Hypothesis Testing of Standard Assumptions in Theoretical Financial Mathematics

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This project investigates a foundational assumption in quantitative finance: whether the log returns of equities and portfolios are normally distributed. Normality is a core assumption in many financial models, yet empirical evidence often reveals deviations due to fat tails, skewness, and market shocks. Our study analyzes historical stock data through rigorous statistical and graphical techniques to assess the validity of this assumption.

Objectives:

The main objectives of the project were:

- Test full-period normality of log returns for 20 widely held stocks.
- Analyze temporal normality using rolling or monthly windows.
- Examine the effect of outlier removal $\pm 3\sigma$ on return distributions.
- Construct and evaluate a portfolio with historically normal return behavior.
- Assess prior project portfolios (high-risk and low-risk) under rolling normality windows.

Methodology

Data used:We collected historical daily closing prices for a diverse set of 20 large-cap equities and ETFs using the yfinance library, covering the period from January 2020 to June 2025. From this data, we computed daily and monthly log returns. The statistical analysis involved applying three complementary normality tests:

- Shapiro-Wilk
- Jaque-Bera
- Anderson-Darling

Both full-period and rolling-window (monthly) tests were performed to assess temporal variations in distributional behavior. Additionally, we implemented a 3-sigma filtering rule to mitigate the influence of extreme outliers.

Key Results

Full-period log returns for individual stocks:

Most assets failed at least one normality test, indicating significant deviations from Gaussian assumptions. Stocks like KO, PG, and UNH exhibited heavy tails and skewness. Only a few stocks (e.g., META, GOOG) showed mild evidence of normality.

Rolling monthly normality tests revealed:

Several stocks exhibited periods of local normality, especially during low-volatility periods post-COVID.

A heatmap and time plots visualized how normality varied over time and across tickers.

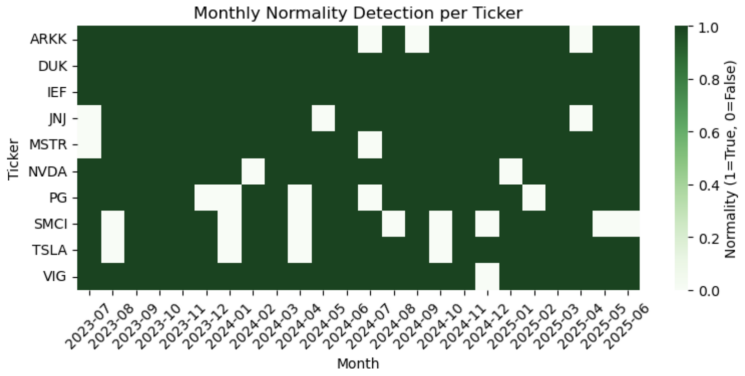
Impact of Outlier Removal:

After removing extreme values (outside ±3 standard deviations), the number of periods classified as "approximately normal" increased significantly. For example:

- DUK, IEF, and VIG passed 100% of filtered monthly periods.
- Volatile assets like TSLA and MSTR showed marked improvement in normality scores post-filtering.

Constructed a "Normality-Optimized" Portfolio:

A time-series heatmap revealed that local normality tends to appear during low-volatility market regimes (e.g., post-COVID), while turbulent periods show substantial departures from Gaussianity.



Mini Project 1 Portfolio Evaluation:

Both high-risk and low-risk portfolios from the previous project were tested using rolling Anderson–Darling windows. Results showed:

- High-risk portfolio: 85.29% of windows passed.
- Low-risk portfolio: 83.69% of windows passed.

This implies even volatile portfolios can exhibit frequent local normality.

Conclusions

- Log returns are rarely normally distributed over long horizons, but local normality can emerge, especially after removing outliers or during stable market periods.
- Filtering techniques and thoughtful portfolio construction can significantly improve the statistical behavior of returns.
- These findings reinforce the importance of diagnostic testing in model assumptions, especially in risk-sensitive applications like VaR, option pricing, and portfolio optimization.