Classification of Poker Hands

Ordinal Response Multicategory Logit Model

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Introduction

- What are the odds of getting a Royal Flush in poker? What about a Straight?
- In order to investigate these ideas, we will use the Poker Hand Data Set provided by the UCI Machine Learning Repository to carry out our analysis.
- Can we fit an adequate model that predicts our poker hand, knowing the 5 cards that we are dealt in a particular game?

Poker Hand Data Set

- This data set contains examples of poker hands, consisting of 5 cards, drawn from a standard deck of 52 cards.
- Each card in the hand is described using it's suit and rank, which gives a total of 10 predictors for a given poker hand.
- The suit of each card is represented by *S1, ... S5*, and the rank is represented by *C1, ..., C5*.
- The suit is given by an ordinal value of $1 - 4$, which represent Hearts, Spades, Diamonds, and Clubs, respectively.
- The rank is given by a numerical value of $1 - 13$, representing Ace, $2, 3$, ..., Queen, and King, respectively.

Class of a Poker Hand

- The last piece of information for each hand is called the *Class*.

Table: Corresponding Poker Hand for Each *Class*.

Class	Corresponding Poker Hand
9	Royal Flush
8	Straight Flush
7	Four of a Kind
6	Full House
5	Flush
4	Straight
3	Three of a Kind
2	Two Pairs
1	One Pair
0	Nothing in Hand

Basics of Poker

- A poker hand is dealt 1 card at a time, without replacement.
- A player receives a total of 5 cards in each hand.
- Once all 5 cards are dealt, the player can have any one of the 10 *Classes* of poker.
- The data set considers a single player with a single poker hand.

Poker Hands

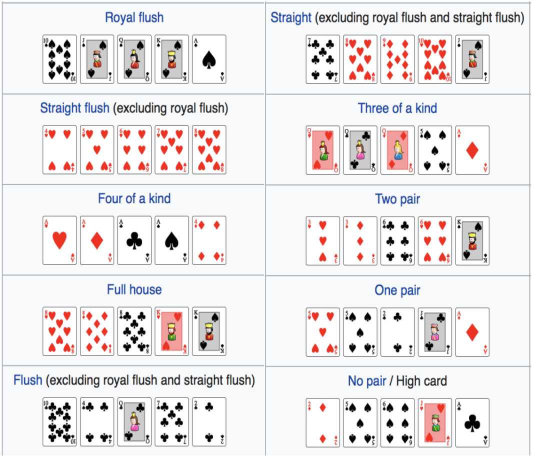


Figure: *Examples of Each Poker Hand*

Known Poker Hand Probabilities and Odds

- Given a standard deck of 52 cards, the number of ways that a player can obtain each hand, along with the probability and the odds of these outcomes, is already known (Wikipedia 2019, UHM Department of Mathematics 2005).
- We can use these values to compare against the probabilities and odds that we obtain from the test and training poker hands provided in the Poker Hand Data set.

Table: Probability and Odds of each Poker Hand

Class	Poker Hand	Distinct Hands	Frequency	Probability	Odds
9	Royal Flush	1	4	0.000153908%	649,737.8 : 1
8	Straight Flush	9	36	0.00138517%	72,192.3 : 1
7	Four of a Kind	156	624	0.0240096%	4,164.001 : 1
6	Full House	156	3,744	0.144058%	693.165 : 1
5	Flush	1,277	5,108	0.19654%	507.802 : 1
4	Straight	10	10,200	0.392465%	253.800 : 1
3	Three of a Kind	858	54,912	2.11285%	46.329 : 1
2	Two Pairs	858	123,552	4.7539%	20.035 : 1
1	One Pair	2,860	1,098,240	42.2569%	1.366 : 1
0	Nothing in Hand	1,277	1,302,540	50.1177%	0.995 : 1
-	Total	7,462	2,598,960	100%	-

Methodology

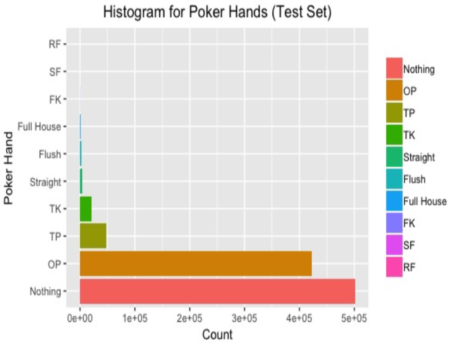
- We will use the *table()* function to get the distribution of *Class* within both the training and test sets.
- We will use the package *ggplot2* in order visualize the probabilities and odds for each *Class* in both the training and test set, as well as to make comparisons to the known values.
- We can fit the proportional odds model on the training set using the *vglm()* function from the ”VGAM” package (Dang 2019, slides 35 - 36, 40 - 42).
- We will use this model since our data is an example of ordered multinomial data. Thus, we want to use an Ordinal Response Multicategory Logit Model to run our analysis (Dang 2019, slides 44 - 47).
- In order to make these predictions, we can use the *predictvglm()* function and our proposed model on the test set (Dang 2019, slides 44 - 48).

