Executive Summary: Advanced Topics in Quantitative Finance

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Overview

This series of research mini-projects systematically explores fundamental and advanced themes in quantitative finance: portfolio optimization under risk constraints, empirical testing of distributional assumptions, analytical investigation of derivative sensitivities, and robust option hedging under stochastic volatility. Through a combination of data-driven analysis, mathematical modeling, and rigorous statistical techniques, these projects collectively build a holistic understanding of risk and return in modern financial markets. Each project is designed to build upon the previous, providing both theoretical depth and practical insights into real-world complexity.

Construction of High and Low Risk Portfolios

Objective:

To design and optimize two contrasting U.S. equity portfolios—one high-risk, one low-risk—maximizing Sharpe ratio while adhering to explicit constraints on beta, volatility, and drawdown.

Methodology:

We collected two years of high-quality daily closing price data for select tickers, aggregated into 5-day log returns to suppress microstructure noise. Annualized means, covariances, betas (vs. SPY), and drawdowns were estimated. The optimization, performed with scipy.optimize, targeted the Sharpe ratio subject to portfolio-specific constraints on weights, beta, volatility, and drawdown.

Results:

- High-Risk Portfolio: Focused on AI compute, electric vehicles, crypto, and innovation themes. Achieved a Sharpe ratio improvement from 0.87 (equal-weight) to 1.05 (optimized), with a portfolio beta of 2.10 and annualized return of 44.3%.
- Low-Risk Portfolio: Built with staples, healthcare, utilities, and duration assets, raising Sharpe from 0.31 to 0.47, with a much lower volatility (10.6%) and annualized return of 5.02%.

The optimization produced two distinct portfolios that meet the respective constraints.

Portfolio	Equal-Weight Sharpe	Optimized Sharpe	β	Volatility σ	Max DD	Annualized return
High-Risk	0.87	1.05	2.10	59%	-59%	44.3%
Low-Risk	0.31	0.47	0.33	10.6%	-9.8%	5.02%

Table 1: Portfolios ouctome

Conclusion:

Thoughtful constraint-driven optimization and thematic diversification extract risk-adjusted alpha, demonstrating that disciplined, data-driven construction outperforms naïve allocations.

Empirical Assessment of Log-Return Normality

Objective:

To critically evaluate the widespread assumption that equity log returns are normally distributed—an axiom at the foundation of most financial theory and risk models.

Methodology:

We analyzed daily and monthly log returns for 20 large-cap stocks and ETFs (2020–2025) using Shapiro-Wilk, Jarque-Bera, and Anderson-Darling tests. Rolling-window tests and 3-sigma outlier filtering were employed to assess the temporal and regime-dependent validity of normality.

Key Findings:

- Full-period log returns of most assets exhibited significant departures from normality (heavy tails, skewness).

Table 2: Normality Test Results

Ticker	Shapiro-Wilk (p)	Jarque-Bera (p)	Anderson-Darling (stat)	Normality
NVDA	0.0	0.0	3.1279	Not Normal
TSLA	0.0	0.0	3.8157	Not Normal
ARKK	0.0	0.0	1.2209	Not Normal
MSTR	0.0	0.0	4.3422	Not Normal
SMCI	0.0	0.0	8.7245	Not Normal
JNJ	0.0	0.0	4.6774	Not Normal
PG	0.0	0.0	4.6009	Not Normal
DUK	0.0655	0.0072	0.4123	Normal
VIG	0.0	0.0	5.6943	Not Normal
IEF	0.3937	0.5287	0.3114	Normal

- Local (windowed) normality was observed during calm, low-volatility periods, especially after outlier removal.
- Constructed portfolios ("normality-optimized") and the previously built high- and low-risk portfolios both exhibited high rates of local normality (>80% of monthly windows post-filtering).

