$$E_D(w) = \frac{1}{2}\sum_{n=1}^{N} r_n \left\{ t_n - w^T \phi(x_n) \right\}^2$$

$$E_D(w) = \frac{1}{2}\left(\Phi \vec{w} - \vec{t}\right)^T R \left(\Phi \vec{w} - t\right)$$

$$= \frac{1}{2}\left(w^T \Phi^T R \Phi w - w^T \Phi^T R t - t^T R \Phi w + t^T R t\right)$$

$$= \frac{1}{2}\left(w^T \Phi^T R \Phi w - 2 t^T R \Phi w + t^T R t\right)$$

$$\nabla_w E_D(w) = \Phi^T R \Phi w - t^T R \Phi = 0$$

$$\Phi^T R \Phi w = t^T R \Phi$$

$$\boxed{w^* = \left(\Phi^T R \Phi\right)^{-1} t^T R \Phi}$$