Fluid Mechanics LAB REPORT-1

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Topic-Flow over Flat Plate (Boundary layer Thickness)

Introduction:

When there is a fluid-solid interaction, a very thin layer of fluid with viscous effects, known as the boundary layer develops. Boundary layer develops due to no slip boundary condition (due to adhesion) at the surface and the thickness of boundary develops due to velocity gradients and wall shear stress. Mathematically, the boundary layer thickness defines when the velocity of boundary layer reaches 99% value of the free stream velocity that is u = $0.99U_\infty$. where U_∞ is the free stream Velocity of the fluid. The boundary layer thickness depends on the velocity gradient, wall shear stress. In Laminar flows boundary layer thickness is less than the turbulent regime because the velocity gradients are more in the turbulent flows compared to laminar flows. This is due to the fluctuating velocity components present in the turbulent flows. The boundary also depends on the surface condition if the surface over which fluid flows is smooth then the boundary layer will be less as compared to rough surface and the transition of laminar to turbulent flows may occur at lower Reynolds number due to surface disturbances.

The experiments involve the study of boundary layer thickness and comparison of boundary layer thickness between smooth surface and rough surface. The glass plate is used as smooth surface and sand paper fixed over plate is used as rough surface. The experiment was performed in the wind tunnel using pitot static tube. The difference in the manometer can be seen with vertical distance from the surface and using Eq. (1) velocity was determined with respect to vertical distance from the surface. The fluid in the manometer is water. The main motive of the experiment is to compare both the boundary layer thickness in case of smooth and rough surface and the compared with the theoretical boundary layer thickness which is given by Blasius. A comparison between log law, power law and inner scaling law profiles have also been presented in this report.

Results and Calculations:

The observations from the experiment were made, the results and comparison are presented below for both the smooth and rough surface.

1) The free stream velocity can be obtained from Eq. (1) by recording the manometer difference Δh:

$$U_\infty = \sqrt{\frac{(2*\rho_{water}*g*\Delta h)}{\rho_{air}}} \qquad \text{Eq. (1)}$$

Where,

Density of Water (ρ_{water}) = 998.2 kg/ m^3

Acceleration due to gravity(g) = 9.81 m/ s^2

Density of air (ρ_{air}) = 1.275 kg/m^3

Kinematic Viscosity $(\upsilon) = 1.515 * 10^{-5}$ m^2/s

Δh = Difference in the manometric height

x = 42.54 Inch =1.080 Meters (Pitot Downstream Location)

Fluid properties are considered at 20 ^{0}C.

From Eq. (1), we can calculate the free stream velocity (U_∞) at $\Delta h = 0.0075$ is 10.74 m/s

From free stream velocity, we can calculate Reynolds number for smooth surface using below Equation

$$Re = \frac{u * x}{\upsilon} \qquad \text{Eq. (2)}$$

Reynolds number is 9.09 * 10^5

The flow can be characterized based on Reynolds number. In this case, the high Reynolds number indicating the flow is turbulent.

Case A) For Smooth surface

The flow is **turbulent** in case of smooth surface because the Reynolds number is much larger than the critical Reynolds number that is 8.78 * 10^5

Case B) For Rough Surface

The calculated Reynolds number is 9.09 * 10^5 in case of rough surface and the flow is in **turbulent** regime.

2) The velocity data and Reynolds number for smooth and rough surfaces are given below in Table (1) and Table (2) with respect to vertical height from the surface.

Table (1) Velocity Data and Reynolds Number for Smooth Surface

Tube Height (mm)	U smooth	Reynolds Numbers
0.5	7.33	6.21 x 10^5
1.5	7.33	6.21 x 10^5
2.5	8.31	7.04 x 10^5
3.5	9.19	7.78 x 10^5
4.5	9.19	7.78 x 10^5
5.5	9.19	7.78 x 10^5
6.5	9.19	7.78 x 10^5
7.5	9.19	7.78 x 10^5
8.5	9.19	7.78 x 10^5
9.5	10.37	8.78 x 10^5
10.5	10.37	8.78 x 10^5
11.5	10.37	8.78 x 10^5
12.5	**10.37**	**8.78 x 10^5**

Table. (2) Velocity Data and Reynolds Number for Smooth Surface

Tube Height (mm)	U rough	Reynolds Number
0.5	6.79	5.75×10^5
1.5	7.33	6.21×10^5
2.5	7.84	6.63×10^5
3.5	8.76	7.42×10^5
4.5	8.76	7.42×10^5
5.5	8.76	7.42×10^5
6.5	9.19	7.78×10^5
7.5	9.19	7.78×10^5
8.5	9.19	7.78×10^5
9.5	10.37	8.78×10^5
10.5	10.37	8.78×10^5
11.5	10.37	8.78×10^5
12.5	10.37	8.78×10^5
13.5	10.37	8.78×10^5
14.5	**10.73**	**9.09×10^5**

From the Table (1) and Table (2), we can conclude that the boundary layer thickness for smooth surface is **12.5 mm** and in case of rough surface it is found to be **14.5 mm** when the velocity reaches the 99% of the free stream velocity. In this case, the free stream velocity is found to be **10.74 m/s.** The boundary layer thickness is more in case of rough surface than in smooth surface this is because there is large velocity gradient and wall shear stresses are present in case of rough surface. The main reason for more wall shear stresses and large velocity gradients are due to surface condition of rough surface. The Fig. (1) and Fig. (2) shows the boundary layer profile development in case of smooth and rough surface.

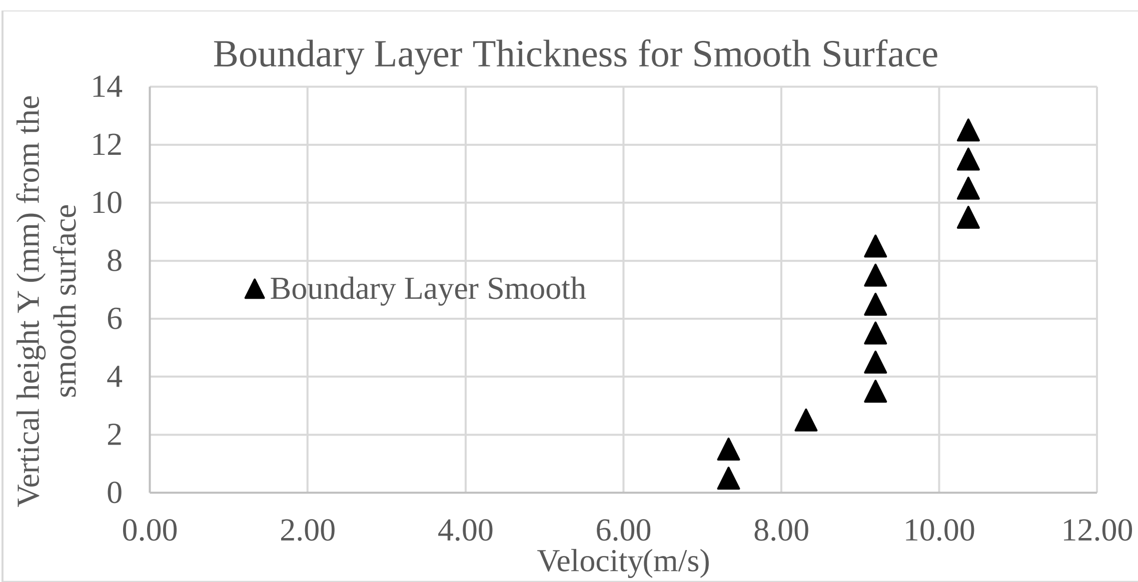


Fig. (1) Boundary Layer Thickness in case of Smooth Surface

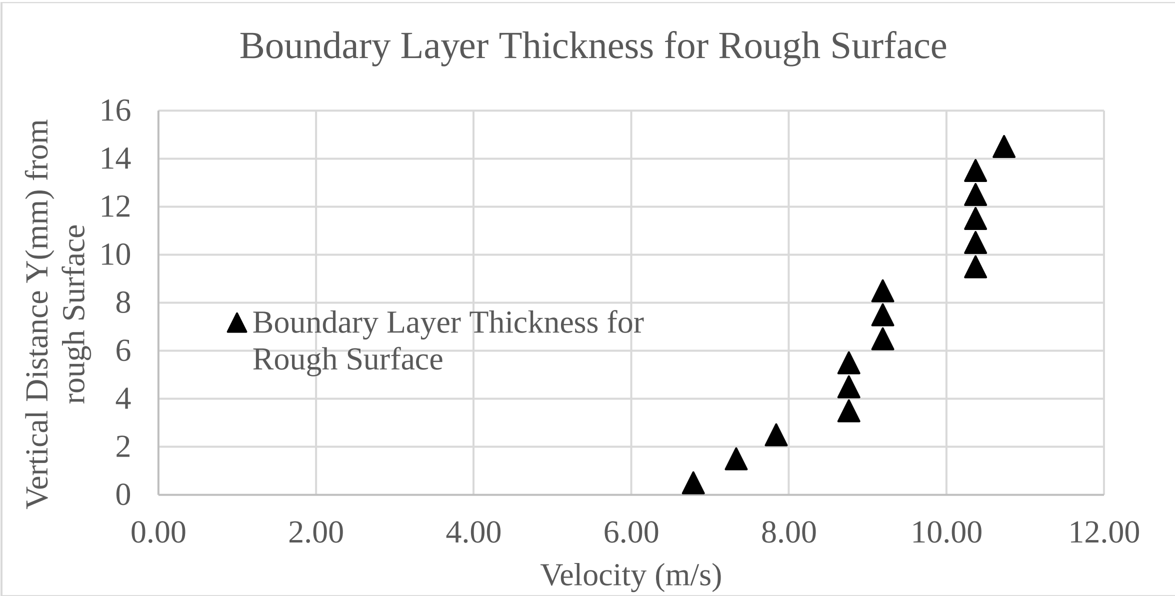


Fig. (2) Boundary Layer Thickness in case of rough Surface

3) In case of smooth surface, the flow falls in the turbulent regime therefore the analytical boundary layer thickness is evaluated using the below equation which is given by Blasius.

