



Western University
Faculty of Engineering

Department of Mechanical and Material Engineering

Project Report-Continuum Mechanics(9611A)

“Comparison of Turbulence Models and Application in CFD”

Literature review on Numerical Simulation on turbulent flow in 90° Bend

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Introduction:

Turbulence is very complex phenomena which is very difficult to define. However, it can be characterized by its features. Some of the features of turbulent flows are listed below:

- a) Randomness of transport variable with respect to time and space.
- b) Strong Mixing: It is due to large momentum transfer due to the fluctuations present in turbulent flows.
- c) There are wide range of length scales and times scales
- d) Eddies and vorticities are the basic entities of turbulent flows which is just a lump of rotating mass.

The characterization of Turbulent flow can be done with help Reynolds number.

- a) For External flows

$$R_{el} \geq 5 * 10^5 \quad \text{Eq. (1)}$$

- b) For Internal flows

$$R_{ed} \geq 2300 \quad \text{Eq. (2)}$$

The Reynold's number is given below:

$$R_{el} = \frac{\rho U l}{\mu} \quad \text{Eq. (3)}$$

Where;

$\rho = \text{Fluid Density}$

$U = \text{Fluid Velocity}$

$l = \text{Characteristic length}$

$\mu = \text{Dynamic viscosity}$

The Eq. (1) and Eq. (2) can only be used with the smooth surfaces. For rough surface, transition to turbulent flow may occur at lower Reynold's number this is due to disturbances created due to surface conditions. In engineering applications, 80% of the flows are turbulent in nature. There is no exact solution are available for turbulent flows. Even for numerical method it is very challenging task for researcher to tackle the wide range of length and time scale that are present in the turbulent flows.

Energy cascading in Turbulent Flows:

The turbulent flows are 3-Dimensional rotating dissipative flows in nature. There are different length scales in turbulent flows and basic entities of turbulent flows are eddies. Therefore, largest eddies which is having a length scale of system gets energy from the mean flow due to instabilities present in the turbulent flows. This largest can not dissipate energy because the inertial effects are more dominant compared to viscous effect due to large Reynolds number. So, it will transfer its energy to the smaller eddies. The smallest eddy will then dissipate energy in the form of damping because in smallest eddy viscous forces are more dominant. An extensive study has been done by Kolmogorov and Richardson to find out different length scale present in the turbulent flows. The Fig. (1) shows the different length scales of eddies present in turbulent flows. This Fig. is the study of Richardson, 1922.

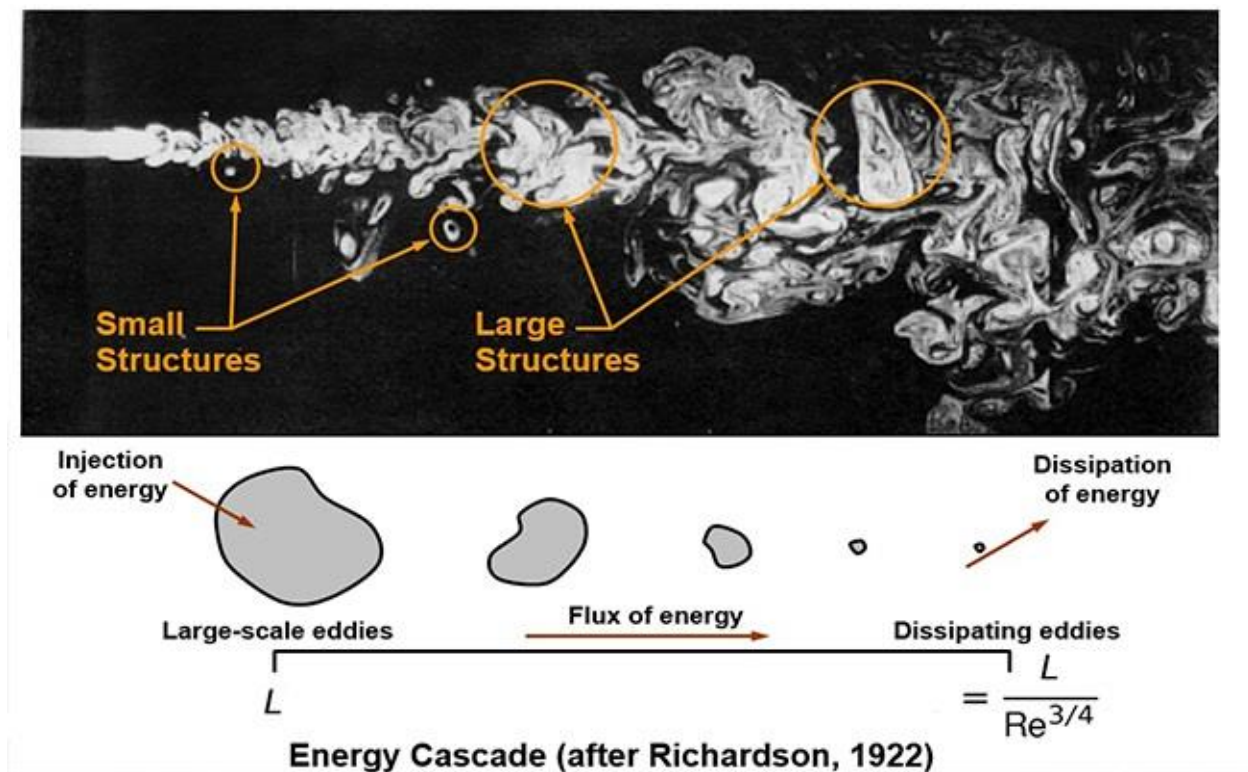


Fig. (1) Energy Cascading

²Mathematical Scale of Smallest Eddy:

In 1941, Kolmogorov gave a mathematical scale of smallest eddy when the inertial forces just be balanced by viscous forces and dissipation of energy starts. The mathematical derivation is given below:

Let us consider,

Scales	Largest Eddy	Smallest Eddy
Length Scale	l	η
Velocity Scale	u	v

1. Rate of extraction of Turbulent Kinetic Energy per unit mass from the Mean Flow:

$$\pi \sim \frac{1/2 \rho u^2}{m} * \frac{1}{t_{scale}} \quad \text{Eq. (4)}$$

2. The time scale of large eddy is given by length scale and velocity scale which is given below:

$$t_{scale} = \frac{l}{u} \quad \text{Eq. (5)}$$

Using Eq. (5) in Eq. (4), we get

$$3. \quad \pi \sim \frac{u^3}{l} \quad \text{Eq. (6)}$$

So, the rate of extraction of turbulent kinetic energy is of the order of $\frac{u^3}{l}$, Where π represents the rate of extraction of turbulent energy by largest eddy from the mean flow.

4. Rate of Dissipation of Kinetic Energy:

$$\varepsilon \sim \nu e_{ij} e_{ij} \quad \text{Eq. (7)}$$

$$\varepsilon \sim \nu \frac{v^2}{\eta^2} \quad \text{Eq. (8)}$$

Where;

e_{ij} = Rate of deformation tensor

Consider that the inertial forces just be balanced by viscous forces, so we can say that Reynolds number is of the order of 1.

$$Re \eta \sim 1 \rightarrow \frac{v \eta}{\nu} \sim 1 \quad \text{Eq. (9)}$$

using Eq. (7) and Eq. (8) in Eq. (9), we get

$$\eta \sim \left(\frac{v^3}{\varepsilon} \right)^{\frac{1}{4}}$$

Eq. (10)

The Eq. (10) represents the length scale of smallest eddy which is also known as Kolmogorov length scale.

For dynamic balance the rate of turbulent kinetic energy should be equal to rate of dissipation of energy, which is given by the following relation using Eq. (4) to Eq. (8)

$$\pi \sim \varepsilon \quad \text{Eq. (12)}$$

$$\frac{\eta}{l} = Re l^{-3/4} \quad \text{Eq. (11)}$$

We know that smallest length scale of smallest eddy. To determine whether we can apply continuum hypothesis to turbulent flows or not. We must check for Knudsen number relation and it must be less than 0.01 to considered the medium to be continuous. There are large number of length scales are present in turbulent flows but the smallest length is η which remains larger than the mean free path λ_m .

The Knudsen number is given by:

$$K_n = \frac{\lambda_m}{l} \quad \text{Eq. (13)}$$

Where;

λ_m = Mean free path

l = length of interest

Vorticity Dynamics in Turbulent Flows:

The turbulent flows consist of eddies and vorticities. So, we have drive the transport equation of vorticity for turbulent flows

The navier stokes equation, assuming there is no body force, is given by:

$$\rho \left[\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla) \vec{u} \right] = \mu \nabla^2 \vec{u} - \nabla p \quad \text{Eq. (14)}$$

The vorticity is given by curl of the velocity vector;

Where, $\nabla \times \vec{u} = \vec{\zeta}$

Taking curl of Eq. (14) on both sides, we get the vorticity transport equation.

$$\rho \left[\frac{\partial \vec{\zeta}}{\partial t} + (\vec{u} \cdot \nabla) \vec{\zeta} \right] = \rho (\vec{\zeta} \cdot \nabla) \vec{u} + \mu \nabla^2 \vec{\zeta} \quad \text{Eq. (15)}$$

The first term in Eq. (15) on left hand side is total derivative of vorticity because it contains local vorticity and convection term of vorticity. The first term of right hand side is known as source of

vorticity and it is difficult to interpret it mathematically and second term is viscous term of vorticity transport equation.

