Homework 2: Independent component analysis

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1 Introduction

Independent component analysis (ICA) seeks to separate mixed (multivariate) signals into its source signals. It is predicated on the notion that the source signals are non-Gaussian and statistically independent from one another. An example often used to demonstrate ICA is that of resolving individual voices in a crowded, noisy room. Similarly to this example, in this assignment we were given a set of audio signals which could be mixed randomly. Then, the objective was to use ICA to resolve and reconstruct the individual audio signals. The inverse of the mixing matrix needed for this reconstruction was obtained using a gradient descent method to maximize the likelihood of the mixed signals given the an approximate source cumulative density function.

2 Methods

First, we assume a cumulative density function for the source signals of the form

$$g(s) = \frac{1}{1+e^{-s}} \tag{1}$$

If the source signals, S, are related to the mixed signals X by the expression

$$S = WX \tag{2}$$

then the following equation represents the likelihood of the data X for a given linear transformation W:

$$l(W) = \sum_{i=1}^{m}(\sum_{i=1}^{m} log\left[g'(w_j^T x^{(i)})\right] + log|W|) \tag{3}$$

The goal is to maximize the likelihood, which equates to finding a stationary point. This can be accomplished via gradient descent.

The 3×44000 matrix of source signals U were first mixed by applying $X = AU$. The elements of 3×3 matrix A were sampled from a uniform distribution on [0,1].

Then, given starting values of W and parameters η and $maxiter$, the following algorithm was used to find a stationary point of (2):

while $||(\Delta W)||_2 > tol$ **and** $iter < maxiter$

$$\begin{aligned} Y &:= WX \\ Z &:= \tfrac{1}{1+e^{-Y}} \\ \Delta W &:= \eta \left[I + (1 - WZ)Y^T) \right] W \\ W &:= W + \Delta W \\ iter &:= iter + 1 \end{aligned} \tag{4}$$

end

The values of the step size parameter η were determined by trial and error and found to work best on the interval [0.001, 0.01]. Since gradient descent is being used to find a maximum of a potentially very non-convex function, the choice of initial point for W is critically important. If it is far from the true value of A^{-1} then the search may converge to another local maximum or may diverge. The performance relative to initial point is discussed in the results.

3 Results

3.1 Initial guess near A^{-1}

The first trial consisted of starting at the initial point $W_0 = 0.9A^{-1}$, with a step size $\eta = 0.001$, $tol = 10^{-9}$, $maxiter = 10^5$. The search performs very well in this case, converging in 21.78s. In Figure 1, the original, recovered, and mixed signals are plotted, demonstrating very good recovery of the signals. Note that the signals have been normalized and offset for easier interpretation of the plots. The arbitary time period [0.162, 0.182] is shown.

3.2 Initial guess further from A^{-1}

For the next trial, the same optimization parameters were used except that the initial starting point was set as $W_0 = A^{-1} + K$ where the elements of K were drawn from a normal distribution

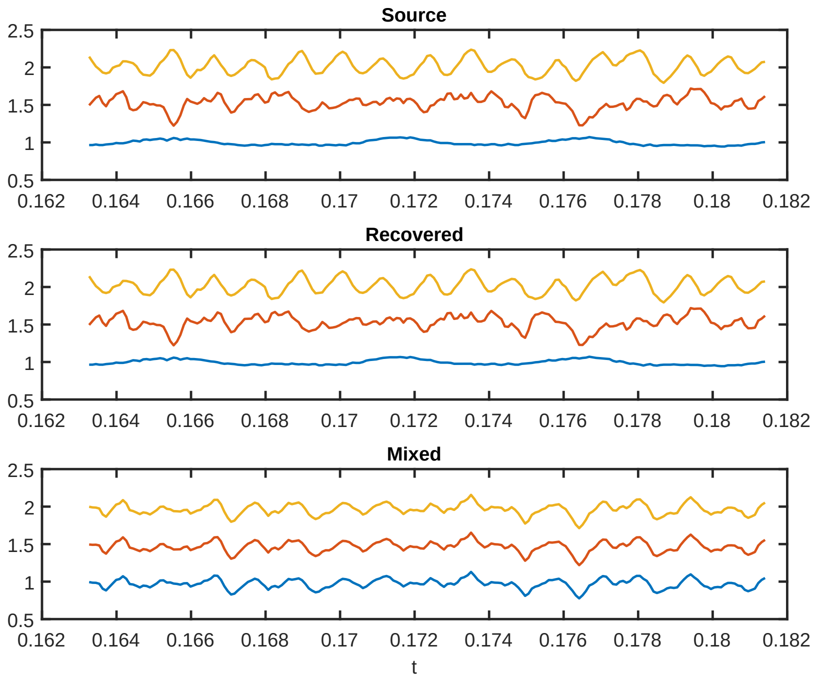

Figure 1: Signal reconstruction starting very close to the true inverse of the mixing matrix

with standard deviation of 0.2.

