Modeling Kinematics of Golf Ball Trajectory With Varied Angular Velocities, Accounting for Magnus Effect

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1 Abstract

When observing the flight characteristics of a ball, it is difficult to properly examine the effects of individual effects such as spin, and drag on trajectory. A computational approach allows us to focus on an individual effect. In this experiment angular velocity was that effect. A variety of angular velocities were used to plot changes in distance traveled in both x, and y directions, as well as output changes in the z direction. The increase in maximum height flown is incredibly small, but the ball floated at higher altitudes for a longer duration of time. The most distinct change was the ball flying farther than eleven meters when zero revolutions per minute's (rpm) performance is compared to 6400 rpm, and more than 26 meters in the rightward direction.

2 Introduction

Isaac Newton first presented the idea of gravity more than 300 years ago as a means to define the natural pulling sensation all objects with mass experience towards larger bodies of mass. Newton through, his research, gave us three laws of motion, most notably, the second where it states: force equals mass times acceleration. Any mass that experiences acceleration, has an associated force. In golf , the ball experiences several different forces. One of these forces being the Magnus force. The Magnus force in the direction of the velocity, v, crossed with the angular velocity, ω, or rpm Nathan:

$$F_m = C_L * \rho * A * (\omega \times v)^2; \tag{1}$$

where C_L is the the coefficient of lift, A is the cross-sectional area of the golf ball, ρ is the density of the fluid the ball is traveling through, in our methods we used the density of air. In the present experiment the effect of rotational angular velocity is examined in order to apply a graphical representation of the distance flown by the theoretical golf ball. Naturally a ball with top spin will fly higher, and farther Adair. Using this knowledge, the method will have a limitation of performance. Other forces are also taken into account during calculations. The force from drag and, intuitively, the force of gravity plays a role in this study. The equation for drag force is as follows:

$$F_d = k * (v \bullet v) * \mu; \tag{2}$$

where v is the vector of net velocity of the wind and initial conditions and is dotted with itself, μ is a unit vector of net velocity in some direction and k is the shorthand representation for:

$$k = C_d * A * \rho; \tag{3}$$

with C_d being the drag coefficient. John McPhee and Gordon Andrews were able to calculate the drag coefficient for a golf ball in their 1987 article as proportionally related to mass where McPhee, Andrews

$$\frac{k}{m} = .25s^{-1}. \tag{4}$$

3 Methods and Modeling

The method is as follows: In order to calculate the iterative steps using some time step, *tstep*, two series methods were used. The first, more general, Euler's method calculates the first iterative step following the initial conditions. Since Verlet's method has an inherently lower error since Euler's is a single integrated series, the main for-loop utilizes Verlet's method.

Euler's Method:

$$V_{i+1} = V_i - A_i * tstep; \tag{5}$$

, where V is the velocity within in an iterative step, i, A is acceleration within an iterative step, and *tstep* is a time step between iterative steps.

Verlet's Method:

$$X_{i+2} = X_{i+1} - X_i + A_{i+1} * tstep^2; \tag{6}$$

, where X denotes a position within a dimension.

For the experiment three trials were conducted: no spin, 1600, 3200 and 6400 rpm.

4 Results

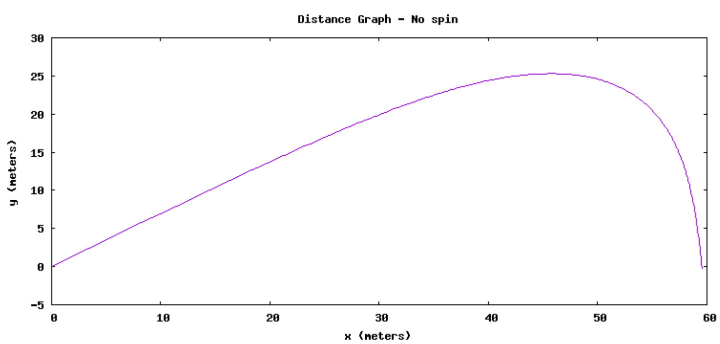

Figure 1: Trial 1: No angular velocity added.