Let us consider an analogy to interpret source of vorticity term in physical sense. Consider a large-scale eddy, eddy is rotating lump of fluid mass, therefore $I\omega$ is the angular momentum of the large eddy,

Take a total derivative of $I\omega$, which is given by;

$$\frac{D(I\omega)}{Dt} = I \frac{D\omega}{Dt} + \omega \frac{DI}{Dt} \quad \text{Eq. (16)}$$

$$\frac{D\omega}{Dt} = \frac{-\omega}{I} \frac{DI}{Dt} + \frac{T_v}{I} \quad \text{Eq. (17)}$$

When compared this equation with vorticity equation

1. $\frac{D\omega}{Dt} \sim \left[\frac{\partial \zeta}{\partial t} + (\vec{u} \cdot \nabla) \zeta \right]$
2. $\frac{-\omega}{I} \frac{DI}{Dt} \sim \rho (\vec{\zeta} \cdot \nabla) \cdot \vec{u}$
3. $\frac{T_v}{I} \sim \mu \nabla^2 \vec{\zeta}$

The second term should have similar physical meaning. Let us understand the physical meaning of second term which is given below

$$\frac{DI}{Dt} = \frac{-I}{\omega} \frac{D\omega}{Dt} \quad \text{Eq. (18)}$$

T_v is neglected because large eddy has large inertial effects

With the expanse of time when smaller eddy takes energy from large eddy its angular speed increases therefor term $\frac{D\omega}{Dt}$ is always positive and $\frac{I}{\omega}$ is also positive term. Therefore, $\frac{DI}{Dt}$ is negative what it represents that with the expanse of time, inertia is decreasing. So, we can say that vortex element will be stretched as shown in figure below.

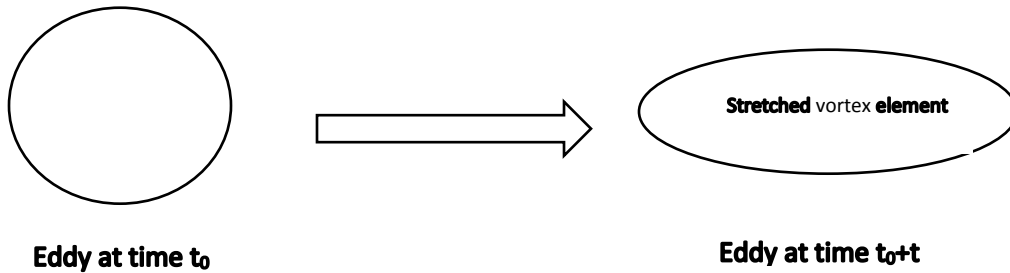


Fig. (2)

Vortex stretching is the source of vorticity. Due to vorticity, interaction with other vortex element will increase results in the intensification of vorticity. Therefore, vortex stretching is the source of vorticity and appears in the vorticity transport equation.

Statistical Representation of Turbulent Flows:

The turbulent flows are highly chaotic, irregular and 3-Dimensional flows. Therefore, it is very difficult to treat them as deterministically. So, statistical techniques are applied for turbulent flows to capture the physics in the turbulent flows.

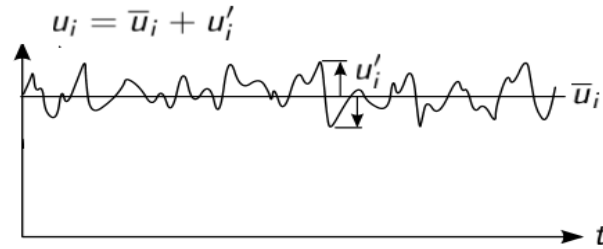


Fig. (4)

In the Fig. (4), this is turbulent flow signal which can be divided into two components that is fluctuating component and mean component. The turbulent flow as shown in Fig. (4) is known as stationary turbulent flows because mean of turbulent flow is stationary.

Instantaneous velocity = Mean + Fluctuations

$$u_i = \bar{u} + u' \quad \text{Eq. (19)}$$

Where;

u_i = Instantaneous velocity

u' = Fluctuation velocity

\bar{u} = Mean Velocity

The Eq. (19) is known as Reynolds decomposition.

Types of Turbulent Flows:

1. Stationary Turbulent Flows:

The turbulent flows are unsteady in nature but we made an average over time the mean component of transport variable is steady as shown in Fig. (4).

2. Homogenous Turbulent flows:

Turbulent statistics are independent of co-ordinate translation.

3. Isotropic Turbulent flows:

Turbulence statistics are independent of rotation, reflections and translation of co-ordinates. The isotropic turbulent flows are also homogenous in nature.

Types of Averaging Techniques:

1. Time Averaging

When the transport variable is averaged over time scale is known as Time Averaging. The selection for time scale should such that it should larger than the individual time scale of fluctuation and much smaller than the full-time scale of the system.

Mathematically;

$$\rightarrow \frac{\int_t^{t+T} u dt}{T} \quad \text{Eq. (20)}$$

2. Space Averaging

When the transport variables are averaged over a spatial co-ordinate system then it is called space averaging.

Mathematically;

$$\rightarrow \frac{\int_x^{x+X} u dx}{X} \quad \text{Eq. (21)}$$

General properties of Turbulent Quantities

For any turbulent quantity f :

$$1) f = \bar{f} + f'$$

$$2) \bar{f} = \lim_{T \rightarrow \infty} \frac{\int_0^T f dt}{T} \rightarrow \lim_{T \rightarrow \infty} \frac{\int_0^T (\bar{f} + f') dt}{T} \rightarrow \bar{f} + \bar{f}'$$

$$\rightarrow \bar{f}' = 0$$

For any two turbulent quantities:

$$1. \bar{f}' = 0$$

$$2. \bar{\bar{f}} = \bar{f}$$

$$3. \overline{f\bar{g}} = \bar{f}\bar{g}$$

$$4. \overline{\bar{f}g'} = 0$$

$$5. \overline{f + g} = \bar{f} + \bar{g}$$

$$6. \overline{f'g'} \neq 0$$

$$7. \frac{\partial \bar{f}}{\partial x} = \overline{\frac{\partial f}{\partial x}}$$

$$8. \overline{\int f dx} = \int \bar{f} dx \quad \text{Eq's. (22)}$$

Averaged Continuity Equation:

1. Continuity Equations for Mean flow and Turbulent Component is derived below using Eq's. (22) Properties;

Continuity equation is given by:

$$\rightarrow \frac{\partial u_j}{\partial x_j} = 0 \quad \text{Eq. (23)}$$

→ Substitute $u_j = \bar{u}_j + u'_j$ and taking average of the equation:

$$\rightarrow \frac{\partial}{\partial x_j} (\bar{u}_j + u'_j) = 0$$

$$\rightarrow \frac{\partial \bar{u}_j}{\partial x_j} = 0 \quad \text{Eq. (24)}$$

The Eq. (23) is continuity Equation for mean component of the flow.

Now, we must find the continuity equation for turbulent fluctuation component. So, for that subtract Eq. (24) from Eq. (23), we get

$$\rightarrow \frac{\partial u'_j}{\partial x_j} = 0 \quad \text{Eq. (25)}$$

The Eq. (25) is the turbulent fluctuation component of Continuity Equation. The continuity equation is driven by assuming constant density flows.

Reynolds average Navier stokes(RANS) equation:

$$\rightarrow \rho \left[\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} \right] = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left(\mu \frac{\partial u_i}{\partial x_j} \right)$$

$$\rightarrow \rho \left[\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} + u_i \frac{\partial u_j}{\partial x_j} \right] = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left(\mu \frac{\partial u_i}{\partial x_j} \right) \quad \text{*Manipulation made by adding, } u_i \frac{\partial u_j}{\partial x_j}$$

$$\rightarrow \rho \left[\frac{\partial u_i}{\partial t} + u_j \frac{\partial}{\partial x_j} (u_i u_j) \right] = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left(\mu \frac{\partial u_i}{\partial x_j} \right) \quad \text{Eq. (26)}$$

Substitute the below components in the Eq. (26)

$$u_i = \bar{u}_i + u'_i, u_j = \bar{u}_j + u'_j, p = \bar{p} + p'$$

And taking the average of the whole equation we get the following equation:

$$\rho \left[\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial}{\partial x_j} (\bar{u}_i \bar{u}_j) \right] = -\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j} \left(\mu \frac{\partial \bar{u}_i}{\partial x_j} \right) - \frac{\partial}{\partial x_j} (\rho \bar{u}'_i u'_j)$$

Term A

Eq. (27)

The Eq. (27) is known as Reynold Averaged navier stoke equation or RANS.

Some of the Deduction of Eq. (27) as follows;

1. The process of averaging has introduced new term (Term A)
2. $\frac{\partial}{\partial x_j} (\rho \overline{u'_i u'_j})$ has a unit of Stress.
3. Velocity fluctuation along x-direction that can interact with fluctuation along y-direction therefore there can be a momentum transfer. So, there is extra component of stress.
4. Total stress is given by
5. $\frac{\partial \tau_{ij}}{\partial x_j} = \frac{\partial}{\partial x_j} \left[\left(\mu \frac{\partial \overline{u_i}}{\partial x_j} \right) - (\rho \overline{u'_i u'_j}) \right]$
6. Near the wall region due to no slip and no penetration boundary condition the viscous shear stresses are more in comparison to turbulent stress component.
7. Term A is known as Turbulent stress and it is a second order tensor and can be represented by the following stress tensor.

$$-\rho \overline{u'_i u'_j} = -\rho \begin{bmatrix} \overline{u'^2_1} & \overline{u'_1 u'_2} & \overline{u'_1 u'_3} \\ \overline{u'_1 u'_2} & \overline{u'^2_2} & \overline{u'_2 u'_3} \\ \overline{u'_1 u'_3} & \overline{u'_2 u'_3} & \overline{u'^2_3} \end{bmatrix}$$

8. In the RANS equations, there are six additional unknowns:
9. $-\rho \overline{u'^2_1}, -\rho \overline{u'^2_2}, -\rho \overline{u'^2_3}, -\rho \overline{u'_1 u'_2}, -\rho \overline{u'_2 u'_3}, -\rho \overline{u'_1 u'_3}$

Closure problem in turbulence necessity of turbulence modelling:

1. In the RANS equation, Reynolds stress gives an additional six terms but there are no explicit governing differential equations for the additional unknowns.
2. 3 Velocity components, 1 pressure term and 6 Reynolds stress terms
3. Therefore, there are 10 unknowns in RANS.
4. But No. of equations are 4 (1 Continuity + 3 momentum)
5. *No. of unknowns > No. of Eqns* → The problem is indeterminate.
6. So, there is a need to close the problem mathematically to obtain the solution. This is known as closure problem in Turbulence.
7. The turbulence modelling tries to represent the Reynolds stresses in terms of the time-averaged velocity components.
8. The common turbulence models are classified, based on the number of additional transport equations that need to be solved along with the “RANS” Equations