Distribution and Probability of Poker Hands (Test Set)

Table: Frequency and Probability of Getting Each Poker Hand in the Test Set

Class	Corresponding Poker Hand	Frequency	Probability
9	Royal Flush	3	0.0003%
8	Straight Flush	12	0.0012%
7	Four of a Kind	230	0.023%
6	Full House	1,424	0.1424%
5	Flush	1,996	0.1996%
4	Straight	3,885	0.3885%
3	Three of a Kind	21,121	2.1121%
2	Two Pairs	47,622	4.7622%
1	One Pair	422,498	42.2498%
0	Nothing in Hand	501,209	50.1209%
-	Total	1,000,000	100%

Visualizing the Test Set Poker Hands



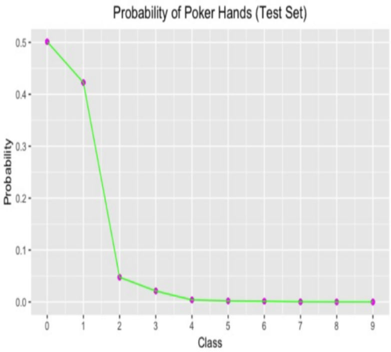


Figure: *Histogram of Frequency and Graph of Probability of Each Poker Hand in the Test Set*

Distribution and Probability of Poker Hands (Training Set)

Table: Frequency and Probability of Getting Each Poker Hand in the Training Set

Class	Corresponding Poker Hand	Frequency	Probability
9	Royal Flush	5	0.019992%
8	Straight Flush	5	0.019992%
7	Four of a Kind	6	0.0239904%
6	Full House	36	0.1439424%
5	Flush	54	0.2159136%
4	Straight	93	0.3718513%
3	Three of a Kind	513	2.05118%
2	Two Pairs	1,206	4.822071%
1	One Pair	10,599	42.37905%
0	Nothing in Hand	12,493	49.95202%
-	Total	25,010	100%

Visualizing the Training Set Poker Hands

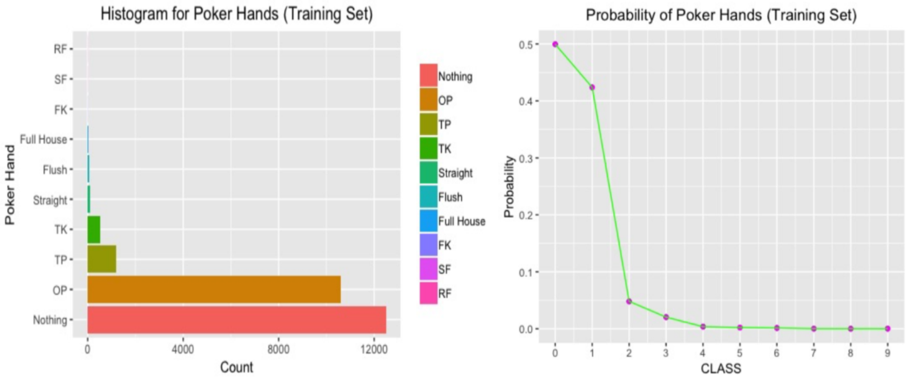


Figure: *Histogram of Frequency and Graph of Probability of Each Poker Hand in the Training Set*

Probability Comparison

- The probabilities for both the test and training set are close to the known probabilities, with the highest discrepancy, 0.17%, being between the training set and the known values when $Class = 9$.

