Ising Models of Ferromagnetic and Paramagnetic Material Using a Monte Carlo Method

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1 Abstract

This paper discusses the modeling and implementation of a Monte Carlo method to determine an Ising model. The model consists of observations of the magnetic dipole moments of atomic spins verse temperature. Ferromagnetic materials are found to retain their original magnetic characteristics for the initial few hundred kelvin before. Paramagnetic materials are found to immediately begin losing their magnetic characteristics. Both transitional regions are exponentially decreasing in magnetic characteristics.

2 Introduction and Background

Ernst Ising was a German mathematician whom the Ising Model is named after. The model is a mathematical model of ferromagnetism in statistical mechanics. The purpose of the model is to calculate observations of the magnetic moment dipoles for electrons in a material as the material experiences a change in temperature. There are three groups of materials, each with different tendencies in an Ising model: ferromagnetic, paramagnetic, and anti-magnetic. Each of these types refer to a a ranger of some real values, we will name these values as J-values. Ferromagnetic are J-values greater than zero, paramagnetic have a J-value of zero, and anti-magnetic J-values less than zero. This paper discusses the former two groups: ferromagnetic and paramagnetic.

An observable magnetic dipole moment of the atomic spin can be found by observing the spin of elementary particles, electrons in our case, and they can have either up or down spin. What this means is that each electron in a lattice is either rotating one direction or the opposite at any given moment and the likelihood of "swapping" changes based on the temperature the material is experiencing. The observable electrons are calculated based on the change in energy if an electron was to "swap" and weighted against a probabilistic value. This allows for a two dimensional x-type magnet with respect to its magnetization and energy at varying temperatures to be mapped.

3 Methods and Modeling

The method used for mapping the magnetic moment is known as a Monte Carlo method. Monte Carlo methods are a general computational algorithm that can be applied to most scenarios, if set up properly for a scenario. The method uses a probabilistic factor by taking a random factor weighted against some factor derived from the scenario. For the Ising model the weighted factor took an exponential relationship inversely relating some change in energy with boltzmann's constant and the current temperature.

$$weight = e\,\frac{\Delta E}{kT} \tag{1}$$

Energy Equation is as follows:

$$E = -J \sum \mu_{i,j}\mu_{i,j} - B\mu_b \sum \mu_{i,j} \tag{2}$$

, where E is energy, $\mu_{i,j}$ is denoting an electron at either spin, B is the magnetic field strength, and μ_b is the bohr magneton.

The model calculated magnetic moments over the range of 0 kelvin to 3000. At each temperature one 100,000 perturbations were applied, or chance for a random electron to swap spin. The resulting Ising models are discussed for ferromagnetic and paramagnetic, respectfully, in the results section below.

4 Results

As previously stated the following figures are both ferro and paramagnetic material. Both cases experience exponential decay their magnetic characteristics after their transition point. For ferro this was around 300-300 kelvin and para began decay immediately. An ideal J-value for ferro was around .01. Values around here are related to specific materials, small increments around .01 can denote very dramatic differences in the output and accuracy of the output. Below are the output graphs of both types of material, both graph the sum of magnetic moments with respect to temperature in kelvin.

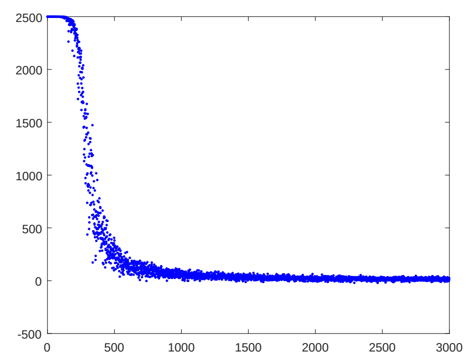

Figure 1:
Ferromagnetic material with a J-value of .01 and a magnetic field of 20 Tesla.

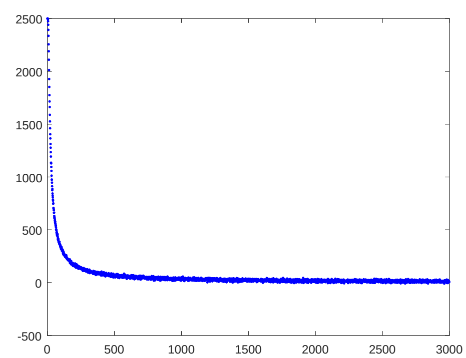

Figure 2:
Paramagnetic material with a J-value of 0 and a magnetic field of 20 Tesla.

5 Conclusion

In conclusion, we found that the the Ising Model is highly dependent on a good J-value to achieve an accurate model. While the J-value is highly sensitive to change, it is useful to determine which material is being computed. Also we concluded that both ferro and para material experience exponential decay in their magnetic material after a transition phase is initiated by reaching a limiting temperature value.

References

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