Problem 1

a.) What is the probability that x = 1?

We find the marginal from the joint.

And so we can see,

b.) What is the probability that x = 1, given y = 1?

From the definition,

From the given table, we can see,

And so

c.) What is the variance of the random variable X?

From the definition,

And so we calculate the expectation of X from its definition,

Using an alternate definition to calculate variance:

So then,

d.) What is the variance of X, given that y = 1?

We find the conditional expectation of X given that Y = y = 1.

Then,

e.) What is ?

From the linearity of expectation, we have,

Where the last term is equal to 3. Then we can calculate the moment generating function of the random variable where ) .

We can find the second and third moments by taking successive derivatives of the m.g.f., and evaluating them at s = 0.

Rewriting our original expression,