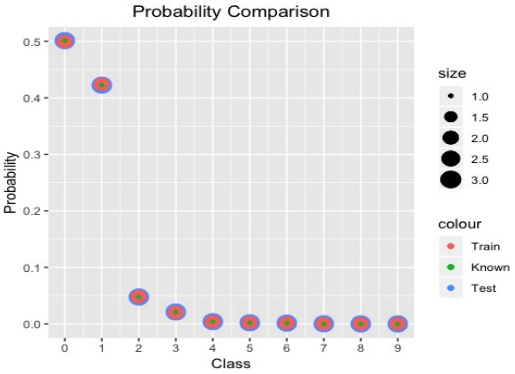


Figure: *Probability Comparisons for Test and Training Set to Known Values*

Odds Comparison

- The odds are close to the known odds for *Class* values from 0 to 7.
- For *Class* $= 8$, the odds for the test set are higher than the known value, while the odds for the training set are lower.
- The odds for the training set and test set are both lower than the known value for the *Class* of 9.

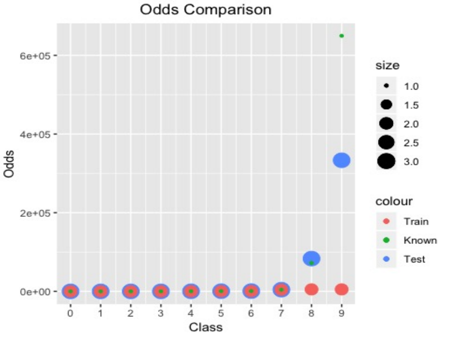


Figure: *Odds Comparisons for Test and Training Set to Known Values*

Fitting a Proportional Odds Model: *vglm*

- We can fit the proportional odds model on the training set using the *vglm()* function from the ”VGAM” package (Dang 2019, slides 35 - 36, 40 - 42).
- Our data is an example of ordered multinomial data.
- We want to use an Ordinal Response Multicategory Logit Model to run our analysis (Dang 2019, slides 44 - 47).
- $m1 = vglm(CLASS\ S1 + C1 + S2 + C2 + S3 + C3 + S4 + C4 + S5 + C5, family = propodds(reverse = FALSE), data = poker\_hands\_train)$

Fitting a Proportional Odds Model: Output

```
> m1 <- vglm(CLASS~S1 + C1 + S2 + C2 + S3 + C3 + S4 + C4 + S5 + C5, family =
↪  propodds(reverse = FALSE), data = poker_hands_train)
>
> summary(m1)

Call:
vglm(formula = CLASS ~ S1 + C1 + S2 + C2 + S3 + C3 + S4 + C4 +
    S5 + C5, family = propodds(reverse = FALSE), data = poker_hands_train)

Pearson residuals:
                       Min        1Q    Median       3Q     Max
logitlink(P[Y<=1])  -1.166 -1.093632 -0.032608 0.979835  1.0448
logitlink(P[Y<=2])  -4.507  0.169423  0.172929 0.368261  0.3842
logitlink(P[Y<=3])  -6.980  0.090532  0.093170 0.111491  1.2234
logitlink(P[Y<=4]) -15.998  0.044937  0.046308 0.047936  1.3636
logitlink(P[Y<=5]) -20.182  0.030109  0.030622 0.031187  5.4339
logitlink(P[Y<=6]) -25.550  0.020807  0.021112 0.021457  7.0133
logitlink(P[Y<=7]) -60.494  0.011602  0.011770 0.011955  5.3419
logitlink(P[Y<=8]) -62.751  0.008860  0.008987 0.009127 24.4133
logitlink(P[Y<=9]) -68.270  0.006842  0.006939 0.007042 24.3813
```

Fitting a Proportional Odds Model: Output Continued

```
Coefficients:
                Estimate Std. Error z value Pr(>|z|)
(Intercept):1 -0.0495686  0.0836927  -0.592   0.5537
(Intercept):2  2.4412451  0.0860272  28.378   <2e-16 ***
(Intercept):3  3.4831885  0.0909992  38.277   <2e-16 ***
(Intercept):4  4.7788260  0.1090881  43.807   <2e-16 ***
(Intercept):5  5.4124210  0.1277055  42.382   <2e-16 ***
(Intercept):6  6.1267866  0.1615713  37.920   <2e-16 ***
(Intercept):7  7.3068854  0.2633894  27.742   <2e-16 ***
(Intercept):8  7.7771275  0.3269150  23.789   <2e-16 ***
(Intercept):9  8.4704740  0.4548318  18.623   <2e-16 ***
S1            -0.0070404  0.0109871  -0.641   0.5217
C1            -0.0017249  0.0032703  -0.527   0.5979
S2             0.0053476  0.0109355   0.489   0.6248
C2             0.0036423  0.0032552   1.119   0.2632
S3             0.0008664  0.0109254   0.079   0.9368
C3             0.0045837  0.0032758   1.399   0.1617
S4            -0.0051708  0.0109883  -0.471   0.6379
C4            -0.0049055  0.0032720  -1.499   0.1338
S5             0.0198628  0.0109653   1.811   0.0701 .
C5             0.0002178  0.0032766   0.066   0.9470
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Fitting a Proportional Odds Model: Output Continued

