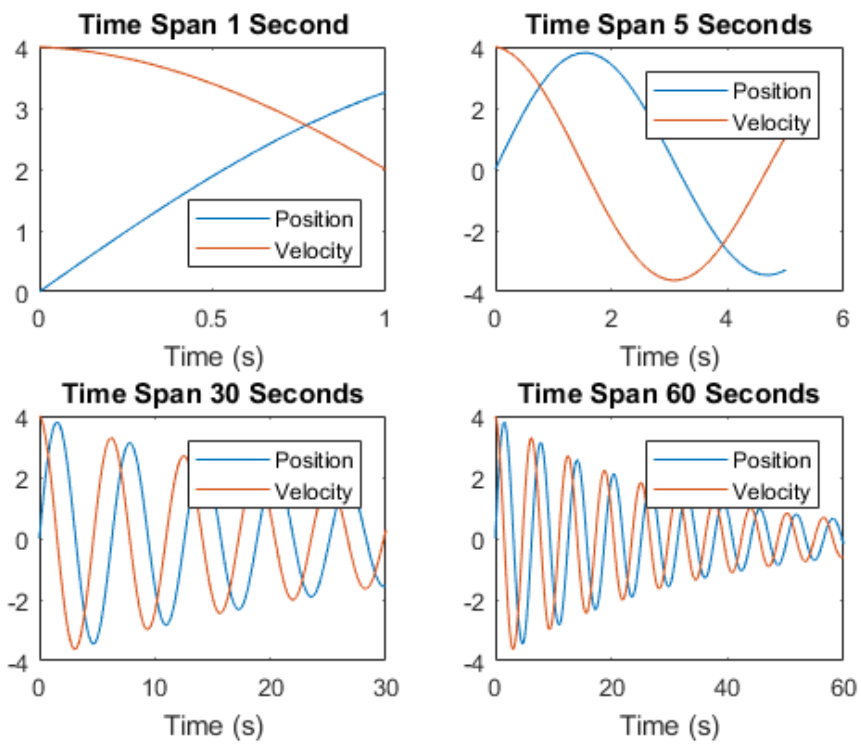
6.)

First graph,  $k = 2$



Second graph,  $k = 4$

John Sigmon  
Js85773



Code:

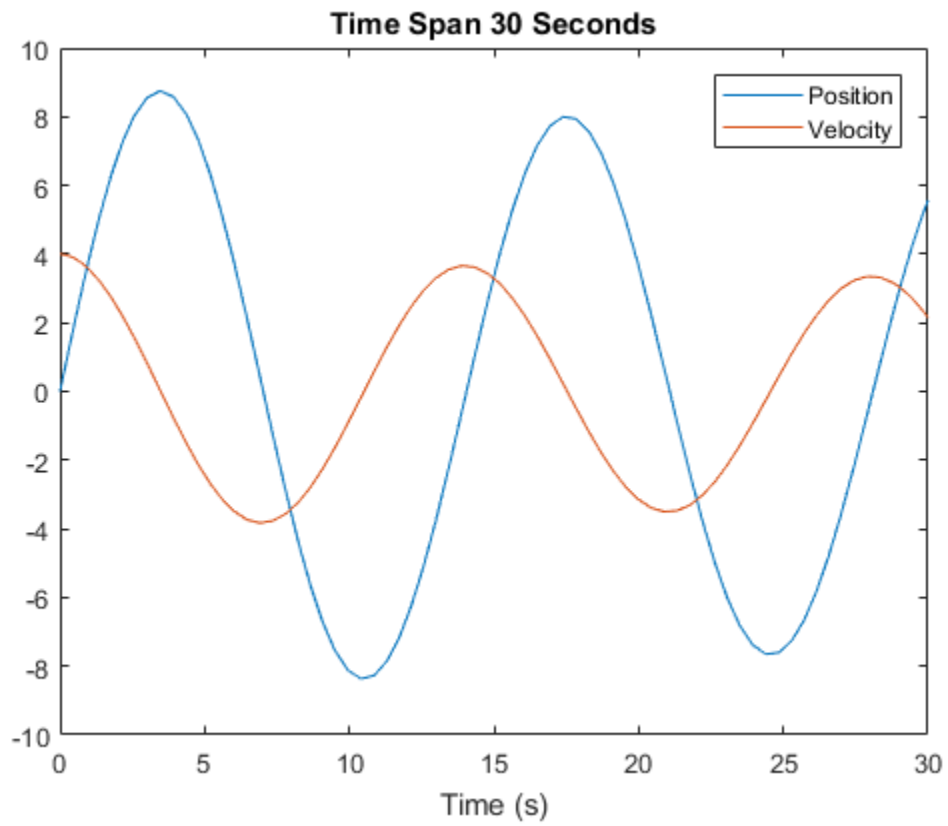
```
1 function zprime = MSD(t,z)
2     k = 4;
3     b = 0.25;
4     m = 4;
5     zprime = [z(2); -k/m*z(1) + -b/m*z(2)];
```

```
1 function hw6
2
3 %first round of subplots for k = 2;
4 subplot(2,2,1);
5 [t,y] = ode45(@MSD,[0 1],[0 4]);
6 plot (t,y(:,1),t,y(:,2))
7 xlabel('Time (s)')
8 legend('Position','Velocity', 'Location','southeast')
9 title('Time Span 1 Second')
10
11 clear t; clear y;
12 subplot(2,2,2);
13 [t,y] = ode45(@MSD,[0 5],[0 4]);
14 plot (t,y(:,1),t,y(:,2))
15 xlabel('Time (s)')
16 legend('Position','Velocity', 'Location','northeast')
17 title('Time Span 5 Seconds')
18
19 clear t; clear y;
20 subplot(2,2,3);
21 [t,y] = ode45(@MSD,[0 30],[0 4]);
22 plot (t,y(:,1),t,y(:,2))
23 xlabel('Time (s)')
24 legend('Position','Velocity', 'Location','northeast')
25 title('Time Span 30 Seconds')
26
27 clear t; clear y;
28 subplot(2,2,4);
29 [t,y] = ode45(@MSD,[0 60],[0 4]);
30 plot (t,y(:,1),t,y(:,2))
31 xlabel('Time (s)')
32 legend('Position','Velocity', 'Location','northeast')
33 title('Time Span 60 Seconds')
```

For the final plot, I changed the mass to 20 kg. As expected it increased the settling time of the system. It also increased the amplitude of the position, and decreased velocity.



John Sigmon  
Js85773



7.) Please see attached sheet.

8.)