$$\delta = \frac{0.382*x}{Rex^{1/5}} \qquad \text{Eq. (3)}$$

On calculation, the analytical boundary layer thickness is found to be **26.39 mm** which is larger than the measured boundary layer in case of smooth surface. On the other hand, the measured boundary layer thickness is **12.5 mm**

At x = 1.080 m and $Re_x = 9.09 * 10^5$

For Rough surface, the theoretical boundary layer thickness for turbulent flow is given by the following equation:

$$\delta = \frac{0.382*x}{{Re_x}^{1/5}} \qquad \text{Eq. (4)}$$

On calculation, the analytical boundary layer thickness is found to be **26.39 mm** which is larger than the measured boundary layer in case of rough surface while the measured boundary layer is **14.5 mm**

At x = 1.080 m and $Re_x = 9.09 * 10^5$

4) The non-dimensional form of boundary layers is presented in Table (3) and Table (4) i.e. (u / U) versus y / δ for smooth and rough surface. The plots for non-dimensionless form of boundary layers for smooth and rough surface are shown in Fig. (3). From the Fig. (3), It is clearly seen that there is large velocity gradient very close to wall in case of rough surface as compared with the smooth surface this is because of more skin friction on rough surface.

However, as the distance increases from the surface, behavior of velocity gradient is almost same. The main reason is that for both the cases the flow is turbulent in nature. Therefore, velocity gradient and wall shear stress are almost same except near the wall region and the boundary layer thickness is more in rough surface as compared to smooth surface.

Table (3) Non-dimensional Boundary layer for smooth surface

u/U Smooth	**Y/delta smooth**
0.68	0.04
0.68	0.12
0.77	0.20
0.86	0.28
0.86	0.36
0.86	0.44
0.86	0.52
0.86	0.60
0.86	0.68
0.97	0.76
0.97	0.84
0.97	0.92
0.97	1.00

Table (4) Non-dimensional data for Rough surface

u/U Rough	**Y/delta rough**
0.63	0.03
0.68	0.10
0.73	0.17
0.82	0.24
0.82	0.31
0.82	0.38
0.86	0.45
0.86	0.52
0.86	0.59
0.97	0.66
0.97	0.72
0.97	0.79
0.97	0.86
0.97	0.93
1.00	1.00

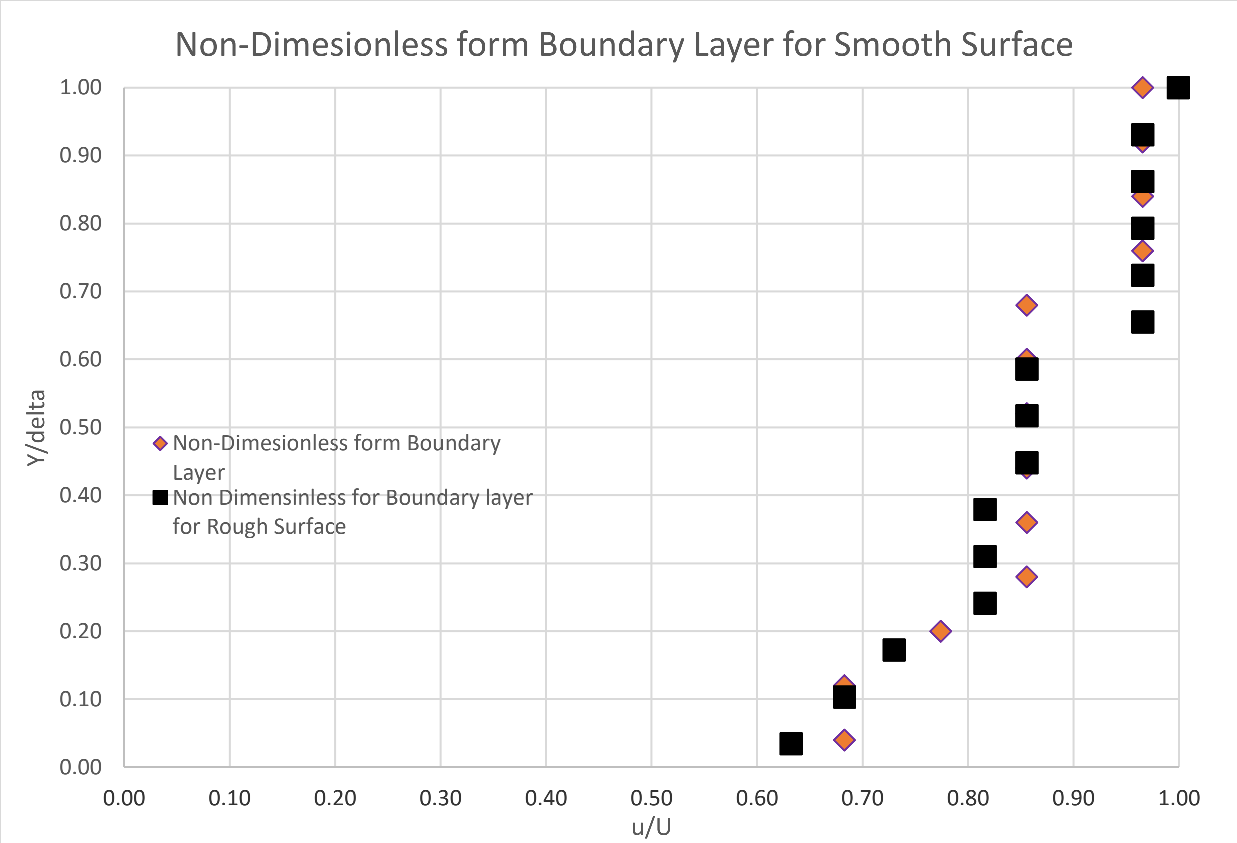


Fig. (3) Normalized Velocity Profile for the Smooth and Rough surface

5) The friction velocity u_* and constant B can be calculated using Eq. (5) and Eq. (6) and from the slope of the curve obtained from the logarithmic profile which is a plot between velocity and ln(y).

$$\frac{u}{u_*} = \frac{1}{k}\ln\frac{(y*u_*)}{\nu} + B \qquad \text{Eq. (5)}$$

$$u = \frac{u_*}{k}\ln(y) + \frac{u_*}{k}ln\frac{u_*}{\upsilon} + (u_* * B) \qquad \text{Eq. (6)}$$

A) For smooth surface:

By using slope of the curve as shown in Fig. (4), the magnitude of the friction velocity is evaluated, $u_* = 0.427$ m/s and by using Eq. (5) and Eq. (6) the value of constant is evaluated, **B=9.52**. where k=0.41, is the von-Karman constant.

B) For rough surface:

By using slope of the curve as shown in Fig. (5), the magnitude of the friction velocity is evaluated, $u_* = 0.510$ m/s and by using Eq. (5) and Eq. (6) the value of constant is obtained, **B=5.29**. where k=0.41, is the von-Karman constant.

Discussion of Result:

The friction velocity u_* represents the shear at the boundary. Therefore, the magnitude of friction velocity is less in case of smooth surface as compared with rough surface. It means there is more shear on the rough surface than smooth surface. This shear represents the turbulence strength and laminar sub layer thickness. The turbulence strength is higher in rough as compared to smooth surface because of higher friction velocity. The value of constant B is lower in case of rough surface as compared with smooth surface because of higher friction velocity.