```
Number of linear predictors:  9

Names of linear predictors: logitlink(P[Y<=1]), logitlink(P[Y<=2]), logitlink(P[Y<=3]), logitlink(P[Y<=4]),
logitlink(P[Y<=5]), logitlink(P[Y<=6]), logitlink(P[Y<=7]), logitlink(P[Y<=8]), logitlink(P[Y<=9])

Residual deviance: 49277.64 on 225071 degrees of freedom

Log-likelihood: -24638.82 on 225071 degrees of freedom

Number of Fisher scoring iterations: 3

Warning: Hauck-Donner effect detected in the following estimate(s):
'(Intercept):4', '(Intercept):5', '(Intercept):6', '(Intercept):7', '(Intercept):8', '(Intercept):9'


Exponentiated coefficients:
       S1        C1        S2        C2        S3        C3        S4        C4        S5        C5
0.9929843 0.9982766 1.0053619 1.0036489 1.0008668 1.0045942 0.9948426 0.9951065 1.0200614 1.0002179
>
> AICvlm(m1)
[1] 49315.64
```

Model and Parameter Interprestation

- The summary of the model with all variables as predictors indicates that some of these variables are not significant, as they have a p-value higher than 0.05.
- However, we will not proceed with any variable selection methods.
- We can not reduce the number of predictors, as the resulting *Class* for a poker hand is directly related to both the rank and suit of each of the 5 cards in hand.
- Therefore, we can use the full model when investigating and making predictions on the test set.

Making Predictions Using Our Model

- We can use this model and results to make predictions about what hand we will have, given the 5 cards we are dealt in the poker game.
- In order to make these predictions, we can use the *predictvglm()* function and our proposed model on the test set (Dang 2019, slides 44 - 48).
- By using the *predict()* function and the proportional odds model, with the *Polr()* function from the *MASS* package, we obtain a distribution for the classification of hands in the test set.

Distribution of Poker Hand Predictions in the Test Set

Table: Expected and Predicted Values for the Frequency of Each Poker Hand in the Test Set

Class	Poker Hand	Expected Frequency	Predicted Frequency
9	Royal Flush	3	0
8	Straight Flush	12	0
7	Four of a Kind	230	0
6	Full House	1,424	0
5	Flush	1,996	0
4	Straight	3,885	0
3	Three of a Kind	21,121	0
2	Two Pairs	47,622	0
1	One Pair	422,498	44,847
0	Nothing in Hand	501,209	955,153
-	Total	1,000,000	1,000,000

Confusion Matrix for Predictions

Table: Confusion Matrix for Prediction on the Test Set

Predicted *Class*	Nothing	1 Pair	2 Pairs	3 of a Kind	Straight	Flush	Full House	4 of a Kind	Straight Flush	Royal Flush
Nothing	478, 607	403, 558	45, 555	20, 229	3, 702	1, 907	1, 361	219	12	3
1 Pair	22, 602	18, 940	2, 067	892	183	89	63	11	0	0
2 Pairs	0	0	0	0	0	0	0	0	0	0
3 of a Kind	0	0	0	0	0	0	0	0	0	0
Straight	0	0	0	0	0	0	0	0	0	0
Flush	0	0	0	0	0	0	0	0	0	0
Full House	0	0	0	0	0	0	0	0	0	0
4 of a Kind	0	0	0	0	0	0	0	0	0	0
Straight Flush	0	0	0	0	0	0	0	0	0	0
Royal Flush	0	0	0	0	0	0	0	0	0	0

Prediction Implications

- We can see that our predicted frequencies do not match the expected value provided in the Poker Hand Test Set.
- The obtained predictions are not accurate.
- When trying to make predictions, the poker hands were allocated to a value for *Class* of 0 or 1, even if the 5 cards were known to give a different poker hand that did not correspond to having Nothing in Hand or One Pair.

Prediction Example

- If we are dealt the 4 of hearts and the 4 of diamonds, we will not get a poker hand corresponding to having Nothing in Hand, a Straight, a Flush, a Straight Flush, or a Royal Flush.
- What more can we say if the third card dealt to us is the 4 of spades? Now, we can not have a poker hand with just One Pair.
- Furthermore, what if the fourth card dealt to us is the 4 of clubs? Now, we can not have a poker hand with just Three of a Kind. We also can not get Two pairs, or a Full House.
- The only hand that is possible, given that our first 4 cards are the 4 of diamonds, hearts, spades and clubs, is Four of a Kind ($Class = 7$), regardless of what the fifth and final card turns out to be.
- When trying to run predictions using our model, we do not obtain these results

Prediction Example Continued

