Problem 1

a.) What is the probability that x = 1?

We find the marginal from the joint.

And we can see,

b.) What is the probability that x = 1, given y = 1?

From the definition,

From the given table, we can see,

And so

c.) What is the variance of the random variable X?

From the definition,

And so we calculate the expectation of X from its definition,

Using an alternate definition to calculate variance:

So then,

d.) What is the variance of X, given that y = 1?

We find the conditional expectation of X given that Y = y = 1.

Then,

e.) What is ?

From the linearity of expectation, we have,

Where the last term is equal to 3. Then we can calculate the moment generating function of the random variable where ) .

We can find the second and third moments by taking successive derivatives of the m.g.f., and evaluating them at s = 0.

Rewriting our original expression,

Problem 2

We have a subspace defined by the span of and , where

We wish to find the projection of 3 points (vectors) onto this subspace. Our points are:

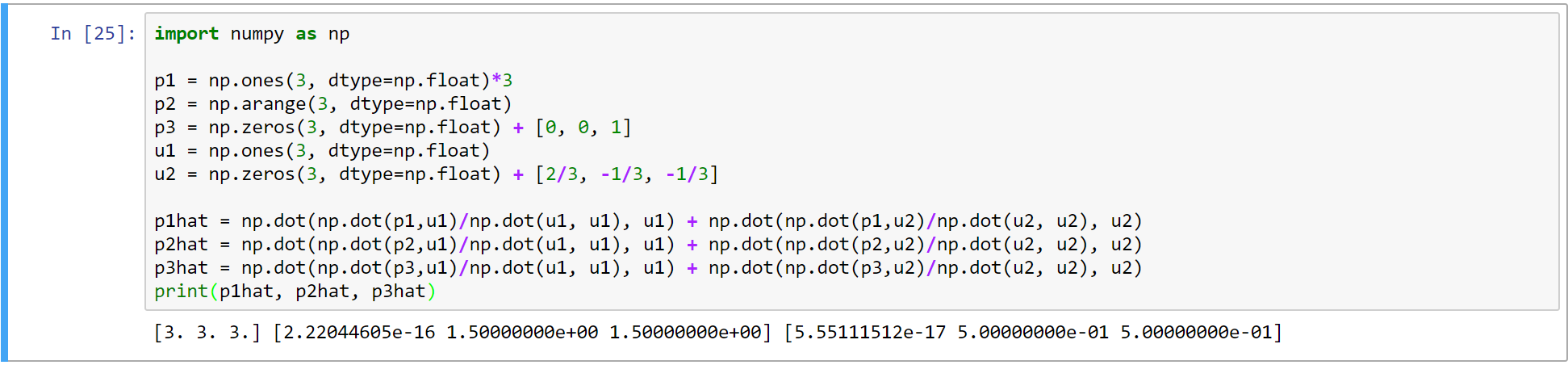
We can call the projections of these points , . If we have an orthogonal basis for W such as , then we can find these projections by the following formula,

(1)

We apply Gram-Schmidt to find the orthogonal basis for W. First, we let . Then to find , we use,

We find that

With our orthogonal basis in hand, we simply apply formula 1 to each p vector. Doing the calculations in python gives the following results:



Problem 3

We know that and therefore . We wish to know the probability of getting 50 heads or fewer in 100 tosses.

This can be represented in a binomial distribution, as the probability of 50 or fewer successes in 100 trials.

Then we know that the expected value of the distribution is:

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From this, the standard deviation is given by:

Using a normal approximation and applying the central limit theorem:

To check our answer, you can also sum the binomial distributions evaluated at 1 through 50:

This calculation was done in python and resulted in the same answer.