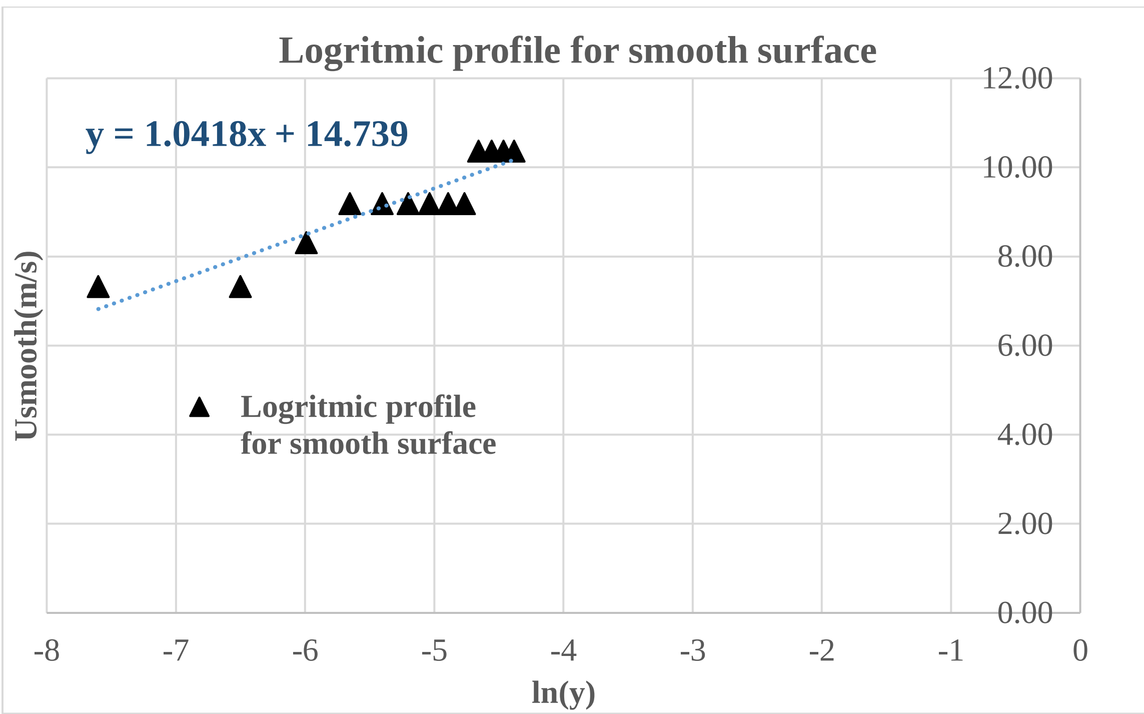


Fig. (4) Logarithmic graph for smooth surface

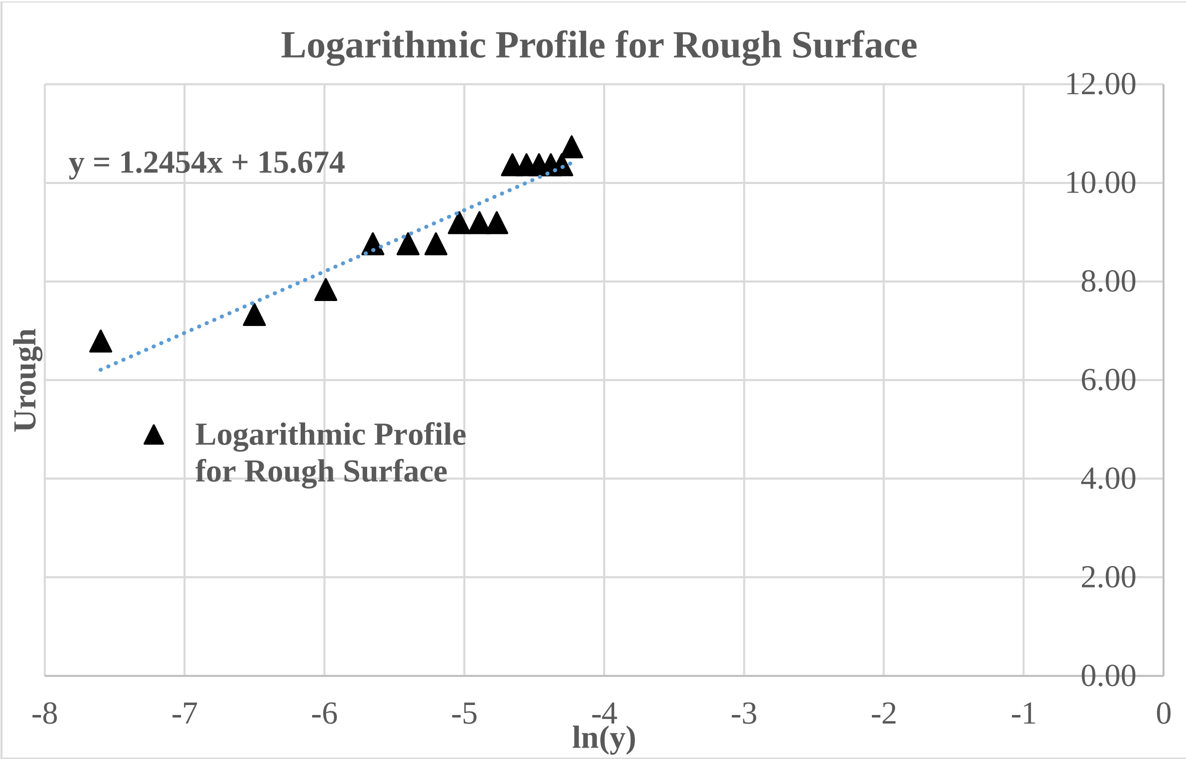


Fig. (5) Logarithmic graph for smooth surface

6) Power law profile for smooth surface can be represented by the following relation for both the smooth and rough surface.

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{\frac{1}{7}} \qquad \text{Eq. (7)}$$

The logarithmic profile for the turbulent can obtained by plotting the curve between $\mathbf{U^+}$ and $\mathbf{Y^+}$ which can be evaluated from the relations as given in Eq. (8) and Eq. (9)

$$\mathbf{U^+} = \frac{\mathbf{u}}{\mathbf{u_*}} \qquad \text{Eq. (8)}$$

$$\mathbf{Y^+} = \frac{\mathbf{y{*}u_*}}{\boldsymbol{\upsilon}} \qquad \text{Eq. (9)}$$

Discussion of Results:

The measured boundary layer profiles show a good agreement with the power law profile as shown in Fig. (6) and Fig. (8). The velocity profile for the power law does not hold valid near the wall. Since, at the wall it predicts infinite velocity gradients and infinite wall shear stress. The log-law profile is shown in Fig. (7) and Fig. (9).

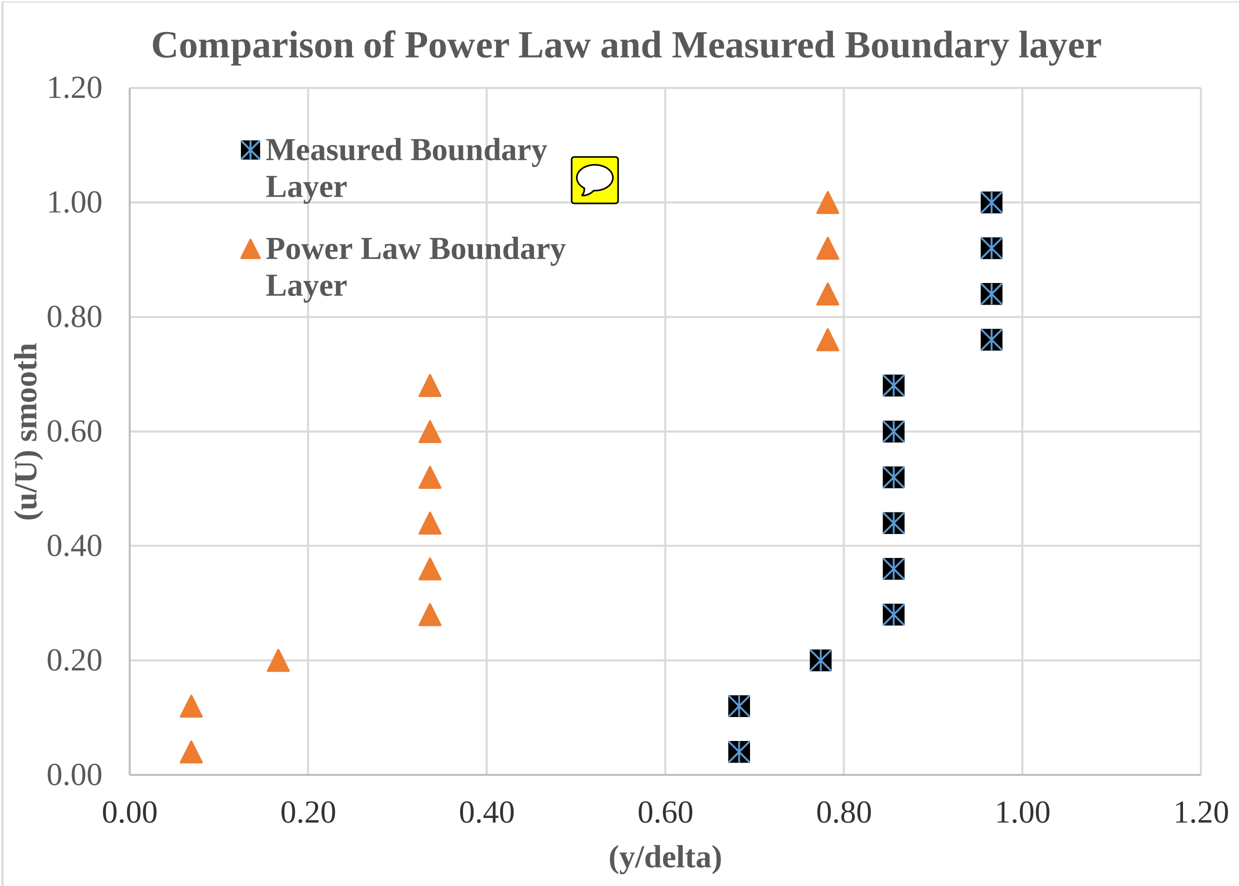


Fig. (6) Comparison Between Power law boundary layer with measured boundary layer