The procedure was still very, fast, attaining a maximum in 18.59s. The results are plotted in Figure 2. Note that a different (true) random mixing matrix A was used here than in the previous example.

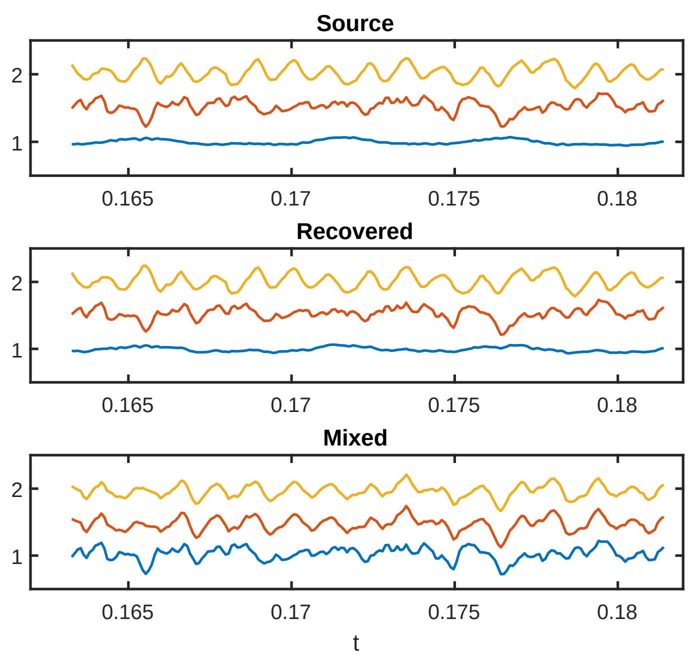

Figure 2: Signal reconstruction starting a small, random distance away from true inverse

3.3 Random initial guess

In this trial, the elements of W_0 were populated by sampling from a uniform distribution on [0, 0.1]. The gradient descent procedure terminated at the maximum number of iterations (10^6) without meeting the convergence tolerance. The signal recovery for the two higher frequency signals is still quite good, as shown in Figure 3. Note that the red reconstructed signal corresponds to the yellow source signal, and the blue reconstructed signal corresponds to the red source signal. Both of these are very close to the true source signals. The lower frequency blue source signal is not able to be recovered as well by the ICA procedure.

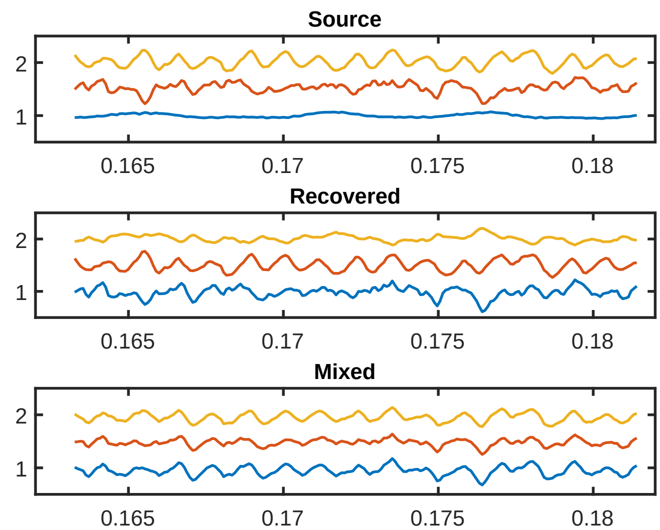

Figure 3: Signal reconstruction starting at a random initial point

4 Conclusions

Independent component analysis can be used to separate a mixed sound signal into source signals. This depends on the validity of the assumption that the source signals are non-Gaussian and statistically independent, both of which were clearly true for this assignment (but may not be in general). Since the objective function is non-convex, the choice of starting point has a great impact on convergence. Poor starting guesses lead to slow convergence and/or convergence to local maxima besides the “true” signal values.