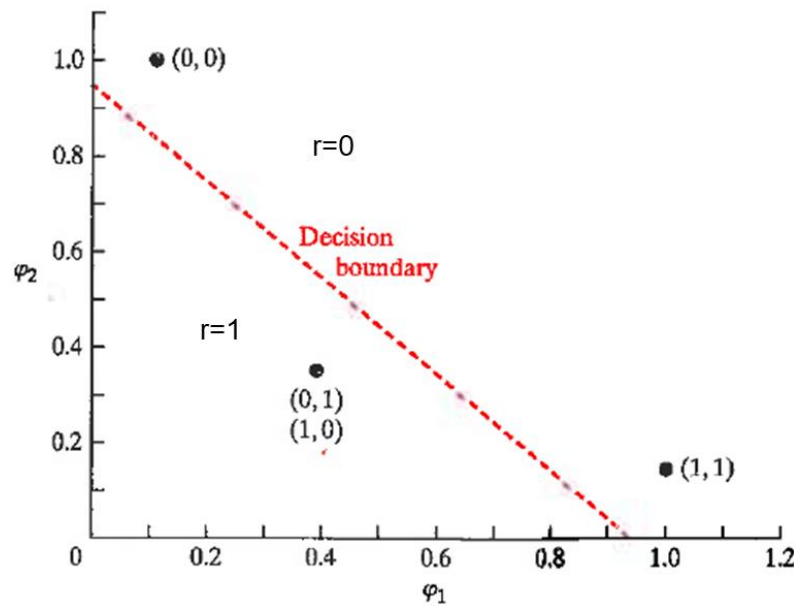


## Part 1



$x_1$	$x_2$	$r$
0	0	0
0	1	1
1	0	1
1	1	0

$$\phi_1 = e^{-|x - [1 \ 1]|^2}$$

$$\phi_2 = e^{-|x - [0 \ 0]|^2}$$

$x$	$\phi_1$	$\phi_2$
(1,1)	1	0.1353
(0,1)	0.3678	0.3678
(0,0)	0.1353	1
(1,0)	0.3678	0.3678

Decision boundary is  $\phi_1 + \phi_2 = 0.94$

Use  $w^T = [1 \ 1]$  to calculate bias and margins

Bias:

$$w^T \phi + b = 0$$

$$w^T \phi = -b$$

$$\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} = -b$$

$$(1 \times \phi_1) + (1 \times \phi_2) = -b$$

$$\phi_1 + \phi_2 = -b$$

$$0.94 = -b$$

$$b = -0.94$$

$r = 0$  margin:

$$d = \frac{|g(1, 0.1353)|}{\|w\|}$$

$$d = \frac{\left| \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0.1353 \end{bmatrix} - 0.94 \right|}{\sqrt{1^2 + 1^2}}$$

$$d = \frac{|1.1353 - 0.94|}{\sqrt{1 + 1}}$$

$$d = \frac{0.1953}{\sqrt{2}}$$

$$d \cong 0.1381$$

$r = 1$  margin:

$$d = \frac{|g(0.3678, 0.3678)|}{\|w\|}$$

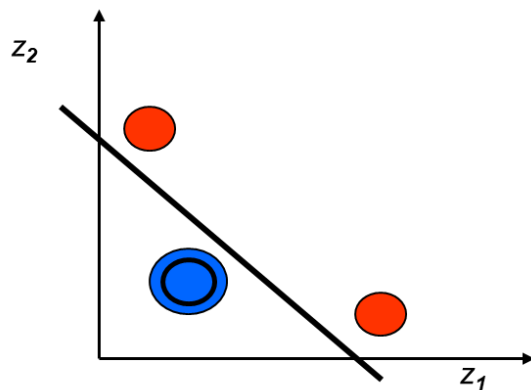
$$d = \frac{\left| \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 0.3678 \\ 0.3678 \end{bmatrix} - 0.94 \right|}{\sqrt{1^2 + 1^2}}$$

$$d = \frac{|0.7356 - 0.94|}{\sqrt{1 + 1}}$$

$$d = \frac{0.2044}{\sqrt{2}}$$

$$d \cong 0.1445$$

## Part 2



Coordinates of features  
in the hidden layer

$$z_1 = \text{sigmoid}(x_1 - x_2 - 0.5)$$

$$z_2 = \text{sigmoid}(-x_1 + x_2 - 0.5)$$

$x_1$	$x_2$	arg <sub>1</sub>	$z_1$	arg <sub>2</sub>	$z_2$	$r$
0	0	-0.5	0.38	-0.5	0.38	0
0	1	-1.5	0.18	0.5	0.62	1
1	0	0.5	0.62	-1.5	0.18	1
1	1	-0.5	0.38	-0.5	0.38	0

Find the bias and equation of the decision boundary with weight vector  $[1 \ 1]$  and equal margins for both groups.

Bias:

$$\frac{|g(0.38, 0.38)|}{\|w\|} = \frac{|g(0.18, 0.62)|}{\|w\|}$$

$$|g(0.38, 0.38)| = |g(0.18, 0.62)|$$

$$\left| [1 \ 1] \begin{bmatrix} 0.38 \\ 0.38 \end{bmatrix} + b \right| = \left| [1 \ 1] \begin{bmatrix} 0.18 \\ 0.62 \end{bmatrix} + b \right|$$

$$|0.76 + b| = |0.8 + b|$$

$$b = -0.78$$

Decision Boundary:

$$w^T z + b = 0$$

$$[1 \ 1] \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} - 0.78 = 0$$

$$(1 \times z_1) + (1 \times z_2) - 0.78 = 0$$

$$z_1 + z_2 - 0.78 = 0$$

$$z_1 + z_2 = 0.78$$

Calculate margins to verify that they are equal.

$r = 0$  margin:

$$d = \frac{|g(0.38, 0.38)|}{\|w\|}$$

$$d = \frac{|0.76 - 0.78|}{\sqrt{1^2 + 1^2}}$$

$$d = \frac{0.2}{\sqrt{2}}$$

$$d \cong 0.014$$

$r = 1$  margin:

$$d = \frac{|g(0.18, 0.62)|}{\|w\|}$$

$$d = \frac{|0.8 - 0.78|}{\sqrt{1^2 + 1^2}}$$

$$d = \frac{0.2}{\sqrt{2}}$$

$$d \cong 0.014$$