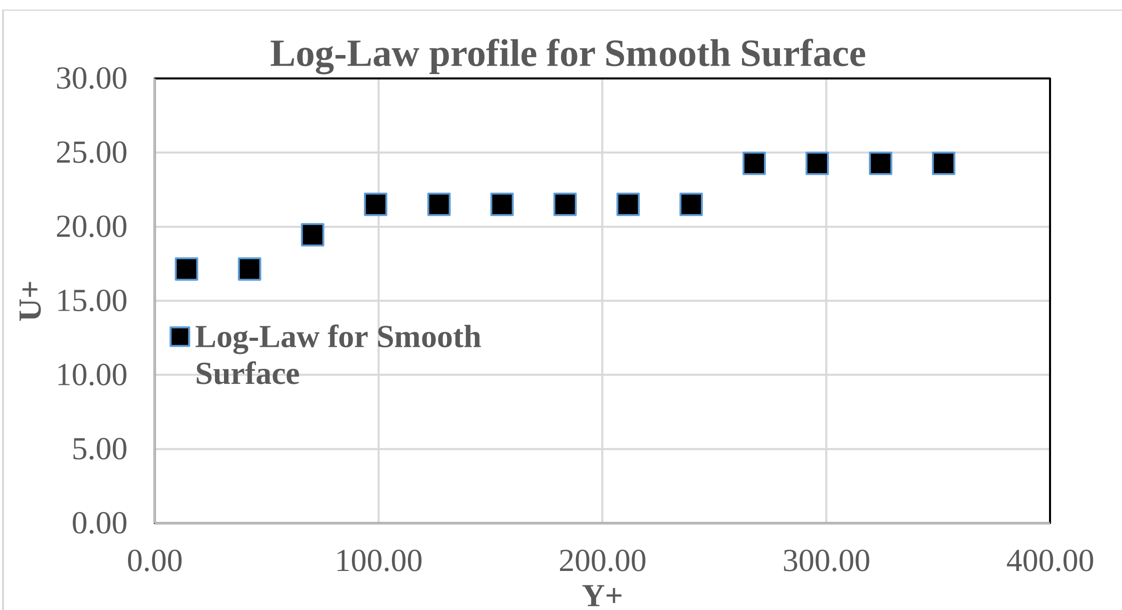


Fig. (7) Log-Law Profile for Smooth Surface

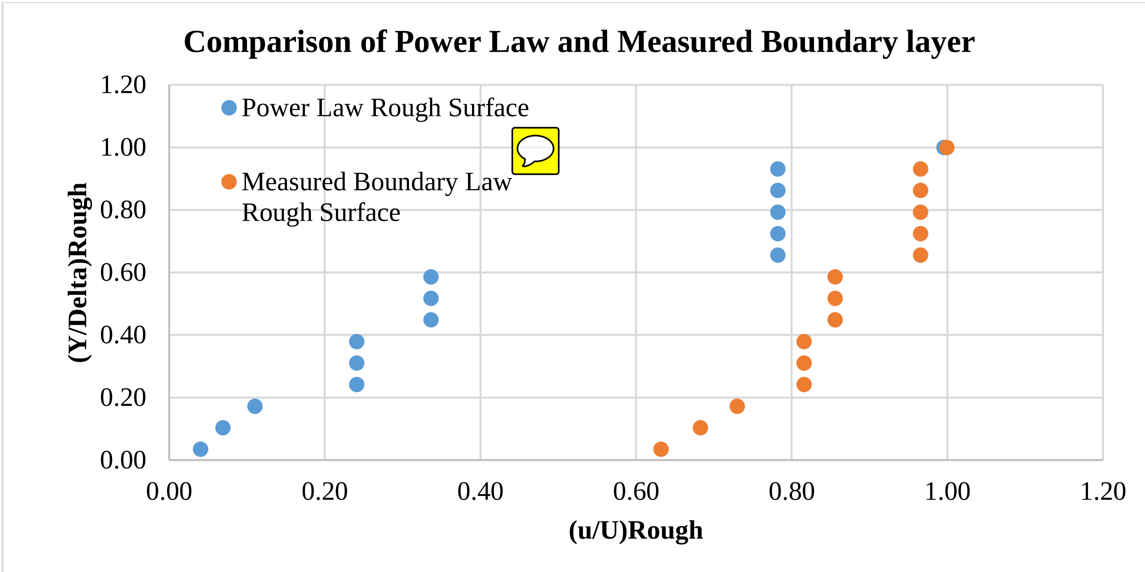


Fig. (8) Comparison Between Power law boundary layer with measured boundary layer

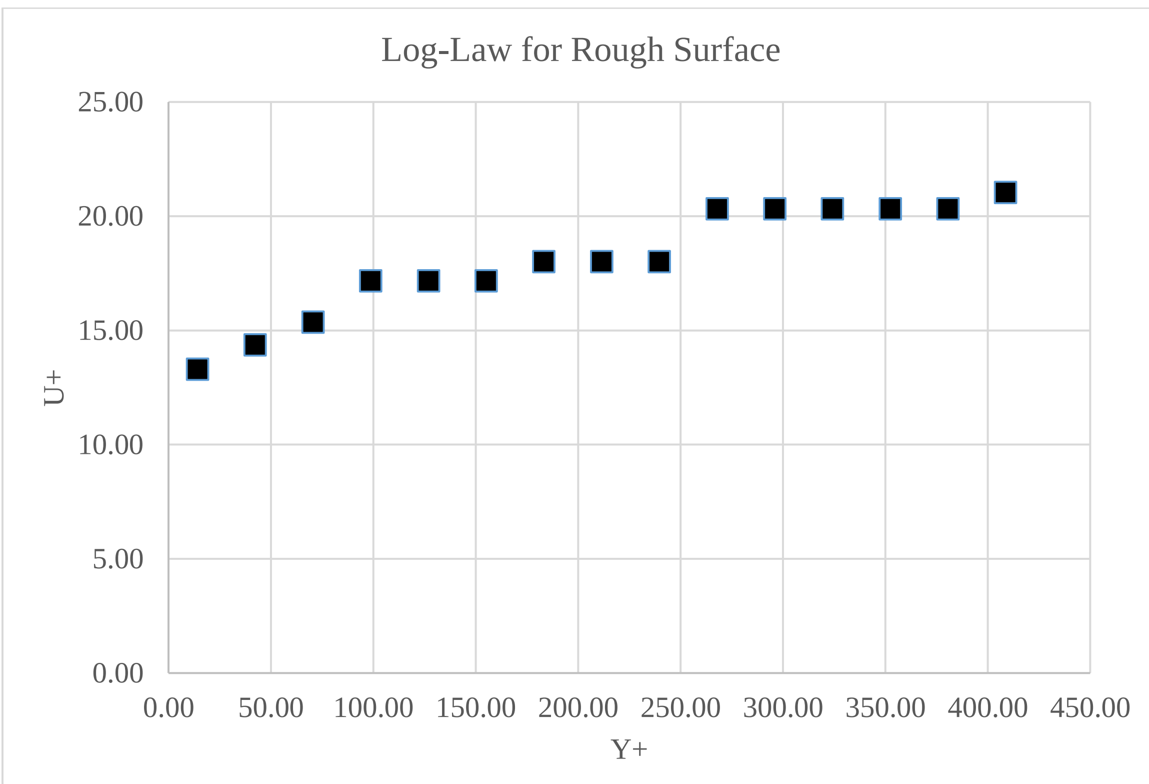


Fig. (9) Log-Law Profile for Smooth Surface

7) Inner wall scaling is the plot between U^+ and Y^+ and the following equations are used for the calculation of U^+ and Y^+