Turbulent Models in Fluent:

In this report, I will only describe turbulent models which are available with FLUENT commercial package and I am not considering any direct numerical simulation techniques. The graphical representation of the models is given below:

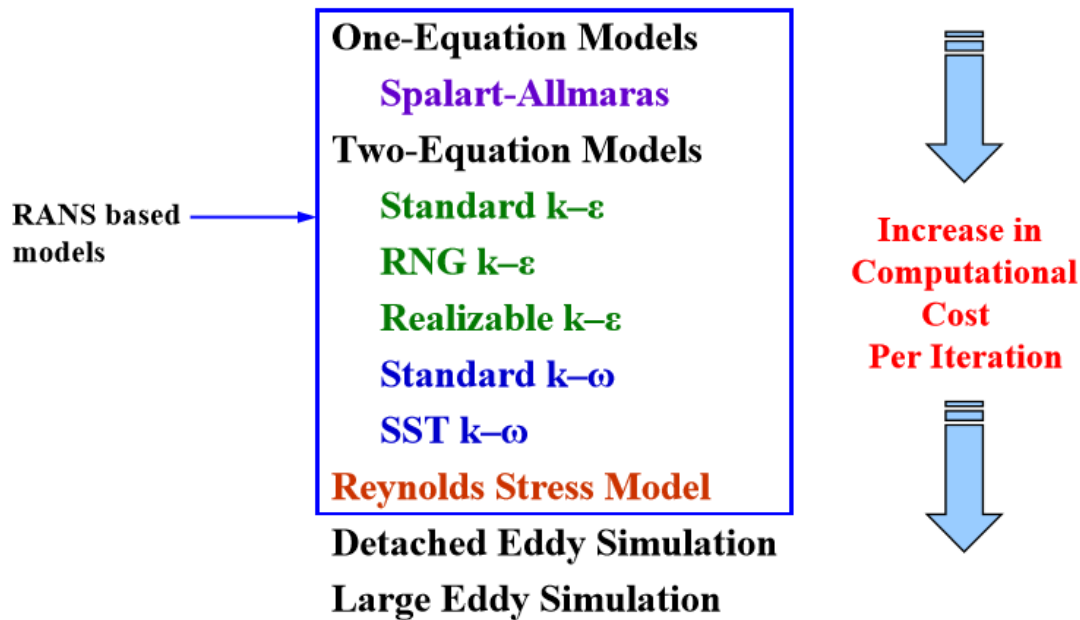


Fig. (5)

All the models available in the fluent are based on RANs as shown in the Fig. (5). There is one equation model that is also known as first order model. Then there are five two equation models which is also known as second order model. The last model is Reynold stress model which is five equation model. The five-equation model is most accurate and robust model but it needs more computational speed as compared to other models. So, it is expensive model.

First order Models:

The first order or One Equation models are based on the comparison of Laminar viscosity with turbulent viscosity. These models are also called Eddy-Viscosity Model(EVM):

The analogy is given by:

$$\text{Laminar: } \left\{ \tau_{ij} = \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \mu \delta_{ij} \frac{\partial u_j}{\partial x_j} \right.$$
$$\text{Turbulent: } \left\{ \tau_{ij}^t = \rho \overline{u'_i u'_j} = \mu_t \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \delta_{ij} \rho k \right.$$

Note:

μ_t is the turbulent viscosity

k is the turbulent kinetic Energy.

* So now the problem is to model the μ_t

The calculation of Turbulent Viscosity is based on dimensional analysis and μ_t can be calculated from a turbulence time scale or velocity scale and a length scale.

1. Turbulent kinetic Energy, $\left(\frac{L^2}{T^2}\right) k = \overline{u'_i u'_i} / 2$
2. Turbulence dissipation rate $\left(\frac{L^2}{T^3}\right), \varepsilon = \nu \frac{\partial w_i}{\partial x_j} \left(\frac{\partial w_i}{\partial x_j} + \frac{\partial w_j}{\partial x_i} \right)$
3. Specific Dissipation rate $\left(\frac{1}{T}\right), \omega = \varepsilon / k$

All the models in fluent has different approach for the calculations of turbulent viscosity;

❑ Spalart-Allmaras:

The Spalart-Allmaras, Solve the transport equation for a modified turbulent viscosity.

$$\mu_t = f(\tilde{\nu}) \quad \text{Eq. (28)}$$

❑ Standard $k - \varepsilon$, RNG $k - \varepsilon$, Realizable $k - \varepsilon$

Standard $k - \varepsilon$, models solve the transport equation for both k and ε

$$\mu_t = f(\rho k^2 / \varepsilon) \quad \text{Eq. (29)}$$

❑ Standard $k - \omega$, SST $k - \omega$

$k - \omega$ Solves the transport Equation for $k - \omega$

$$\mu_t = f(\rho k / \omega) \quad \text{Eq. (30)}$$

Spalart Allmaras Model

In this model, the transport Equation for modified turbulent viscosity is given below:

$$\frac{\partial}{\partial t}(\rho \tilde{v}) + \frac{\partial}{\partial x_i}(\rho \tilde{v} u_i) = G_v + \frac{1}{\sigma_{\tilde{v}}} \left[\frac{\partial}{\partial x_j} \left\{ (\mu + \rho \tilde{v}) \frac{\partial \tilde{v}}{\partial x_j} \right\} + C_{b2} \rho \frac{\partial \tilde{v}^2}{\partial x_j} \right] - Y_v + S_{\tilde{v}} \quad \text{Eq. (31)}$$

Where,

- G_v is the Generation of turbulent viscosity and Y_v is the dissipation of turbulent viscosity that occurs near- wall region due to no penetration and viscous damping.
- $\sigma_{\tilde{v}}$ and C_{b2} are the constants
- In the Spalart Allmaras model kinetic energy is not calculated therefore the term $\frac{2}{3} \delta_{ij} \rho k$ in the turbulent viscosity is neglected.

