Results from the three simulation studies:

Note: for all of these studies, I considered a grid free-flowing if it did not reach gridlock within 1000 iterations. The vast majority of grids that do this are indeed free-flowing for the small grid sizes being checked.

I originally ran all of these trials 100 times instead of 10, to get smoother data, but this took an extremely long time.

1. Chance of Reaching Gridlock (by density)

    a. Method: I tested only a 10x10 grid, mainly so I could take a lot of data points and thus get a good answer. The density *p* required to get gridlock, as opposed to a free-flowing grid, will vary based upon the size of the grid, but for a 10x10 grid that transition density is about 0.5. I tested a 10x10 grid for gridlock 10 times for each density between 0.01 and 1.00, increasing by factors of 0.01. I then found the percentage of grids that reached gridlock for each of the 100 possible densities, and plotted the percentage of 10x10 grids reaching gridlock for a certain density against the densities of the grids.
    b. Results: As I mentioned earlier, the transition between free-flowing traffic and gridlock occurs around a density of 0.5. This transition is not immediate at any particular density, rather it occurs over a range of densities, as my figures indicate. Theoretically it would be possible for a density 0.01 grid to lock up, if it simply happened to generate with far more spaces filled than the average density 0.01 grid. Thus, the transition is continuous, not occurring at any specific density – grids begin locking up around a density of 40%, and by a density of around 60%, almost all grids lock up.

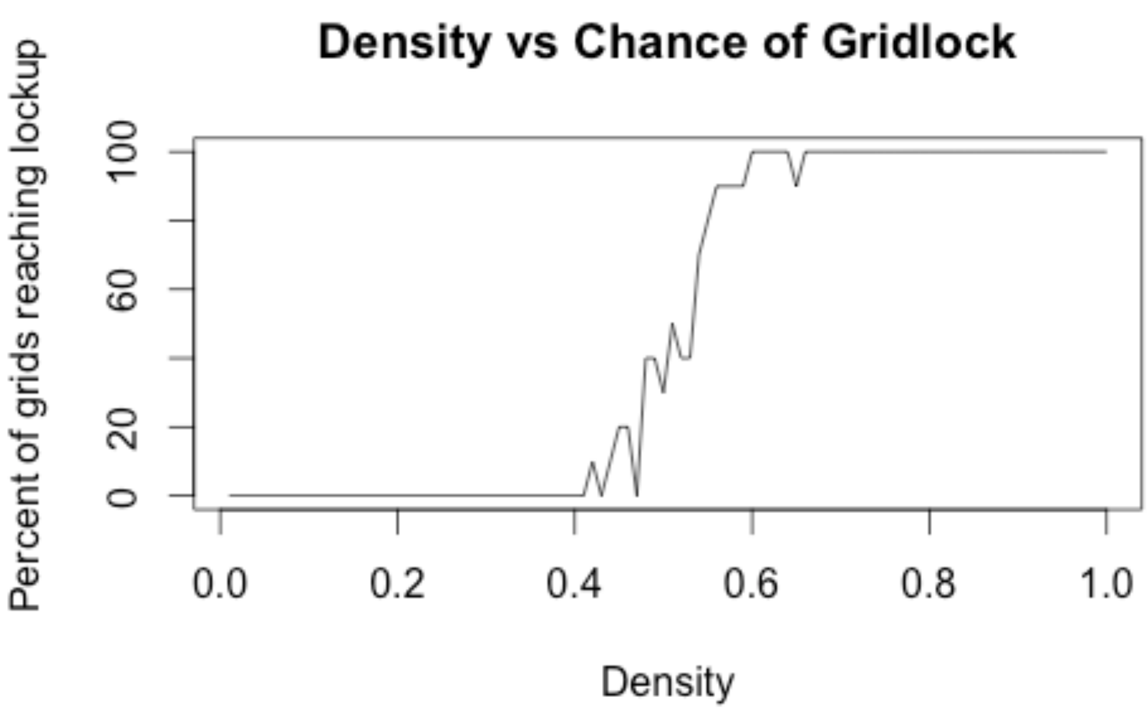
2. Number of Steps taken before Gridlock (by density)

