6.20 $\{x_i\}_{i=1}^N$

$$p(x \mid a,b) \sim \frac{d\phi}{dx} = \frac{b}{b^2+(x-a)^2}$$

* We drop the conditioning of $x \in [-1,1]$ to avoid cluttering the notation.

but since $p(\cdot \mid a,b)$ is a probability density we know

$$p(x \mid a,b) = \frac{1}{\underbrace{\int_{-1}^{1} p(x)\,dx}_{= -\arctan\left(\frac{a-1}{b}\right) + \arctan\left(\frac{a+1}{b}\right) = C(a,b)}} \cdot \frac{b}{b^2+(x-a)^2} \ * = C(a,b)^{-1} \frac{b}{b^2+(x-a)^2}$$

$$p(\{x_i\} \mid a,b) = \prod_{i=1}^{N} p(x_i \mid a,b) = \prod_{i=1}^{N} C(a,b)^{-1} \frac{b}{b^2+(x_i-a)^2}$$

From Bayes theorem we know

$$p(a,b \mid \{x_i\}) = \frac{p(\{x_i\} \mid a,b)\, p(a,b)}{p(\{x_i\})}$$

for our prior we let $p(a,b)$ be uniformly distributed on $-1 < a < 1$ $\quad 0 < b < \infty$.

$p(\{x_i\})$ is irrelevant as it cancels out.

as long as (a',b') is in $[-1,1] \times (0,\infty)$ otherwise its 0

We consider $\dfrac{p(a',b' \mid \{x_i\})}{p(a,b \mid \{x_i\})} = \dfrac{p(\{x_i\} \mid a',b')\, p(a',b')}{p(\{x_i\} \mid a,b)\, p(a,b)} = \dfrac{p(\{x_i\} \mid a',b')}{p(\{x_i\} \mid a,b)}$

$$= \frac{b'}{b} \frac{C(a,b)}{C(a',b')} \frac{b^2+(x_i-a)^2}{b'^2+(x_i-a')^2}$$