$$U^+ = \frac{u}{u_*}$$

$$Y^+ = \frac{y * u_*}{v}$$

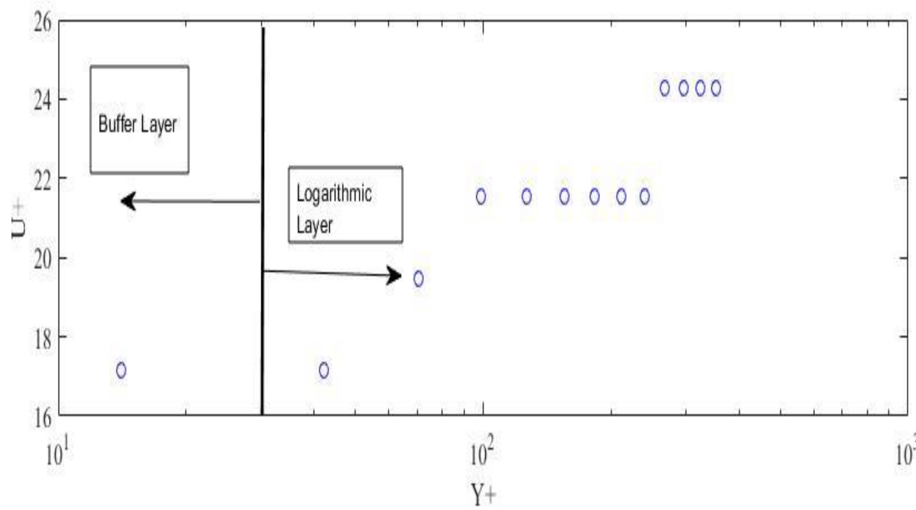


Fig. (10) Inner scaling law profile for smooth surface

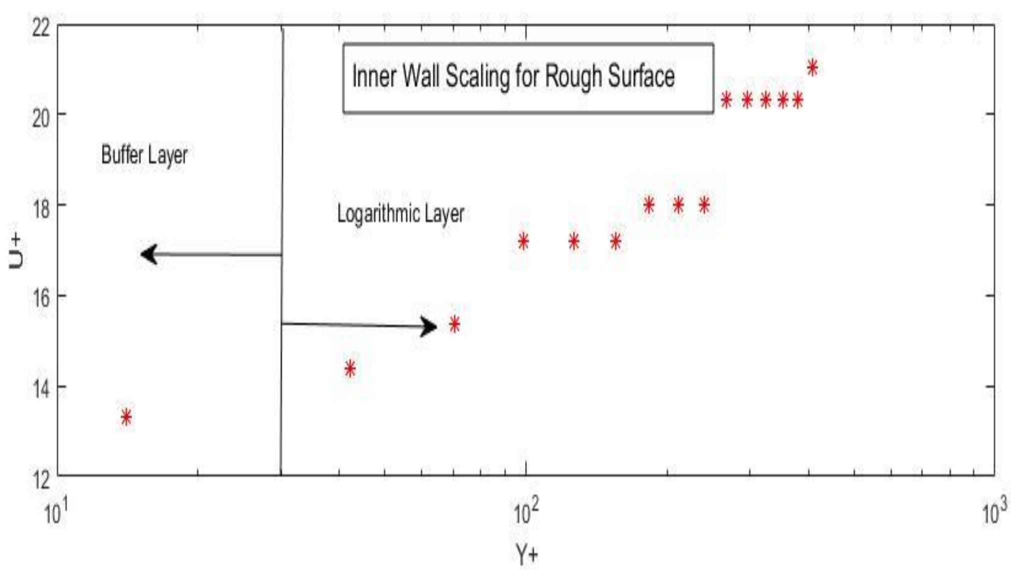


Fig. (11) Inner scaling law profile for smooth surface

Comparison of Profiles:

Both the profiles follow the similar trend. However, there is more shear in case of rough surface due to rough surface elements. This leads to more friction velocity in case of rough surface and lower friction velocity in case of smooth surface.

8) Three are basically three zones in inner wall scaling. However, Fig. (10) and Fig. (11) depicts mainly two zones for smooth as well as rough surface because of low values of Y^+. Four possible cases are given below and zones are differentiated based on Y^+ value. The Y^+ is a vertical non-dimensionless distance from the wall.

Case 1:

When $Y^+ < 5$, this zone is called viscous sub layer where viscous stresses are more dominant than the turbulent stresses because of no slip and no penetration boundary condition.

Case 2:

When $5 \leq Y^+ \leq 30$, this zone is called buffer layer. In this zone the flow is neither linear or logarithmic.

Case 3:

When $30 < Y^+ \leq 3800$, this is called logarithmic zone.

Case 4:

When $Y^+ > 3800$, this is called wake region.

9) Calculated wall shear stresses for both the smooth and rough surfaces are given below:

Case 1: Smooth Surface

$$\tau_{wsmooth} = \rho * {u_*}^2$$

$$\tau_{wsmooth} = 1.275 * \; 0.427^2$$

$$\tau_{wsmooth} = 0.232 \; pa$$

Case 2: Rough Surface

$$\tau_{wrough} = \rho * {u_*}^2$$

$$\tau_{wrough} = 1.275 * \; 0.510^2$$

$$\tau_{wrough} = 0.331 \; pa$$

Comparison between wall shear stresses

The wall shear stresses are calculated above and it is found that the wall shear stress is less in smooth surface that is **0.232 pa** as compared with rough surface that is **0.331 pa**. The main reason for higher wall shear stress in rough surface is due to high turbulence strength that leads to high to large velocity gradients and large friction velocity. Large velocity gradients are present in the rough surface because of geometric conditions and rough element size.

10) Tabulated Values are given below:

	Reynolds number	Flow type	Theoretical boundary layer thickness (m)	Measured boundary layer thickness (m)	% difference	Friction velocity u* (m/s)	Constant B	Wall shear stress (Pa)
Test 1 (smooth)	$8.78*10^5$	Turbulent	0.02657	0.0125	52.95	0.427	9.52	0.232
Test 2 (rough)	$9.09*10^5$	Turbulent	0.02639	0.0145	45.054	0.510	5.29	0.331

Discussion of Results:

From the above table, it can be clearly seen that in both the cases the flow is turbulent the downstream location of pitot static tube is 1.080 meters from the leading edge. But in rough surface, more turbulence strength can be seen because of surface disturbances originated from rough elements. The measured boundary layer is 12.5 mm in case of smooth surface. On the other hand, it is found to be 26.57 mm theoretically which leads to 52.95 percentage difference. While, in rough surface the percentage difference is 45.054.

The large percentage difference has been seen in measured boundary layer when compared with theoretical boundary layer. The main reason of large percentage difference may be due to random or systematic error that are commonly seen in experimentation. Random errors can be controlled by statistical method but systematic error are difficult to control. In case of smooth surface, the last reading shows a 97% of the free stream velocity and further readings may be obtained at higher vertical height to match a 99% of the free velocity this will decrease the percentage difference between the analytical and experimental calculated boundary layer thickness.

Provided Velocity Data:
As the sampling frequency of velocity data is 1250 Hz and at one y value, there are 20000 velocity data readings. So, the total time scale for generating the velocity data points becomes 16 seconds.

12) Extracting data from MATLAB by using following code:

```
>> filename='jagdata.xlsx';
>> Z=vel;
>> sheet=1;
>> jagrange='A2:AW20001';
>> xlswrite (filename, Z, sheet, jagrange)
>> f=y;
>> jagrange1='A1:AW1';
>> xlswrite(filename, f, sheet, jagrange1)
```

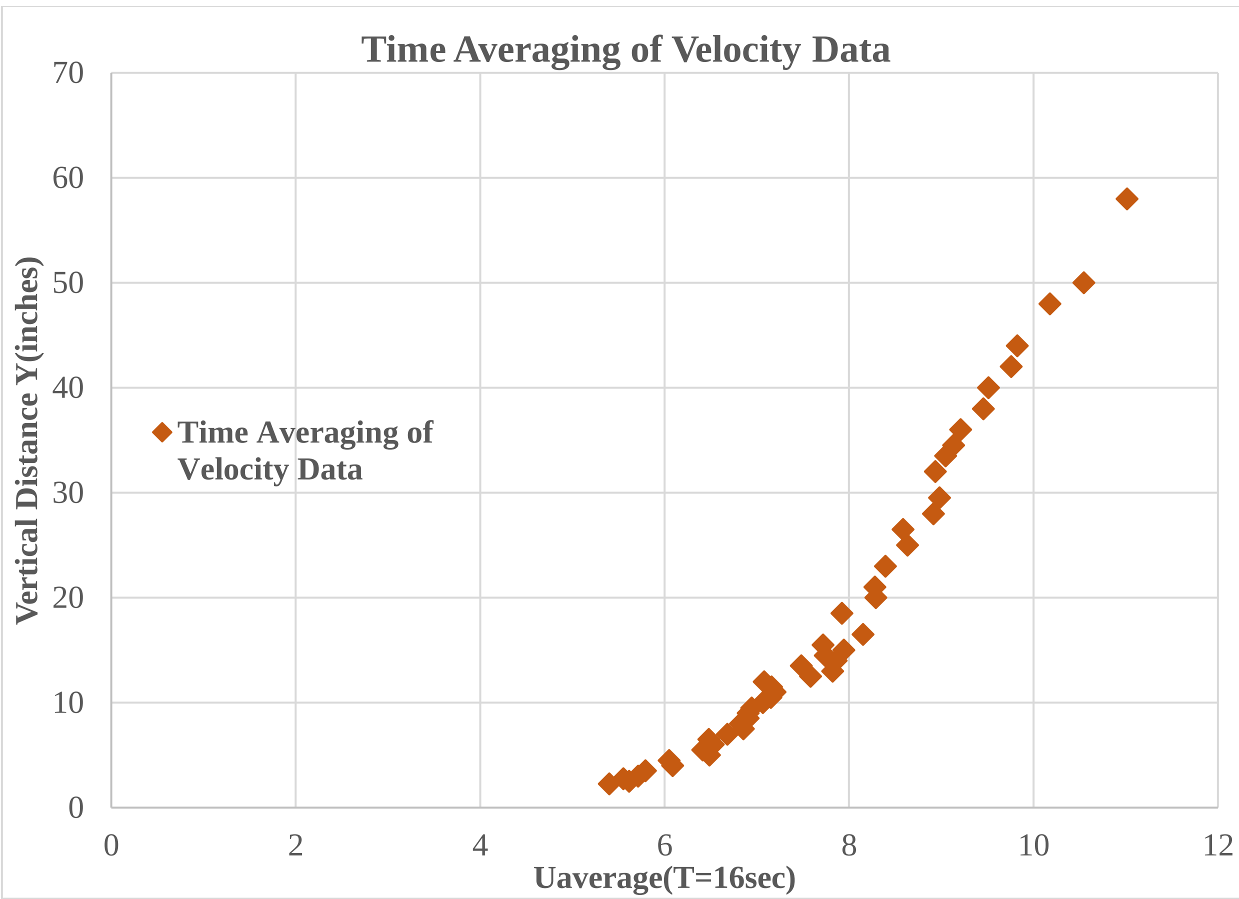


Fig. (12) Time-averaged mean stream wise velocity

Calculated U average with respect to the vertical height from the surface is tabulated below:

Vertical height Y(in)	U(average)
2.25	5.40
2.50	5.62
2.75	5.55
3.00	5.71
3.50	5.79
4.00	6.09
4.50	6.05
5.00	6.49
5.50	6.41
6.00	6.53
6.50	6.48
7.00	6.68
7.50	6.85
8.00	6.82
8.50	6.91
9.00	6.90
9.50	6.94
10.00	7.07
10.50	7.15
11.00	7.20
11.50	7.16
12.00	7.08
12.50	7.58
13.00	7.82
13.50	7.48
14.00	7.86
14.50	7.74
15.00	7.94
15.50	7.72
16.50	8.15
18.50	7.92
20.00	8.29
21.00	8.28
23.00	8.40
25.00	8.63
26.50	8.58
28.00	8.92

29.50	8.98
32.00	8.93
33.50	9.05
34.50	9.13
36.00	9.21
38.00	9.45
40.00	9.51
42.00	9.76
44.00	9.82
48.00	10.18
50.00	10.54
58.00	11.01

13) Time-averaged of stream-wise velocity in both inner and outer layer scaling is shown in Fig. (14) and Fig. (16)

Friction velocity is calculated from slope of linear fit of the curve as shown in Fig. (13) which is used to plot the inner and outer scaling profiles.

From Fig. (11), $u_*/ k = 1.5625$, k= 0.41

Friction velocity $u_* = 0.64$ m/s

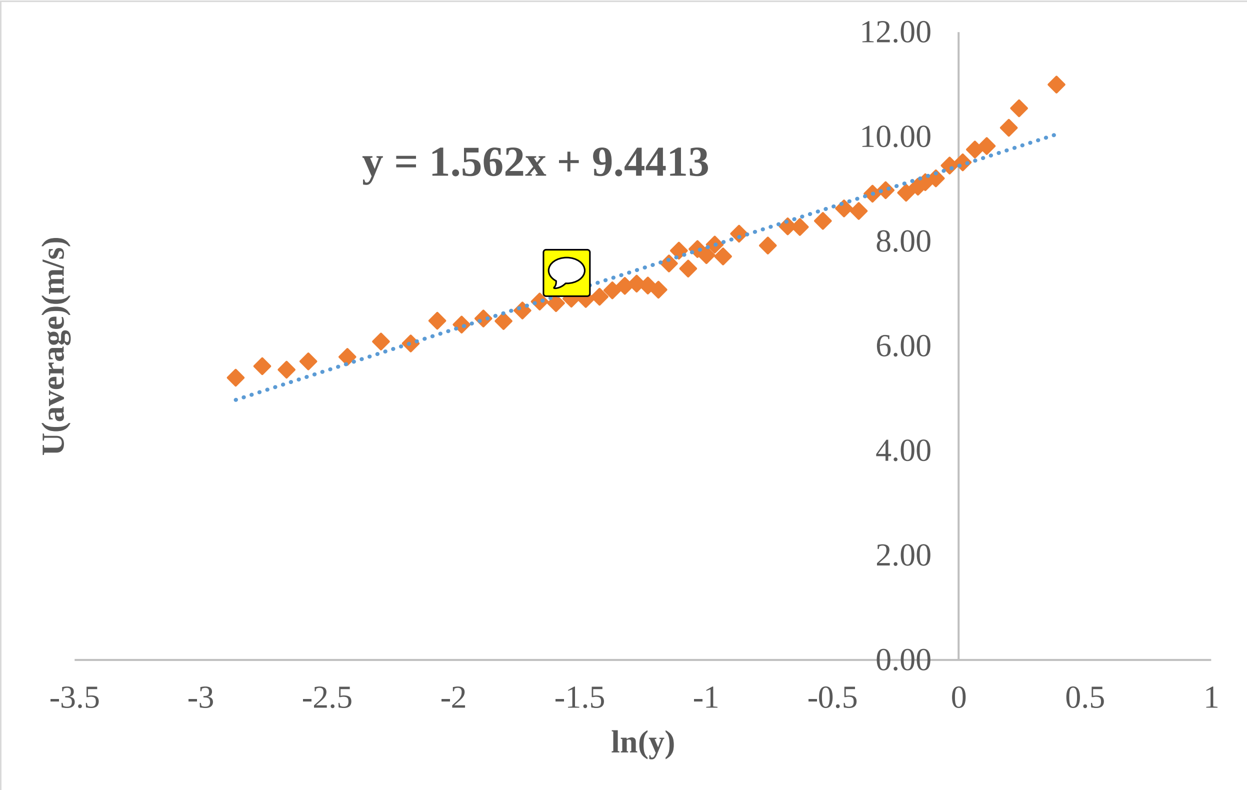


Fig. (13) Logarithmic y plot versus u stream velocity

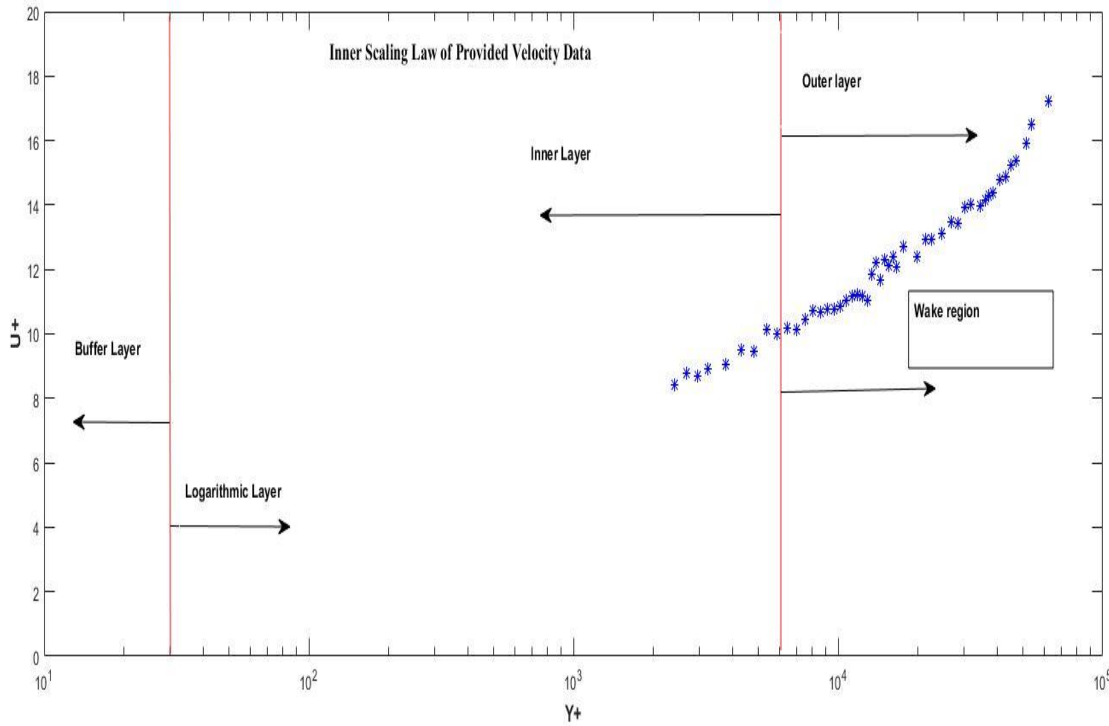
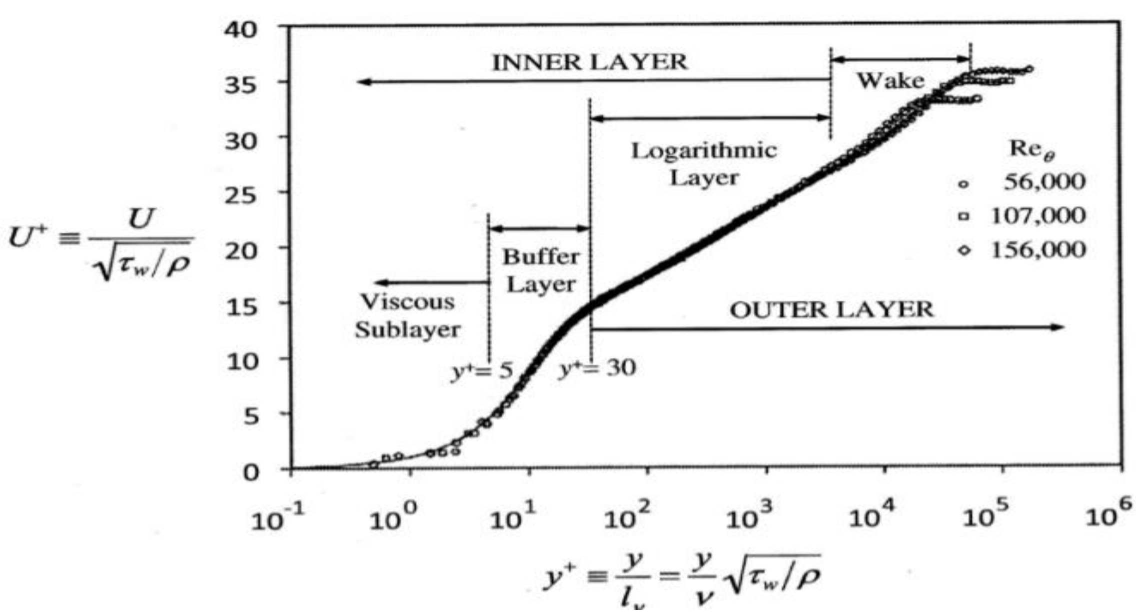


