Phenomenological Aspects of D-Branes

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Abstract

A general overview is presented on string phenomenology, emphasizing the role played by D-branes. A general discussion of the main challenges for string phenomenology is followed by recent progress made in constructing realistic models from D-branes and anti-branes at singularities and also from intersecting D-branes. Some possible cosmological implications of these classes of string models are also mentioned.

Contents

1 Introduction

The promise of string theory to include all observed particles and interactions in a unified and consistent way has yet to be realised explicitly. The standard model of particle physics [1] has a very rich structure and reproducing it from string theory represents a very important challenge for this theory.

String theory [2, 3] solves the problem of quantum gravity but the theory itself is not yet properly defined, especially in its nonperturbative aspects, despite much progress being made during the past several years. The efforts towards approaching the fundamental questions regarding the proper formulation of the theory may be called 'string nuomenology', borrowing the term from Kant, in contrast with 'string phenomenology' in which the effort is towards making contact of the theory with low-energy physics. This the subject of these lectures.

Without a full nonperturbative formulation it may still be too early to approach the phenomenological implications of the theory but there are several important reasons to do it. One is historical. The development of physics, and science in general, does not actually follow a logical line. We do not need to know all the details of a theory before trying to extract its physical implications. One of the prime examples to support this is the development of the Weinberg-Salam model in the late 1960's before knowing that gauge theories are renormalisable. Also, we already know a great deal about the theory so it is natural to explore its possible physical implications. Moreover, the study of the compactifications of the theory have actually lead to some of the most remarkable discoveries regarding the theory itself, including the discovery of duality symmetries and mirror symmetry. Finally it is important to keep in mind the real motivation for the development of string theory: rather than being a mathematical physics subject, its main virtue is to provide a fundamental theory of nature, which eventually will have to be confronted with the experiments.

These lectures were prepared having in mind students and some postdocs who are familiar with string theory and D-branes but less familiar with the standard model of particle physics and its possible realisation from string theory. Time and space constraints limited the presentation, but hopefully this may at least serve as a guide to non-experts through the relevant topics, the main achievements so far, and well defined open questions. We will start with a short reminder of the current status of the standard model, including its open problems. Next, as a matter of basic background, we will briefly

review relevant aspects of string phenomenology before the emergence of D-branes [4-82]. This discussion will be mostly based on ref. [4], to which we refer for further details and a more complete set of references regarding the earlier work. In chapter 4 we discuss the phenomenological aspects of D-branes [83-139], concentrating mostly on the possible ways of deriving the standard model, or its realistic extensions, from D-branes. We point out the main differences with previous constructions as well as their general possible implications. We finish with a short discussion of D-brane cosmology [139-155], in particular we present a concrete way of deriving inflation from D-branes which not only uses the structure of the models discussed in the previous section but also illustrates a possible direction towards eventually setting some of these models to test against cosmological observations.

2   The Standard Model for String Theorists

Let us start by recalling the characteristic properties of the standard model[1]. It is formulated in $3 + 1$ dimensions, three spatial and one time. It describes $3 + 1$ interactions: three gauge interactions corresponding to the strong, weak and electromagnetic interactions by means of a gauge symmetry $SU(3)_c \times SU(2)_L \times U(1)_Y$ broken to $SU(3)_c \times U(1)_{EM}$ at a scale of 10^2 GeV. The remaining interaction is gravity, described by Einstein's General Relativity with basic scale determined by the Planck scale $M_{Planck} = 10^{19}$ GeV. Finally the matter content comes in $3 + 1$ families: 3 families of spin 1/2 quark and lepton fields plus one scalar Higgs particle responsible for the symmetry breaking and whose vacuum expectation value (VEV) determines the mass of all massive particles of the model. Of the 3+1 dimensions, interactions and families it is usually the 1 that is less understood (time, gravity and the Higgs). We summarise in the table the spectrum of the standard model.

To better appreciate the rich structure of the standard model, and to have an idea of how difficult is to obtain such a model from string theory we will review here some of its most relevant properties.

- *Non-uniqueness.* The non-gravitational part is only one in an infinite number of consistent quantum field theories which can be considered in $3 + 1$ dimensions by having arbitrary gauge symmetry and matter content. There is no reason, other than current experimental evidence, to prefer this model over any other consistent quantum field theory.

THE STANDARD MODEL		
INTERACTIONS	$SU(3)_c \times SU(2)_L \times U(1)_Y$	

MATTER	QUARKS		
	Q_L	d_R	u_R
	$(\mathbf{3}, \mathbf{2}, \frac{1}{6})$	$(\bar{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})$	$(\bar{\mathbf{3}}, \mathbf{1}, \frac{1}{3})$
(3 FAMILIES)	LEPTONS		
	L	e_R	ν_R
	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{1}, 0)$

SYMMETRY BREAKING	HIGGS
	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$

Table 1: The standard model of elementary particle physics. With 3 gauge interactions mediated by particles of spin 1 plus gravity, mediated by a spin 2 particle. Matter comes in 3 families of spin 1/2 particles (quarks and leptons) plus one Higgs particle of spin 0. The subscripts on the gauge symmetry groups refer to 'color', 'left' and 'hypercharge', respectively. The 'left' stands for the left-handed character of the weak interactions. The electric charge is given by $Q = T_3 + Y$ with T_3 the third generator of $SU(2)_L$. The numbers in parentheses label the representations of the corresponding particles under the three gauge symmetries. Here Q_L includes the left handed parts of up and down quarks whereas L includes the left handed parts of the neutrinos and electrons. The right handed neutrino was not usually included in this list.

Something important to keep in mind when faced with the degeneracy of string models.

- *Chirality.* The standard model is chiral. This can be easily seen from the spectrum. For instance the left handed quarks are doublets under the $SU(2)_L$ symmetry whereas the right-handed quarks are singlets under $SU(2)_L$. The same applies to the leptons. Furthermore, taking a look at the spectrum we can see that out of the infinite possible representations of the corresponding groups, the only ones that seem to be realised in nature are those which are (bi-)fundamentals.

- *Anomaly free.* Despite its chiral structure, the standard model is anomaly free. This refers to the fact that the triangular Feynman diagram with fermions running in the loop and gauge fields or gravitons as external legs, vanishes identically. This is a nontrivial consistency check of the model. If the diagrams had not cancelled, that would have reflected a breaking of the gauge symmetries by quantum effects, rendering the model inconsistent. The conditions of anomaly cancellation, depending on the external legs simply impose that for $SU(3)$ the number of triplets equals the number of conjugate triplets, which can be easily checked. For mixed gravitational-gauge anomalies the conditions become $\mathrm{Tr}Y^3 = \mathrm{Tr}T_R Y^2 = \mathrm{Tr}Y = 0$ where Y is the hypercharge and T_R is the third generator of $SU(2)_L$. It is a useful exercise to check that these conditions are all satisfied with the fermion spectrum of the standard model. It is worth pointing out that the anomalies cancel for each family separately.

- *Couplings.* There are some 20 arbitrary parameters characterizing the standard model. These correspond to couplings which can be classified as: (i) gauge couplings (ii) Yukawa couplings (iii) Higgs self-couplings, besides the kinetic terms for each of the fields. The gauge couplings of matter fields are determined by the minimal coupling defined by the gauge covariant derivative. There are three different gauge couplings corresponding to the three gauge group factors g_1, g_2, g_3. They all run according to the renormalisation group equations. At the current experimental scale (10^2 GeV) they are all different with the $U(1)_{EM}$ and $SU(3)_c$ fine structure constants ($\alpha_i = g_i^2/4\pi$) taking the values $\alpha_{EM} = 1/127.922 \pm 0.027$ and $\alpha_3 = 0.1200 \pm 0.0028$ respectively. At this scale, the weak mixing angle is known with great precision to be

$$\sin^2\theta_W = g_1^2/(g_1^2 + g_2^2) = 0.23113 \pm 0.00015. \tag{1}$$

With the current spectrum of the standard model, the running of the three gauge couplings is very different, for instance the coupling of $U(1)_Y$ increases with energy whereas the one for $SU(3)_c$ decreases or is asymptotically free, which is a good indication of why quarks and gluons live in the confinement phase, although a proper explanation of confinement is one of the most important problems of the standard model. The different running of the couplings raised the hope that they could eventually meet at a larger scale. However the precision

measurements of these couplings at LEP and other experiments shows clearly that with the spectrum of the standard model the three of them *will not* meet at a single point, as it would have been expected in grand unified theories (GUT's).

The Yukawa couplings take the form

$$\lambda_1 Q_L d_R H, \quad \lambda_2 Q_L u_R \bar{H}, \quad \lambda_3 L e_R H \tag{2}$$

with the couplings λ_i being generation dependent. Once the Higgs gets a VEV, $\langle H \rangle \neq 0$ it induces masses for all the fermions. Notice that direct mass terms are forbidden for all spin $1, 1/2$ particles in the standard model from the gauge symmetries and therefore the VEV of the Higgs is the source of all the masses (with the sole exception of a Majorana mass for the right handed neutrinos $M\nu_R\nu_R$). An important point to make is that the couplings λ_i are generation dependent and therefore the corresponding mass matrix has to be diagonalised. The masses of quarks and leptons present an interesting hierarchy increasing with the family. The masses of the up quarks, down quarks and leptons are known to be in the ranges:

$$
\begin{aligned}
m_u &= 0.9 - 2.9 \text{ Mev} & m_c &= 530 - 680 \text{ MeV} & m_t &= 168 - 180 \text{ GeV} \\
m_d &= 1.8 - 5.3 \text{ Mev} & m_s &= 35 - 100 \text{ MeV} & m_b &= 2.8 - 3.0 \text{ GeV} \\
m_e &= 0.51 \text{ MeV} & m_\mu &= 105.6 \text{ MeV} & m_\tau &= 1.777 \text{ GeV}
\end{aligned}
$$

Notice the large range of masses that spreads over more than 5 orders of magnitude, with the top mass much larger than the rest. Moreover, there is recent evidence for nonzero neutrino masses from solar, atmospheric and laboratory neutrino experiments. These masses tend to be as small as a few eV, enhancing the range of masses to 11 orders of magnitude. A big challenge for a theorist to explain. The preferred scenario to explain the smallness of the neutrino masses is the 'seesaw' mechanism in which the right-handed neutrinos ν_R have a large Majorana mass $M \nu_r\nu_R$, $M \gg M_{EW}$, which combined with a standard mass term ($\sim M_{EW}$) for the left-handed neutrinos ν_L give rise to a non-diagonal mass matrix for the neutrinos with one eigenvalue of order M and the other M_{EW}^2/M, for $M \gg M_{EW}$, this could explain the smallness of neutrino masses. Notice that this mechanism works only for neutrinos since they are the only particles that can have a Majorana mass.

After diagonalising the quark and lepton mass matrix, the couplings of quarks to the electroweak gauge fields are not diagonal in this basis. The mixing between families is determined by the Cabibbo-Kobayashi-Maskawa matrix which is known to take the form, with 90% confidence:

$$|V_{CKM}| = \begin{pmatrix} 0.9741 - 0.9756 & 0.219 - 0.226 & 0.0025 - 0.0048 \\ 0.219 - 0.226 & 0.9732 - 0.9748 & 0.038 - 0.044 \\ 0.004 - 0.014 & 0.037 - 0.044 & 0.9990 - 0.9993 \end{pmatrix}$$

$$(3)$$

Its particular elements can be determined by studying the weak decays of the corresponding quarks or from deep inelastic neutrino scattering. An important part of the CKM matrix is that its (13) component is complex and is proportional to one arbitrary phase $\exp i\delta_{13}$ which is experimentally constrained to be $\delta_{13} = 59^o \pm 13^o$. The existence of this phase illustrates the fact that the discrete symmetry CP is broken by the weak interactions of the standard model.

Finally the Higgs self couplings are given by a renormalisable quartic potential of the form $V(H) = (|H|^2 - k^2)^2$, with k constant, which has the well known Mexican hat shape. This triggers the breaking of the gauge symmetry $SU(2)_L \times U(1)_Y$ to the electromagnetic $U(1)_{EM}$ at the electroweak scale $M_{EW} \sim 10^2$ GeV. This is also the source for the masses of the gauge bosons: $M_Z = 91.1876 \pm 0.0021$ Gev and $M_W = 80.423 \pm 0.039$ GeV, coming from the minimal coupling of the Higgs to gauge bosons. The experimental discovery of the Higgs is essentially the only missing piece of the standard model. Current experimental bounds indicate that (if it exists) its mass should lie in the range $45\text{GeV} \leq m_H \leq 191$ GeV and it is hoped to be detected within the present decade, either at the Tevatron or LHC.

- *Baryon and lepton number*. The gauge symmetries restrict the renormalisable Lagrangian of the standard model in such a way that there are two global symmetries, which are accidentally conserved. These correspond to lepton and baryon number. Notice that if we add nonrenormalisable gauge invariant couplings to the Lagrangian, these global symmetries cease to be conserved (that is why they are only accidental symmetries of the renormalisable Lagrangian). One such generic

term is $\frac{1}{M_{cutoff}}L^2H^2$ which is a dimension 5 operator (remember that fermions like L have dimension 3/2 and bosons like H have dimension 1.). This operator carries two units of lepton number and therefore breaks that symmetry. The scale of breaking will be determined by the cut-off mass scale M_{cutoff} in the coupling. Baryon number conservation would imply a stable proton. The current limit on its lifetime is of order 10^{33} years, from the Super-Kamiokande experiment. Finally we should point out that independent of this, nonperturbative effects break baryon number, but at zero temperature these are usually very much suppressed.

- *Naturalness problems.* The standard model is not natural. By this we mean that some of the couplings allowed by the symmetries are very small without any explanation. In particular the strong interactions symmetry $(SU(3)_c)$ allows a term of the form $\theta F_{\mu\nu}\tilde{F}^{\mu\nu}$ with $F_{\mu\nu}$ the gluon field strength. This term violates the discrete symmetry CP. Experimental constraints limit the value of the parameter θ to be $\theta \leq 10^{-8}$ and there is no good explanation, within the standard model, for the smallness of this parameter. This is the strong CP problem. A popular proposal to solve this problem is to introduce an axial Peccei-Quinn symmetry for which the Goldstone mode is an axion field a that couples to gauge fields as

$$\frac{a}{M_{PQ}}\ F_{\mu\nu}\tilde{F}^{\mu\nu}. \tag{4}$$

Astrophysical and cosmological constraints set a bound on the axion scale $M_{PQ} \sim 10^{11}$ GeV. A nonperturbatively generated potential for a can fix it to its minimum at $a = 0$ explaining the smallness of the effective θ parameter.

A more serious naturalness problem refers to the Higgs mass. The Higgs is the only particle allowed to have a mass term in the standard model Lagrangian (besides the right handed neutrinos). Unlike the fermion and gauge fields, there are no symmetries preventing the Higgs from acquiring a mass. Therefore assuming that the standard model is valid up to a cut-off scale $M_{cutoff} \gg 10^3$ GeV, all loop corrections will tend to make the Higgs mass of the order of M_{cutoff}. In the extreme case $M_{cutoff} = M_{Planck}$ we are left with the hierarchy problem of explaining why the low standard model scale is stable against radiative

corrections. The hierarchy problem can be separated in two parts, first why $M_{EW} \ll M_{Planck}$, second why this hierarchy is stable under quantum corrections.

Finally, once we include gravity we have to consider the cosmological constant problem, which is arguably the biggest puzzle for fundamental physics given that this constant Λ is essentially the vacuum energy and naturally the scale of gravity would set it to be $\Lambda \sim M_{Planck}^4$ which is a huge number compared with the observed value which sets it to $\Lambda \leq 10^{-120} M_{Planck}^4$. Probably more seriously is that this puzzle goes on scale by scale. Even at scales of the standard model and QCD the generated vacuum energy is many orders of magnitude bigger than the observed value of Λ ($\Lambda \leq 10^{-56} M_{EW}^4$. This problem has become more difficult by the recent observations that a dark energy dominates the energy density of the universe, that seems to indicate that Λ is not exactly zero.

- *The standard model is not supersymmetric.* We are so much used to the idea of supersymmetry, especially from string theory, that it is hard to admit that our only experimentally tested model is actually non supersymmetric. If the world were supersymmetric the Higgs would have been a partner of a fermion field for which the mass term is forbidden and therefore there would not be the naturalness part of hierarchy problem. To keep this positive feature, despite the absence of supersymmetry, it is expected that supersymmetry is a good symmetry down to scales slightly above the standard model scale ($\sim 10^3$ GeV), where it is softly broken, protecting in this way the Higgs mass to be arbitrarily large and solving the second part of the hierarchy problem. This is still the best solution of the hierarchy problem[7]. It also fits well with string theory that has supersymmetry incorporated. Therefore the hierarchy problem is converted into the problem of explaining supersymmetry breaking. If supersymmetry is relevant at low-energies then it has to be only the minimal number of supersymmetries $\mathcal{N} = 1$, since this is the only one that admits chiral fermions. A minimal supersymmetric extension of the standard model, the MSSM, duplicates the number of particles by adding a fermionic partner to each boson and a bosonic partner to each fermion.

Anomaly cancellation in the MSSM requires at least the addition of a second Higgs doublet, which is also good to give masses to the up

quarks. The extra number of particles change the running of the gauge couplings in such a way that now the three of them tend to meet at a single point, unlike the case for the standard model. This result has been seen as probably the major experimental indication in favour of low-energy supersymmetry and gauge coupling unification. This is consistent with the existence of grand unified theories (GUT) at high energies based on a simple group ($SU(5), SO(10), E_6$, etc.) that breaks to the standard model scale at some high scale M_{GUT}. The scale of unification is of order $M_{GUT} = 10^{16}$ GeV.

Contrary to the standard model case, the supersymmetric extensions do not usually preserve baryon and lepton number. Terms violating these symmetries appear in the MSSM even at the renormalisable level, such as

$$u_R d_R d_R, \ LQ_L d_R, \ LLe_R, \ LH \tag{5}$$

where we are using the same letters used for the original particles to identify the corresponding superfield. Extra symmetries have to be imposed to forbid these terms, the standard one is R-parity

$$R = (-1)^{2S+3(B-L)} \tag{6}$$

with S the helicity and B, L baryon and lepton number, respectively. Under R-parity, the standard model particles do not transform and their superpartners change by a sign. This symmetry forbids the terms above but it allows higher order operators that could give rise to proton decay. If supersymmetric grand unified theories are suitable, they predict baryon number violation in such a way that the proton is unstable with a lifetime just above the current experimental bound $\geq 10^{33}$ years. This prediction will be accessible to experiments in a few years time.

An important problem of GUT's is the 'doublet triplet splitting'. A typical example for the GUT group $SU(5)$ and the standard model Higgs is included in the fundamental representation, after the symmetry is broken to $SU(3) \times SU(2) \times U(1)$ the **5** of SU(5) decomposes into a $(\mathbf{3}, \mathbf{1}) + (\mathbf{1}, \mathbf{2})$ of $SU(3) \times SU(2)$. The doublet is the Higgs that has to stay light but the triplet should acquire a heavy mass. It is very difficult to achieve this in practice. This is a remnant of the hierarchy problem, since in this case the two members of a multiplet have to acquire very different masses (10^{16} GeV against 10^2 GeV).

- *Cosmology.* The standard model of particle physics fits very well with the standard model of cosmology, controlling the relevant cosmological scales where different phase transitions occur, from the very early universe, with a quark-gluon plasma all the way up to nucleosynthesis and the formation of atoms. Cosmology also points towards the need to extend the model by the requirement of dark matter candidates, the explanation of baryogenesis, the possible origin of dark energy and inflation, or any other early universe scenario that could solve the horizon and flatness problem, providing also an explanation for the density perturbations observed in the cosmic microwave background (CMB). Supersymmetric models provide natural dark matter candidates. R parity, for instance, imply that an even number of supersymmetric partners of the standard model particles are present in any interaction. Then, the lightest of these fields is stable becoming a very good candidate for cold dark matter. Deriving the existence of a symmetry like R-parity from a fundamental theory, is an interesting challenge.

- *Quantum gravity.* The main reason we know that the standard model is not a complete theory is because it includes gravity only at the classical level. String theory is the best proposed solution for a quantum description of gravity. It also incorporates gauge symmetries and matter fields. This makes the search of realistic string models the best guide to try to go beyond the standard model.

3   Pre D-Brane String Phenomenology

Let us start by reviewing the main aspects of string phenomenology before D-branes were introduced and later describe how the discovery of D-branes has changed this understanding.

We know that there are five consistent string theories, all existing in ten dimensions, namely, type I open strings with gauge symmetry $SO(32)$, type IIA and IIB closed strings and the two closed heterotic strings with gauge symmetries $E_8 \times E_8$ and $SO(32)$. The yet unknown extension of string theory, M-theory, adds a further theory to these five, which has the massless spectrum of 11-dimensional supergravity g_{MN}, C_{MNP}, Ψ_M. We present in table (3), the massless spectrum of each of the five supersymmetric string theories. Comparing this table with table 1 is a way to appreciate the difficulty to obtain a realistic string model.

MASSLESS SPECTRUM OF STRING THEORIES				
THEORY	DIMENSION	SUPERCHARGES	BOSONIC SPECTRUM	
Heterotic $E_8 \times E_8$	10	16	$g_{\mu\nu}, B_{\mu\nu}, \phi$ $A_\mu^{i\bar{j}}$ in adjoint representation	
Heterotic $SO(32)$	10	16	$g_{\mu\nu}, B_{\mu\nu}, \phi$ $A_\mu^{i\bar{j}}$ in adjoint representation	
Type I $SO(32)$	10	16	NS-NS	$g_{\mu\nu}, \phi$
			$A_\mu^{i\bar{j}}$ in adjoint representation	
			R-R	$C_{(2)}$
Type IIB	10	32	NS-NS	$g_{\mu\nu}, B_{\mu\nu}, \phi$
			R-R	$C_{(0)}, C_{(2)}, C_{(4)}$
Type IIA	10	32	NS-NS	$g_{\mu\nu}, B_{\mu\nu}, \phi$
			R-R	$C_{(1)}, C_{(3)}$

Table 2: Massless bosonic spectrum for the five consistent string theories in 10-dimensions. In type I, II, the spectrum is splitted into NS-NS corresponding to states built out of two bosonic states in the NSR formulation of the theories and R-R to those constructed from two fermionic states. The number of supercharges gives rise to the number of supersymmetries in 10 dimensions. $\mathcal{N} = 2$ corresponds to 32 supercharges leading to $\mathcal{N} = 8$ supersymmetry in 4-dimensions, upon dimensional reduction, whereas $\mathcal{N} = 1$ corresponds to 16 supercharges leading to $\mathcal{N} = 4$ supersymmetry in 4-dimensions, upon dimensional reduction. The R-R sector states $C_{(n)}$ are antisymmetric tensors of rank n. The fermionic spectrum is simply two gravitini for the $\mathcal{N} = 2$ models and one gravitino plus the corresponding gaugino partners of the gauge fields, for the $\mathcal{N} = 1$ models.