Two Equation Model:

□ $k - \varepsilon$ Models

- Standard
- RNG
- Realizable

All the above three models have similar forms with two transport equation of k and ε .

The main difference is method of

1. Different Approach for calculating the turbulent kinetic energy.
2. Different method to calculate the turbulent Prandtl number
3. Difference in the generation and dissipation term in ε equation

The Fig. (6) shown below is taken from Fluent software. The Fig. (6) represents the different viscous models that are available in FLUENT Software. For turbulent models like $k - \varepsilon$, we have to select near wall treatment model also because the near wall region flow behaviour is very different. Flow near the wall region is kind of laminar because the near wall region viscous stresses are dominant because of no fluctuation component present near the wall because of no-penetration boundary condition.

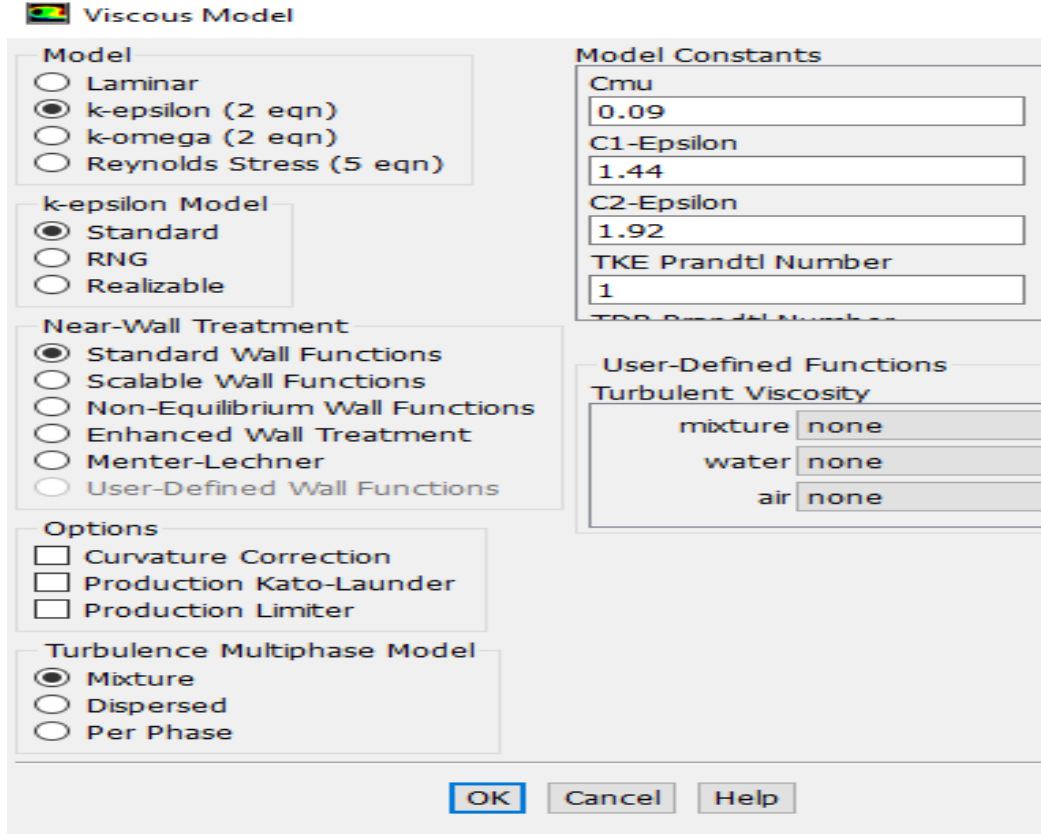


Fig. (6)

¹Transport Equation for k – ε Models:

Transport Equation for Turbulent Kinetic Energy **k**:

$$\frac{\partial}{\partial t}(\rho k) + \frac{\partial}{\partial x_i}(\rho k u_i) = \frac{\partial}{\partial x_j} \left(\mu + \frac{\mu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_j} + G_k + G_b - \rho \varepsilon - Y_m + S_k \quad \text{Eq. (32)}$$

Transport Equation for Dissipation **ε**:

$$\frac{\partial}{\partial t}(\rho \varepsilon) + \frac{\partial}{\partial x_i}(\rho \varepsilon u_i) = \frac{\partial}{\partial x_j} \left(\mu + \frac{\mu_t}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon}{\partial x_j} + C_{1\varepsilon} \frac{\varepsilon}{k} (G_k + C_{3\varepsilon} G_b) + C_{2\varepsilon} \rho \frac{\varepsilon^2}{k} + S_\varepsilon \quad \text{Eq. (33)}$$

Where;

G_k is the Generation of Turbulence Kinetic Energy

G_b is the Generation of Turbulence Kinetic Energy due to buoyancy

Y_m is the contribution of fluctuating dilatation in compressible turbulence to the overall dissipation rate

$C_{1\varepsilon}$, $C_{2\varepsilon}$, $C_{3\varepsilon}$ are constant and σ_k , σ_ε are the Prandtl numbers for k and ε respectively.