Fig. (14) Time-averaged mean stream velocity inner scaling plots



[1]Fig. (15) Inner scaling Profiles

Outer Scaling:

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[1] Taken from Lecture Notes given by Prof. Kamran Siddiqi (Boundary Layer)

Boundary layer thickness: 1.47 m

Free stream velocity: 11 m/s

$\frac{U-u}{u_*} = F(\frac{y}{\delta})$ Eq. (10)

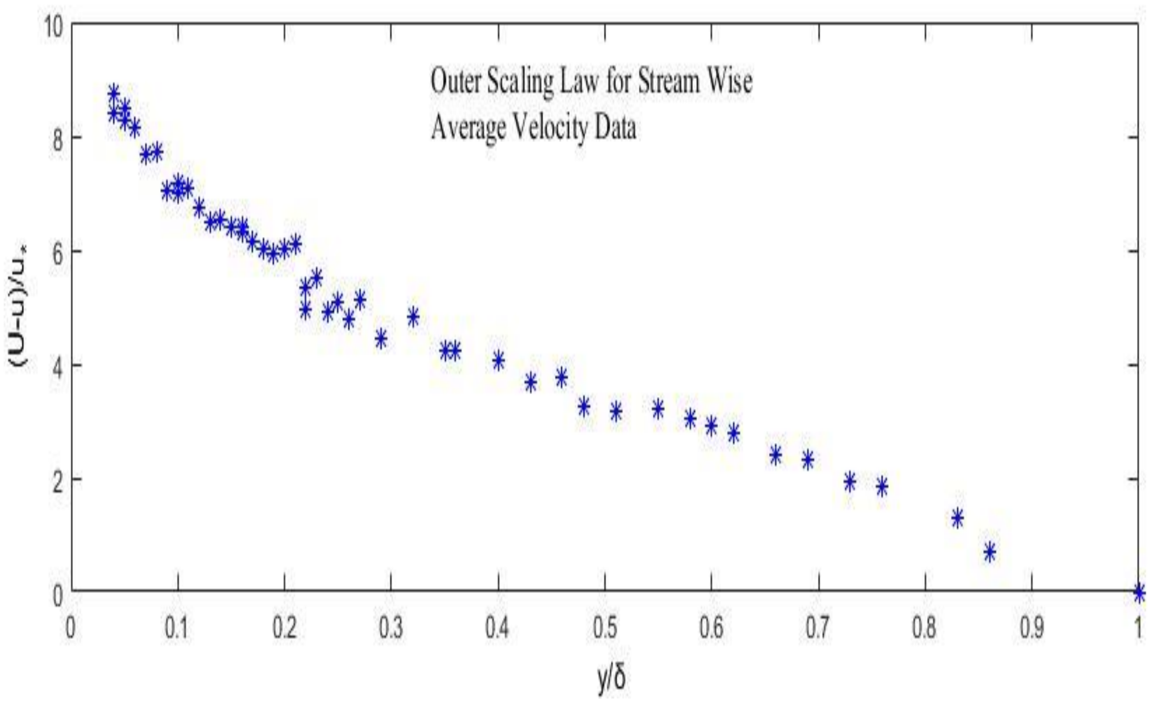

Fig. (16) Time-averaged mean stream velocity outer scaling plots

14) In inner scaling layer,

If $Y^+ <= 5$, This zone is called viscous sub-layer where laminar viscosity is dominant.

If $5<Y^+<30$, It leads to Buffer layer

If $Y^+> 100$ results in the Logarithmic layer

If Y+>3800 it leads to wake region

Therefore, from the above graph, the Y+ value starts from 2414.57.

15) Turbulence intensity is given by the given below expression

$I = \frac{\sigma}{U_{mean}}$ Eq. (11)

The standard deviation was calculated by the following formula:

$$\sigma = \sqrt{\frac{\sum(u_i - U_{mean})^2}{n}} \qquad \text{Eq. (12)}$$

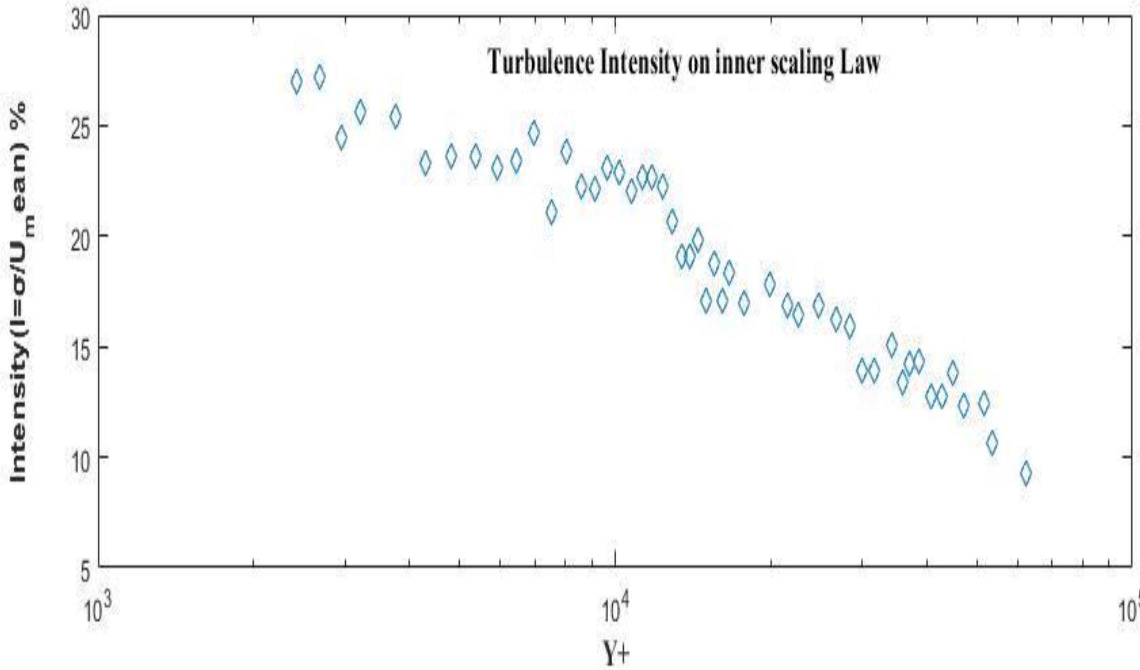

Fig. (17) Turbulent intensity profile in inner layer scaling

Reynold stress can be expressed as $\tau = \rho(u'u')_{mean}$

$$u' = u_i - \bar{u} \qquad \text{Eq. (13)}$$

$\rho = 1.2754\ kg/\ m^3$

Velocity data is provided and Y^+can be calculated.