Out of the five string theories, the one that attracted most of the attention was the heterotic $E_8 \times E_8$ theory because it was the most promising for phenomenology: upon compactification to 4D it gives rise to chiral $\mathcal{N} = 1$ supersymmetric models with an observable sector, coming from the first E_8 which contains the standard model symmetry $SU(3) \times SU(2) \times U(1)$ and several families of matter fields. The second E_8 gives rise to a hidden sector, which fits perfectly with the attempts of supersymmetric model building prior to string theory. There, a hidden sector was proposed to break supersymmetry at an intermediate scale $\sim 10^{12}$ GeV and gravity plays the role of messenger of supersymmetry breaking to the observable sector, which feels the breaking of supersymmetry just above the electroweak scale $\sim 10^3$ GeV.

The other four string theories were much less interesting, there was even a no-go theorem proving the impossibility to obtain the standard model out of type II theories [9].

Let us briefly recall the properties of compactifications. The standard paradigm was that the properties of the compact 6d manifold would determine the low-energy physics. The manifold being a Calabi-Yau threefold complex space with holonomy group $SU(3)$ implies $\mathcal{N} = 1$ supersymmetry in 4d, as well as permitting the existence of chiral fermions. In the simplest cases, the observable E_8 symmetry group gets broken to E_6 which was a natural grand unified group, with the hidden E_8 unbroken. Also the number of families is given by the topological Euler number of the manifold. A great effort was dedicated to the study and classification of these manifolds, as well as the calculation of the Yukawa couplings. However the metric on these manifolds is not known and the explicit description of string theory in such complicated background is also unknown, except for some special limits. We will describe here only simple compactifications that lead to models with realistic properties.

3.1 Realistic Heterotic Models

The geometrical approach to 4D string model building can be described *à la* Kaluza-Klein in the sense of viewing the 10D spacetime as a product of 4D Minkowski spacetime and a small 6D compact manifold.

3.1.1 Toroidal Compactifications

In order to construct string models in less than 10D as well as to understand the heterotic string construction, we need to consider the simplest compactifications which correspond to the extra dimensions being circles or their

higher dimensional generalization.

Let us first see the case of a circle. This means that the 10D space is represented by flat 9D spacetime times a circle S^1. We know that a circle is just the real line identifying all the numbers differing by $2\pi R$, where R is the radius of the circle. So the only difference with the flat space are the boundary conditions. The solution of the wave equations of free string theory can be written in terms of left and right moving momenta: $p_R = m/2R - nR$ and $p_L = m/2R + nR$, m is an integer reflecting the fact that the momentum in the compact direction has to be quantized in order to get single-valued wave function. The integer n however refers to the fact that the string can wind several times around the compact dimension and is thus named the 'winding number'. The mass formula is then:

$$M^2 = N_R + N_L - 2 + \frac{m^2}{4R^2} + n^2 R^2, \qquad N_R - N_L = mn. \qquad (7)$$

This shows several interesting facts. First, for $n = 0$ and varying m, we obtain an infinite tower of massive states with masses proportional to $1/R$; these are the standard 'momentum states' of Kaluza-Klein compactifications in field theory. In particular the massless states with $n = m = 0$ and one oscillator in the compact direction are vector fields in the extra dimensions giving rise to a $U(1)_L \otimes U(1)_R$ Kaluza-Klein gauge symmetry. The states with $n \neq 0$ are the winding states and are purely of string origin; they represent string states winding around the circle, they have mass $\sim R$. Second, there are special values of m and n which can give rise to extra massless states. In particular for $m = n = \pm 1$ we can see that at the special radius $R^2 = 1/2$ in units of α', there are massless states with a single oscillator $N_R = 1, N_L = 0$ corresponding to massless vectors which in this case generate $SU(2)_R \times SU(2)_L$. This means that the special point in the 'moduli space' of the circle $R^2 = 1/2$ is a point of enhanced symmetry. The original $U(1)_R \times U(1)_L$ Kaluza-Klein symmetry of compactification on a circle gets enhanced to $SU(2)_R \times SU(2)_L$. This is an effect of string origin, because it depends crucially on the existence of winding modes ($n \neq 0$). The third interesting fact about this compactification is that the spectrum is invariant under the following 'duality' transformations [11]:

$$R \leftrightarrow \frac{1}{2R} \qquad m \leftrightarrow n. \qquad (8)$$

This is also a stringy property. It exchanges small with large distances but at the same time it exchanges momentum (Kaluza-Klein) states with winding

states. This symmetry can be shown to hold not only for the spectrum but also for the interactions and therefore it is an exact symmetry of string perturbation theory.
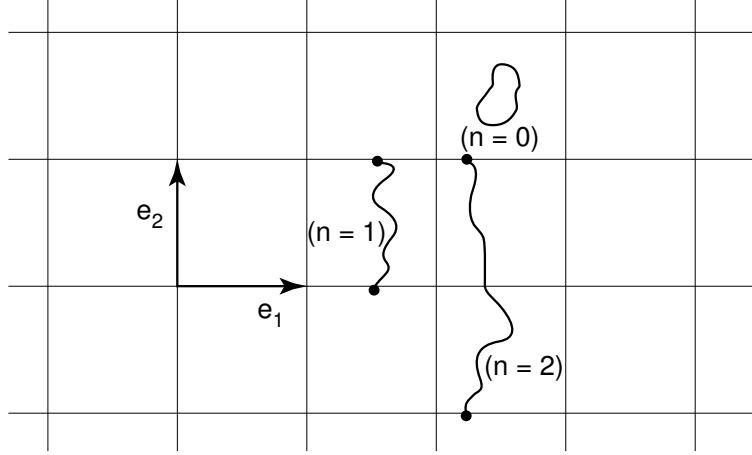

Figure 1: A 2D torus T^2 defined by the identification of points on $\mathbf{R}^2$ by elements of the lattice defined by $\mathbf{e_1}$ and $\mathbf{e_2}$. We display examples of a closed string on $\mathbf{R}^2$ which is also closed on T^2 ($n = 0$), also a string closed on T^2 but not on $\mathbf{R}^2$, winding around the torus once($n = 1$) and twice ($n = 2$).

Let us now extend the compactification to two dimensions, ie the 26D spacetime is the product of flat 24D spacetime and a 2D generalization of a circle, the torus T^2. Again the only difference with flat space is for the boundary conditions. The two compact dimensions are identified by vectors of a 2D lattice, defining the torus T^2. Out of the three independent components of the compactified metric G_{11}, G_{22}, G_{12} and the single component of B_{MN} namely B_{12} we can build two complex 'moduli' fields:

$$U \equiv \frac{G_{12}}{G_{22}} + i\,\frac{\sqrt{G}}{G_{22}}$$
$$T \equiv 2\left(B_{12} + i\,\sqrt{G}\right). \tag{9}$$

U is the standard modular parameter of any geometrical 2D torus and it is usually identified as the 'complex structure' modulus. T is the 'Kähler structure' modulus (since T^2 is a complex Kähler space) and its imaginary part measures the overall size of the torus, since G is the determinant of the

2D metric. It plays the same role as R did for the 1D circle. In terms of T and U we can write the left- and right-moving momenta as:

$$p_L^2 = \frac{1}{2U_2 T_2} \|(n_1 - n_2 U) - T(m_2 + m_1 U)\|^2$$

$$p_R^2 = \frac{1}{2U_2 T_2} \|(n_1 - n_2 U) - T^*(m_2 + m_1 U)\|^2 \tag{10}$$

The mass formula, depending on $p_L^2 + p_R^2$, again shows that there are enhanced symmetry points for special values of T and U. It also shows the following symmetries:

$$U \to \frac{aU + b}{cU + d}, \qquad T \to \frac{aT + b}{cT + d}, \qquad T \leftrightarrow U. \tag{11}$$

Where a, b, c, d are integers satisfy $ad - bc = 1$. The first transformation is the standard $SL(2, \mathbf{Z})_U$ 'modular' symmetry of 2D tori and is independent of string theory; it is purely geometric. The second transformation is a stringy $SL(2, \mathbf{Z})_T$ named T-duality and it is a generalization of (8) for the 2D case. Again this is a symmetry as long as we also transform momenta m_1, m_2 with winding n_1, n_2. The third symmetry exchanges the complex structure U with the Kähler structure T and it is called 'mirror symmetry'. This statement generalises to more complicated compactification such as Calabi-Yau manifolds but in this case mirror symmetry exchanges the complex and Kähler structures of two different Calabi-Yau spaces. If U and T each parametrise a complex plane $SL(2, \mathbf{R})/O(2)$, the duality symmetry implies that they can only live in the fundamental domain defined by all the points of the product of complex spaces $SL(2, \mathbf{R})/O(2) \otimes SL(2, \mathbf{R})/O(2) \cong O(2, 2, \mathbf{R})/(O(2) \times O(2))$ identified under the duality group $SL(2, \mathbf{Z})_U \times SL(2, \mathbf{Z})_T = O(2, 2, \mathbf{Z})$.

This statement gets generalized to higher dimensions. In general, compactification on a d-dimensional torus has the moduli space $\mathcal{M} = O(d, d, \mathbf{R})/O(d) \times O(d)$ with points identified under the duality group $O(d, d, \mathbf{Z})$. For the heterotic string with 16 extra left moving coordinates describing the $E_8 \times E_8$ or $SO(32)$ gauge symmetry $\mathcal{M} = O(d + 16, d, \mathbf{R})/O(d + 16) \times O(d)$ with a similar modification to the duality group. The left- and right- moving momenta p_L, p_R live on an even, self-dual lattice of signature $(22, 6)$, which is usually called the Narain lattice $\Lambda_{22,6}$[10]. This generalizes the $\Lambda_{2,2}$ lattice defined by the integers $m_1, m_2; n_1, n_2$ of eq. (10).

We can easily verify in this case that the dimension of $\mathcal{M}$ is $d(d + 16)$ corresponding to the number of independent components of G_{mn}, B_{mn} and the Wilson lines A_m^I with $m, n = 1 \cdots d; I = 1, \cdots 16$. The Wilson lines

correspond to non-vanishing constant gauge configurations which are non trivial because of the noncontractible loops of the torus. For each loop γ the Wilson line is given by

$$W^I \;=\; \int_\gamma A^I_m dx^m \tag{12}$$

For $d = 6$ we have a 4D string model with a moduli space of dimension $132 = 6 \times 7/2 + 6 \times 5/2 + 16 \times 6$, each number corresponding to the number of independent components of G_{mn}, B_{mn} and A^I_m respectively. To this we have to add the dilaton field ϕ which, together with the spacetime components of the antisymmetric tensor $B_{\mu\nu}$, can be combined into a new modular parameter:

$$S \equiv a + i\, e^\phi. \tag{13}$$

Here the axion field a is defined as $\nabla_\mu a \;=\; \epsilon_{\mu\nu\rho\sigma} \nabla^\nu B^{\rho\sigma}$. S parametrises again a coset $SL(2,\mathbf{R})/O(2)$. This was one of the motivations for the later proposal of S duality [12].

3.1.2 Orbifold Compactifications

We have then succeeded in constructing 4D super-string models from toroidal compactifications and understand the full class of these models given by the moduli space $\mathcal{M}$. Unfortunately, all of these models have $\mathcal{N} = 4$ supersymmetry and therefore they are not interesting for phenomenology, because they are not chiral. To obtain a chiral model we should construct models with at most $\mathcal{N} = 1$ supersymmetry. If we still want to use the benefits of free 2D theories, we should construct models from flat space and modify only the boundary conditions. We have already considered identifications by shift symmetries of a lattice defining the tori. We still have the option to also use rotations and consider 'twisted' boundary conditions [13]. As an example let us start with the torus T^2 discussed before. If we make the identification $X^i \rightarrow -X^i$ we are constructing the orbifold $O^2 \equiv T^2/\mathbf{Z}_2$, shown in figure 2, where the $\mathbf{Z}_2$ twist is rotation by π. This space *is not* a manifold because it is singular at the points left fixed by the rotation $\{(0,0), (0,1/2), (1/2,0), (1/2,1/2)\}$. Notice that, for instance, the point $(1/2, 1/2)$ is fixed because it is transformed to $(-1/2, -1/2)$ which is identical to the original point after a lattice shift. In general, the discrete group of rotations defining the orbifold is called the point group $\mathcal{P}$, whereas the non-Abelian group including the rotations and also the translations of the lattice Λ, is the space group $\mathcal{S}$. So usually a torus is defined as $T^d \equiv \mathbf{R}^d/\Lambda$ and an orbifold $O^d \equiv T^d/\mathcal{P} \equiv \mathbf{R}^d/\mathcal{S}$.

It is important to notice that the orbifold twist transforms the original spectrum of states of the torus in two ways. It projects out all the states which are not invariant under the twist, reducing the number of states in the spectrum. It also increases the number states in another direction, since now it includes the so called 'twisted sector'. An open string around a fixed point with its two end-points lying at points which are identified under the orbifold is not included in the spectrum of states of the torus but it is a valid closed string in the orbifold. See figure 2.
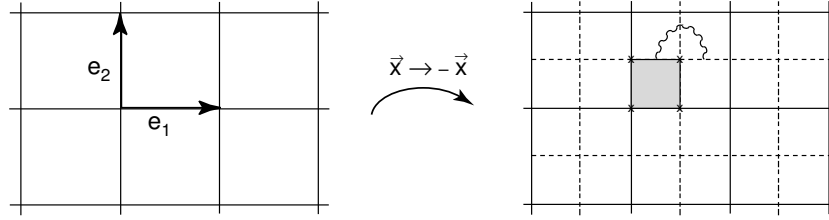

Figure 2: Starting from the two-torus T^2, we generate the orbifold $O^2 \equiv T^2/\mathbf{Z}^2$, 'the ravioli', by the identification $\vec{x} \leftrightarrow -\vec{x}$. O^2 is singular at the four 'fixed points' shown. Besides the momentum and winding states of the torus, the orbifold spectrum also has 'twisted' states, corresponding to strings closed in O^2 but not on T^2. The twisted states are attached to fixed points, we display one example.

We can easily construct 4D strings from orbifold compactifications in which the 10D spacetime of the heterotic string is the product of 4D flat spacetime and a six-dimensional orbifold O^6. A simple way to do it is to describe the six extra dimensions in terms of three complex coordinates z_k $k = 1, 2, 3$. We can define the 6d orbifold twist to be determined by the vector (v_1, v_2, v_3) corresponding to the twist

$$z'_k = \exp\left(2\pi i v_k\right) z_k, \qquad k = 1, 2, 3 \tag{14}$$

with $v_k = l_k/M, l_k \in \mathbf{Z}$. The requirement of having only one supersymmetry left implies that $v_1 + v_2 + v_3 = 0 \bmod 1$.

The heterotic string is particularly interesting because we can extend the action of the point group to the 16D lattice of the gauge group by embedding the action of the orbifold twist in the gauge degrees of freedom defined by the $E_8 \times E_8$ lattice, say. This can easily be done in two ways:

(i) Perform a homomorphism of the point group action in the gauge lattice by *shifting* the lattice vectors by a vector $V = (1/M)L$ where M is the order of the point group and L is any lattice vector in 16D.

(ii) Perform the homomorphism by twisting also the gauge lattice by an order M rotation belonging to the Weyl group of the corresponding gauge group.

In the absence of Wilson lines both embeddings are equivalent. The first one is definitely simpler to handle and has proven more efficient at generating realistic models. It is important to notice that these embeddings are restricted by modular invariance of the one-loop partition function which is the $SL(2, \mathbf{Z})$ symmetry of the world-sheet torus of the one-loop string expansion. Modular invariance for instance restricts the shifts to satisfy, for a $\mathbf{Z}_M$ orbifold, $M \sum v_i = 0 \bmod 2$ but also:

$$M \left(V^2 - v^2 \right) = 0 \bmod 2 \tag{15}$$

These embeddings on the gauge degrees of freedom allow us to break the gauge group, reduce the number of supersymmetries and generate $\mathcal{N} = 1$ chiral models in 4D as desired. The reason for this is the following: using the embedding by a shift V, we start with the spectrum of the toroidal compactification and have to *project out* all the states that are not invariant by the orbifold twist. For the gauge group, only the elements satisfying $P \cdot V \in \mathbf{Z}$ remain, where $P \in E_8 \otimes E_8$, breaking the gauge group to a subgroup of the same rank. The four gravitinos of the $\mathcal{N} = 4$ toroidal compactification also transform and depending on the orbifold twist they are reduced to only one or two invariant states, indicating that there is only $\mathcal{N} = 2$ or $\mathcal{N} = 1$ supersymmetry. Actually there are only four twists $\mathbf{Z}_M$ leading to $N = 2$ $(v_1, v_2, v_3) = 1/M(1, -1, 0)$ for $M = 2, 3, 4, 6)$ and some twenty $\mathbf{Z}_M$ (for which non of the v_k's vanish) or $\mathbf{Z}_M \times \mathbf{Z}_N$ twists leading to $\mathcal{N} = 1$ supersymmetry [14], which are the phenomenologically interesting ones. For each of these twists we can have several (~ 10) different embeddings on the gauge degrees of freedom.

One of these embeddings is called the standard embedding because it acts identically in the gauge degrees of freedom as in the 6D space, This means that the only non-vanishing components of V are $V = (v_1, v_2, v_3, 0, \cdots 0)$ and $V^2 = v^2$ is trivially satisfied. This embedding also describes compactifications of the type II strings and is distinguished because in the 2D world-sheet, the corresponding model has two supersymmetries on the left-movers and two supersymmetries on the right-movers, the corresponding models are called $(2, 2)$ models. All other embeddings do not have supersymmetry in the left moving side and are called $(0, 2)$ models.
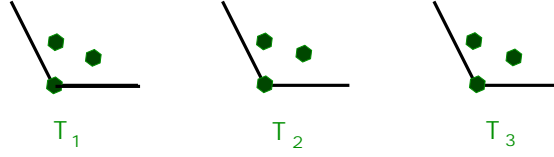
Figure 3: The $\mathbf{Z}_3$ orbifold constructed from a product of three two-torii each twisted by the action of $\mathbf{Z}_3$. There are three fixed points at each torus making then a total of $3^3 = 27$ fixed points on which the space is singular.

On top of all these embeddings we can also add the Wilson lines [15] $A_m^I, m = 1, \cdots 6, I = 1, \cdots, 16$, mentioned in the previous section. The order of the orbifold and modular invariance impose similar restrictions on the Wilson lines A_m^I as for the orbifold shift V. However, in some cases the Wilson lines may be continuous. This increases the number of possible consistent models by a large amount, which we can only estimate between millions and billions because some may turn out to be actually equivalent.

Both classes of embeddings mentioned above can be obtained by starting with the Narain lattice of toroidal compactifications $\Lambda_{22,6}$ and twist it in a consistent manner. This already takes into account the Wilson lines (which were already present, parametrising $\Lambda_{22,6}$) and also allows for the possibility of performing left-right asymmetric twists, the so-called asymmetric orbifolds [16]. This extra degree of freedom increases the number of possible models.

We can see now how a vast amount of heterotic string orbifold models can be generated. There are many classes of models; different classes are differentiated by the choice of original 6D toroidal lattice, the orbifold point group, the embeddings V^I and the discrete Wilson lines given by A_m^I. But each of these *discrete* choices allows for a variation of different *continuous* parameters such as the moduli fields (like S, T, U), the continuous Wilson lines (that correspond to charged untwisted sector moduli fields) and there are also charged twisted-sector moduli fields[17]. All the continuous parameters can be seen as flat potentials for fields in the effective field theoretical effective action.

Only a few of these classes of models include quasi-realistic models. As an example [18], the model based on the $\mathbf{Z}_3$ orbifold with embedding V^I and non-vanishing Wilson lines A_m^I given by:

$$V = \frac{1}{3} \left(1, 1, 1, 1, 2, 0, 0, 0 \right) \times (2, 0, \cdots, 0)$$

$$A_1 = A_2 = \frac{1}{3}(0,0,\cdots,2) \times (0,1,1,0\cdots,0)$$

$$A_3 = A_4 = \frac{1}{3}(1,1,1,2,1,0,1,1) \times (1,1,\cdots,0) \qquad (16)$$

with $A_5 = A_6 = 0$[1] breaks $E_8 \times E_8$ to $SU(3) \times SU(2) \times U(1) \times U(1)^7 \times SO(10)$ with three families of quarks and leptons. This can be easily checked by taking all the root vectors P^I of $E_8 \times E_8$ and keep only those that satisfy $P \cdot V = 0$, $P \cdot A_m = 0$. The extra $U(1)^7$ symmetry can be broken by the standard Higgs mechanism which, in string theory requires the existence of flat directions among some charged matter fields; these can be analyzed in the models at hand because there are general 'selection rules' forbidding couplings not invariant under the action of the point and space groups. The $SO(10)$ remains as a hidden sector (in the sense that it only has gravitational strength couplings with the observable $SU(3) \times SU(2) \times U(1)$ sector). This is an example of a quasi realistic model. The structure of Yukawa couplings can be analyzed leading to very realistic properties and problems such as very fast proton decay can be avoided [2] However, there are extra doublets in the model that give rise to unrealistic values of Weinberg's angle. This may in principle be solved by contemplating the existence of intermediate scales, but at this point the model stops being stringy. It also has the drawback that without knowing details about supersymmetry breaking many of the low energy parameters cannot be determined. There are variations of this model that allow for an extra $U(1)$ symmetry at low energies, implying a relatively light Z' particle.

There are several models in the literature with similar properties as this one, showing that it is possible to get models very close to the standard model of particle physics. Essentially all the realistic models built so far are either based on the $\mathbf{Z}_3$ [18] or $\mathbf{Z}_2 \times \mathbf{Z}_2$ orbifolds [20, 21], the main reason for this is that in $\mathbf{Z}_3$ it is natural to get 3 families coming from the untwisted sector

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[1] Notice that $A_1 = A_2$ is imposed because the two nontrivial cycles of the first two-torus are identified under the orbifold twist, the same happens to the other pairs.

[2] It is worth mentioning that this model belongs to a class of models found in [18]. A companion model in the original article looked better since it incorporated, in a natural way, a solution to the doublet-triplet splitting problem mentioned in the introduction. The mechanism was that the triplets were projected out by the orbifold projection but the doublets were invariant. A similar mechanism was previously proposed by Witten in the context of Calabi-Yau compactifications. However a detailed study of the flat directions for that model proved that it was not viable and the one presented here was better. It so happens that in this case even though the triplets are not projected out, they naturally obtain a mass at generic points of the flat directions. This is a different solution to the doublet triplet problem.

and for $\mathbf{Z}_2 \times \mathbf{Z}_2$ it is natural to get 3 families but from the twisted sector. Both classes of models share several interesting properties and are still worth studying. The other orbifolds may or may not lead to three families. These approaches to build realistic models are being continuously considered for more than 15 years already and, even at present, some interesting new models are emerging. However it is fair to say that there is no single model that could be considered fully realistic.