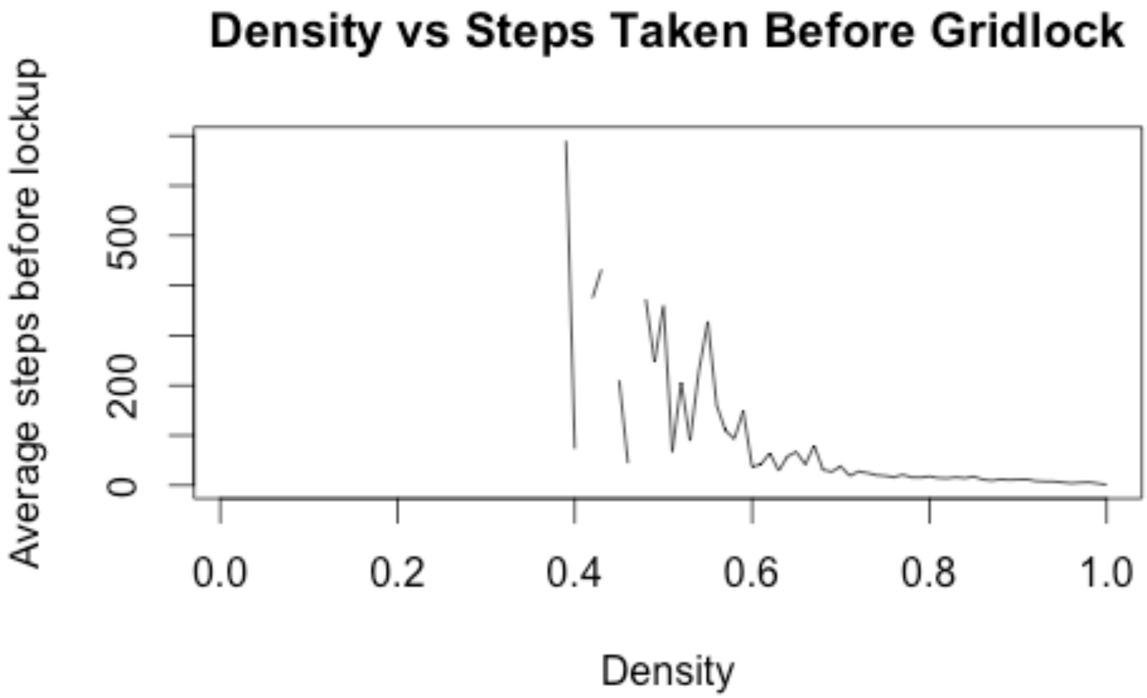
    a. Method: I performed roughly the same general test as in #1, checking a 10x10 grid 10 times for each of 100 densities, attempting to find the number of steps necessary for a grid to lock up. I set grids that did not lock up to appear as NAs, and then plotted the average number of steps to achieve gridlock against the densities of the grids.
    b. Results: For a 10x10 grid, there are no results before a density of about 0.4, as the grid will never lock up. The white spaces on the graph indicate densities at which no grids reached gridlock – fairly common for low densities, but not occurring past a density of 50%. From there the average number of steps taken falls to 0 as density approaches 1, since a grid with 100% density is instantly gridlocked. Interestingly, though my code allows a matrix to run 1000 steps before assuming it to be free-flowing, there is only one density for which matrixes take, on average, greater than 500 steps to lock up. This shows the effectiveness of 1000 steps as a cutoff point.

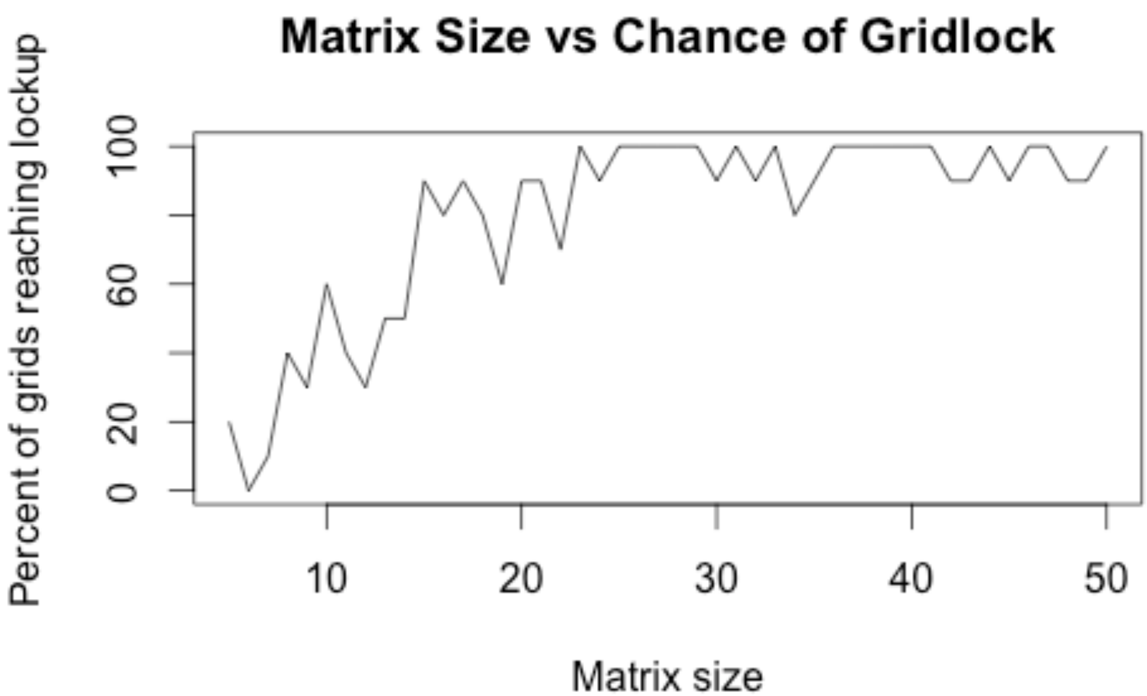
3. Chance of Reaching Gridlock (by grid size)

a. Method: For this test I kept density at 0.5 and varied the size of the grid, from 5x5 to 50x50, checking for gridlock 10 times for each grid size. I then found the percentage of grids that reached gridlock for each of the 100 possible sizes, and plotted the percentage of grids reaching gridlock for a certain size against the size of the grids.
b. This to me is the only simulation where only running 10 times for each data point created some strong adverse effects in terms of smoothness. Nevertheless, the relationship is clear: as the grid increases in size, it tends to lock up more often, as the chances that it will be arranged in a free-flowing solution decrease. This originally seemed somewhat counterintuitive – shouldn't a larger matrix have more room for free-flowing streams to occur, even if most of it is in gridlock? - but made sense as I considered the fact that free-flowingness really depends on a grid's ability to avoid any portion falling into gridlock in the first place, and thus bringing down the rest of the grid. With a larger grid, it's more likely that at least some part will eventually hit gridlock, and bring all the other parts into gridlock along with it.

Graphical Results:




Density vs Steps Taken Before Gridlock
Average steps before lockup
0
200
500
0.0
0.2
0.4
0.6
0.8
1.0
Density


Matrix Size vs Chance of Gridlock
Percent of grids reaching lockup
0
20
60
100
10
20
30
40
50
Matrix size