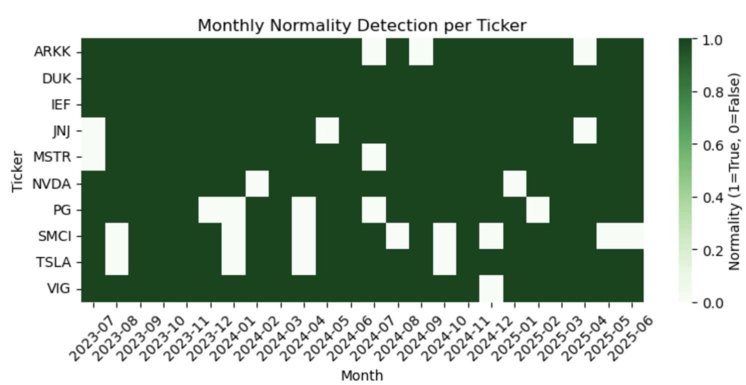

Figure 1: Normality test pass/fail heatmap across tickers and time

Heatmap showing monthly rolling normality test outcomes. Green indicates periods of local normality, emphasizing the regime-dependent nature of statistical assumptions in finance.

Conclusion:

Log returns rarely adhere to normality over long periods, but local normality can emerge under stable regimes or after filtering. These results reinforce the need for robust, diagnostic testing of statistical assumptions, especially in applications such as VaR, pricing, and optimization.

Analytical and Numerical Exploration of Option Sensitivities (Greeks)

Objective:

To analyze how Black–Scholes prices of European call and put options respond to changes in time-to-maturity and underlying spot price, with focus on the Greeks theta (time decay) and delta (spot sensitivity).

Methodology:

Python implementations of closed-form and numerical derivatives enabled us to plot option values and sensitivities across a range of maturities and spot prices.

Findings:

- Theta: Both call and put options experience a sharp spike in time-decay (theta) as expiry approaches, especially near-the-money. For long maturities, theta stabilizes near zero.
- Delta: Call and put deltas display the characteristic S-curve, with maximum sensitivity and gamma risk at-the-money.

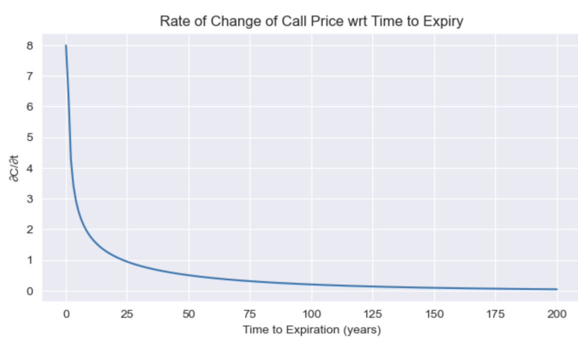

Figure 2: Call and put theta as a function of time to expiry, highlighting the sharp increase in time decay near expiration.

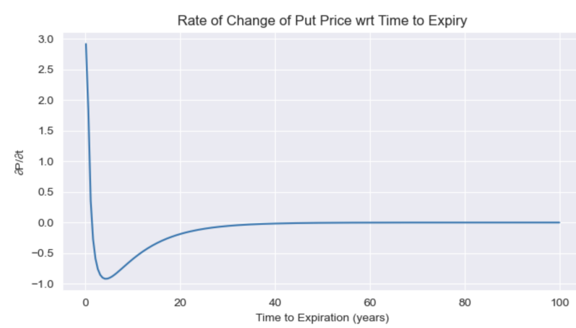

Figure 3: Delta for calls and puts as a function of spot price, illustrating S-shaped sensitivity and peak gamma at-the-money.

Conclusion:

Understanding the behavior of option Greeks is critical for effective risk management and hedging. The results illustrate how time decay and directional risk evolve as market conditions change, with practical implications for hedging and portfolio management.

Hedging Performance under Non-Constant Volatility

Objective:

To assess how stochastic volatility affects the performance of option hedging, comparing delta and vega-hedging strategies under Black–Scholes, Heston, and GARCH models.