3.1.3 Calabi-Yau Compactifications

As we mentioned before, a more general geometrical way of describing $\mathcal{N} = 1$ models is by means of Calabi-Yau manifolds. This may be seen as follows. In general the toroidal orbifolds have singularities at their fixed points. These singularities may be 'blown-up' in well defined mathematical ways. The smooth manifold obtained in this way is a Calabi-Yau manifold. Mathematically, these are 6D complex manifolds with $SU(3)$ holonomy or equivalently vanishing first Chern class. They were actually the first standard Kaluza-Klein compactification considered in string theory, leading to chiral 4D models and generically gauge group $E_6 \times E_8$, with E_8 a hidden gauge group.

This is in principle a more general way at obtaining compactifications with $N = 1$ supersymmetry. Since Calabi-Yau manifolds are understood mostly from their topological properties it is more difficult to extract explicit information from those compactifications. However, over the years, some remarkable results have been obtained in this direction, both from the phenomenological and the mathematical points of view. A more detailed discussion of these manifold is beyond the scope of these lectures.

The drawback of compactifications on Calabi Yau manifolds is that they are highly nontrivial spaces and we cannot describe the strings on such manifolds, contrary to what we did in the case of free theories such as tori and orbifolds. In particular we cannot compute explicitly the couplings in the effective theory, except for the simplest renormalisable Yukawa couplings.

On the other hand, Calabi-Yau manifolds have been understood much better during the past few years and have lead to some beautiful and impressive results. In a way they are more general than orbifolds because an orbifold is only a particular singular limit of a Calabi-Yau manifold. Also there are other constructions of these manifolds which are not related to orbifolds. They can be defined as hypersurfaces in complex (weighted) projective spaces $\mathbb{P}^4_{(k_0,k_1,k_2,k_3,k_4)}$ where k_i's are the weights of the corresponding

coordinates for which there is the identification $z_i \cong \lambda^{k_i} z_i$. The hypersurface is defined as the vanishing locus of a polynomial of the corresponding coordinates. For instance the surface defined as:

$$P \equiv z_1^{12} + z_2^{12} + z_3^6 + z_4^6 + z_5^2 = 0 \qquad (17)$$

defines a Calabi-Yau manifold with weights:$(1, 1, 2, 2, 6)$. The relation $\sum_i k_i = d$ where d is the degree of the polynomial ensures that the surface is a Calabi-Yau manifold. The manifold is guaranteed to be smooth if both the polynomial P and its derivatives do not vanish simultaneously. Larger classes of manifolds can be constructed by considering intersections of hypersurfaces in higher dimensional projective spaces, the so-called complete intersection Calabi-Yau manifolds (CICY). Actually, it is known in the mathematical literature, that *all* Calabi-Yau spaces can be defined as (intersection of) hypersurfaces in weighted projective spaces. Large numbers of these manifolds have been classified, although the full classification is not complete. The mathematical estimate is that there are of the order of ten thousand Calabi-Yau manifolds. The corresponding string models are of the type $(2, 2)$. Some of the highlights of Calabi-Yau compactifications are the following:

(i) There are classes of moduli fields, generalizing the fields T and U of the two-torus mentioned before. The number of these fields is given by topological numbers known as Hodge numbers $h_{i,j}, i+j \leq 3$. They correspond to the number of complex harmonic forms that can be defined in the manifold with i holomorphic indices and j anti-holomorphic indices. Then the number of complex structure fields (U) is given by $h_{2,1}$ and the number of Kähler structure fields (T) is given by $h_{1,1}$. Many of the $h_{2,1}$ forms correspond to the coefficients of different monomials which can be added to the defining polynomial that still give rise to the same space, other forms correspond to the blowing up of possible singularities. Many of the $h_{2,1}$ forms correspond to polynomial deformations of the defining surface, but others are related with the reparation of singularities. For Calabi-Yau manifolds we have $h_{1,1} \geq 1$, therefore there is always a special Kähler class deformation which can be thought as the overall size of the corresponding manifold, it is usually called, the Kähler form. All the other Hodge numbers of Calabi-Yau manifolds are fixed ($h_{3,0} = h_{0,3} = 1, h_{1,0} = h_{0,1} = 0, h_{0,2} = h_{2,0} = 0$). In particular there is always one unique $(3, 0)$ form, Ω that characterizes a given Calabi-Yau manifold.

(ii) The gauge group in 4D is $E_6 \times E_8$. The matter fields transform as 27's or $\bar{2}$7's of E_6. The number of each is also given by the Hodge numbers $h_{1,1}, h_{1,2}$ and the number of generations is then topological: $N_g \equiv h_{1,1} - h_{1,2} = \chi/2$ where χ is the Euler number of the manifold. This is one of the most appealing properties of these compactifications since they imply that topology determines the number of quarks and leptons.

(iii) Mirror symmetry [23]. Similar to the 2D toroidal compactifications, it has been found that there is a mirror symmetry in Calabi-Yau spaces that exchanges the moduli fields T_m and U_a $m = 1, \cdots h_{1,1}$, $a = 1, \cdots, h_{2,1}$. This means that for every Calabi-Yau manifold $\mathcal{M}$, there exists another manifold $\mathcal{W}$ which has the complex and Kähler structure fields exchanged, *ie* $(h_{1,1}, h_{2,1}) \leftrightarrow (h_{2,1}, h_{1,1})$ and opposite Euler number χ. The mirror symmetry of the two-torus described previously, is only a special example on which the manifold is its own mirror. Mirror symmetry is not only a nontrivial contribution of string theory to modern mathematics, but it has very interesting applications for computing effective Lagrangians as we will see in the next chapter. it also relates the geometrical modular symmetries associated to the U_a fields of the manifold $\mathcal{M}$ to generalized, stringy, T-duality symmetries for the mirror $\mathcal{W}$ and vice-versa (see for instance [24]).

(iv) Even though these string models are not completely understood in terms of 2D CFTs, special points in the moduli space of a given Calabi-Yau are CFTs, such as the orbifold compactifications mentioned before. In fact some people believe that there is a one to one correspondence between string models with $(2, 2)$ supersymmetry in the worldsheet and Calabi-Yau manifolds. There is also a description of CFTs in terms of effective Landau-Ginzburg Lagrangians which are intimately related to Calabi-Yau compactifications as two phases of the same 2D theory [25]. In particular the potential of that Landau-Ginzburg theory is determined by the polynomial P defining the Calabi-Yau hypersurface.

(v) There are also few models with three generations that, after the E_6 symmetry breaking, could lead to quasi-realistic 4D strings. One of these models was analyzed in some detail [22]. Usually, E_6 models also lead to the existence of extra Z' particles of different kinds. They have been thoroughly studied because of the potential experimental

importance of detecting an extra massive gauge boson (see for instance ref. [26]).

(vi) Although most of the Calabi-Yau models studied so far correspond to standard embedding in the gauge degrees of freedom ($(2,2)$ models), there is also the possibility of constructing $(0,2)$ models by performing different embeddings, similar to the orbifold case. This increases substantially the number of string models of this construction [27].

3.2 $\mathcal{N} = 1$ Supersymmetric Effective Actions

We know that consistent 4D chiral string models lead to $N = 1$ supersymmetry. For phenomenological purposes, we are interested in finding the effective action for the light degrees of freedom, that means we want to integrate out all the heavy degrees of freedom at the Planck scale M_p and compute the effective couplings among the light states (massless at the Planck scale). This will be a standard field theoretical action with $N = 1$ supersymmetry.

The on-shell massless spectrum of these models have the graviton-gravitino multiplet $(G_{\mu\nu}, \Psi_\mu)$, the gauge-gaugino multiplets $(A_\mu^\alpha, \lambda^\alpha)$ and the matter and moduli fields which fit into $N = 1$ chiral multiplets of the form (z, χ) except for the dilaton field which together with the antisymmetric tensor $B_{\mu\nu}$ belong to a linear $N = 1$ multiplet $L = (\phi, B_{\mu\nu}, \rho)$. The most general couplings of supergravity to one linear multiplet and several gauge and chiral multiplets is *not* yet known, although some progress towards its construction has been done recently [28, 29]. Nevertheless, as we mentioned in the previous chapter, this field can be dualized to construct the chiral multiplet (S, χ_S) with $S = a + i\,e^\phi$ and $\nabla_\mu a \equiv \epsilon_{\mu\nu\rho\sigma}\nabla^\nu B^{\rho\sigma}$. After performing this duality transformation we are left with a $N = 1$ supergravity theory coupled only to gauge and chiral multiplets. Although, the partial knowledge about the Lagrangian in terms of L is enough to understand most of the results we will mention next [29], we will only mention the approach with the chiral field S instead of L, which is the most commonly used in the literature.

The general Lagrangian coupling $N = 1$ supergravity to gauge and chiral multiplets was constructed by Cremmer *et al*, [30], it depends on three arbitrary functions of the chiral multiplets:

(1) The Kähler potential $K(z, \bar{z})$ which is a *real* function. It determines the kinetic terms of the chiral fields

$$\mathcal{L}_{kin} = K_{z\bar{z}}\partial_\mu z \partial^\mu \bar{z} \tag{18}$$

with $K_{z\bar{z}} \equiv \partial^2 K/\partial z \partial \bar{z}$. K is called Kähler potential because the manifold of the scalar fields z is Kähler, with metric $K_{z\bar{z}}$.

(2) The superpotential $W(z)$ which is a *holomorphic* function of the chiral multiplets (it does not depend on $\bar{z}$) W determines the Yukawa couplings as well as the F-term part of the scalar potential V_F:

$$e^{K/M_p^2} \left\{ D_z W \, K_{z\bar{z}}^{-1} \bar{D}_z \bar{W} - 3 \frac{|W|^2}{M_p^2} \right\}, \tag{19}$$

with $D_z W \equiv W_z + W K_z/M_p^2$. Here and in what follows, the internal indices labelling different chiral multiplets z_i are not explicitly written.

(3) The gauge kinetic function $f_{ab}(z)$ which is also *holomorphic*. It determines the gauge kinetic terms

$$\mathcal{L}_g = \mathrm{Re} \, f_{ab} F^a_{\mu\nu} \, F^{\mu\nu\,b} + \mathrm{Im} \, f_{ab} F^a_{\mu\nu} \, \tilde{F}^{\mu\nu\,b} \tag{20}$$

it also contributes to gaugino masses and the gauge part of the scalar potential.

$$
\begin{aligned}
V_D &= \left(\mathrm{Re} \, f^{-1} \right)_{ab} (K_z, T^a \, z) \left(K_{\bar{z}}, T^b \, \bar{z} \right) \\
V &= V_F + V_D
\end{aligned}
\tag{21}
$$

These three functions are arbitrary for a generic $\mathcal{N} = 1$ supersymmetric model, but in string theory we should be able to compute them for each specific model. General non-renormalisation theorems can be applied to the holomorphic functions W and f. The main reason is that the dilaton field ϕ is always present in string models and its vev is the loop counting parameter. In $\mathcal{N} = 1$ 4D models it joins an axion-like field a having a Peccei-Quinn symmetry $a \to a+$ constant, to form the complex (chiral) multiplet S, because of the existence of the Peccei-Quinn symmetry W cannot depend on a and, because a only appears through S, then W cannot depend on S and so it does not depend on the loop-counting-parameter ϕ, therefore it is not renormalised. A similar argument shows that f does not have corrections beyond one-loop. The Peccei-Quinn symmetry is usually broken by nonperturbative effects therefore they will contribute to corrections to W and f. For K there are no simplifications and very little is known beyond some tree level calculations.

The general structure of these functions of matter multiplets Q, moduli T and dilaton S is the following:

$$W(S, T, Q) = W_{tree}(T, Q) + W_{np}(S, T, Q)$$
$$f(S, T, Q) = -iS + f_{1-loop}(T, Q) + f_{np}(S, T, Q)$$
$$K(z, \bar{z}) = (K_{tree} + K_{1-loop} + \cdots) + K_{np} \tag{22}$$

Therefore we can see that the lack of control on the perturbative corrections to K is the main source of ignorance of the full perturbative 4D effective actions. So far this is a restatement of the general results known for $\mathcal{N} = 1$ supersymmetric models. In string theory the challenge is then to compute each of the arbitrary functions given above, especially for realistic models.

The problem raised in this section is: given a 4D string model, calculate the functions K, W, f. In order to do that let us separate the fields into the moduli U, T, the dilaton S, and the matter fields charged under the gauge group Q^I. Most of the structure of the couplings depends on the model. But there are some couplings which are model independent. To extract them, the best procedure is to use all the symmetries at hand. For this let us remark that 4D strings are controlled by two perturbative expansions. One is the expansion in the sigma-model (2D worldsheet) which is governed by the expectation value of a modulus field T (the size of the extra dimensions), whereas proper string perturbation theory is governed by the dilaton field S.

3.2.1 Tree-level Couplings

Let us consider first the couplings generated at string tree-level and also tree-level in the sigma-model expansion [32, 33]. Besides the 4D Poincaré symmetry, supersymmetry and gauge symmetries which determine the Cremmer *et al* Lagrangian, we also can use the 'axionic' symmetry: $B_{MN} \rightarrow B_{MN} +$ closedform This is a symmetry which for the 4D fields would imply that S, T, U can be shifted by an arbitrary real constant. There are also two scaling properties of the 4D Lagrangian $S \rightarrow \lambda S$, $G_{\mu\nu} \rightarrow \lambda G_{\mu\nu}$ for which the Lagrangian scales as $\mathcal{L} \rightarrow \lambda \mathcal{L}$. Also, given a scale Λ, define $\tau = \kappa \Lambda^4$ where κ gives Newton's constant in 10D. The transformations $S \rightarrow \tau^{-1/2} S$, $T \rightarrow \tau^{1/2} T$ with similar transformations for the other fields, imply that the Lagrangian should scale as $\mathcal{L}(\kappa) \rightarrow \tau^{-1/2} \mathcal{L}(\Lambda^{-4})$. These scaling properties are not symmetries of the Lagrangian but of the classical field equations and so they can be used to restrict the form of the *tree-level* effective action only.

Using these symmetries we can extract the full dependence of the effective action on the dilaton field S, which is the most generic field in all

compactifications. We conclude that at tree-level in both expansions [33]:

$$
\begin{aligned}
K(S, T, U, Q^I) &= -\log\left[i(S - S^*)\right] + \hat{K}(T, U, Q) \\
W(S, T, U, Q^I) &= y_{IJK} Q^I Q^J Q^K \\
f_{ab}(S, T, U, Q^I) &= -iS\,\delta_{ab}
\end{aligned}
\tag{23}
$$

with $\hat{K}$ still undetermined. This is however a very rough approximation. What we really want is to know these functions at tree-level in the string expansion but *exact* in the sigma model expansion. This should be achievable because many of the 4D models are exact 2D CFTs as we saw in the previous chapter. We can still extract very useful information from equation (23). As we said above, the axionic symmetries imply that to *all orders* in sigma-model expansion the superpotential does not depend on T, U and it is just a cubic function of the matter fields Q^I. This is important for several reasons: First, we know the field T comes from the internal components of the metric and controls the loop expansion of the worldsheet action. If W does not depend on T it means that it cannot get any corrections in sigma model perturbation theory![27]. Therefore the only T, U dependence of the (exact) tree-level superpotential is due to nonperturbative effects in the worldsheet, in particular all nonrenormalisable couplings in the superpotential are exponentially suppressed ($\sim e^{iT}$) [34]. A way to see that there are nonperturbative world-sheet corrections to the string tree-level superpotential is to realize that the axionic symmetry shifting T by an imaginary constant, is broken by nonperturbative world-sheet effects to $T \to T + n$, $n \in \mathbf{Z}$. This is nothing but one of the $SL(2, \mathbf{Z})_{T,U}$ transformation for toroidal orbifold compactifications ($a = b = d = 1, c = 0$ in eq. (10)). Therefore the only conditions these symmetries impose on W is that it should transform as a modular form of a given weight ($W \to (cT+d)^{-3} W$ for the simplest toroidal orbifolds with T the overall size of the compactification space)[35]. In fact, explicit calculations for specific orbifold models show that

$$
W_{tree}(T, Q^I) = \chi_{IJK}(T)\, Q^I Q^J Q^K + \cdots
\tag{24}
$$

with $\chi(T)$ a particular modular form of $SL(2, \mathbf{Z})$ or any other duality group and the ellipsis represent higher powers of Q, exponentially suppressed. The identification of $\chi(T)$ with modular forms was a highly nontrivial check of the explicit orbifold calculations which were performed in refs. [36] without any relation (nor knowledge) of the underlying duality symmetry $SL(2, \mathbf{Z})$. This kind of symmetry also sets strong constraints to the higher order, non-renormalisable, corrections to W, since each matter field Q transforms in a

particular way under that symmetry ($Q \to (cT + d)^n Q$ with n the modular weight of Q). There are also other discrete symmetries, as those defined by the point group $\mathcal{P}$ and space group $\mathcal{S}$ of an orbifold which have to be respected by the superpotential W. These 'selection rules' are very important to find vanishing couplings and uncover flat directions which can be used to break the original gauge symmetries and construct quasi-realistic models.

Second, and more important, the superpotential above does not depend on S which is the string loop-counting parameter, and therefore W_{tree} does not get renormalized in string perturbation theory![37]. This means that we only need to compute W at the tree level and it will not be changed by radiative corrections, except wave function renormalisation. This is the string version of the standard non-renormalisation theorems of supersymmetric theories. Also for $Q = 0$ the superpotential vanishes, independent of the values of S, T, U ($W(S, T, U, Q = 0) = 0$)! There are no self couplings among the 'moduli' fields and therefore they represent flat directions in field space (see for instance [38]). Notice that due to the non-renormalisation theorems, this result is exact in string perturbation theory!. The only possibility we have to lift this vacuum degeneracy is by nonperturbative string effects.

The quantity we have less information on, even at tree-level, is the Kähler potential $\hat{K}(T, U, Q)$. It has been computed only for several simple cases. For instance in the simplest possible Calabi Yau compactification ($h_{1,1} = 1, h_{2,1} = 0$) a consistent truncation from the 10D action gives [32, 33]:

$$K = -\log\left[-i(S - \bar{S})\right] - 3\log\left[-i(T - \bar{T} + Q\bar{Q})\right] \tag{25}$$

Curiously enough, the second term appeared in the so-called 'no-scale models' studied before string theory [39]. This form holds also for the untwisted fields of orbifold compactifications, but the dependence on the twisted fields is not known. It also gives the appropriate result in the large radius limit of Calabi-Yau compactifications although it gets non-perturbative world-sheet corrections relevant at small radii. In order to find the exact tree-level Kähler potential, the best that has been done so far is to write the Kähler potential as an expansion in the matter fields [71]:

$$
\begin{aligned}
K &= -\log\left[-i(S - \bar{S})\right] + K^M(T, \bar{T}, U, \bar{U}) + K^Q(T, \bar{T}, U, \bar{U})Q\bar{Q} \\
&+ Z(T, \bar{T}, U, \bar{U})(QQ + \bar{Q}\bar{Q}) + \mathcal{O}(Q^3),
\end{aligned}
\tag{26}
$$

and compute the moduli dependent quantities K^M, K^Q, K^E. This has been done explicitly for some $(2, 2)$ orbifold compactifications. For instance for

factorized orbifolds, that is orbifolds of a 6D torus which is the product of three 2D tori T^2, the dependence on the corresponding moduli fields is given by

$$
\begin{aligned}
K^M &= -\sum_a \log\left[-i(T_a - \bar{T}_a)\right] - \sum_m \log\left[-i(U_m - \bar{U}_m)\right], \\
K^Q &= \prod_{a,m} \left[-i\left(T_a - \bar{T}_a\right)\right]^{n_m^I}\left[-i\left(U_m - \bar{U}_m\right)\right]^{n_a^I},
\end{aligned}
\tag{27}
$$

and $Z(T, \bar{T}, U, \bar{U}) = 0$, giving rise to the Kähler potential:

$$
\begin{aligned}
K &= -\log\left[-i(S - \bar{S})\right] - \sum_a \log\left[-i(T_a - \bar{T}_a)\right] - \sum_m \log\left[-i(U_m - \bar{U}_m)\right] \\
&\quad + \sum_I |Q_I|^2 \prod_{a,m} \left[-i\left(T_a - \bar{T}_a\right)\right]^{n_m^I}\left[(-iU_m - \bar{U}_m)\right]^{n_a^I}
\end{aligned}
\tag{28}
$$

Where the fractional numbers n_m^I, n_a^I are the 'modular weights' of the fields Q_I with respect to the duality symmetries related to the moduli T_a or U_m. For instance, under T duality, the fields Q^I transform as:

$$
Q^I \rightarrow (c_m T_m + d_m)^{n_m^I} Q^I
\tag{29}
$$

3.2.2 Loop Corrections

We have seen there is good understanding of some of the tree-level couplings of 4D string models. Also non-renormalisation theorems guarantee that the superpotential computed at tree-level is exact at all orders in string perturbation theory. This powerful result depends crucially on the fact that the superpotential is a holomorphic function of the fields, so if by the Peccei-Quinn symmetry cannot depend on the imaginary part of the dilaton field S, then it cannot depend on the real part of S either. This fact cannot be used for the Kähler potential, which in general will be corrected order by order in string perturbation theory. This is then the least known part of any string theory effective action. On the other hand the gauge kinetic function f is also holomorhic and we know it exactly at the tree-level ($f = S$). Since this function determines the gauge coupling itself, it is very interesting to consider the loop corrections to f.

Explicit one-loop corrections to f have been computed, especially for some orbifold models. First it was shown that string loop diagrams reproduce the standard running of the gauge couplings in field theory, as expected.

More interesting though, was to find the finite corrections given by threshold effects which include heavy string modes running in the loop. These corrections will be functions of the moduli fields, such as the geometric moduli T, U but also other moduli like the continuous Wilson lines of orbifold models.