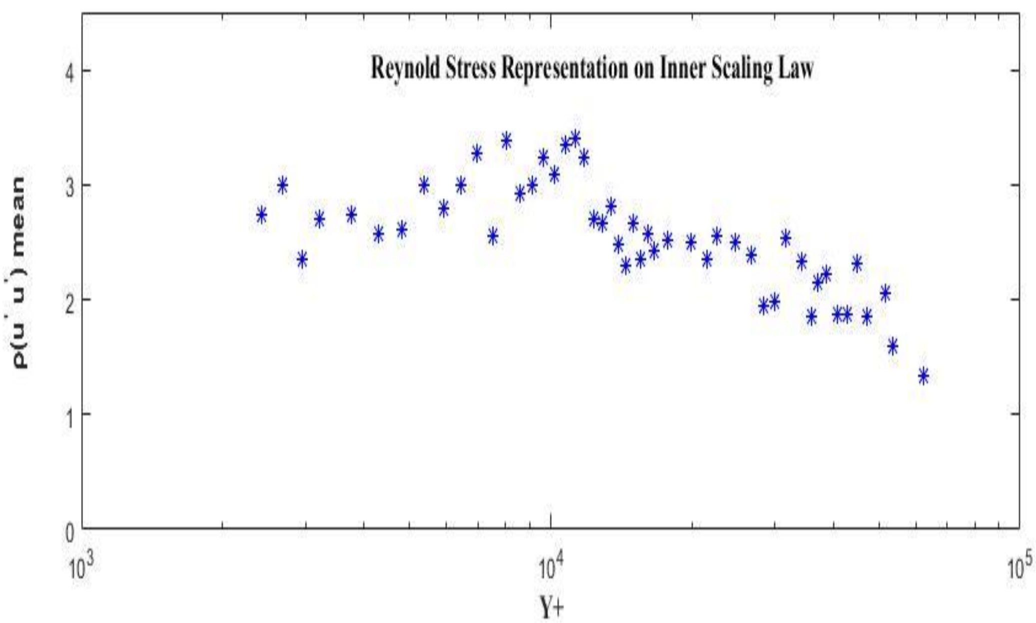

Fig. (18) Reynold stress profile in inner layer scaling

Discussion of Results:

Inner scaling profile of calculated velocity data is shown in Fig. (14) and Fig. (15) shows the inner scaling profiles given in literature. The calculated inner scaling profile follow the similar trend in logarithmic zone as well as in wake region. The first reading is taken from significant height from the surface that is 2.25 inch that is first y+ comes near about 2414.57 that's why the plot does not give the complete scenario of the viscous sub layer and buffer layer. However, when compared with literature curve it follows the same behaviour. While Fig. (16) shows the behaviour of profiles on outer scaling and it is also found that it follows the similar trend with figure shown in literatures.

The turbulent intensity and Reynold stress follows the similar trend as shown in Fig. (15) and Fig. (16) respectively. We know that very close to the wall the viscous stress is significant and Reynold stress is negligible due to no slip boundary condition. But, here the first reading is taken from significant height from the height where the Reynolds stresses are very much dominant and this is depicted by higher Reynolds stress at first reading and similar trend can be seen in turbulence intensity plot. From the Fig. (18), the Reynold stress is maximum when y+ is around 4500 that is the edge of the logarithmic profile. Further increase in the height leads to decrease in the Reynolds stress because effect of stresses weakens as we move towards free stream velocity. on the other hand, with the increase of height from the surface turbulent intensity decreases continuously as shown Fig. (17). The maximum turbulent intensity is found to be **27%** and minimum turbulent intensity is found to be approximately **9%.** However, for most of the Engineering application the turbulent intensity is found to be less than 10% and 30% or above turbulent intensity is found in atmosphere or in oceans.

Power Spectral density: Power spectral density is obtained at two different heights at 3.5 inch. and 9 inch. using "MATLAB" coding as follows:

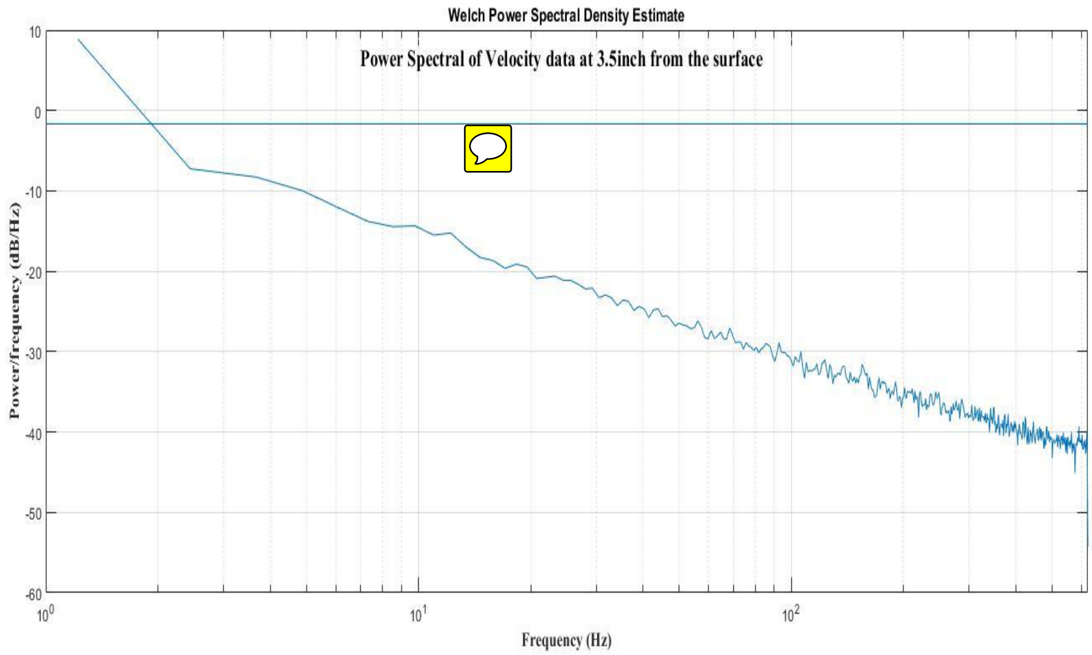

Fig. (17) Power spectral density at 3.5 inch

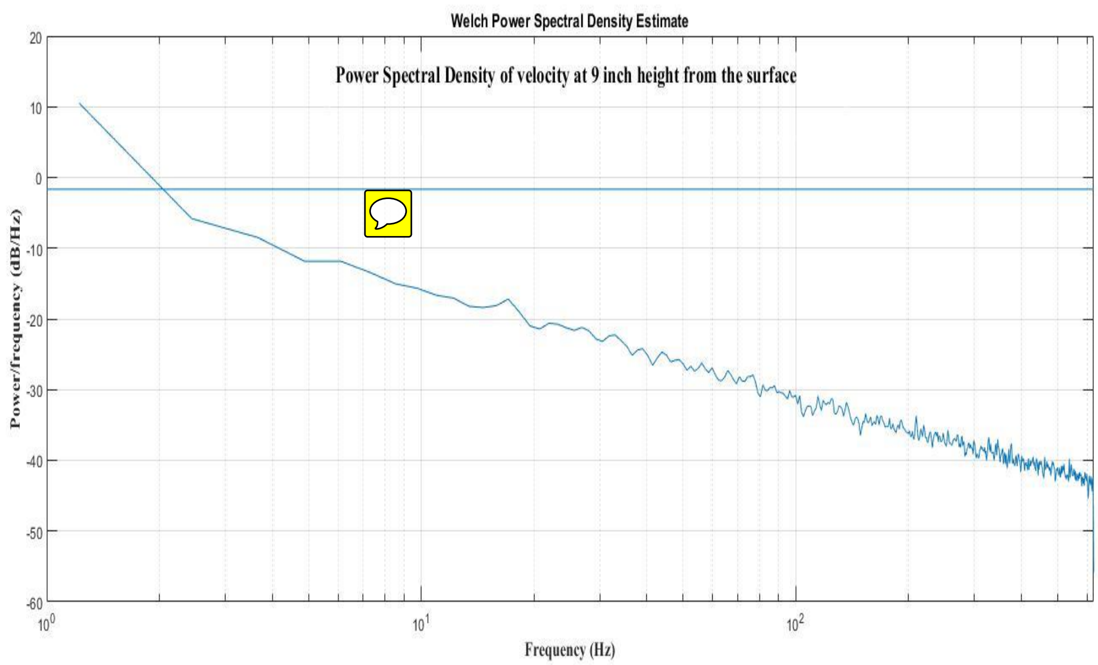

Fig. (18) Power spectral density at 9 inch

MATLAB Code:

Power spectral density plot at 3.5 inch of height as shown in Fig. (17)

```
>> pwelch(newdata35, 1024, [ ], [ ], 1250);
>>hold on;
>>stline=-5/3*ones(1, 20000);
>>Plot(stline)
```

Power spectral density plot at 9.0 inch of height as shown in Fig. (18)

```
>> pwelch(newdata9, 1024, [ ], [ ], 1250);
>>hold on;
>>stline=-5/3*ones(1, 20000);
>>Plot(stline)
```

Where,

NFT = 2^n, where n = 10 and frequency = 1250 Hz

newdata35 is a vector name having velocity data points at 3.5 inch

newdata9 is a vector name having velocity data points at 9 inch

By using the NFT, 20 power spectral density is obtained and average amplitude is plotted at different heights.

Comments:

Power spectral density is used to capture the amplitude(attenuation) in the signal which comes from velocity data at 3.5 inch and 9 inch at 1250 sampling frequency as shown in Fig. (17) and Fig. (18). The power spectral density is plot between power spectral vs sampling frequency. The highest amplitude is about 10 Db/Hz.

References:

[1] Lab Manual, Western University, London Ontario, pp.1-9

[2] Davidson, P.A, "Turbulence, An introduction for Scientists and Engineers" Edition 2004, pp. 126-145

[3] Schlichting. H, Gersten.K," Boundary Layer Theory" Edition 2000 Springer Link, pp.29-45