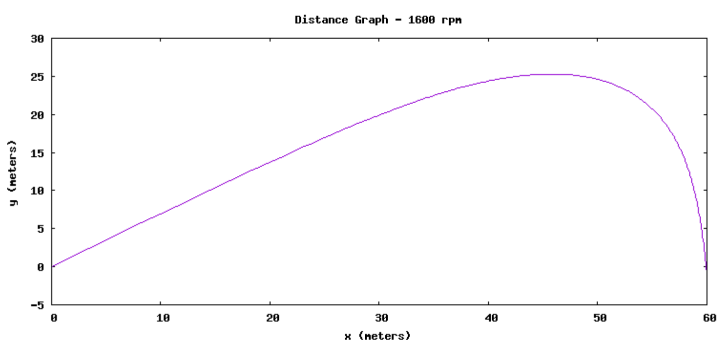

Figure 2: Trial 2: 1600 rpm used to calculate angular velocity.

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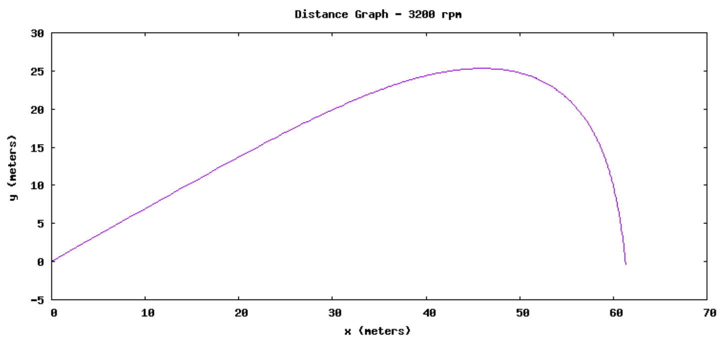

Figure 3: Trial 3: 3200 rpm used to calculate angular velocity.

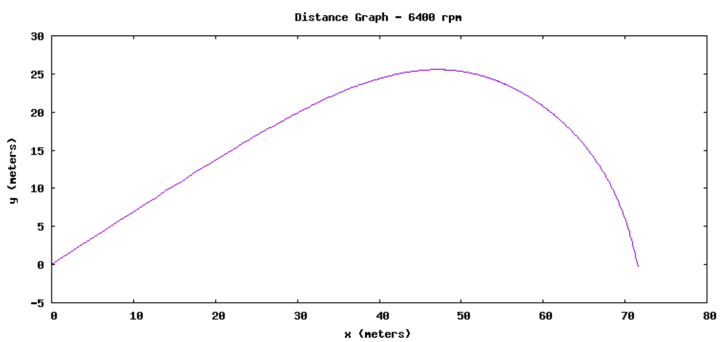

Figure 4: Trial 4: 6400 rpm used to calculate angular velocity.

As shown between the four figures the ball flies farther as the topspin is set faster (or higher rpm). Topspin is characterized at a counterclockwise rotation, if the positive y-axis is oriented orthogonal going away from the surface of the Earth. In addition, a rpm of zero results in only a rightward movement of 8.6 meters, and at 6400 rpm, 35 meters. The positive z-axis is defined as the space to the right of forward movement. This baseline z-axis shift is because of the drag force from fluid resistance and extended by Magnus force. It is to be noted that the average pro golfer only achieves a spin of 3200 to 4500 rpm on a golf ball. So 6400 rpm stands as an extraneous situation, to simply amplify the results to better see the effect of the Magnus force on flight.

5 Conclusion

In conclusion, the Magnus force results in significantly farther distances in high rpms, roughly 6000. Within the average human performance the Magnus force contributes a marginal increase, with regard to the initial velocity that was chosen in this method. Also it is important to note the addition of movement in the z-axis, even 8.6 meters, which would be impossible in an experimental test, is a significant distance if attempting to hit the ball into the hole.

References

[1] Robert K. Adair. *The Physics of Baseball* 1987-1989. Department of Physics, Yale University and National Baseball League.

[2] Alan M. Nathan. *The effect of spin on the flight of a baseball* 2007. Department of Physics, University of Illinois.

[3] John J, McPhee, Gordon C. Andrews. *Effect of sidespin and wind on projectile trajectory, with particle application to gold* 1988. Department of Mechanical Engineering, University of Waterloo.