For factorized orbifold models, the explicit dependence of the one-loop corrections on the T_i, U_i moduli fields takes the form [71]:

$$f_a = k_a\, S - \sum_i \frac{\alpha_a^i}{4\pi^2} \log \eta(M^i) + \text{constant} \tag{30}$$

where k_a are the Kac-Moody levels of the corresponding gauge groups, The coefficients α_a^i are group theoretical quantities depending on the Casimir $T(\Phi)$ of the representation of the matter fields Q and on the Casimir of the adjoint $T(G_a)$ as well as on the modular weights of the fields Q^I.

$$\alpha_a^i = \sum_I T_a(Q^I)(1 - 2n_I^i) - T(G_a). \tag{31}$$

M^i refers to the set of both moduli U, T for each of the three tori, and η is the Dedekind function:

$$\eta(T) \equiv e^{-\pi i T/12} \prod_{n=1}^{\infty} \left(1 - e^{2\pi i n T}\right). \tag{32}$$

Notice that although $\eta(M)$ transforms in a simple way under $SL(2, \mathbf{Z})$ transformations, the function f is not invariant under T, U duality. This is just as well because the quantity we need to be invariant is not f but the physical (or canonical) gauge coupling which depends not only on f but also on the Kähler potential at tree-level. Notice the singularity structure of f. Using arguments about singularities and invariance of the full gauge coupling, it is possible to extract the expression of the function f in more complicated cases such as Calabi-Yau compactifications, for which the string computation is not possible and the duality group is not as simple as $SL(2, \mathbf{Z})$ [47]. Also, additional effects entering equation (29) have been computed, including its dependence on continuous Wilson lines [71].

The knowledge of loop corrections to f is not only important for studying questions of gauge coupling unification and supersymmetry breaking by condensation of hidden sector gauge fermions. It is also important because there is also a non-renormalisation theorem for f stating that *there are no further corrections to f beyond one-loop* [48]. This is again as in standard supersymmetric theories [51]. The only thing to keep in mind is that, for

f, it is important to state clearly that we are working with the 'Wilson' effective action rather than the 1PI effective action. In this case the gauge kinetic function is holomorphic and does not get renormalised beyond one loop [51]. On the other hand, the 1PI gauge coupling is *not* holomorphic and does get corrections from higher loops but, since it gives the physical coupling, it is invariant under duality symmetries.

3.3 General Properties of the Models

Let us recapitulate in this section what we can say about perturbative heterotic string models which is independent of the model. This is the closest we can get to string predictions and help us in approaching general questions, differentiating the generic issues from those of a particular model. Since the full nonperturbative formulation of string theory is not yet available, we have to content ourselves mostly with predictions of string perturbation theory, assuming that the corresponding string model is given by a CFT.

   (i) String models *predict* the existence of gravity and gauge interactions. This point has to be emphasized since it is the first theory that makes these fundamental predictions for interactions we experience in the every day life.

  (ii) There are other fields which survive at low energies: charged matter fields Q, candidates to be basic building blocks of matter but also the dilaton field S and the moduli T. We have to mention that, although as yet there is no 4D model without moduli fields, there is no general theorem implying their existence. In that sense the dilaton is the most generic modulus field, with a flat potential in perturbation theory.

 (iii) There is only one arbitrary parameter α' fixed to be close to the Planck scale M_p. All other parameters of the effective action are determined by expectation values of fields such as the dilaton and the moduli. In particular the gauge coupling is given at tree level by the VEV of S.

 (iv) The existence of spacetime supersymmetry is needed for consistency, although $\mathcal{N} = 1$ is selected for phenomenological reasons. There is a general requirement for a CFT to lead to $\mathcal{N} = 1$ spacetime supersymmetry: It has to have $(0, 2)$ supersymmetry in the world-sheet (2D) (plus a quantization condition on the charges of the $U(1)$ group mixing the two supersymmetries).

(v) There are no global internal symmetries in 4D string models, besides the already mentioned Peccei-Quinn symmetry of the S field and some accidental global symmetries (like baryon and lepton numbers in the standard model). This introduces very strong constraints on string models compared with standard field theory models.

(vi) There are generically some *discrete* symmetries in string models. Some infinite dimensional such as T-duality which in the simplest version takes the form of an $SL(2, \mathbf{Z})$ transformation

$$T \to \frac{a\,T + b}{c\,T + d} \tag{33}$$

with a, b, c, d integers satisfying $ad - bc = 1$ There are also finite dimensional discrete symmetries, such as those inherited from the twist defining orbifold constructions, which are seen as discrete gauge symmetries in the 4D effective theory. These can in principle be useful for model building, hierarchy of masses etc. There are however some couplings that vanish in string theory and *cannot* be explained in terms of symmetries of the effective 4D theory, these are called 'string miracles' since from the 4D point of view they seem to break the criterium for naturalness. T-duality symmetries restrict very much the form of the effective action and quantities such as Yukawa couplings have to be modular forms of a given duality group. These symmetries are valid to all orders in string perturbation theory and are thought to be also preserved by nonperturbative effects. Matter fields Q^I are assigned special quantum numbers, the modular weights n, according to their transformation properties under the duality group. For a $SL(2, \mathbf{Z})^m$ group for instance we have:

$$Q^I \to (ic_l\, T_l + d_l)^{n_I^l}\, Q^I, \quad l = 1, \cdots, m. \tag{34}$$

Since fermions transform non-trivially under these symmetries, there may be 'duality anomalies' which have to be cancelled for consistency. This imposes strong constraints on the possible spectrum of the corresponding string model.

(vii) There is unification without the need of a GUT. If the gauge group is a direct product of several groups we have for the heterotic string:

$$k_1\, g_1^2 = k_2\, g_2^2 = \cdots = \frac{8\pi}{\alpha'} G_N \equiv g_{string}^2. \tag{35}$$

Where k_i are special stringy constants known as the Kac-Moody levels of the corresponding gauge groups (for the standard model groups it is usually assumed that $k_2 = k_3 = 1, k_1 = 5/3$), g_i are the gauge couplings and G_N is Newtons constant. We can see there is a difference with standard GUTs in field theory for which we compute the unification scale by finding the point where the different string couplings meet. In heterotic string theory, the unification scale is given in terms of the string coupling g_{string} and the Planck scale.More precisely: $M_{string} \sim 5.27 \times 10^{17} g_{string}$ Gev. For $g_{string} \sim \mathcal{O}(1)$ this shows a discrepancy with the 'observed' value of the unification scale given by the experiments $M_{GUT} \sim 2 \times 10^{16}$ GeV. For the Weinberg angle, on the other hand, assuming the MSSM spectrum, it gives $\sin^2 \theta_W = 0.2337 - 0.25(\alpha_3(M_Z) - 0.119) \pm 0.0015$ differing from the experimental value of $\sin^2 \theta_W = 0.23113 \pm 0.00015$ just 2% [59]. Therefore the string 'predictions' are very close to the experimental value, which is encouraging, but differ by several standard deviations from it. The mismatch of the scale by a factor of 20 between the MSSM and the string scales is usually called the string unification problem[3]. The situation looks better for simple supersymmetric GUT's which have good agreement with experiment but there, the unification scale is put in by hand. Several ideas have been proposed to cure this problem in string theory, including large values of threshold corrections, intermediate scales, extra particles, changing the values of Kac-Moody levels etc, with no compelling solution yet (for discussions of this issue see for example ref. [58].).

(viii) There are usually fractionally charged particles in 4D string models. In fact it can be shown that we cannot have simultaneously $k_2 = k_3 = 3k_1/5 = 1$ in the standard model and only integer charged particles, because if that is the case the standard model gauge group would be enhanced to a full level-one $SU(5)$. This 'problem' can be evaded in models where the fractionally charged particles are heavy string states, it has also been proposed that those particles could confine at intermediate energies and be unobservable [60].

(ix) There are 'anomalous' $U(1)$ groups in most of the models, but there is also a counter-term in the action cancelling the anomaly and generating

---

[3]Notice that this does not refer to general string models, only those that may have the same spectrum at low-energies as the MSSM, which is usually not the case.
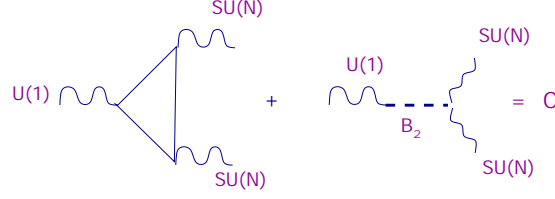
Figure 4: The Green-Schwarz mechanism for anomaly cancellation in $4D$. The triangular graph is cancelled by a tree-level coupling of the 2-form. This induces a Fayet-Iliopoulos term in the effective $4D$ effective action.

a Fayet-Iliopoulos kind of term :

$$\frac{1}{S+\bar{S}}\left|\frac{Trq^a}{48\pi^2}\frac{1}{(S+\bar{S})^2}+\sum q_I^a|Q_I|^2\right|^2,\qquad(36)$$

where q_I^a are the anomalous charges of the scalar fields Q_I. This term is responsible to break the would-be anomalous group, by fixing the value of a combination of the matter fields Q_I, breaking the $U(1)$ and usually other gauge groups, but not supersymmetry (although this has not been shown in general). A combination of the fields Q_I and the dilaton S still remains massless and plays the role of the new dilaton field.

There are further model independent results which refer to nonperturbative string effects and will be discussed next.

3.4   Supersymmetry Breaking and Vacuum Degeneracy

As we discussed previously there are two main problems of string perturbation theory namely, the enormous vacuum degeneracy and supersymmetry breaking. There were two lines of research towards addressing these problems.

(i) To use a particular field-theoretical nonperturbative effect which is the condensation of gauginos induced after the asymptotically free hidden sector becomes strongly interacting at lower energies ($\sim 10^{12}$ GeV).

(ii) To consider the supersymmetry breaking sector as a black box and study its possible implications assuming that a combination of the moduli fields T and the dilaton S are responsible for breaking supersymmetry, but without specifying how.

In the first approach the gaugino condensation idea is as follows. In heterotic strings there is usually a hidden and an observable group. The observable one includes the standard model and the hidden one is such that all the standard model particles are singlet under it. If the hidden sector is asymptotically free then the gauge coupling constant increases with decreasing energy. At a given energy, determined by the beta function of the corresponding hidden sector, the gauge coupling becomes so strong that nonperturbative effects may induce the condensation of a pair of hidden sector gaugino fields ($\langle \lambda \lambda \rangle \neq 0$). This will induce new terms in the effective action that may or may not break supersymmetry. If the gauge coupling is a constant the gauginos can condense but do not break supersymmetry. However for field dependent gauge couplings, the gaugino condensation induces a potential for the S and T fields since

$$\langle \lambda \lambda \rangle \sim \Lambda_G^3 \sim M_s^3 e^{-a/g^2} \sim e^{-a(-iS+f(T))} \tag{37}$$

where we have used the information about the gauge kinetic function from the previous section. Here Λ_G is the renormalisation group invariant scale of the hidden sector and the constant a is given by the corresponding beta function which is model dependent. The size of a will determine the effective scale of Λ_G.

The first interesting point about this is that this is a natural way to induce a hierarchy of scales since the Λ_G will be exponentially suppressed as compared to the string scale scale. For explicit models it can naturally be an intermediate scale $\Lambda_G \sim 10^{11}$ GeV. The second observation is that through the gauge kinetic function we have now a nontrivial potential for the fields S and T which may help solving the problem of lifting the vacuum degeneracy. Actually, for the explicit dependence on T in terms of the Dedekind function above and from T duality, it is known that T can be fixed at a point close to but different from the self-dual point $T = 1$. This is important because minima at self-dual points would not break supersymmetry. The minimum on T does break supersymmetry. If the scale of Λ_G is intermediate $M_I \sim 10^{11}$ GeV, this is a realisation of the so-called gravity mediated supersymmetry breaking in the sense that the observable sector will know about supersymmetry breaking only through gravitational strength interactions and the splitting of the masses in the observable sector will then take the form:

$$\Delta m = M_I^2 / M_{Planck} \sim 10^3 \text{ GeV}. \tag{38}$$

which is the desired value for supersymmetry to solve the hierarchy problem.

What about the dilaton S?. An exponential potential for S will naturally have a runaway behaviour without a minimum in the S direction. However in most cases the hidden sector group is actually a product of groups. In this case the potential will be a sum of exponentials

$$W(S) \; = \; \sum_k c_k \exp\left(i a_k S\right) \tag{39}$$

which can conspire to fix the dilaton at a finite value corresponding to weak coupling, and the expected unification scale value ($\mathrm{Re}\, S \sim 1/25$). depending on the relative values of the constants a_i ($\sim 1/N$) for each $SU(N)$ factor. This is not a generic situation but it is achievable. These are very good news. This has been named 'the racetrack scenario' since it depends on a sum of exponentials to get the local minimum. There are some negative points abut this scenario also:

(i) The cosmological constant, given by the value of the potential at the local minimum, is very large and negative.

(ii) The dilaton potential is runaway and there has to be some fine tuning in order to obtain a nontrivial minimum besides the minimum at zero coupling and $-iS \to \infty$.

(iii) There are at least two serious cosmological problems for the gaugino condensation scenario. First, it was found under very general grounds, that it was not possible to get inflation with the type of dilaton potentials obtained from gaugino condensation [77]. The second problem is the so-called 'cosmological moduli problem' which applies to any (non-renormalisable) hidden sector scenario including gaugino condensation [78]. In this case, it can be shown that if the same effect that fixes the vev's of the moduli, also breaks supersymmetry, then: the moduli and dilaton fields acquire masses of the electroweak scale ($\sim 10^2$ GeV) after supersymmetry breaking [78]. Therefore if stable, they over-close the universe, if unstable, they destroy nucleosynthesis by their late decay, since they only have gravitational strength interactions. At present there is no satisfactory explanation of this problem and it stands as one of the unsolved generic problems of string phenomenology.

The runaway behaviour of the dilaton has been argued to be a generic problem for string models [72]. The reason for this is that S being the string

coupling, we know that for $-iS \to \infty$ the theory becomes free and then the scalar potential has to vanish. This was used in [72], to argue that strings have to be strongly coupled in order to develop another minimum, unless some parameters conspire to fix S at weak coupling. The racetrack scenario overcomes this problem by a discrete tuning of parameters given by having similar values of coefficients $a - I$ above (which amounts to take two groups of similar rank).

Finally notice that once the dilaton gets a potential and a mass, the field S is no longer dual to a linear multiplet L including the two-index antisymmetric tensor. Still there is a duality with a massive superfield but this time including a *massive* three-index antisymmetric tensor, which is known to be dual to a massive scalar. Then the formulation with the field S is still valid and the physics in both formalisms is equivalent [76].

Soft SUSY Breaking Terms

It was found many years ago that some terms can be added to a supersymmetric Lagrangian that do not respect supersymmetry but still keep the soft ultraviolet behaviour of the theory. These are the soft-breaking terms. They are naturally generated after supersymmetry breaking in general supersymmetric models and correspond to the following terms

(i) Scalar masses, implying that the scalars such as squarks, become usually heavier than the fermions of the same multiplet. These are terms in the Lagrangian of the form $m_I^2 \left| Q^I \right|^2$.

(ii) Gaugino masses $M_a \lambda^a \lambda^a$ splitting the gauge multiplets.

(iii) Cubic A terms. Cubic scalar terms in the potential related to Yukawa couplings and controlled by arbitrary dimensionful coefficients (A) of order the gravitino mass.

(iv) The B-term. A quadratic term in the potential for the scalars of the form $B\mu H\bar{H}$ where $H, \bar{H}$ represent the Higgs fields and μ is a constant that gives rise to a term in the original superpotential $W = \mu H \bar{H}$ which is allowed by all the symmetries of the minimal supersymmetric standard model. Since μ is a dimension-full parameter it causes a problem to introduce it in the supersymmetric Lagrangian since it has to be of the order of the gravitino mass and there is no reason that a term in the supersymmetric Lagrangian knows the scale of the

breaking of supersymmetry.This is known as the μ problem and several solutions have been proposed. Depending on the proposed solution there is an expression for the parameter B after supersymmetry breaking. In particular, if $Z \neq 0$ in eq. (25), it can be seen that the μ term is generated after supersymmetry breaking. Some calculations have shown that there are models for which $Z \neq 0$.

In this section we will follow this strategy: Treat the supersymmetry breaking mechanism as a black box, but based on the experience with gaugino condensation, use that the end result of this mechanism is to induce non-vanishing values to the auxiliary fields of the moduli or dilaton fields. Therefore we can parametrise our ignorance of the particular breaking mechanism by working with general values of these auxiliary fields. Let us for simplicity treat a single modulus field T and the dilaton S. But the analysis has been done in more general cases [80, 57, 81, 82]. We can then write the goldstino field (Goldstone fermion eaten by the gravitino in the process of supersymmetry breaking) as a linear combination of the fermionic components of S and T [82]:

$$\tilde{\eta} = \sin\theta\,\tilde{S} + \cos\theta\,\tilde{T} \tag{40}$$

where the goldstino angle θ mixing $\tilde{S}$ and $\tilde{T}$ describes the relative contribution of S and T to the breaking of supersymmetry. The general procedure for extracting the soft breaking terms is clear. We start with the supersymmetric Lagrangian and substitute in it the non-vanishing auxiliary fields ($\sim e^{K/M_p^2} K_{z\bar{z}}^{-1/2} D_z W$). Perform the so-called flat space limit in which $M_p \to \infty$ with fixed gravitino mass $m_{3/2}$ (representing the non-vanishing VEV's of the auxiliary fields and parametrising the breaking of supersymmetry. We end up with the following values for the soft breaking parameters [82]:

$$
\begin{aligned}
m_I^2 &= m_{3/2}^2\left(1 + n_I \cos^2\theta\right) \\
M^a &= \sqrt{3}m_{3/2}\sin\theta \\
A_{IJK} &= -\sqrt{3}m_{3/2}\left(\sin\theta + \cos\theta(n_I + n_J + n_K)\right)
\end{aligned}
\tag{41}
$$

From this we can extract several conclusions. The dilaton dominated scenario for which $\sin\theta = 1$, the soft breaking parameters are *universal*. This is a very appealing result explaining one of the less justified assumptions of the minimal supersymmetric standard model. On the other hand, this scenario is so restrictive that it is relatively easy to rule it out. In particular

an analysis of charge and colour breaking restrictions within the MSSM rule out this scenario if the string scale is close to M_{Planck}. The importance of this scenario is that eqs (40) for $\sin \theta = 1$ are valid in general and not only for orbifold models. For arbitrary mixing angle, the soft breaking terms are not necessarily universal (unless special values of the modular weights are taken). In that case we have to confront problems with flavour changing neutral currents [82]. Another conclusion we can extract from the form of the soft breaking terms is that for negative modular weights, we can get tachyons in the spectrum, for certain values of the mixing angle. The same condition that avoids tachyons, implies that the gaugino masses have to be bigger than the scalar masses, unless both vanish; in that case loop corrections may be important to determine the relative masses.

4   D-Brane Phenomenology

As we have seen, the present understanding of string theory indicates that all the different 10-dimensional string theories (types I, IIA, IIB and two heterotic) happen to be different manifestations of a single M- theory, which also includes 11D supergravity. It has also led to a prime role played by high dimensional surfaces known as D-branes, giving support to the idea that our 4-dimensional world could itself be a brane embedded in a higher dimensional spacetime.

The techniques used to construct realistic models have been very much increased. We may start from any of the six theories and obtain something realistic.

1. *Start from 11D supergravity.* If we impose the conditions on a 7D manifold to give rise to a $\mathcal{N} = 1$ supersymmetric model, it requires the holonomy group to be G_2 [83]. Therefore 7D manifolds of G_2 holonomy naturally extend the Calabi-Yau 6D manifolds of $SU(3)$ holonomy in looking for geometrical constructions. These manifolds are much less understood but it has been shown that it is not possible to obtain chiral fermions except for singular points. General and elegant results exist already for these constructions. But there are no explicit models constructed in this way so far. This is a major open question concerning string model building.

    A simpler way to obtain $\mathcal{N} = 1$ models from 11D supergravity is the Horava-Witten construction in which the 11th dimension is compactified on an interval (a $S^1/\mathbf{Z}_2$ orbifold). This provides the strong

coupling realisation of the heterotic $E_8 \times E_8$ string. Each E_8 lives at each of the endpoints of the interval which are 10D surfaces. Further compactification on Calabi-Yau manifolds give rise to interesting chiral models. Some explicit models have been constructed although their phenomenological properties are difficult to extract given the mathematical complexity of the mechanism and the fact that we do not actually know what the full completion of 11D supergravity is. Still interesting general results have been obtained in this direction [84].

2. *Starting from type I,IIA,IIB strings.* The main new ingredient in these models is the existence of D-branes, which are the surfaces in which the endpoints of open strings are constrained to live when Dirichlet boundary conditions are used. D-branes can carry gauge and matter fields within. To get chiral models the branes have to be placed at singular points of a manifold, or to intersect at nontrivial angles, with chiral matter living only at the intersection. The compact dimensions can be Calabi-Yau manifolds in their orbifold limits as in the heterotic case. A new ingredient is that the worldsheet can be orbifolded due to orientation symmetry (parity) Ω in the worldsheet of type IIB strings giving rise to orientifold constructions. Fixed 'points' of these twists correspond to orientifold fixed planes which are fixed surfaces where the orientation of the strings change. They are novel stringy objects trapped at singular points usually with negative tension and positive RR charge. Consistency conditions amount to having tadpole cancellations. The typical example corresponds to the open string for which a 10-form RR field can have a linear coupling $\int C_{10}$ with coefficient $N - 32$ *ie*:

$$(N - 32) \int C_{(10)} \tag{42}$$

This term would give rise to a tadpole diagram for that field which cannot be cancelled except if the coefficient vanishes. This is what requires $N = 32$ setting the gauge symmetry to $SO(32)$. One way to say this is that the orientifold twist gave rise to orientifold planes with nonzero RR charge. To cancel that charge we need to introduce 32 D-branes. These include branes and their image under the twist on each side of the orientifold plane. The group is orthogonal rather than unitary at the location of the orientifold plane. Similar examples occur in compactifications. The conditions of tadpole cancellation are sufficient conditions to get non-abelian anomaly cancellation in the low-energy effective theory. Abelian groups can be anomalous with the

anomaly cancelled by a Green-Schwarz mechanism. In these models there are many antisymmetric tensors which can cancel the anomalies of several $U(1)$'s, as opposed to the perturbative heterotic case where only one anomalous $U(1)$ is allowed.

Furthermore, type IIB compactifications can also be described in terms of F-theory. In this case the two scalar fields of the 10D theory, the dilaton and axion, are combined into one complex field $S = a + i\phi$, which realises S-duality by an $SL(2, \mathbf{Z})$ transformation as in eq. (10). S can be considered as the modular parameter of a 2-torus. Therefore a four complex dimensional Calabi-Yau manifold is constructed from a local product of this torus with a six-dimensional (non Calabi-Yau) space B_3 under which the type IIB theory is compactified. The four-fold is then said to be an elliptically fibered Calabi-Yau. In particular, B_3 itself can be thought as a local product of K_3 and $\mathbf{P}_1$. These compactifications naturally incorporate D7 branes, which are given by the points in P_1, say, where the elliptic fibration degenerates.