RNG K- ϵ Model

1. The RNG model has additional terms in its transport equation that helps to improve the robustness and can be applicable to wide range of flow regimes. The RNG model is suitable for highly compressible fluids.
2. The swirl effect also included in the transport equation in RNG Model. Therefore, RNG Model can be applied to flows where swirling effects are dominant and to capture the detailed physics of swirling flows.
3. The RNG model provides new analytical formula for Prandtl number which gives better results as compared to standard model.
4. The standard model is only for high Reynolds number. So, the standard model can not predict the physics of low Reynolds number. While in RNG model, they provide an analytically-derived differential for effective viscosity that accounts for lower Reynolds number also.

The above features make the RNG model best for computational techniques for large engineering application as it is also not very expensive.

k – ω Model

1. This model solves the transport equation for Dissipation rate and specific dissipation rate. This model is best when the physics of near wall zone is required for analysis. Because in this model there is no need to select near wall treatment function with standard model. Because, near wall treatment is inbuilt in it. For $y^+=1$, it will capture all the details of near wall region using two-layer zone.
2. The models do not contain the term which are unknown at the near wall zone.
3. This model is well suited for Boundary layer flows with pressure gradient. For Example, Flow over cylinder, sphere.

Fluent has two models in K- ω

Standard K- ω

1. This model finds its application in aerospace and turbomachinery industry.
2. There are different sub models are available which accounts for the effects of compressibility, transitional flows.

SST K- ω (Menter, 1994)

1. The SST K- ω model uses mixing algorithm function to gradually transition from the standard K- ω model near the wall (Because, they are highly suitable for boundary layer) to high Reynolds number version of k- ϵ in the outer portion of the boundary layer. So, this model can be applied where we have to capture boundary layer and out of boundary layer physics.

2. This models includes a modified turbulent viscosity to account for the transport effects of the principal turbulent shear stress.

Large Eddy Simulation Model:

1. The basic concept of Les model is to neglect small scale eddies to decrease the computational time.
2. Les model is best for high end applications where Reynolds average navier stokes model fails. For example:
 - Combustion
 - Mixing
 - External Aerodynamics (Flow around bluff bodies, Flow over cylinder)

Results and Discussion on Literature Review:

To find out which model is best for analysis of flow separation in a 90° Pipe bend. ³Kim et al. 2014 compared different turbulent models with experimental studies done by Sudo et al. (1998). The experimental studies were done by ⁴Sudo et al. (1998) on 90° pipe bend with $R_c/D = 2$. His team used hot wire anemometer to perform an analysis on stream-wise velocity (U_s) and circumferential velocity (U_c) at different length of the pipe. The air used as a working fluid and test was performed on Reynolds number of 6×10^4 which satisfies the condition of turbulent flows because in pipe flows the Reynolds should greater the 2300.

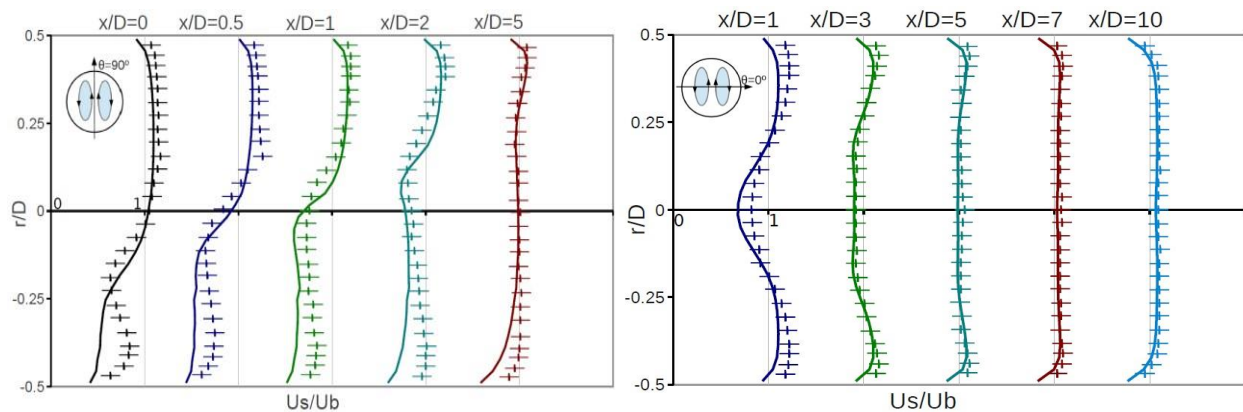


Fig. (6) & Fig (7)

The fig shows the comparison between experimental measured stream-wise velocity component and numerical studies done by Kim at el. (2014). In the Fig. (6) and Fig. (7) results are obtained using Standard k-Epsilon model. Where Solid lines represents numerical results and plus sign represents Experimental data in the Fig. (6) and Fig. (7). In the Fig. (6) the study was performed at 90°. While, in Fig. (7) the study was done on straight section at 0° portion of the pipe.