Strategy	Mean	Std Dev	Sharpe	VaR 5%	CVaR 5%	KS p-value
Delta + Constant σ	0.003	0.426	0.007	-0.665	-1.007	0.0049
Delta + Heston σ	-0.095	1.093	-0.087	-2.087	-2.654	0.4036
Delta + GARCH σ	-55.858	4.807	-11.621	-63.773	-65.939	0.2602
Vega + Constant σ	-1.497	0.551	-2.718	-2.508	-2.752	0.1833
Vega + Heston σ	-2.706	1.485	-1.822	-4.766	-5.157	0.0000
Vega + GARCH σ	15.971	19.672	0.812	-20.169	-45.205	0.0000

Table 3: Advanced Risk and Performance Statistics for Each Strategy

Methodology:

Option pricing and hedging simulations were performed across 1,000 Monte Carlo paths under constant and stochastic volatility (Heston and GARCH). Both delta and vega hedging strategies were analyzed using advanced statistical metrics: Sharpe ratio, Value-at-Risk (VaR), Conditional VaR (CVaR), Q-Q plots, and Kolmogorov–Smirnov tests.

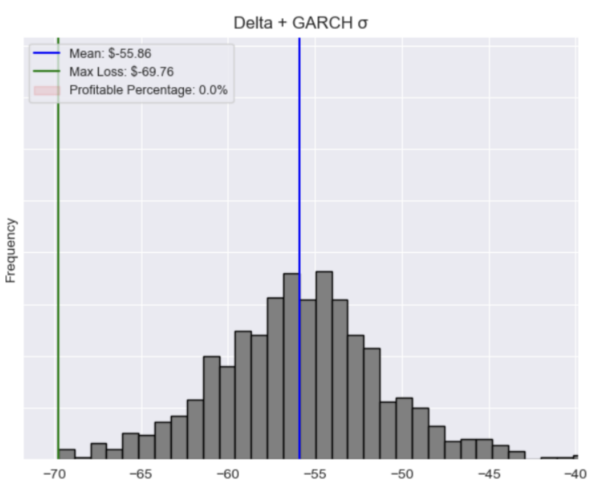

Figure 4: Delta + GARCH σ

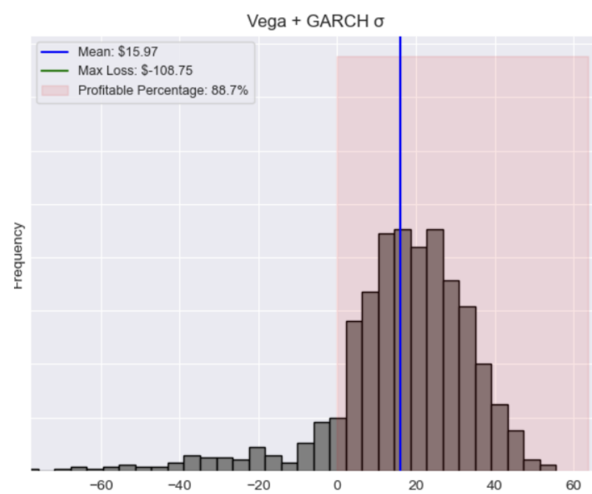

Figure 5: Vega + GARCH σ

Figure 6: Profit Distributions: Delta vs Vega under GARCH Volatility

Key Findings:

- Delta Hedging: Performs reliably under constant volatility but suffers significant losses and dispersion under GARCH volatility (Sharpe $\sim$ -11.6).
- Vega Hedging: Under GARCH, achieves the highest mean and Sharpe ratio ($\sim$ 0.81) but exposes the portfolio to severe tail risk (CVaR -$45.2).
- Significant deviations from normality were observed, especially in the profit/loss tails of vega-hedged portfolios under stochastic volatility.
- Pairwise t-tests confirm the statistical significance of differences between hedging strategies.

Conclusion:

The effectiveness of hedging strategies is acutely sensitive to the assumed volatility model. Robustness to volatility dynamics and careful strategy selection are essential, as naive delta hedging may prove hazardous in the presence of regime shifts or persistent volatility clustering.

Conclusion and synthesis

Together, these mini-projects offer a comprehensive, layered view of modern quantitative finance:

- They highlight the necessity of empirical validation for statistical assumptions.
- Demonstrate how model selection and constraint engineering can drive portfolio outperformance.
- Emphasize the critical role of advanced Greeks and dynamic hedging strategies in managing risk under realistic, non-idealized market conditions.

The consistent theme across all work is the indispensable availability of diagnostic rigor and adaptability, whether optimizing portfolios, stress testing models, or designing robust hedging strategies in the presence of real-world market complexities. This integrated approach is vital for practitioners seeking both theoretical depth and practical edge.