The major ingredient in these new constructions is the fact that our world can be a brane (either a D-brane or one of the fixed surfaces in the Horava Witten construction or possibly a surface at the singularity of a G_2 holonomy manifold where chiral fermions can live). The brane world scenario has been subject to intense investigation during the past five years and new interesting mechanisms have been proposed to solve longstanding problems with the standard model, such as the hierarchy problem, gauge coupling unification, neutrino masses, the strong CP problem, etc. without necessarily referring to string theory. Here we will concentrate only on string theoretical realisations.

One of the interesting properties of this scenario is that it allows for a fundamental scale of nature to be much below the Planck scale and therefore closer to energies available in experiments. However until recently there were no explicit realisations of this scenario within string theory, with low-energy fundamental scale. We review here the progress made in that direction during the past few years.

4.1 The Brane World versus Kaluza-Klein

It is important to realise the difference between the brane world and the better known Kaluza-Klein scenario. In Kaluza-Klein all fields feel the extra
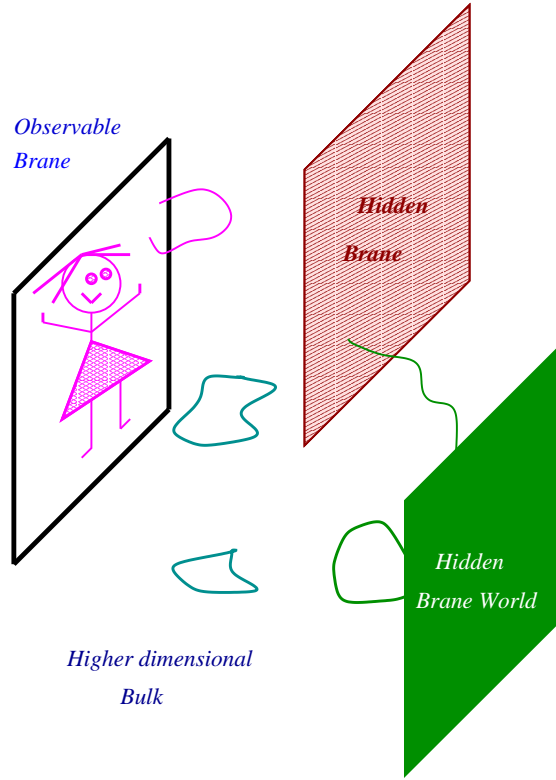
Figure 5: A cartoon version of a typical brane world. Our Universe fits into a surface (the observable brane) inside a higher dimension space (the bulk). There are other branes which are hidden to us because they only couple to the bulk through gravity. The open strings representing the matter fields have their endpoints attached to the brane (a D-brane) whereas gravity only comes from the closed strings which are free to move in the bulk.

dimensions whereas in the brane world, only a subset of the fields (gravity and moduli fields in string theory) feel all the extra dimensions. Thus, the brane-world is a variation of a Kaluza-Klein theory for which the extra dimensions are felt only by a subset of the fields. The typical case is that the Standard Model fields are constrained to live inside a low dimensional surface, or brane, of the high dimensional spacetime, but gravity lives in the full spacetime [85].

This simple fact has very important physical implications regarding the possible values of the fundamental scale. An explicit way to see the difference is comparing the low-energy effective actions for perturbative heterotic strings and type I and II strings. In the perturbative heterotic case, both

gravity and the gauge fields live on the full 10-dimensional spacetime corresponding to a standard Kaluza-Klein scenario. The low-energy effective action in 10 dimensions takes the form:

$$S = -4 M_s^8 \int d^{10}x \sqrt{-g} \; e^{-2\phi} \left(\mathcal{R} + \frac{1}{4} M_s^{-2} F_{MN}^2 + \cdots \right), \qquad (43)$$

where $M_s = 1/\sqrt{\alpha'}$ is the string scale and ϕ is the dilaton field. Upon compactification to 4-dimensions each of the two terms in the action above will have a volume factor coming from the integration of the 6 extra dimensions (assuming a factorized geometry $\mathcal{M}_{10} = \mathcal{M}_4 \times \mathcal{M}_6$). We can then compare with the 4D effective action for gravity and Yang-Mills gauge fields:

$$S_{4D}^{eff} = - \int d^4x \sqrt{-g} \left(M_{Planck}^2 \mathcal{R}_4 + \frac{1}{4g_{YM}^2} F_{\mu\nu}^2 + \cdots \right) \qquad (44)$$

Now we can compare both actions. This gives us an expression for the gravitational and gauge couplings (the numerical coefficients of each of the two terms above) of the form:

$$M_{Planck}^2 = 4e^{-2\phi} M_s^8 V_6 \;, \qquad \alpha_{YM}^{-1} = \frac{1}{16\pi} e^{-2\phi} M_s^6 V_6 \qquad (45)$$

where V_6 is the overall volume of the extra dimensions and remember that $\alpha_{YM} = g_{YM}^2/(4\pi)$. Taking the ratio of those expressions the volume and dilaton factors cancel and we obtain

$$M_{Planck}^2 = \frac{1}{64\pi} \alpha_{YM}^{-1} M_s^2. \qquad (46)$$

Therefore for α_{YM} not much different from $1/25$ (as expected) we have to have the fundamental scale M_s to be of the same order of magnitude as the gravitational scale $M_{Planck} \sim 10^{19}$ GeV. This was the old belief that the string scale was constrained to be very close the Planck scale.

Things are very different in the brane world scenario as we can see for the case of the type I, II strings. For a configuration with the standard model spectrum belonging to a Dp-brane, the low energy action in 4-dimensions takes the form:

$$S = - \int \frac{d^4x}{2\pi^7} \sqrt{-g} \left(M_s^8 V_6 \; e^{-2\phi} \mathcal{R}_4 - \frac{1}{4} V_{p-3} M_s^{p-3} \; e^{-\phi} F_{\mu\nu}^2 + \cdots \right) \; (47)$$

There are two differences to point out with the heterotic case. First, the power of the dilaton is different for the gravity and the gauge kinetic terms.

Second, the total volume $V_6 = r^6$ enters on the gravity part but only the volume of the $p - 3$ cycle of the internal manifold, V_{p-3}, that the p branes wrap around, appears in the gauge part of the action. In particular, for a D3-brane, *there is no* volume factor contribution to the gauge coupling. Comparing the coefficient of the Einstein term with the physical Planck mass M_{Planck}^2 and the coefficient of the gauge kinetic term with the physical gauge coupling constant $\alpha_p(\sim 1/24$ at the string scale), we find the relations:

$$M_{Planck}^2 = \frac{M_s^8}{(2\pi)^7} V_6 e^{-2\phi} , \qquad \alpha_{YM}^{-1} = 4\pi M_s^{p-3} V_{p-3} e^{-\phi} \qquad (48)$$

from which we can easily see that if the Standard Model fits inside a D3-brane, for instance, we may have M_s substantially smaller than M_{Planck} as long as the volume of the extra dimensions are large enough. This without affecting the value of the gauge coupling. In the heterotic case, setting the volume very large it would make the gauge coupling extremely small, which is unrealistic.

Notice that in type I we could have the gauge fields on a D9 brane, still feeling all the dimensions. The volume factor would then appear in the gauge coupling, but there is still a difference with the heterotic case, since taking the ratio as in (46) would still leave a e^{phi} factor in front. This means that for weak enough string coupling $e^{-\phi}$, the string scale can be hierarchically smaller than the Planck scale in type I (we would expect only a few orders of magnitude, for instance we may have $M_s = M_{GUT}$ in this way but no $M_s = M_{EW}$, say.).

The requirement of having large extra dimensions reformulates the hierarchy problem into that of explaining naturally the existence of a large volume.

Given the fact that we do not have a way to fix the size of the extra dimensions we can take advantage of our ignorance and follow a bottom-up approach considering different possibilities motivated by phenomenological inputs. Several scenarios have been proposed depending on the value of the fundamental scale. The four main scenarios at present correspond to

1. $M_s \sim M_{Planck}$. This is just the old perturbative heterotic string case corresponding to compactification scale close to the Planck scale. There is nothing wrong with this possibility. Research over the years has shown it is difficult to obtain, in this case, explicit string models with gauge coupling unification at this scale.

2. $M_s \sim M_{GUT} \sim 10^{16}$ GeV. Obtained for $r \sim 10^{-30} cm$ in the expression above (48), for $r^6 = V_6$. This proposal [86] was made precisely to

'solve' the gauge coupling unification problem in string theories. This requires a compactification scale of order 10^{14} GeV.

3. $M_s \sim M_I \sim 10^{10-12}$ GeV. If the world is a D3-brane we can see that this scale is obtained from the equations above for $r \sim 10^{-23} cm$. This proposal [89, 90] was based on the special role played by the intermediate scale M_I in different set-ups beyond he standard model. Examples of these include the scale of supersymmetry breaking in gravity mediated supersymmetry breaking scenario and the scale for the axion field introduced to solve the strong CP problem. This then allows one to identify the string scale with the supersymmetry breaking scale and opens up the room for non supersymmetric string models to be relevant at low-energies, solving the hierarchy problem. Explicit models realising this scenario will be discussed in the next section.

4. $M_s \sim M_{EW} \sim 10^3$ GeV. This is obtained for overall radius $r \sim 10^{-12} cm$ above and if only two of the six dimensions were large this would have given us the famous $r \sim 1mm$, quoted as the extreme case of the brane world scenario (lengths above this value would have been observed by deviations from Newtonian gravitational interactions.) This is the most popular scenario [87] due to its proximity with experiment. It has opened up a completely new approach towards looking for physics beyond the standard model at present and future experiments, especially after the work of Arkani-Hamed, Dimopoulos and Dvali [88] where a detailed analysis was done about the possible experimental, astrophysical and cosmological constraints of this scenario which range from comparisons with Van der Waals forces in molecules to over-cooling of supernovae, especially supernova 1987a. Concrete string models realising this scenario do exist. We will discuss some such attempts in the next section.

Notice that only the first scenario was possible following the standard Kaluza-Klein approach in the perturbative heterotic string models. The brane world opened up the possibility of the next three scenarios as well as any other scale in the range $M_{EW} < M_s < M_{Planck}$. An important question is how to realise these scenarios in explicit string models.
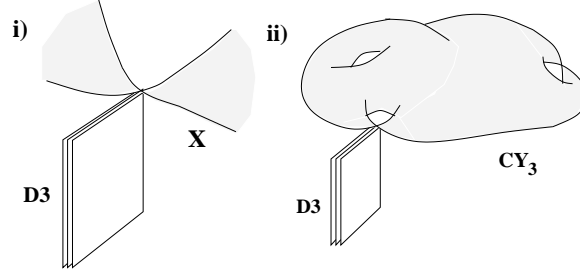
Figure 6: Pictorial representation of the bottom-up approach to the embedding of the standard model in string theory. In step **i)** the standard model is realized in the world-volume of D3-branes sitting at a singular non-compact space X (in the presence of D7-branes, not depicted in the figure). In step **ii)** this local configuration is embedded in a global setting, like a compact Calabi-Yau threefold. Many interesting phenomenological issues depend essentially only on the local structure of X and are rather insensitive to the details of the compactification in step **ii)**. The global model may contain additional structures (like other branes or anti-branes) not shown in the figure.

4.2   Realistic D-brane models

Let us see how we can build realistic models inside D-branes. D-branes have several interesting properties. The first one we should note is that for a single Dp-brane in type II string theory, the open string ending on that brane gives rise to a $U(1)$ gauge boson, $9 - p$ scalars and the corresponding fermions. These can be seen from a Kaluza Klein reduction of the $U(1)$ vector multiplet from 10 to the $p + 1$ dimensions of the D brane world volume. The scalars correspond to the Goldstone modes of the part of the Poincaré symmetry broken by the presence of the brane. Similarly the fermions are goldstinos for supersymmetry. The D-brane breaks half of the supersymmetry. Therefore in flat space, after toroidal compactification, one D-brane carries the spectrum of $\mathcal{N} = 4$ supersymmetric vector superfield.

Furthermore, D-branes are BPS states for which a no force condition applies. This means that two parallel D-branes do not interact with each other. The reason for this is that both have a positive tension and, therefore, are naturally attracted to each other by gravitational interactions. Also the exchange of the dilaton field naturally leads to an attractive interaction. However, both branes are also charged under antisymmetric Ramond-Ramond fields for which the interaction is repulsive, given that both branes have the

same charge. Therefore the combined action of the three interactions cancels exactly if the branes are BPS.

This calculation can be done explicitly, the interaction amplitude corresponds to the exchange of closed strings between the two branes. The amplitude can be computed by calculating the one-loop open string amplitude corresponding to a cylinder [3]:

$$
\mathcal{A} = 2 \int \frac{dt}{2t} \mathrm{Tr} e^{-tH} = 2T_p \int \frac{dt}{2t} \left(8\pi^2\alpha' t\right)^{-(p+1)/2} e^{-\frac{Y^2 t}{2\phi^2\alpha'}} \left[Z_{NS} - Z_R\right] \equiv A_{NS} - A_R
$$
(49)

With

$$
Z_{NS} = \frac{-16 \prod_n \left(1 + q^{2n}\right)^8 + q^{-1} \prod_n \left(1 + q^{2n-1}\right)^8}{\prod_n \left(1 - q^{2n}\right)^8}
$$

$$
Z_R = \frac{q^{-1} \prod_n \left(1 - q^{2n-1}\right)^8}{\prod_n \left(1 - q^{2n}\right)^8}
$$
(50)

Here $q \equiv e^{-t/4\alpha'}$ and t is the proper time parameter for the cylinder. H is the Hamiltonian for each sector of the theory. It is easy to see that $Z_{NS} = Z_R$. Therefore the interaction potential just vanished. The cancellation between R-R and NS-NS sectors is a reflection of the BPS condition for the D-branes.

This gives rise to an interesting phenomenon which is essentially (but not quite) the inverse of the Higgs effect. Open strings with both end points on one brane give rise to the $U(1)$ gauge field for that brane. In the presence of a second brane, besides having the second $U(1)$ there are also pairs of strings with endpoints on each of the two branes. These correspond to massive states with mass proportional to the separation of the branes. We may identify one string with a particle like W^+ (of the standard model) and the string with opposite orientation with W^-. The important point is that when both branes overlap these particles become massless and enhance the $U(1) \times U(1)$ symmetry to the full $U(2)$ symmetry. This is then the way to obtain non-abelian supersymmetric gauge theories on the brane. Matter then naturally exists on the brane.

This is interesting but unrealistic, mainly because of the lack of chirality of the spectrum of $\mathcal{N} = 4$ supersymmetric theories. We should look for ways to obtain chiral fermions on the brane. Notice that changing the compactification space from a torus to orbifold or Calabi-Yau does not improve the situation since this is a local issue at the location of the brane.

There are two known concrete ways to obtain a chiral spectrum on the D-brane.
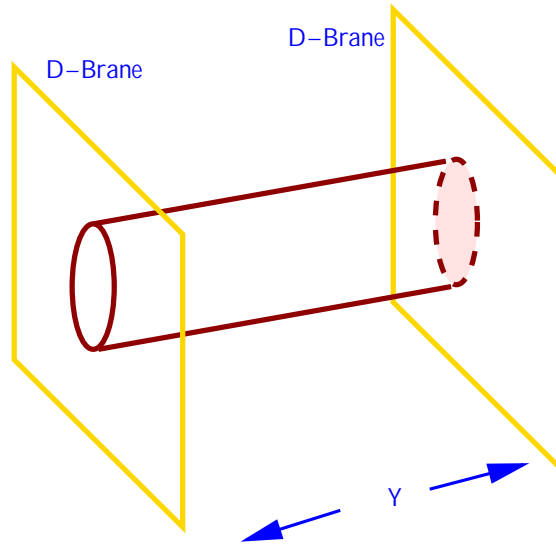
Figure 7: Cylinder interaction between two branes. It can be regarded in two dual ways, as a tree-level exchange of closed strings, valid at large distances or as a one-loop exchange of open strings, dominant at small distances.

- D-branes at singularities.

- Intersecting D-branes.

In principle there is a third way to build realistic models which is closer to the line followed in the heterotic string, namely, start with a type II string, perform an orientifold twist that will give rise to the type I string theory with gauge group $SO(32)$. Then compactifying on an orbifold (orientifold of type II) or general Calabi-Yau, gives rise to chiral models in 4d. This method happens to be a particular case of the methods mentioned above after the use of T-dualities.

We will discuss briefly each of the two constructions since each has very interesting properties. The main point we will try to make is that both constructions can be considered 'modular' in the sense that we can locally obtain the standard model without having to know all the details of the compactification. This is of greatest importance because it alleviates the 'needle in the haystack' problem of string phenomenology. We can follow a bottom-up approach to construct the standard model from D-branes instead of looking at random by particular compactification that could give rise to
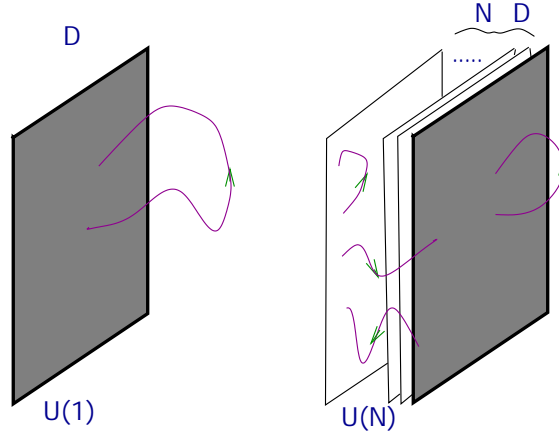
Figure 8: The BPS nature of D-branes allows them to for stacks of D-branes. One single D-brane carries a $U(1)$ supersymmetric multiplet inside, given in terms of open strings ending on the brane. A stack of N branes includes also strings ending on different branes. When the branes overlap these correspond to further massless states, enhancing the symmetry to $U(N)$.

the standard model.

We will present a bottom-up approach for building a realistic model. Most of the important details of the models, such as the gauge group, chirality, number of families, etc., will depend only on the structure of the singularity that the branes sit at or at the way the branes intersect. This can happen inside all sorts of spaces and therefore we can keep the main properties whether we are talking about a complicated Calabi-Yau or a simpler toroidal orbifold compactification. This makes the models considered more robust. This is probably the main practical advantage of brane model building over heterotic string ones.

4.2.1 D-branes at singularities

Let us see explicitly how the spectrum of D-branes at a $\mathbf{Z}_N$ singularity, like the fixed points of orbifolds, becomes chiral. Let us consider a D3 brane in flat 10D space with the extra six dimension modded out by a $\mathbf{Z}_N$ twist θ [4]. In complex coordinates we have:

$$(z_1, z_2, z_3) \to \left(\alpha^{l_1} z_1, \alpha^{l_2} z_2, \alpha^{l_3} z_3 \right) \tag{51}$$

---

[4]In this section we are referring to N for the order of the twist, n for the number of overlapping D-branes and $\mathcal{N}$ for the number of supersymmetries.

with $\alpha = e^{2\pi i/N}$. The l_i's are related to the v_i's introduced earlier by $\frac{1}{N}(l_1, l_2, l_3) = (v_1, v_2, v_3)$. We know that for $l_1 + l_2 + l_3 = 0$ this twist leaves an $\mathcal{N} = 1$ supersymmetry in the bulk. If we have a stack of n D-branes the original gauge group is $U(n)$. But we can extend the action of the twist to the gauge degrees of freedom (as we did in the heterotic case). These are represented by the Chan-Paton matrix $\lambda^i_j, i, j = 1, \cdots n$, associated to the endpoints of the open strings and which belongs to the adjoint of $U(n)$. Therefore the action of the twist θ on λ^i_j can be written as

$$\lambda \rightarrow \Gamma_\theta \lambda \Gamma_\theta^{-1} \tag{52}$$

where Γ_θ has to be of order N and can be diagonalised to take the simple form.

$$\Gamma_\theta \;=\; \begin{pmatrix} I_{n_0} & & & & & & \\ & \alpha I_{n_1} & & & & & \\ & & \alpha^2 I_{n_2} & & & & \\ & & & . & & & \\ & & & & . & & \\ & & & & & . & \\ & & & & & & \alpha^{N-1} I_{n_{N-1}} \end{pmatrix} \tag{53}$$

Here I_{n_k} is the identity matrix in n_k dimensions and the integers n_k have to satisfy the constraint $\sum_k n_k = n$. The information in Γ_θ can also be encoded in a vector of the type:

$$V \;=\; \frac{1}{N}\left(0^{n_0}, 1^{n_1}, \cdots, (N-1)^{n_{N-1}}\right) \tag{54}$$

very much as the shift vectors used in the heterotic case before. Here, 0^{n_0} means $0, 0, \cdots 0$, n_0 times, and so on.

Let us then now see how the original vector multiplet of $\mathcal{N} = 4$ transforms under the action of the twist defined by θ and Γ_θ . We can write the $\mathcal{N} = 4$ multiplet in terms of the $\mathcal{N} = 1$ multiplets as: $V \equiv (A_\mu, \lambda)$ is the $\mathcal{N} = 1$ vector and $\Phi_a \equiv (\phi_a, \psi_a)$ $a = 1, 2, 3$ are the complex chiral superfields. Notice that the index a refers to the three extra complex dimensions and therefore Φ_a feel both the action of θ and Γ_θ whereas V only feels the action through Γ_θ.

Remember that we have to keep only the states which are invariant under the twist. This means that $V = \Gamma_\theta V \Gamma_\theta^{-1}$. This breaks the gauge group to:

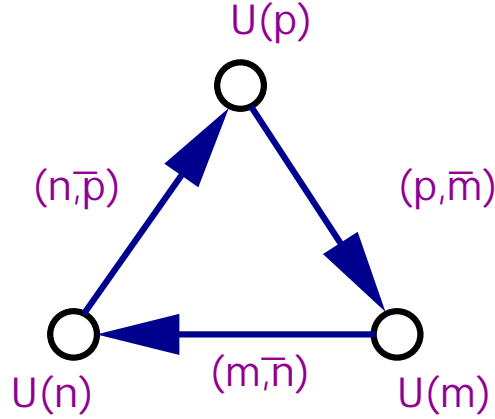$$U(n) \rightarrow U(n_0) \times U(n_1) \times \cdots \times U(n_{N-1}) \tag{55}$$

Figure 9: A quiver diagram for $\mathbf{Z}_3$. Each circle represents a $U(n)$ gauge group, the arrows represent bifundamentals under the corresponding gauge group, the direction of the arrow stating which is the fundamental and which is the anti-fundamental. The closed triangle shows that there should be an invariant cubic superpotential built from the product of the three fields. Notice that since the multiplicity is 3 for $\mathbf{Z}_3$ there should actually be three identical lines joining every pair of circles.

with the number of factors equals the order of the twist N. That means that if we want three factors we should have a $\mathbf{Z}_3$ twist and so on.

