Modeling a Vibrating Manifold Within a Confined Space Using Runge-Kutta 4

Jeremiah Simmons

November 3, 2016

1 Abstract

Predicting planetary movement has many uses that are significant to astronomers and astrophysicists. The ability to predict, gives us the opportunity to prepare for and study astronomical phenomenons such as, planetary alignments, when will rare sightings such as comets be visible, and more relatable, with next year being 2017: eclipses. The model developed for this computational experiment depicts celestial movement for the planets and Sun for an arbitrary amount of time. Allowing for an estimation of position for any planet at a desired time and desired time step. For planets within the asteroid belt a smaller time step is preferable to ensure minimal error caused the close proximity of the Sun's large mass.

2 Introduction

Civilization has always been amazed by the sky. Celestial bodies were attributed to deities and woven into almost every culture. Often being tied to astrology, religion, early calenders, and navigation. Today our fascination with space has not diminished, but simply we, as a race, are more aware of how the universe works. Rather than attributing the work of gods and goddesses to the movement of our universe, we know how natural physical laws govern the inter-workings of our cosmic home. Issac Newton first discovered the basic mathematical processes of how gravity works back in the 1600s, which is outlined in his work: Principia Mathematica. For general gravitational force interactions the equation:

$$F_{grav} = Gm_1m_2/R^2 \tag{1}$$

where G is the gravitational constant, m_1 and m_2 are the mass of the two bodies that force is being calculated for, and R is the radius between the center of both objects. G in SI units is roughly:

$$6.77 * 10^{-11} \tag{2}$$

Solving for an acceleration is simply done by using substituting Newton's second law in (1): The acceleration of an object as produced by a net force is directly proportional to the magnitude of the net force, in the same direction as the net force, and inversely proportional to the mass of the object. The resultant equation now has the form, given by some minor algebra:

$$F = ma \tag{3}$$

$$F_{grav} = m_2a \tag{4}$$

$$m_2a = Gm_1m_2/R^2 \tag{5}$$

$$a = Gm_1/R^2 \tag{6}$$

Additionally, the following kinematic equations allow for insight on how acceleration relates to position and velocity.

$$v_{velcity} = v_{initial} + at_{tstep} \tag{7}$$

$$x_{position} = v_{velocity}t_{tstep} + x_{initial} \tag{8}$$

3 Methods and Modeling

In order to calculate the iterative steps using some time step, $tstep$, two series methods were used. The first, more general, Euler's method calculates the first iterative steps following the initial conditions. Since Verlet's method has an inherently lower error since Euler's is a single integrated series, the main for-loop utilizes the former method.

Euler's Method:

$$V_{i+1} = V_i - A_i * tstep; \tag{9}$$

, where V is the velocity within in an iterative step, i, A is acceleration within an iterative step, and *tstep* is a time step between iterative steps.

Verlet's Method:

$$X_{i+2} = X_{i+1} - X_i + A_{i+1} * tstep^2; \tag{10}$$

, where X denotes a position within a dimension.

For the purpose of the planetary bodies, an entire loop was desired by all bodies. A *tstep* of 1 Earth day was used, and since Pluto takes roughly 250 Earth years to complete one orbit the total iteration of time covered is 91250 days, or 91250 *tsteps*, minimum.

4 Results

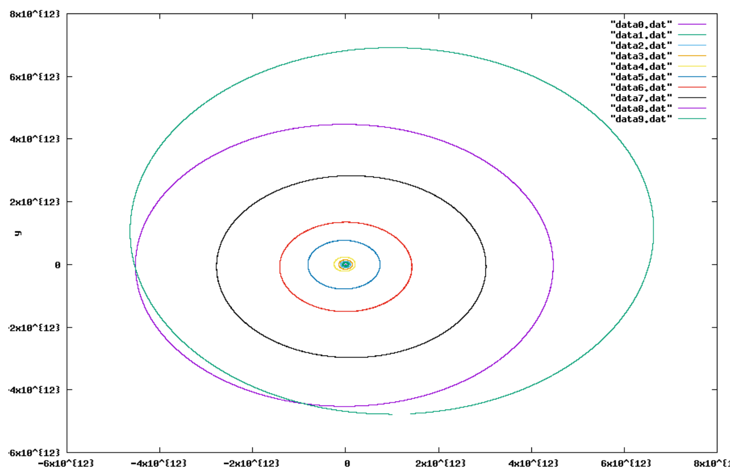

Figure 1: 250 Earth orbits, and the equivalent orbits of the other planetary bodies for the entire Sol system.

