Stat 133, Fall 2014
Homework 6: Simulation Study of the BML Traffic Model
Due Friday, November 7th.

The Problem

The following text is adapted from NSF Nuggets Web page `http://www.mathinstitutes.org/nuggets/traffic_gridlock.html`. See also the description at `http://en.wikipedia.org/wiki/BihamMiddletonLevine_traffic_model` and use the visualizations at `https://www.jasondavies.com/bml/` as a reference in your work.

The BML traffic model is perhaps the simplest system exhibiting phase transitions and self-organization. Moreover, it is an underpinning to extensive modern studies of traffic flow. The general belief is that the system exhibits a sharp phase transition from free flowing to fully jammed, as a function of initial density of cars. The existence of this sharp transition has been cited in the scientific literature several hundreds of times.

However, D'Souza discovered intermediate stable phases, where jams and free flowing traffic coexist. The geometric structure of such phases is highly regular, with bands of free flowing traffic intersecting at jammed wavefronts that propagate smoothly through the space. These intermediate states have a crisp, well defined geometric structure, which is a consequence of the finite size and aspect ratio of the underlying lattice.

The model is summarized as follows:

- Two species of particles, "blue" and "red", are initialized at random sites of a 2-dimensional r-by-c lattice
- On even timesteps, all blue particles simultaneously attempt to move one site to the north. They succeed unless the site they wish to occupy is non-empty, in which case they are "blocked" and stay stationary for that timestep.
- On odd timesteps the red particles undergo an analogous process, only attempting to move towards the east.
- When a particle reaches the edge of the grid, it moves to the opposite edge, e.g. when it arrives at the top of the grid, it's next move is to the bottom of the grid.
- Particles are initialized at random locations, in accordance with a uniform density. After the random initialization step, the dynamics are fully deterministic.

See for images and more information about the model.

The BML Experiment

The following questions and comments are meant to help you organize your thoughts and code. When writing the R code to carry out a BML traffic experiment use the template given in the file `bml_functions.R`.

- You will need a function that initializes the system.
- You will need to create functions that move one timestep.
- What are the input parameters for one experiment?
- What are the output values that your experiment should return? Remember, you want to conduct an analysis of the results from many experiments. With this in mind, what information do you think would be helpful to return?
- As you write the functions for the experiment, be sure to test them out to confirm that they are doing what you think they should be doing. Make use of functions such as `browser()` to step through a function, and `image()` to visualize each step.
- Consider what your stop criteria should be.
- As you proceed with piecing the experiment together, you may find that you will need to rewrite one or more functions. That's OK - we often find better ways to do things after we have had a chance to wrestle with the problem.
- Once you have all of the pieces in place to run one experiment, try it out with a very small grid. Use the return value to study what you found and see if it makes sense.

The BML Simulation Study

Use your BML experiment to conduct a simulation study of the behavior of the BML model. Below are some questions to answering. Put the code you use to run the simulation study in the file `bml_simulation.R`.

1. For what values of p, the density of the grid, did you find free flowing traffic and traffic jams? Did you find any cases of a mixture of jams and free flowing traffic?

2. How many simulation steps did you need to run before observing this behavior?
3. Does the transition depend on the size or shape of the grid?

To answer these questions, run the experiment for different values of the input parameters. For each set of input parameters that you examine, run the experiment multiple times to understand the variability in the process. For example, does a $p = 0.8$ always end in a traffic jam or does it only end in a traffic jam 70% of the time? Answer these questions in the text file `bml_comments.txt` or `bml_comments.pdf`. You are encouraged to include 1-4 plots to support your observations, either as a separate file (`bml_figures.pdf`) or embedded in the pdf file with your comments.

Your BML Results

In your repository on git please make sure that you have the files:

- `bml_functions.R` : The basic functions to initialize the system, to do one timestep, and to do a single run.
- `bml_simulation.R`: The code you ran when studying the system (you will call the functions defined in `bml_functions.R` and run the experiment for different input values and save appropriate output.
- `bml_comments.txt or bml_comments.pdf` : Write a paragraph or two describing the results of your simulation study and save in a text or pdf file. You can include tables if you wish.
- `bml_figures.pdf` : Your plots, can be included in the file `bml_comments.pdf`.

NOTE: This is a large project. You have two weeks to complete it. You may need to use a computer that is more powerful that your laptop to carry out the simulation study. In section you will be able to sign up for an account on the Statistics Department compute servers.