The surviving chiral superfields satisfy $\Phi_a = \alpha^{l_a} \Gamma_\theta \Phi_a \Gamma_\theta^{-1}$. The first factor being the action of θ. Therefore remembering that λ carries adjoint indices (which are composed of fundamentals and anti-fundamentals) we can easily see that the remaining matter fields transform as:

$$\sum_{a=1}^{3} \sum_{i=0}^{N-1} \left(\mathbf{n}_i, \bar{\mathbf{n}}_{i+l_a} \right) \tag{56}$$

where the sum in the index of the last term is understood to be mod N. Here $\mathbf{n}_i$ means the fundamental of $U(n_i)$.

This is a typical spectrum in this class of models. The matter fields tend to come in bifundamentals of the products of groups. These can be arranged into 'quiver' diagrams, see figure (4.2.1). These diagrams are made out of one node per group factor, *ie* the *ith* node corresponding to the group $U(n_i)$. There are also arrows joining the nodes. An arrow going from the *ith* to the *jth* node correspond to a chiral field in the representation $(n_i, \bar{n}_j)$. A closed triangle of arrows would indicate the existence of a gauge invariant cubic superpotential for those fields.
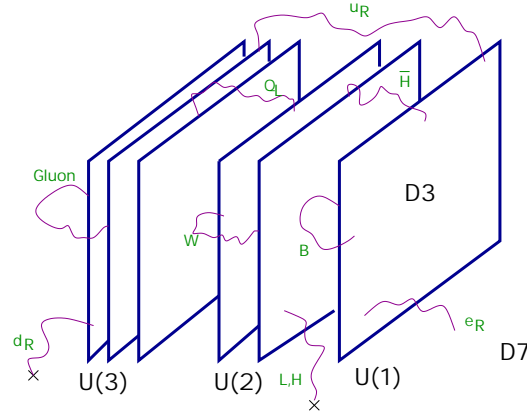
Figure 10: The standard model at a singularity. A configuration of six D3 branes giving rise to the standard model gauge group and matter fields. A pictorial open string representation is given of each of the particles of the standard model. D7 branes need to be introduced for tadpole (anomaly) cancellation. All the D3 branes are actually overlapping.

From (56) we can extract a very simple but powerful conclusion: Only for $\mathbf{Z}_3$ will we get the matter spectrum in three identical copies or families. The reason is that only for that case we have $l_1 = l_2 = l_3 \bmod N$, since $l_1 = l_2 = 1$ and $l_3 = -2 = 1 \bmod 3$. Other twists given by $(1/N, 1/N, -2/N)$ will give rise to two families. Therefore three is not only the maximum number of families for this class of models but it is obtained only for one twist, the $\mathbf{Z}_3$ twist. This is a rather remarkable result.

If we want to have the standard model we need to have $n_0 = 3, n_1 = 2, n_2 = 1$ to get the gauge group in $\mathbf{Z}_3$ to be $U(3) \times U(2) \times U(1)$. The spectrum will then be:

$$3\left[(\mathbf{3}, \mathbf{2}) + (\mathbf{1}, \mathbf{2}) + (\bar{\mathbf{3}}, \mathbf{1})\right] \tag{57}$$

where we are omitting the $U(1)$ quantum numbers. This gives the 3 families of left handed quarks, right handed down quarks and leptons, just as in the standard model. However we can easily see that we are missing at least the right handed down quarks. Actually the spectrum as it is, is anomalous. What happens from the string theory point of view is that there are uncancelled tadpoles for twisted sector fields. We will take care of this later, but until then we can explore general properties of the model as it stands now.

First there are actually three $U(1)$'s. Only one combination of them is

anomaly free and it is defined in general (for any N) by:

$$Q_Y = -\left(\frac{1}{3}Q_3 + \frac{1}{2}Q_2 + \sum_{s=1}^{N-2} Q_1^{(s)}\right) \tag{58}$$

In a generic orbifold all other $N-1$ additional $U(1)$ factors will be anomalous and therefore massive due to a version of the Green-Schwarz mechanism, with mass of the order of the string scale. We can easily check that this $U(1)$ does correspond to hypercharge, as expected since hypercharge is essentially the only non-anomalous $U(1)$ with the spectrum of the standard model. For instance, fields transforming in the $(\mathbf{3}, \mathbf{2})$ representation have Q_Y charge $-\frac{1}{3} + \frac{1}{2} = \frac{1}{6}$, as corresponds to left-handed quarks. Fields transforming in the $(\mathbf{3}, \mathbf{1})$ (necessarily with charge -1 under one of the $Q_1^{(s)}$ generators) have a Q_Y charge $-\frac{1}{3} + 1 = -\frac{2}{3}$, as corresponds to right-handed U quarks, etc.

It is worth noticing that normalization of this hypercharge $U(1)$ depends on the order of the twist N. In fact, by normalizing $U(n)$ generators such that $\mathrm{Tr}T_a^2 = \frac{1}{2}$ the normalization of the Y generator is fixed to be

$$k_1 = 5/3 + 2(N - 2) \tag{59}$$

This amounts to a dependence on N in the Weinberg angle, namely

$$\sin^2 \theta_W = \frac{g_1^2}{g_1^2 + g_2^2} = \frac{1}{k_1 + 1} = \frac{3}{6N - 4} \tag{60}$$

Thus the weak angle decreases as N increases. Notice that the $SU(5)$ result $3/8$ is only obtained for a Z_2 singularity. However in that case the D3 brane spectrum is necessarily vector-like and hence one cannot reproduce the SM spectrum. For the interesting case for us, $N = 3$, we find $\sin^2 \theta_W = 3/14$.

To cancel these tadpoles at the singularity it is necessary to have not only the starting D3 branes but also D7 branes. There are three types of D7 branes that can be introduced depending which 2 of the 3 complex dimensions they contain. The consistency conditions (tadpole cancellation) can be written as:

$$\mathrm{Tr}\Gamma_{\theta,7_3} - \mathrm{Tr}\Gamma_{\theta,7_1} - \mathrm{Tr}\Gamma_{\theta,7_2} + 3\mathrm{Tr}\Gamma_{\theta,3} = 0 \tag{61}$$

It can be shown that these conditions are equivalent to non-Abelian anomaly cancellation in the effective field theory. Notice that without the D7 branes we could not have had the standard model on the D3 brane (only groups like $U(3)^3$ for instance).

The D7 branes will have extra (hidden) gauge group and matter fields living on the D7 brane which can be obtained on a similar way, with a matrix $\Gamma_{\theta,7_i}$ with $i = 1, 2, 3$ labelling the different D7 branes, acting on the gauge degrees of freedom of the D7 branes. There are also massless matter fields living at the intersection of the D7 and the D3 branes, corresponding to open strings with one endpoint on the D3 branes and the other on the D7 branes. These will complete the spectrum of the standard model and render the model anomaly free at the singularity. We show in the table a particular example of configuration of D3 and D7 branes with the full spectrum, just as an illustration.

Matter fields	Q_3	Q_2	Q_1	$Q_{u_1^r}$	$Q_{u_2^r}$	Y
33 sector						
$3(3, 2)$	1	-1	0	0	0	1/6
$3(\bar{3}, 1)$	-1	0	1	0	0	-2/3
$3(1, 2)$	0	1	-1	0	0	1/2
37$_r$ sector						
$(3, 1)$	1	0	0	-1	0	-1/3
$(\bar{3}, 1; 2')$	-1	0	0	0	1	1/3
$(1, 2; 2')$	0	1	0	0	-1	-1/2
$(1, 1; 1')$	0	0	-1	1	0	1
7$_r$**7**$_r$ sector						
$3(1; 2)'$	0	0	0	1	-1	0

Table 3:   Spectrum of $SU(3) \times SU(2) \times U(1)$ model. We present the quantum numbers under the $U(1)^9$ groups. The first three $U(1)$'s come from the D3-brane sector. The next two come from the D7$_r$-brane sectors, written as a single column with the understanding that e.g. fields in the **37**$_r$ sector are charged under the $U(1)$ in the **7**$_r$**7**$_r$ sector.

A similar model can be constructed choosing $n_0 = 3, n_1 = 2, n_2 = 2$ with:

$$\Gamma_\theta \;=\; diag\left(\mathbf{I_3}, \alpha\mathbf{I_2}, \alpha^2\mathbf{I_2}\right) \tag{62}$$

giving rise to a left-right symmetric gauge model with group
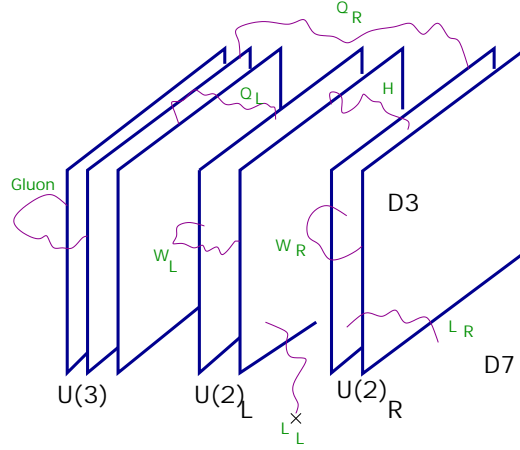
$$U(3) \times U(2)_L \times U(2)_R \tag{63}$$

Figure 11: The standard model at a singularity. A configuration of six D3 branes giving rise to the standard model gauge group and matter fields. A pictorial open string representation is given of each of the particles of the standard model. D7 branes need to be introduced for tadpole (anomaly) cancellation. All the D3 branes are actually overlapping.

with three families of

$$3\left[(\mathbf{3},\mathbf{2},\mathbf{1}) + (\bar{\mathbf{3}},\mathbf{1},\mathbf{2}) + (\mathbf{1},\mathbf{2},\mathbf{2}) + (\mathbf{1},\mathbf{2},\mathbf{1}) + (\mathbf{1},\mathbf{1},\mathbf{2})\right] + \cdots \qquad (64)$$

Again of the extra $U(1)$'s it survives the anomaly free $U(1)_{B-L}$, the other ones acquire a mass by the Green-Schwarz terms.

This model happens to be very interesting for several reasons. It is the easiest to construct, the higgses and leptons can be easily distinguished since they belong to different representations. Most importantly it happens to lead to gauge coupling unification with a precision comparable to that of the MSSM case. Even including two-loop corrections. The reasons to achieve unification are: first the $U(1)_{B-L}$ coupling is normalised at a nonstandard value of 32/3 (which corresponds to the hypercharge normalisation of 11/3 instead of the standard 5/3 of GUT's. Second, the Higgs multiplets come in three families. Therefore for a scale of $SU(2)_R$ breaking of at most one order of magnitude above the standard model, this model provides the first concrete example of successful gauge coupling unification from string theory with logarithmic running. This is quite remarkable. The unification scale happens to be of order 10^{11} GeV.

Another remarkable property is the existence of precisely R-parity after the breaking of $SU(2)_R$. This comes from the fact that there is a $B - L$

symmetry surviving at low energies, together with a stringy $\mathbf{Z}_2$ symmetry under which all the particles coming from strings with endpoints at a D3 and a D7 brane, change sign, whereas all the other ones are inert. Therefore the standard phenomenological explanation for proton stability in supersymmetric models is realised at a string level in this model. This seems to be the first time that R parity comes out in string models. The special properties of this model makes it very appealing. The structure of Yukawa couplings have been partially explored with the possibility to incorporate a hierarchy and a pattern of neutrino masses. However, their presence is not compelling and depends on higher order terms in the superpotential.

This is not the full story for these models. Remember we are building them step by step from a bottom-up approach. There is an extra source of anomalies. So far we have concentrated only on a singularity in flat space modded out by the action of $\mathbf{Z}_N$. If we compactify the extra dimensions the total RR charge of the D7 branes has to cancel, since there is no place for the RR flux to escape in a compact space. This will force us to add anti D7 branes (branes with opposite RR charge) living at different fixed points or consider other sources of tadpole cancellation.

The effect of the anti D7 branes is to break supersymmetry, since they preserve that half of the supersymmetry that the D-branes break therefore if both are present the full supersymmetry is broken. If the anti-branes are trapped at different fixed points from the branes then only the bulk fields can mediate the breaking of supersymmetry to the observable brane. This is a realisation of gravity mediated supersymmetry breaking scenario. This is another remarkable feature of the LR symmetric model. In order to obtain a realistic spectrum of supersymmetric particles the scale of supersymmetry breaking has to be intermediate $\sim 10^{11}$ GeV. This is precisely the same energy where the gauge couplings unify. Therefore if the string scale is intermediate we get both, gauge coupling unification at the string scale and low-energy supersymmetry breaking solving the hierarchy problem. Furthermore, the model contains antisymmetric tensor fields attached to the fixed points which couple to the gauge fields (after a duality transformation) precisely as the axion field. We knew that the axion scale was constrained to be also of the order of 10^{11} GeV. Therefore gauge unification, supersymmetry breaking and the solution of the strong CP problem point precisely at the same scale $M_I = 10^{11}$ GeV. It is left to explain why the size of the extra dimensions are such to give the string scale to have this intermediate value.

Once supersymmetry is broken there appear potentials for the moduli fields (dilaton, etc.). Fixing all those fields is not under control at the mo-
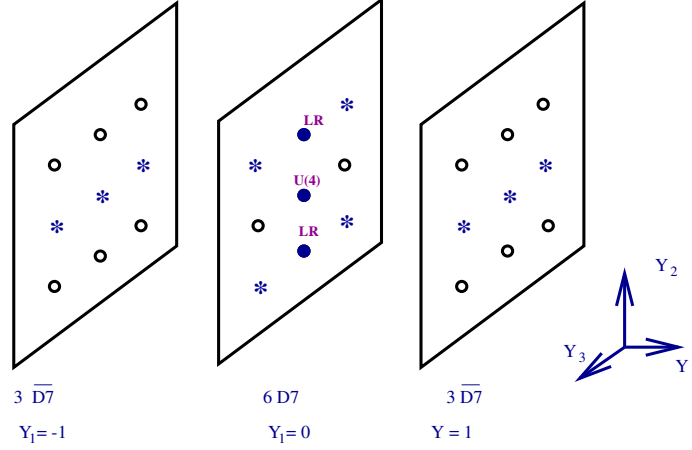
Figure 12: A global configuration with all RR tadpoles cancelled. Each rectangle represents a stack of D7 branes or anti-branes, each covering 9 of the 27 fixed points of the $\mathbf{Z}_3$ orbifold. We label the fixed points as (l, m, n), $l, m, n = -1, 0, 1$. At the point $(0, 1, 0)$ we have a stack of D3 branes with the realistic left-right (LR) symmetric model. The same model is at the mirror point under the orientifold twist. The orientifold fixed point has (besides the O_3 plane) eight D3 branes giving rise to a $U(4)$ symmetry. The ten fixed points marked with a $*$ have one single D3 brane, giving rise to $U(1)$ groups. The anti D7 branes cannot move to annihilate the D7 branes due to the presence of D3 branes in their fixed points. Both types of branes are needed to cancel tadpoles. The spectrum at each D3 brane is supersymmetric but the global configuration is not supersymmetric due to the anti-branes. Supersymmetry breaking is gravity mediated.

ment in these models (but see the next section). We can say that these potentials correspond essentially to NS tadpoles, which, unlike RR tadpoles do not point out to any inconsistency. We only need to understand better their potential and see if there is any way of obtaining a stable minimum.

This instability due mainly to the lack of supersymmetry should not be considered as a major problem. The option is to build a supersymmetric model with the same properties but without introducing anti-branes. This is actually possible to do, and has been done explicitly for the LR symmetric model above in terms of F-theory. In particular F-theory allows that cancellation without the introduction of anti-branes and therefore preserves supersymmetry. This is an example of the modular structure of the model that can be lifted to non-supersymmetric and supersymmetric realisations with the same local structure of gauge group, and spectrum. The supersymmetric realisation, although not suffering from NS tadpoles, leaves open the question of supersymmetry breaking.

Finally let us remark that there are large classes of models that can be constructed in this way. Singularities beyond the simplest $\mathbf{Z}_N$ singularities discussed here have been considered in [98]. These include non-abelian twist groups G, orientifold singularities and conifold singularities. For the case of non-abelian singularities, an interesting model was proposed in [101]. They consider a singularity of the type $\mathbf{C_3}/G$ with $G = \Delta_{27}$ which is one of the non-abelian discrete groups of $SU(3)$, thus preserving supersymmetry on the D3 brane, like in the $\mathbf{Z}_N$ cases. One of the interesting properties of this model is that there is no need to introduce the D7 branes to cancel the local tadpoles. The gauge group on the D3 branes is $U(3)^2 \times U(1)^9$ which can further break to the standard model with three families (due partially to the $\mathbf{Z}_3$ subgroup of Δ_{27}). The compactification of the model to obtain realistic gravity in 4D has not been considered yet.

4.2.2 Intersecting Branes

A probably more flexible construction of realistic models is the intersection of branes. This also gives rise to chiral fermions in a way very similar to the branes at singularities. The overall picture is the following. We know by now that an open string carries the indices in the adjoint representation of $U(n)$. The adjoint can be seen as the product of fundamental and anti-fundamental, therefore one index transforms as the fundamental and the other as the anti-fundamental. When the two end points of the open string lie on the same stack of branes then we have particles, like gauge bosons,
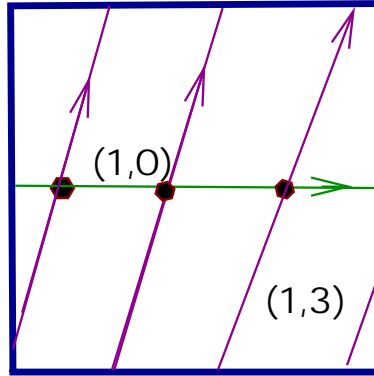
Figure 13: Intersection of two branes wrapping cycles of a two-torus. Since one brane wraps three times one cycle and the other brane wraps once the orthogonal cycle then they intersect three times, providing a natural explanation for family replication.

in the adjoint. But when they lie on different stacks of branes it gives rise to bi-fundamentals. This is what happens at the intersection of two branes. The states corresponding to open strings ending on each of the two branes correspond to bifundamentals that can naturally lead to a chiral spectrum.

A way to obtain the standard model group and spectrum is to intersect several stacks of branes. One stack of three correspond to the strong interactions, it can intersect a stack of two D-branes corresponding to $SU(2)_L$. At the intersection we will have then the quark doublets. At a different point the stack of two D-branes will intersect one brane carrying a $U(1)$ and the leptons will be at the intersection and so on. See the figure as an illustration of a simple realisation. In this simple setting, the gauge group is $U(3) \times U(2) \times U(1)^2$. This provides the standard model group plus three extra $U(1)$'s. Again, the extra $U(1)$'s, anomalous or not, tend to acquire a mass of the order of the string scale after coupling to RR fields, as in the Green-Schwarz mechanism.

To obtain family replication we can use the fact that the starting point are D4, D5 or D6 branes wrapping nontrivial cycles of the compact extra dimensional space. In the simplest case let us consider the intersection of two D4-branes in type IIA theory, wrapping around different cycles of a two-torus. See figure. Depending of the way they wrap the cycles the two branes can intersect several times, at each intersection point we will get identical spectrum. We can then choose these angles such that they intersect three times to give rise to three families of quarks and leptons.

In a more realistic setting we can consider D6 branes of type IIA theory. If we regard the extra six dimensions as products of three two-torii, the D6 branes wrap three-cycles which can be taken to be products of three one-cycles for each of the torii. The ath brane will wrap each of the two one-cycles of the Ith torus (n_a^I, m_b^I) times. The wrapping numbers n_a^I and m_b^I determine the angles between the intersecting branes. In general the intersection number is given by the product of the intersections in each two-torus:

$$I_{ab} = \left(n_a^1 m_b^1 - m_a^1 n_b^1\right) \times \left(n_a^2 m_b^2 - m_a^2 n_b^2\right) \times \left(n_a^3 m_b^3 - m_a^3 n_b^3\right) \qquad (65)$$

In the figure we see an example of two branes intersecting in a torus. One brane is of the $(1, 0)$ type, meaning it wraps only once about one (horizontal) cycle. The other brane is of $(1, 3)$ type wrapping once the horizontal cycle and three times the vertical one. The number of intersections is clearly three coinciding with $n^1 m^2 - n^2 m^1$ $(3 = 1 \times 3 - 0 \times 1)$. Negative signs for the (n_a^I, m_b^I) numbers reflect opposite orientation.

Tadpole cancellation requires that the number of fundamentals of a given group equals the number of anti-fundamentals $\sum N_b I_{ab} = 0$. This is in general identical to non-Abelian anomaly cancellation. The exception is $SU(2)$ for which gauge theory does not distinguish between fundamentals and anti-fundamentals but tadpole cancellation in string theory does. Therefore if we want a model with three copies of left-handed quarks $Q_L = (\mathbf{3}, \mathbf{2})$ we have already 9 doublets of $SU(2)$. To cancel tadpoles we would then need to have as many anti-doublets, therefore implying necessarily at least 6 more doublets than in the standard model. Remember that the existence of extra doublets was generic in previous models.

If we want to construct a realistic model we just have to consider the numbers of branes and intersection numbers that cancel tadpoles. A simple example is given in the table which leads precisely to three families of the standard model group (times several $U(1)$'s, most of them massive), plus 6 extra doublets and singlets. This is remarkably close to the standard model, taking into account the simplicity of the set-up.

This model as well as all toroidal models is non-supersymmetric. The reason for this is that cancellation of RR charges for BPS states would imply also cancellation of the tension, but the branes have nonzero tension that cannot be cancelled. Then there are NS tadpoles and we have to worry about stability. Also the string scale has to be very close to the standard model for the model to be successful since there is no other way to solve the hierarchy problem. The models tend to have scalar fields which are tachyonic. They

Number of D-branes	(n^1, m^1)	(n^2, m^2)	(n^3, m^3)
$N_1 = 3$	(1,2)	(1,-1)	(1,-2)
$N_2 = 2$	$(1, 1)$	$(1, -2)$	$(-1, 5)$
$N_3 = 1$	$(1, 1)$	$(1, 0)$	$(-1, 5)$
$N_4 = 1$	$(1, 2)$	$(-1, 1)$	$(1, 1)$
$N_5 = 1$	$(1, 2)$	$(-1, 1)$	$(2, -7)$
$N_6 = 1$	$(1, 1)$	$(3, -4)$	$(1, -5)$

Table 4: An intersecting D-brane model close to the standard model. The last three columns refer to the wrapping numbers for each of the three two-torii.

may be used to play the role of the standard model Higgs field, which would be a very interesting possibility. Another interesting property of these constructions is that the Yukawa couplings have natural hierarchy. The point is that the magnitude of these couplings depend on the separation of the corresponding field to the Higgs. The family that is closer to the Higgs is the heaviest with a natural hierarchy of families to emerge.