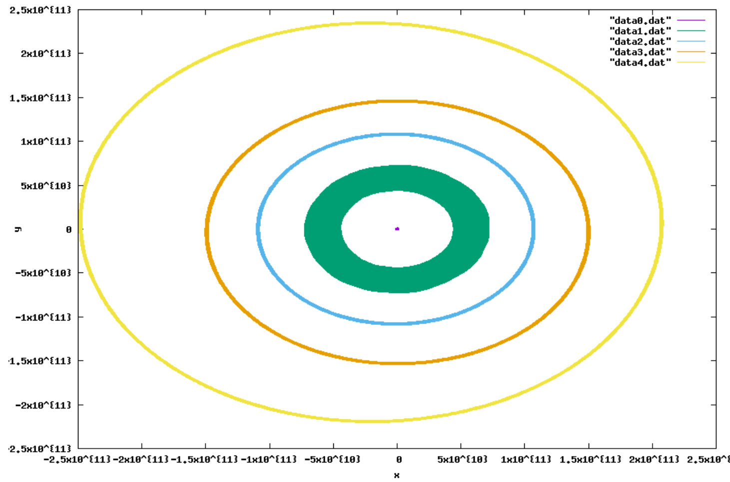

Figure 2: 250 Earth orbits, and the equivalent orbits of the other planetary bodies for the Sun, Mercury, Venus, Earth, and Mars.

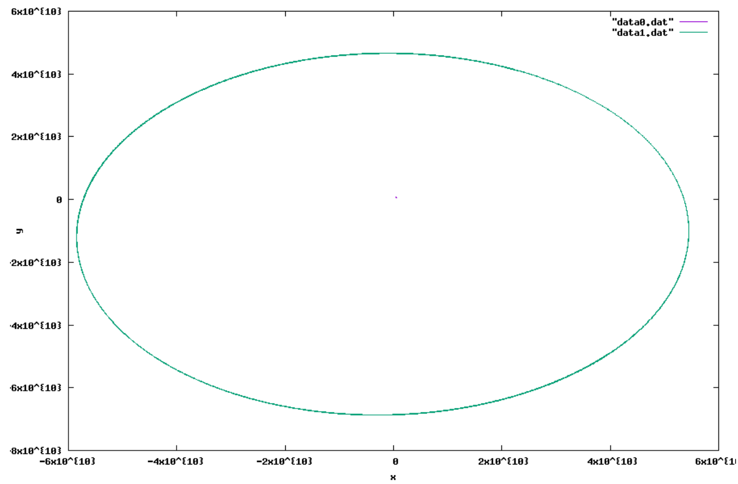

Figure 3: One Earth year with *tstep* of one hour, observing the Sun, and Mercury only.

5 Conclusion

By using a simplified model of the solar system allows additional material, e.i. asteroids, moons, and other debris, to appear negligible for the purposes of developing stable orbits among the planetary bodies. Mercury shows unusual oscillations within the position vector in Figure 1 and 2. This is expected since Mercury is so close to the Sun and the *tstep* of one day is very large and may create those non-uniform patterns over a long period of time. When the *tstep* is dropped significantly we can see a clearly uniform orbit for Mercury in Figure 3. Therefore confirming the model is not lacking inherently due to precision of the code or approximations, rather to the initial conditions for how often in time the positions are calculated. This is easily accounted for by adjusting the *tstep* to be smaller as the position of any arbitrary planet in relation to the Sun shrink, such as was done with Figure 3 to isolate the Sun, and Mercury.

References

[1] Alan B. Chamberlin, Ryan S. Park. *Solar System Dynamics* 2016. Nasa, Jet Propulsion Laboratory, California Institute of Technology.

[2] Issac Newton. *Principia Mathematica* 1687.

[3] Wikipedia. *list of Solar System Objects by Size* 2016. Wikimedia Project.