- When predicting the outcome of this example poker hand, given that we get the 4 of hearts as our first card ($S1 = 1, C1 = 4$), the 4 of diamonds as our second card ($S2 = 3, C2 = 4$), the 4 of spades as our third card ($S3 = 2, C3 = 4$), the 4 of clubs as our fourth card ($S4 = 4, C1 = 4$) and some arbitrary card, say the 3 of hearts, as our fifth card ($S5 = 1, C5 = 3$), we obtain the following.

Table: Predicted Probability for Example Poker Hand

Class	Poker Hand	Probability
9	Royal Flush	0.0206%
8	Straight Flush	0.0206%
7	Four of a Kind	0.02472%
6	Full House	0.14831%
5	Flush	0.22244%
4	Straight	0.38301%
3	Three of a Kind	2.11115%
2	Two Pairs	4.9533%
1	One Pair	42.93125%
0	Nothing in Hand	49.18462%

Predict Example Continued

- As we can see, the model gives a probability other than 0% for each *Class* of poker hands.
- Meanwhile, as we discussed in the previous analysis, the only possible outcome with this example hand is to get Four of a Kind, corresponding to a *Class* of 7.
- Therefore, we should see a value for the probability of all other poker hands of 0% and a value of 100% for the *Class* of 7, corresponding to having Four of a Kind in hand.

Limitations and Challenges

- The training data set of $25,000$ instances was largely biased towards poker hands corresponding to having Nothing in Hand ($Class = 0$) and One Pair ($Class = 1$), having more than $20,000$ instances of those two alone, as compared to only 5 being Royal Flush ($Class = 9$).
- With the overall probability of predicting One Pair or Nothing being around 92.33%, any model like the proportional odds model or multinomial model gives predictions that allocate the poker hands to these two *Classes*.

Limitations and Challenges: Some Possible Solutions

- Inculcating Support Vector Machine or Neural Networks with many hidden layers is one potential solution to this problem.
- One of the main things to note is that the model should have the ability to differentiate between Three of a Kind and Full House, as well as One Pair and Full House (Bhat and Selvam 2016).
- To overcome some of these limitations and successfully classify the poker hands, we can add additional observations for *Classes* from 2 to 9 to reduce or remove this bias and increase the size of the training data set (Bhat and Selvam 2016).

Conclusion

- We were able to calculate the probability and odds of having one of the 10 *Classes* of poker hands for both the training and test sets provided in the Poker Hand Data set and compare them to the known values for a standard deck of 52 cards.
- Since the training and test set are ordered multinomial data, we fit a proportional odds model using the *vglm* function. The model contained all 10 predictors, the suit and rank corresponding to each of the 5 cards in a particular poker hand.
- This model did not prove to be accurate in making predictions for the poker hands in the test set, since we had a disproportional data set, with a large proportion being in $Class = 0$.

Future Work

- Future work includes finding a package or function in R that is able to take into account the disproportionate data in the test set.
- With such a function in R, the goal would be to find a way to make more reliable predictions and accurate classifications of the poker hands.

Key References

1. Bhat, Gautam and Selvam, Kaviarsan 2016, *NN-based Poker Hand Classification and Game Playing*, Boston University, https://pdfs.semanticscholar.org/905a/0f27e520b3f2627863f9fa94a87fda7060cd.pdf, Accessed on 30, Apr 2019.
2. Dang, S 2019, *Multinomial Regression*, lecture notes, Regression II MA532, Binghamton University, delivered 10, Apr 2019.
3. Dua, Dheeru and Graff, Casey 2017, *UCI Machine Learning Repository, Poker Hand Data Set*, University of California, Irvine, School of Information and Computer Sciences, https://archive.ics.uci.edu/ml/datasets/Poker+Hand, Accessed on 28, Apr 2019.
4. UHM Department of Mathematics 2005, *5-CARD POKER HANDS*, http://www.math.hawaii.edu/~ramsey/Probability/PokerHands.html, Accessed on 28, Apr 2019.
5. Wikipedia 2019, *Poker probability*, https://en.wikipedia.org/wiki/Poker_probability, Accessed on 28, Apr 2019.

Thank you!