If we want to get just the standard model spectrum or have some realisation of supersymmetry, we have to go beyond toroidal compactifications. The simplest possibility is to consider orientifold twists. For a simple $\mathbf{Z}_2$ twist acting on the three two-torii as changing the sign to only one of the coordinates, we have on each of the torus two fixed lines (so a total of $2^3 = 8$). These are orientifold planes. Their role (besides providing source of RR charge) is to force the inclusion of the reflected wrapping brane. This is usually denoted by a^* brane to distinguish it from the original a brane. See figure (14). Notice the change in orientation for the mirror brane. This has an important effect, at the intersection of one brane with one mirror brane the matter fields transform like $(\mathbf{n}, \mathbf{n})$ rather than $(\mathbf{n}, \bar{\mathbf{n}})$. Therefore to eliminate the problem of the extra doublets we may just look for a configuration with say 2 copies of $(\mathbf{3}, \bar{\mathbf{2}})$ coming from the intersection of two standard stacks of branes, and one copy of $(\mathbf{3}, \mathbf{2})$ coming from the intersection of a stack of branes with a stack of mirror branes. The tadpole conditions then require the need of only 3 $(= 6 - 3)$ extra doublets which can be the leptons of the standard model, without the problem of the extra doublets mentioned before.

A simple realisation of this proposal includes only 4 stacks of branes, as
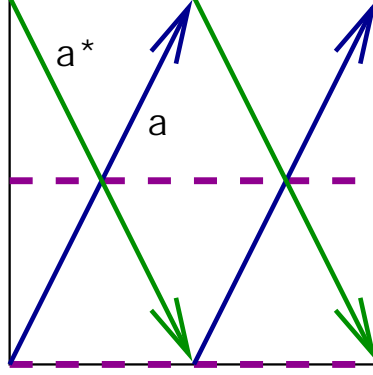
F. Quevedo

Figure 14: The effect of the orientifold. On a two-torus, the dashed lines are orientifold fixed planes O4. One D4 brane wrapping twice the torus is represented by a. The orientifold construction implies we have to include also its image under the $\mathbf{Z}_2$ twist, which we label as a^*, having the orientation reversed. This explains the fact that at the intersection of a brane with the mirror of a second brane, instead of having standard bifundamentals $(\mathbf{n}, \bar{\mathbf{m}})$ we have representations of the type $(\mathbf{n}, \mathbf{m})$.

Label	Multiplicity	Gauge Group	Name
stack a	$N_a = 3$	$SU(3) \times U(1)_a$	Baryonic brane
stack b	$N_b = 2$	$SU(2) \times U(1)_b$	Left brane
stack c	$N_c = 1$	$U(1)_c$	Right brane
stack d	$N_d = 1$	$U(1)_d$	Leptonic brane

Table 5: Brane content yielding the SM spectrum.

in the figure. 15 The spectrum is summarized in Table 1.

We have four stacks of branes : the *baryonic* stack contains three parallel branes giving rise to the QCD interactions, and the *left* stack contains two parallel branes yielding the electroweak $SU(2)_L$ SM interactions. In addition there is the *right* and *lepton* stacks containing each a single brane. These four stacks of branes intersect in the compact six dimensions (plus Minkowski) and at the intersections chiral fermions with the quantum numbers of the SM appear. Thus, for example, the right-handed U-quarks occur at three different intersections of the *baryonic* stack with the *right* stack (see fig.15).

Each stack of branes comes along with a unitary gauge group so that the initial gauge group is $SU(3)_{QCD} \times SU(2)_L \times U(1)_a \times U(1)_b \times U(1)_c \times U(1)_d$. A linear combination of these four $U(1)$'s may be identified with the stan-
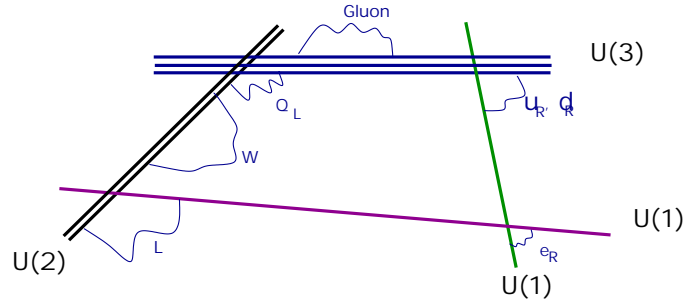
Figure 15: The building block to have just the standard model from intersecting D-branes. This is a set of four stacks of D branes. The QCD stack contains three D-branes giving rise to $U(3)$. The 'left' stack contains two branes giving rise to $U(2)$, the intersection of these two stacks contains the left-handed quarks. Two more stacks, each containing one single brane, is needed to have the spectrum of the standard model.

dard hypercharge and at some level the rest of the $U(1)$'s should become massive. In the class of D6-brane models of ref.[109] and D5-brane models of ref.[118] the charges of quarks and leptons with respect to these $U(1)$'s is shown in Table 6. Here the asterisk denotes the 'orientifold mirror' of each

Intersection	Matter fields		Q_a	Q_b	Q_c	Q_d	Y
(ab)	Q_L	$(3, 2)$	1	-1	0	0	1/6
(ab*)	q_L	$2(3, 2)$	1	1	0	0	1/6
(ac)	U_R	$3(\bar{3}, 1)$	-1	0	1	0	-2/3
(ac*)	D_R	$3(\bar{3}, 1)$	-1	0	-1	0	1/3
(bd)	L	$3(1, 2)$	0	-1	0	1	-1/2
(cd)	N_R	$3(1, 1)$	0	0	1	-1	0
(cd*)	E_R	$3(1, 1)$	0	0	-1	-1	1

Table 6: Standard model spectrum and $U(1)$ charges. The hypercharge generator is defined as $Q_Y = \frac{1}{6}Q_a - \frac{1}{2}Q_c + \frac{1}{2}Q_d$.

given brane, which must always be present in this kind of orientifold constructions (see [109] for details). Note that $U(1)_a$ and $U(1)_d$ can be identified with baryon number and (minus) lepton number respectively. On the other hand $U(1)_c$ can be identified with the third component of right-handed weak isospin. Finally, $U(1)_b$ is an axial symmetry with QCD anomalies, very much like a PQ-symmetry. It is easy to check from the above fermion spectrum

that $U(1)_b$ and $3U(1)_a - U(1)_d$ linear combination have triangle anomalies whereas $U(1)_a + 3U(1)_d$ and Q_c are both anomaly-free. In fact the standard hypercharge may be written as a linear combination of these two symmetries:

$$Q_Y = \frac{1}{6}Q_a - \frac{1}{2}Q_c + \frac{1}{2}Q_d \ .$$ (66)

The above mentioned anomalies of two $U(1)$ linear combinations are cancelled by a generalized 4-dimensional Green-Schwarz mechanism which may be summarized as follows. In Type II string theory there are some closed string 'Ramond-Ramond'(RR) modes coupling to the gauge fields. In particular in the D6- and D5-brane models here discussed there are four RR two-form fields B_i with couplings to the $U(1)_\alpha$ field strengths:

$$\sum_i c_i^\alpha \ B_i \wedge \mathrm{Tr}(F^\alpha) \ i = 1,2,3,4 \ ; \ \alpha = a,b,c,d$$ (67)

and in addition there are couplings of the Poincaré dual scalars (representing the same degrees of freedom) η_i of the B_i fields:

$$\sum_i d_i^\beta \eta_i \mathrm{Tr}(F^\beta \wedge F^\beta),$$ (68)

where F^β are the field strengths of any of the gauge groups. The combination of both couplings, by tree-level exchange of the RR-fields, cancels the mixed $U(1)_\alpha$ anomalies $A_{\alpha\beta}$ with any other group G_β as:

$$A_{\alpha\beta} \ + \ \sum_i c_i^\alpha d_i^\beta \ = \ 0 \ .$$ (69)

The coefficient c_i^α and d_i^α may be computed explicitly for each given D-brane configuration and we will give their values for D6- and D5-brane models below. Note that for given α, β, both c_i^α *and* d_i^β have to be non-vanishing for some i in order to cancel the anomalies.

The couplings in (67) give masses to some linear combinations of $U(1)$'s. Indeed, after a duality transformation the $B \wedge F$ couplings turn into explicit mass terms for the Abelian gauge bosons given by the expression:

$$M_{\alpha\beta}^2 = g_\alpha g_\beta M_s^2 \sum_{i=1}^3 c_i^\alpha c_i^\beta, \quad \alpha, \beta = a,b,c,d.$$ (70)

where the sum runs over the four RR-fields present in the models. Here we have normalized the gauge boson kinetic functions to one.

Furthermore, even though these gauge bosons become massive, the $U(1)$'s are not all anomalous. Therefore there is one combination that gets a mass only because of the coupling to an antisymmetric tensor: $B \wedge F$ without the anomaly cancelling term. The net effect of this is that the corresponding gauge boson gets a mass not by the Higgs mechanism (there are no scalars getting a VEV) but by a stringy realisation of the Stuckelberg mechanism.

Let us see how this is realised: Consider the following Lagrangian:

$$\mathcal{L} = \frac{3}{\gamma^2} \left(\partial_\mu B_{\nu\rho} \right)^2 + \frac{1}{g^2} \left(\partial_\mu A_\nu \right)^2 + c\, \epsilon^{\mu\nu\rho\sigma} B_{\mu\nu}\, F_{\rho\sigma}, \qquad (71)$$

where all the indices are assumed antisymmetrized and γ, g, c are arbitrary constants. This corresponds to the kinetic term for the fields $B_{\mu\nu}$ and $F_{\mu\nu}$ together with the $B \wedge F$ term. We will now proceed to dualize this Lagrangian in two equivalent ways. First we can rewrite it in terms of the field $H_{\mu\nu\rho}$ imposing the constraint $H = dB$ by the standard introduction of a Lagrange multiplier $c\eta$ in the following way:

$$\mathcal{L} = \frac{1}{\gamma^2} \left(H_{\mu\nu\rho} \right)^2 + \frac{1}{g^2} \left(\partial_\mu A_\nu \right)^2 - c\, \epsilon^{\mu\nu\rho\sigma} H_{\mu\nu\rho}\, A_\sigma - c\eta \epsilon^{\mu\nu\rho\sigma} \partial_\mu H_{\nu\rho\sigma}. \quad (72)$$

Notice that integrating out η implies $d^*H = 0$ which in turn implies that (locally) $H = dB$ and then we recover (71). On the other hand, integrating by parts the last term in (72) we are left with a quadratic action for H which we can solve immediately getting

$$H^{\mu\nu\rho} = \frac{c\gamma^2}{2} \epsilon^{\mu\nu\rho\sigma} \left(A_\sigma + \partial_\sigma \eta \right). \qquad (73)$$

Plugging this back into (72) we get:

$$\mathcal{L} = \frac{1}{g^2} \left(\partial_\mu A_\nu \right)^2 - \frac{c^2 \gamma^2}{2} \left(A_\sigma + \partial_\sigma \eta \right)^2 \qquad (74)$$

which is clearly a mass term for the gauge field A_μ after eating the scalar η. Notice that this is like the Stuckelberg mechanism where we do not need a scalar field with a vacuum expectation value to give a mass to the gauge boson, nor we have a massive Higgs-like field at the end.

Furthermore we can understand this mechanism in a dual way in which it is not the gauge field that eats a scalar but is the antisymmetric tensor

that eats the gauge field to gain a mass. This can be seen as follows: Start now with the first order Lagrangian:

$$\mathcal{L} = \frac{3}{\gamma^2} \left(\partial_\mu B_{\nu\rho}\right)^2 + \frac{1}{g^2} \left(F_{\mu\nu}\right)^2 + c \, \epsilon^{\mu\nu\rho\sigma} B_{\mu\nu} \, F_{\rho\sigma} - Z_\mu \, \epsilon^{\mu\nu\rho\sigma} \partial_\nu F_{\rho\sigma}. \quad (75)$$

Again, Z_μ here is a Lagrange multiplier forcing the condition $F = dA$. Similar to the previous case, integrating over Z_μ gives back the original Lagrangian, (71) but integrating by parts the last term and solving the quadratic equation for $F_{\mu\nu}$ (now an arbitrary field) gives the dual Lagrangian:

$$\mathcal{L} = \frac{3}{\gamma^2} \left(\partial_\mu B_{\nu\rho}\right)^2 - \frac{3g^2}{2} \left(c B_{\mu\nu} + \partial_\mu Z_\nu\right)^2. \quad (76)$$

We can then clearly see that this is the Lagrangian for a massive two-index antisymmetric tensor, which gains a mass after eating the vector Z_μ. This is completely equivalent to the massive vector above since in four dimensions a massive vector and a *massive* two-index tensor have the same number of degrees of freedom (3).

We would like to emphasize that this mechanism requires the presence of the Green-Schwarz term $B \wedge F$ but not necessarily the anomaly cancelling term $(\eta F \wedge F)$. Therefore as long as a $U(1)$ field has a Green-Schwarz coupling $B \wedge F$, it does not have to be anomalous in order to get a mass.

In most previous models found in string theory it was generally the case that it was only anomalous $U(1)$'s that had a Green-Schwarz term and therefore those were the ones becoming massive. However in the recent models [109, 115, 116] there are non anomalous $U(1)$'s that can also become massive. Those $U(1)$ bosons get a 'topological mass' induced by the Green-Schwarz term with no associated massive scalar field in the spectrum. Note that this is totally different from what happens when a $U(1)$ becomes massive in the standard Higgs mechanism. Indeed in the latter case in addition to the massive gauge boson there is always an explicit scalar field, the Higgs field. Such a field is not present in the mechanism we discussed.

Let us finally recall that, as emphasized in [109], the gauge group is broken to a global symmetry and therefore symmetries like baryon and lepton number remain as perturbative global symmetries, giving a simple explanation for proton stability in models with a low string scale. Baryon number will then be broken by QCD nonperturbative effects as usual, but at zero temperature these effects are very small. This guarantees naturally the stability of the proton without a large scale to suppress its decay.

A detailed study of the precision constraints on the existence of these extra gauge bosons was carried out in [127]. This puts a constraint of order 10 TeV on the string scale and put lower bounds on the masses of the extra gauge bosons which could be detected in the near future.

The orbifolding procedure helps also to get supersymmetric and quasi-supersymmetric models. The main reason for obtaining supersymmetric models is the fact that orientifold planes carry negative tension, therefore allowing for the possibility of BPS states. In [111] supersymmetric three family models were constructed, which have the big advantage of being stable in the sense that there are no NS tadpoles left uncancelled. The explicit models constructed so far are not realistic since they involve also some exotic matter beyond the standard model. It is an interesting challenge to find more realistic supersymmetric models in this class. These models have also been argued to be duals of G_2 holonomy manifold construction. The study of more models of this class could lead to string model building with a larger string scale that may be consistent with the observation of gauge coupling unification at large scales $M_{GUT} \sim 10^{16}$ GeV.

Finally there are some models which are not supersymmetric but in an interesting way. Each intersection preserves one supersymmetry but this supersymmetry is different from the one preserved in the other intersections. Therefore the full model is not supersymmetric. This is an explicit realisation of an idea proposed in [124] in order to alleviate the cosmological constant problem. The idea there was to separate the scale of the multiplet-splitting to the scale of the cosmological constant, after supersymmetry breaking. If different branes break different supersymmetries then interactions between pairs of branes are enough to generate a multiplet splitting of the order of M_{EW}. But in order to get a nonzero cosmological constant, all the branes have to couple with each other, with the couplings being suppressed by higher powers of the Planck mass. The larger the original number of supersymmetries the bigger the suppression for the cosmological constant.

It is interesting that these intersecting brane models have precisely this property. They have not been used to alleviate the cosmological constant since that is related to the NS tadpoles and stabilisation of the potential, however it has been used to obtain some cancellations. In particular loop corrections to scalar masses appear only at two-loops. This has been argued to be important in order to keep a stable hierarchy between a string scale of some 10 TeV and M_{EW}. Models of this type with the spectrum of just the MSSM model plus an extra non supersymmetric sector have been constructed. These models have name quasi [118] or pseudo supersymmet-

ric [125].

Furthermore, the study of how supersymmetry breaking is transmitted to the observable sector in this setting has been studied from effective field theories in 4d where extended supersymmetries are non-linearly realised [125]. This is an important issue since it is yet to be seen if this way of breaking supersymmetry is actually equivalent to the soft terms induced by F and D terms of moduli and dilaton fields.

4.3   Fluxes, Warping and Moduli Fixing

So far we have seen several realistic models that in order to work need a particular value of the string scale $M_s \ll M_{Planck}$, either $M_s \sim M_{EW}$, $M_s \ sim M_I = 10^{11}$ GeV or $M_s \sim M_{GUT} = 10^{16}$ GeV. The value of the string scale is determined (in a bottom-up approach) from the corresponding value of the volume of the extra dimension. But we have left the fixing of the volume as well as the other moduli, as an open question. Can we find a stringy way to fix the moduli? Could the hierarchy $M_{EW} \ll M_{Planck}$ emerge naturally after we fix them? Remember we needed a very large volume in order to allow for small values of M_s. There are no full answers to these questions but significant progress has been achieved recently which we will briefly review in this section. The main ingredient for this progress is the inclusion of fluxes of antisymmetric tensor fields (RR or NS-NS).

First we will recall the interesting proposal of Randall and Sundrum regarding the hierarchy between M_{EW} and M_{PLanck}. So far we have considered the geometry of compactified spacetimes in terms of a direct product of spaces $M_4 \otimes M_6$. This structure is reflected in the metric taking a fully diagonal form:

$$ds^2 \ = \ \eta_{\mu\nu} \ dx^\mu dx^\nu \ + \ g_{mn}(y) dy^m dy^n \tag{77}$$

where as usual the coordinates x^μ $\mu = 0, \cdots 9$ are the 4D Minkowski spacetime and y^m $m = 1, \cdots 6$ are the internal space coordinates.

It has been known for some time that this metric can be easily generalised by adding a 'warp factor' $A(y)$ of the form:

$$ds^2 \ = \ e^{2A(y)} \eta_{\mu\nu} \ dx^\mu dx^\nu \ + \ e^{-2A(y)} g_{mn}(y) dy^m dy^n \tag{78}$$

This warp factor is allowed by the symmetries of the system and does not have to be restricted to be a constant. The important observation of Randall and Sundrum, working in a simple context of 5D gravity theories with 3-branes, is that the warp factor can play a role regarding the hierarchy of

scales. The point is that when deriving the 4D Planck mass and gauge coupling we need to compute the volume of the extra dimensions, but now in the determinant of the metric there is a contribution from the warp factor. Therefore starting from 10D, the Planck scale in 4D will be given by:

$$M_{Planck}^2 \sim M_s^8 V_6 \tag{79}$$

with V_6 the 'effective' six-dimensional volume given in terms of the warp factor:

$$V_6 = \int d^6 y \sqrt{-g_6}\, e^{-2A(y)} \tag{80}$$

Therefore depending on the functional form of $A(y)$, determined by the solutions of Einstein's equations in the presence of the branes, we may have a natural mechanism to obtain a large hierarchy. In particular in the 5D example of Randall and Sundrum, $A(y)$ was linear in the compactification scale, giving rise to an exponential suppression factor. This is a very elegant way to explain the hierarchy, compared with the case without warp factor in which a large volume has to be postulated without justification. Here a factor of order ten in the compactification scale may give rise to the right hierarchy between M_{Planck} and M_{EW}. Notice that this mechanism to generate the hierarchy can be seen at the same level of the gaugino condensation mechanism mentioned before, in which an exponential factor, given by the scale of condensation, naturally gives rise to a hierarchy. This does not solve the naturalness part of the hierarchy problem though, the one that supersymmetry does in the MSSM. Moreover, the hierarchy is not explained before the scenario provides a mechanism to fix the size of the extra dimension. This is not included in the Randall-Sundrum scenario.

We may wonder if the field equations of the different string theories allow for the existence of a nontrivial warp factor. This was investigated very generally in ref [132] for type IIB string compactifications. Starting with the 10-dimensional effective action for type IIB in the Einstein metric:

$$S_{IIB} = M_s^8 \int d^{10}x \sqrt{-g} \left\{ \mathcal{R} - \frac{\partial_M S \partial^M \bar{S}}{2(\mathrm{Im}S)^2} - \frac{G_{(3)} \cdot \bar{G}_{(3)}}{12\mathrm{Im}S} - \frac{\tilde{F}_{(5)}^2}{4 \cdot 5!} \right\}$$
$$+ \frac{M_s^8}{4i} \int \frac{C_{(4)} \wedge G_{(3)} \wedge \bar{G}_{(3)}}{\mathrm{Im}S} + S_{loc} \tag{81}$$

Where $G_{(3)} = F_{(3)} - SH_{(3)}$, with $F_{(3)}, H_{(3)}$ the field strength of the RR and NS-NS forms $C_{(2)}$ and $B_{(2)}$ respectively. Also $S = C_{(0)} + ie^{-\phi}$ is essentially

the same complex dilaton field defined in the previous sections. And the self dual form is given by:

$$\tilde{F}_{(5)} \;=\; F_{(5)} - \frac{1}{2}C_{(2)} \wedge H_{(3)} + \frac{1}{2}B_{(2)} \wedge F_{(3)}. \tag{82}$$

Finally S_{loc} is the action due to the presence of the localised objects, such as D-branes and O-planes.

If there are no localised sources, the equations of motion imply a constant warp factor and vanishing fluxes for the corresponding antisymmetric tensors. However when negative tension objects are present, such as orientifold planes, the conditions allow the existence of warp factors but also of fluxes of antisymmetric tensor fields. Actually the warp factor is determined by the value of the 5-form $F_{0123m} = \partial_m \alpha$ with :

$$e^{4A(y)} \;=\; \alpha \tag{83}$$

Furthermore the same equations imply a condition for the complex three-form: The condition reads that $G_{(3)}$ has to be imaginary self-dual, or :

$$*_6 G_{(3)} \;=\; iG_{(3)} \tag{84}$$

This condition can be understood as a minimisation condition of a scalar potential and implies that some of the moduli have to be fixed. Remember that S is in the definition of G and therefore it will also get fixed. The scalar potential comes from the term $\frac{G_{(3)} \cdot \bar{G}_{(3)}}{12\mathrm{Im}S}$ in the effective action.

This is quite remarkable since we can see that the inclusion of non-vanishing fluxes for antisymmetric tensor fields imply the appearance of warp factors, needed to generate the hierarchy, but it also provides a mechanism to fix some of the moduli. In particular the dilaton field. It is a model dependent statement how many moduli are fixed and if their value is consistent with the supergravity approximation. In principle we should be able to adjust the value of the fluxes to find desired weak coupling solutions.