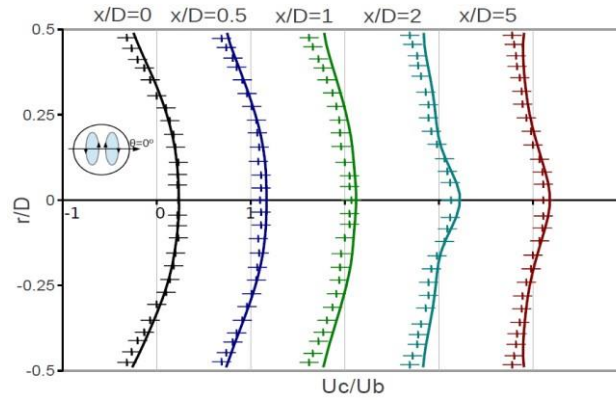
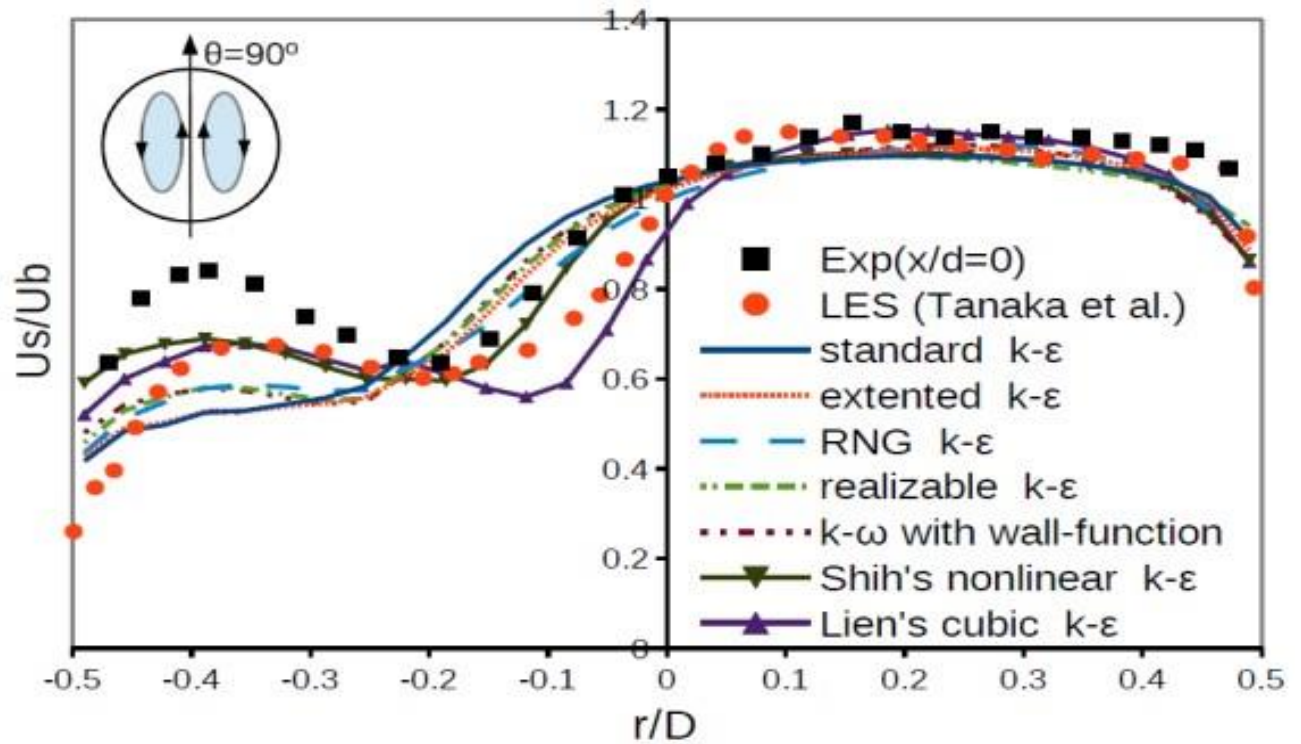


Fig. (8)

The Fig. (8) Shows results of experimental data and numerical solution for circumferential velocity. In the Fig. (6), Fig. (7) and Fig. (8) the numerical results are in good agreement with experimental studies.



In the above figure Kim. Et al (2014) used different turbulent models at $r/D=0$ at the symmetry line of the pipe along with experimental results to find out which turbulent model gives best result. From the above figures, it can be clearly seen that the RNG k-Epsilon gives best result when compared with experimental results on stream-wise velocity components.

Conclusion:

To find out which turbulent gives best results depends on the problem. From the overall perspective, the Reynold stress model is best turbulent model but it is expensive because it requires high computational speed to get results quickly. For engineering application, the most widely used turbulent model is RNG K-epsilon model.

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