There is one modulus which is not fixed by this mechanism. This is the overall Kähler structure modulus, or T field of the previous sections. Its Kähler potential being

$$K(T, T^*) \;=\; -3\log\left[-i(T - T^*)\right] \tag{85}$$

is as in the heterotic models, of the no-scale type. Therefore allowing the possibility of breaking supersymmetry with zero cosmological constant.

Actually, in terms of a 4D effective field theory, these potentials can be derived in a very elegant way from a superpotential proposed by Gukov-Vafa and Witten [134]:

$$W = \int_{\mathcal{M}_6} \Omega \wedge G_{(3)} \tag{86}$$

where Ω is the unique $(3, 0)$ form of the compact manifold. Plugging this superpotential in the general expression for the scalar potential of $\mathcal{N} = 1$ supergravity, and using the Kähler potential for the T field, we are left with a positive definite potential for the rest of the fields, which is the standard situation in no-scale models.

$$V = e^K \, K^{i\bar{j}} D_i W \, \bar{D}_j \bar{W} \tag{87}$$

Where the indices run over all the moduli fields except for T. The minimum is at the solutions of $D_W = 0$. This reproduces the condition for the imaginary self-duality of $G_{(3)}$. Furthermore, if we want to preserve supersymmetry, we should further have $D_T W = W = 0$ which would imply that the components $G_{\bar{k}l\bar{m}}$ vanish. This condition is usually referred to as primitive.

If $D_T W \neq 0$ we will have broken supersymmetry with zero cosmological constant as in no-scale models. However, as usual, this is only a tree level result and the flat direction for T is expected to be lifted by quantum corrections. We may wonder if it is possible to fix T also. Actually, after fixing all the other moduli we may consider effects such as gaugino condensation mentioned above. Now the gauge coupling will depend only on the T field and therefore having a racetrack scenario may be used to fix T (remember that in the heterotic case it was used to fix S).

This was actually pursued in reference [133]. They found a minimum for T with negative cosmological constant, as usual. Furthermore, they considered the addition of anti D3 branes to modify the scalar potential and lift the minimum towards a positive value giving rise to de Sitter space. This is a very preliminary study but it is encouraging that it seems that combining all the techniques available at the moment can actually fix the moduli fields, and generate a hierarchy by a warp factor.

4.4   General Properties of the Models

We may try now to compare the different classes of models obtained in this section with those of heterotic compactifications.

1. *No more heterotic domination.* In the past only heterotic models were viable. Now there are realistic models from each of the five possible string theories and also from the 11D limit of M-theory. This is very encouraging. On the other hand the number of possible realistic models has substantially increased not improving the vacuum degeneracy problem. The modular structure of the models, built from a bottom-up approach make the search for realistic models more systematic and many explicit models share the same building blocks with the local properties such as the gauge symmetry, number of families, Weinberg angle, unification scale, etc.

2. *No more Planck scale domination.* Perturbative heterotic models are attached to the Planck scale in the sense that the string scale is of the order of the Planck scale. The D-brane models may have the string scale taking any value, from 1 TeV to the Planck scale.

3. *No more Calabi-Yau domination.* The structure of heterotic models was completely determined from the properties of the underlying Calabi-Yau manifold. In D-brane models, the more relevant properties such as the gauge group, chirality, and number of families, are local. Singularities and intersection points are the source of chirality. There may be chiral models even in toroidal compactifications. Furthermore, the presence of fluxes may indicate realistic compactifications which are not Calabi-Yau. The hope that the full classification of Calabi-Yau manifolds and possible gauge embeddings would be a way to obtain all phenomenologically interesting models is not clear at the moment. We also need the different ways that D-branes can be accommodated at singular points and furthermore we need to know all the ways they intersect each other. Remember also that intersecting branes provide chiral models even in toroidal compactifications.

4. *Gauge coupling unification.* This is a weak point for most models built so far. The evidence usually quoted for gauge coupling unification is based on having the spectrum of the MSSM. At the moment there is no single model with the spectrum of the MSSM and string scale of the GUT size. Within perturbative heterotic strings there are recent claims for obtaining the spectrum of the MSSM at low energies but the fundamental scale cannot be lowered to the GUT scale from the natural Planck scale of those models. In intersecting brane models, the most realistic need the fundamental scale to be close to electroweak and

so no gauge coupling unification. In the left-right model of branes at singularities, the unification of the gauge couplings is achieved with accuracy as good as the MSSM, but then the apparent unification of the MSSM supported by the precision experiments may look only as an accident in these models. It may be interesting to construct explicit realistic models with strings at the GUT scale an the spectrum of the MSSM at low-energies.

5. *Not only one anomalous $U(1)$.* In heterotic models there is only one antisymmetric tensor field to cancel the $U(1)$ anomalies. In D-brane models there are many of these fields so there may be many anomalous $U(1)$'s becoming massive from the Green-Schwarz mechanism. Furthermore some of these fields can combine into a QCD axion field. In particular for intermediate scale strings these couple precisely with the right strength.

6. *Beyond the Higgs mechanism.* In heterotic, as in standard spontaneously broken gauge theories only the Higgs mechanism is responsible to generate masses for gauge bosons. In D-brane models there may be gauge bosons which are not anomalous having the coupling $B \wedge F$ that gives them a mass (after dualisation), without any scalar getting a VEV.

7. *Extra Z's.* Many string models predict extra Z' gauge bosons. The most studied ones are the ones derived from E_6 models based on the Calabi-Yau compactifications of heterotic strings. In D-brane models there are other $U(1)$'s that could survive at low-energies, such as baryon and lepton number. If the string scale is close to the standard model these particles may be suitable to experimental search in the next decade.

8. *Global Symmetries* In heterotic models there are no global symmetries, besides some accidental and possible PQ type symmetries associated to antisymmetric tensors. In D-brane models there may survive global symmetries after a gauge boson becomes massive without a scalar field getting a VEV. Baryon number conservation may come from this. The symmetry will be broken by quantum effects, like instantons, which are usually negligible at zero temperature.

9. *Not only $\mathcal{N} = 1$ supersymmetry.* Realistic heterotic models were $\mathcal{N} = 1$ supersymmetric to solve the hierarchy problem. D-brane models may
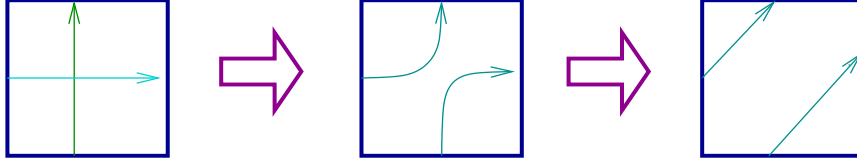
F. Quevedo

Figure 16: The decay of two intersecting branes into a single one wrapping around the torus. Notice that the original system has a $U(1) \times U(1)$ symmetry and the final one has only $U(1)$. The process not only breaks the gauge symmetry but it also reduces the rank of the gauge group, similar to the Higgs mechanism. The field responsible for the breaking of the symmetry is an open string tachyon.

be non-supersymmetric, with anti-branes or intersecting branes providing the stringy mechanism for supersymmetry breaking. The issue of stability of the models is left unsolved at the moment. There are also supersymmetric models in both constructions, which share similar properties as the heterotic models, leaving the problem of supersymmetry breaking unsolved.

10. *Soft breaking terms.* The source of supersymmetry breaking in most of these models is stringy. It is yet to be seen if it can be represented in the effective 4d theory in terms of non-vanishing VEV's for auxiliary fields of the dilaton and moduli fields. In general classes of type I models the assumption that this is the case gives rise to soft terms different from the heterotic case. In particular the dilaton dominated scenario which was ruled out in the heterotic case it is not in models with intermediate scale string theories.

11. *Moduli fixing.* Fixing the dilaton VEV was an outstanding problem in heterotic models. In the models with RR and NS fluxes mentioned before, it is relatively easy to fix the dilaton. Other moduli fields, like the complex structure moduli are also fixed by fluxes. Fixing the Kähler structure moduli, in particular the overall T field has proven more difficult. But fluxes combined with the previous mechanisms such as gaugino condensation have been considered recently to fix also these moduli. Playing with the number of branes and anti-branes the vacuum may be adjusted to give a positive cosmological constant. However the main problem of explaining the smallness of the cosmological constant still remains.

12. *Not afraid of tachyons.* Tachyons may be present in some models with the possible application of being the Higgs field. An issue of hierarchy of scales between the string and electroweak scale has to be raised before this works efficiently. The open string tachyon also plays an important role in cosmology as we will see next.

5 D-Brane Cosmology

Cosmology is becoming a precise science and could be the main way to eventually test the ideas of string theory. From the phenomenological point of view we should add to the experimental constraints that a particular, realistic, string model should satisfy, to incorporate a realistic cosmology. In principle string models should explain dark matter, give rise to a realistic baryogenesis scenario, provide a mechanism to get inflation or any alternative to it and dark energy,including perhaps the biggest challenge for string phenomenology which is the explanation of the smallness of the cosmological constant.

5.1 Inflation from D-brane interactions

Here we will briefly review the recent attempts to derive inflation from D-branes. The idea is very simple and appealing. When we have a non-supersymmetric brane configuration, the attractive force between two branes can be seen as a potential for the brane separation. The potential has naturally two parts. The first one is just the sum of the brane tensions, which, assuming all closed string moduli have been fixed, is a positive constant piece. The second part is the interaction potential which takes the Newtonian form and varies depending on the dimensionality of the branes and their relative angles. The potential takes the following form:

$$V \; = \; A \; - \; BY^{-n} \tag{88}$$

where n is model dependent and A and B are computable constants.

We can wonder if it is possible to obtain inflation from this potential.

Let us briefly review the conditions for a potential to give rise to inflation and solve the flatness and horizon problems of big-bang cosmology.

The conditions for inflation to be realised can be summarised in two useful equations, known as the slow roll conditions:

$$\epsilon \; \equiv \; \frac{M_{Planck}^2}{2} \; \left(\frac{V'}{V}\right)^2 \; \ll \; 1 \;, \tag{89}$$

$$\eta \equiv M_{Planck}^2 \frac{V''}{V} \ll 1 \ . \tag{90}$$

The parameters ϵ and η have become the standard way to parametrise the physics of inflation. If the first condition is satisfied the potential is flat enough as to guarantee an exponential expansion. If the second condition is satisfied the friction term in the corresponding scalar field equation dominates and therefore implies the slow rolling of the field on the potential, guaranteeing the inflationary period lasts for some time.

If the potential were a constant we would be in a de Sitter universe expansion and the amount of inflation would be given by the size of H (since $a(t) \sim e^{Ht}$). More generally the number of e-foldings is given by:

$$N(t) \equiv \int_{t_{init}}^{t_{end}} H(t')dt' = \int_{\psi_{init}}^{\psi_{end}} \frac{H}{\dot{\psi}}d\psi = \frac{1}{M_{Planck}^2} \int_{\psi_{end}}^{\psi_{init}} \frac{V}{V'}d\psi \ . \tag{91}$$

A successful period of inflation required to solve the horizon problem needs at least $N \geq 60$. The recipe to a successful model of inflation is then to find a scalar field potential V satisfying the slow roll conditions in such a way that the number of e-foldings exceeds 60 (slightly smaller values are sometimes allowed depending on the scale that inflation occurs). It is of no surprise that many potentials have been proposed that achieve this. For a collection of models see for instance the book of Liddle and Lyth in [141]. Usually getting a potential flat enough requires certain amount of fine tuning unless there is a theoretical motivation for the potential.

Probably the most relevant property of inflation is that it can provide an explanation for the density perturbations of the CMB and therefore indirectly account for the large scale structure formation. Quantum fluctuations of the scalar field give rise to fluctuations in the energy density that at the end provide the fluctuations in the temperature observed at COBE [141, 142, 143]. Furthermore most of the models of inflation imply a scale invariant, Gaussian and adiabatic spectrum which is consistent with observations. This has made inflation becoming the standard cosmological paradigm to test present and future observations.

The typical situation is that any scale, including the perturbations, will increase substantially during inflation whereas the Hubble scale remains essentially constant. Therefore the scale will leave the horizon (determined essentially by H^{-1}) and the fluctuations get frozen. After inflation, the Hubble scale will increase faster and then the scales will re-enter the horizon. The amplitude of the density perturbation ($\delta\rho/\rho$) when it re-enters the

horizon, as observed by Cosmic Microwave Background (CMB) experiments is given by:

$$\delta_H = \frac{2}{5}\mathcal{P}_{\mathcal{R}}^{1/2} = \frac{1}{5\pi\sqrt{3}} \frac{V^{3/2}}{M_p^3 \, V'} = 1.91 \times 10^{-5} \,, \qquad (92)$$

where $\mathcal{P}_{\mathcal{R}}$ is the power spectrum computed in terms of the two-point correlators of the perturbations. Here the value of δ_H is implied by the COBE results [144].

In order to study the scale dependence of the spectrum, whatever its form is, one can define an effective *spectral index* $n(k)$ as $n(k) - 1 \equiv \frac{d \ln \mathcal{P}_{\mathcal{R}}}{d \ln k}$. This is equivalent to the power-law behaviour that one assumes when defining the spectral index as $\mathcal{P}_{\mathcal{R}}(k) \propto k^{n-1}$ over an interval of k where $n(k)$ is constant. One can then work $n(k)$ and its derivative by using the slow roll conditions defined above [142, 141], and they are given by

$$n - 1 = \frac{\partial \ln \mathcal{P}_{\mathcal{R}}}{\partial \ln k} \simeq 2\eta - 6\epsilon \,, \qquad \frac{dn}{d \ln k} \simeq 24\epsilon^2 - 16\epsilon\eta + 2\xi^2 \,. \qquad (93)$$

where $\xi^2 \equiv M_P^2 \frac{V' V'''}{V^2}$. Showing that for slow rolling ($\eta, \epsilon \ll 1$) the spectrum is almost scale invariant ($n \sim 1$).

The gravitational wave spectrum can be calculated in a similar way. The gravitational spectral index n_{grav} is given by

$$n_{grav} = \frac{d \ln \mathcal{P}_{grav}(k)}{d \ln k} = -2\epsilon \,. \qquad (94)$$

Therefore we have a simple recipe to check if any potential can give rise to inflation: compute the parameters ϵ, η, check the slow roll conditions, if they are satisfied we can right away find the spectral indices and the COBE normalisation (92) puts a constraint on the parameters and scales of the potential.

We can now check if the D-brane potentials satisfy these requirements. Generically this is not the case, the potentials are not flat enough as to give rise to inflation. To get inflation we have to be in a configuration that a brane and an anti-brane are close to the antipodal points of toroidal compactifications or that branes intersect at very small angles. If this is the case in both cases we can obtain inflation. This is already remarkable given the fact that it had proven very difficult to derive inflation from string theory. Furthermore the standard fine tuning of potentials to get inflation has been transformed into a property of brane configurations and it is left as
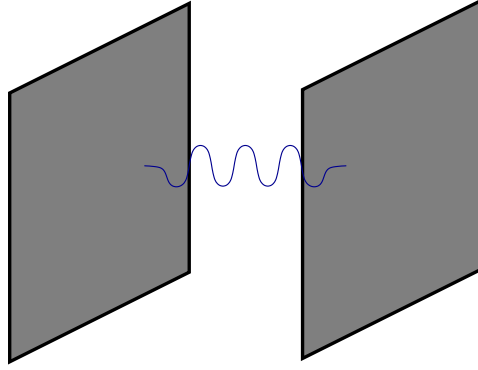
Figure 17: An open string state becomes tachyonic at a critical inter brane separation.

a task to understand why those configurations can be achieved. One simple explanation for this is that we may imagine a brane gas configuration for which most interacting D-branes will not give rise to inflation but if one arrangement of two D-branes is distributed in such a way that it gives rise to inflation, it will dominate over the rest and will become our observable universe. A natural selection explanation.

For the brane/anti-brane to obtain the minimum number of efoldings $N \geq 60$ and the COBE normalised value of the density fluctuations we can easily see that $\delta_H \approx 10^{-5}$ is obtained for a compactification radius $r_\perp^{-1} \approx 10^{12}$ GeV corresponding to an intermediate string scale $M_s \approx 10^{13}$ GeV. The spectral index $n \approx 1 - 3/N$ is in the favoured range. Similar results can be obtained for the intersecting branes for which $M_s \approx 10^{15}$ GeV.

5.2 Tachyon condensation

A more difficult problem is the so-called graceful exit problem or how to end inflation once it has started. Here is where the stringy property of the configuration comes out with a natural explanation. It so happens that before the branes collide, at a critical distance, a open string mode with endpoints at each of the two branes becomes massless. At distances smaller than this the field is tachyonic, representing an instability as the maximum of the Mexican hat potential forcing then the configuration to relax to the minimum of the potential in the tachyon direction. In the effective field theory, the potential can be seen as a two-field potential with one field being the brane separation and the second field being the tachyon. This mechanism
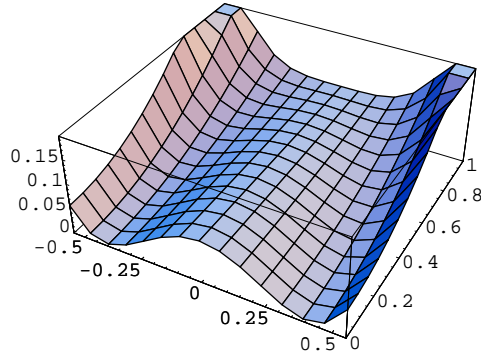
Figure 18: A standard potential realising hybrid inflation. One field behaves as the inflaton. At a critical value the second field becomes tachyonic and drives the end of inflation.

was proposed in the past to end inflation and comes under the name of hybrid inflation. It is remarkable that string theory has all the ingredients to realise hybrid inflation.

The tachyon potential has been subject to intense investigation in recent times. It has many interesting properties that have been verified by use of string field theory and related techniques. At the overlapping point, the height of the potential should be twice the value of the branes tensions. the minimum being supersymmetric with zero energy, reflecting the annihilation of the brane and anti-brane.

An important issue is the existence of topological defects of the tachyon potential, corresponding to D branes (of two dimensions less than the original system). Actually it has been claimed that all D branes can be obtained in this way. This has allowed a proposal that a gas of branes is a natural outcome of the process of brane annihilation. The conditions for inflation may be satisfied only in very rare occasions but once realised, as usual, inflation dominates. Furthermore, we may even discuss a theory of initial conditions in terms of spontaneous creation of brane/anti-brane pairs.

There are experimentally testable properties of these models. Given the precision results from WMAP, the predictions for the spectral index can be subject to direct test. At the moment they fall in the right regime. Furthermore, the creation of topological defects allows cosmic strings but no domain walls nor monopoles, which is just as well since those objects are dangerous for over-closing the universe. Cosmic strings are very much

constrained from the CMB spectrum since they cannot contribute more than a few percent to the density perturbations. So far, both brane/anti-brane models and intersecting brane models are consistent with these constraints. The main number to compare is the value of the string tension as in terms of the Newtons constant $G\mu \leq 10^{-7}$.

There are open questions on this proposal. The main one is related to initial conditions. In practice is the assumption that all the moduli have been fixed. This may not be that difficult after the recent progress, reported in the previous section, of using fluxes, but it remains as the main question before success can be claimed about getting inflation from string theory. Finally there is the issue about reheating. It is not clear how the process of tachyon condensation gives rise to reheating. The main problem here is that the tachyon potential does not seem to have, at first sight, the standard form of a double well potential.

6   Conclusions

Phenomenological aspects of string theory have evolved considerably with steady progress made during the past two decades.

Perturbative heterotic models still provide very interesting realisations of realistic string models, with a large unification scale, hidden sector group to break supersymmetry, a topological explanation for the number of families, in terms of the Euler number of the corresponding Calabi-Yau manifold, etc.

D-branes have opened a new way at looking string phenomenology that it is only being uncovered recently with the explicit construction of models based on branes at singularities and intersecting brane models. These provide explicit string realisations of the brane-world scenario with the fundamental scale either intermediate or close to TeV for non-supersymmetric models and higher for the supersymmetric ones. Actually the models incorporate novel string theoretical mechanisms to break supersymmetry by means of anti-branes, arbitrary intersecting angles and fluxes of antisymmetric RR or NS-NS fields. Moreover, the study of these fluxes has opened the possibility to obtain a hierarchy of scales as well as fixing many of the moduli fields. Furthermore, a particular robust scenario of brane inflation has emerged that complements the phenomenological issues of the models with cosmological ones. This may provide new possibilities for these models to be confronted with experiments.

There are many open questions left. Still the main problem is supersymmetry breaking. Finding mechanisms to break it is not as difficult as to what

to do after it is broken. In particular, instabilities due to NS tadpoles have to be addressed in non-supersymmetric models. For supersymmetric models we must rely on field theoretical mechanisms, like gaugino condensation, to be the source for its breaking, just delaying the issue. In both cases we have to face the most difficult question regarding the value of the cosmological constant after supersymmetry has been broken.

One further open question is the vacuum degeneracy problem. We all would have liked that the unique fundamental theory would have a unique vacuum state that corresponds precisely to the world we live in. All the indications we have so far is that that expectation is too naive and we will have to accept the multitude of (at least discrete) degenerate vacua, of which only a few could be consistent with our world. A full classification of vacua in a 'string vacuum project' may not be impossible to look after in the long run. We have already a large amount of information about many classes of vacua that a systematic approach mixing basic formalism with computer power to fully classify, say all $\mathcal{N} = 1$ vacua, may be eventually reachable. This may be a way to extract predictions of classes of models that could be confronted with experiment.

We are definitely living exciting times in string theory, where not only strings but also higher-dimensional objects are emerging as different pieces of an underlying fundamental theory. It is interesting to remark that the extended objects were originally introduced to represent elementary particles and now, in the brane-world scenario, they can even represent different different universes. On the other hand the current optimism for this theory should be taken with the right perspective, none of these achievements can substitute a solid experimental test of the theory, something which may still be very far in the future. However we might be lucky and some discoveries at current and future accelerators, as well as some possible astrophysical results, could provide important clues on the validity of string theory. In particular the idea that the string scale may be as low as the TeV scale may have dramatic experimental consequences. Even if that scale is larger, it does not have to be as large as the Planck scale and we can say that, at present, the hopes of eventually testing string theory are not unreasonable.

In any case the idea that our Universe is a brane inside a higher dimensional space has changed our view of the physical world. We have seen how the standard scenario where matter lives on the brane and gravity in the bulk is naturally realised in terms of D-branes or also in the Horava-Witten realization of the heterotic string in M-theory. The brane-world scenario is allowing several communities of physicists (formal string theo-

rists, string phenomenologists, model builders, hard core phenomenologists, collider experimentalists, table-top experimentalists, cosmologists and astrophysiscists) to communicate with each other more than ever in the past. This is a positive signature of a healthy science. We hope all this combined effort will be fruitful in the not too far a future.

Acknowledgements

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