Part 1

Find a linear discriminant that separates the patterns in the logical OR dataset.

- Choose $\boldsymbol{w}$ that points toward the (1, 1) data point.
- Use (0, 0.6) to calculate the bias.

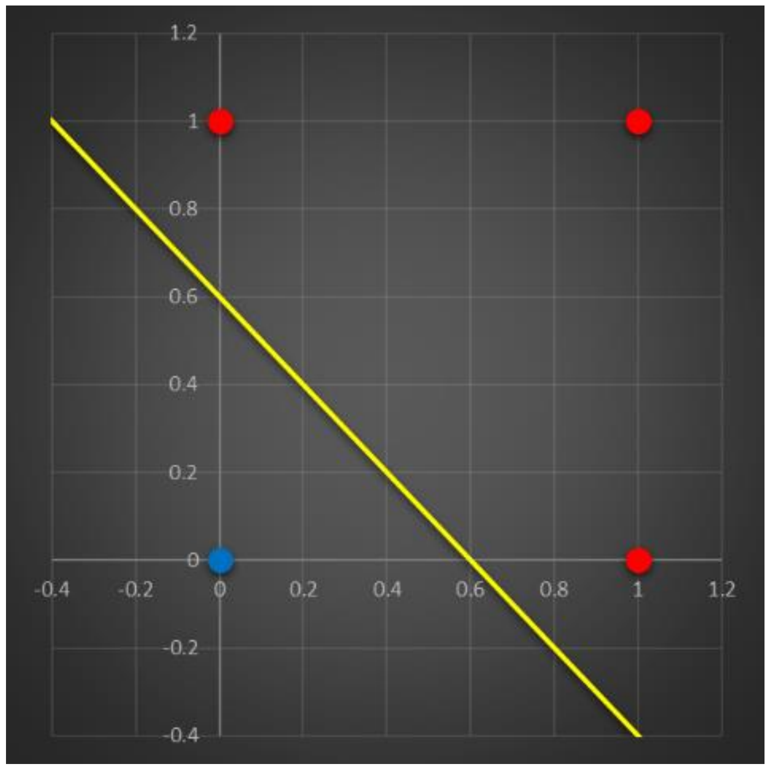
1. Find the equation of the decision boundary and draw it to the attribute-space diagram.

Bias:

$$\boldsymbol{w}^T\boldsymbol{p} + b = 0$$
$$\boldsymbol{w}^T\boldsymbol{p} = -b$$
$$-b = \boldsymbol{w}^T\boldsymbol{p}$$
$$-b = [1 \quad 1]\begin{bmatrix}0\\0.6\end{bmatrix}$$
$$-b = 0.6$$
$$b = -0.6$$

Decision boundary:

$$\boldsymbol{w}^T\boldsymbol{p} + b = 0$$
$$[1 \quad 1]\begin{bmatrix}x_1\\x_2\end{bmatrix} - 0.6 = 0$$
$$(1 \times x_1) + (1 \times x_2) - 0.6 = 0$$
$$x_1 + x_2 - 0.6 = 0$$
$$x_1 + x_2 = 0.6$$



2. Calculate the margins of the open circle and closed circles patterns.

Open-circles margin:

$$d = \frac{|g(0,0)|}{||\boldsymbol{w}||}$$
$$d = \frac{\left|[1 \quad 1]\begin{bmatrix}0\\0\end{bmatrix} - 0.6\right|}{\sqrt{1^2 + 1^2}}$$
$$d = \frac{|0 - 0.6|}{\sqrt{1+1}}$$
$$d = \frac{0.6}{\sqrt{2}}$$
$$d \cong 0.42$$

Closed-circles margin:

$$d = \frac{|g(0,1)|}{||\boldsymbol{w}||}$$
$$d = \frac{\left|[1 \quad 1]\begin{bmatrix}0\\1\end{bmatrix} - 0.6\right|}{\sqrt{1^2 + 1^2}}$$
$$d = \frac{|1 - 0.6|}{\sqrt{1+1}}$$
$$d = \frac{0.4}{\sqrt{2}}$$
$$d \cong 0.28$$

Part 2

Use PLA with **specified initial weights and bias** to find a weight vector and bias that separates the 2 points below in 3D attribute space.

- Assume the output of the perceptron is
  $$hardlim(\boldsymbol{w}^T\boldsymbol{p} + b)$$

Training Set

$$\left\{\mathbf{p}_1 = \begin{bmatrix}-1\\1\\-1\end{bmatrix}, t_1 = \begin{bmatrix}1\end{bmatrix}\right\} \quad \left\{\mathbf{p}_2 = \begin{bmatrix}1\\1\\-1\end{bmatrix}, t_2 = \begin{bmatrix}0\end{bmatrix}\right\}$$

Initial Weights

$$\mathbf{W} = \begin{bmatrix}0.5 & -1 & -0.5\end{bmatrix} \qquad b = 0.5$$

1. Report the resulting weight vector, bias, and the equation of the decision boundary.

Point Classification Checks	Adjustments
$a = hardlim(\boldsymbol{w}^T\boldsymbol{p}_1 + b)$ $a = hardlim\left(\begin{bmatrix}0.5 & -1 & -0.5\end{bmatrix}\begin{bmatrix}-1\\1\\-1\end{bmatrix} + 0.5\right)$ $a = hardlim(-0.5 - 1 + 0.5 + 0.5)$ $a = hardlim(-0.5)$ $a = 0$ $\boldsymbol{P}_1$ is misclassified, $t_1 = 1$ and $a = 0$ $e = t_1 - a = 1 - 0 = +1$	$\boldsymbol{w}_{new} = \boldsymbol{w}_{old} + e\boldsymbol{p}_1$ $\boldsymbol{w}_{new} = \begin{bmatrix}0.5\\-1\\-0.5\end{bmatrix} + \begin{bmatrix}-1\\1\\-1\end{bmatrix}$ $\boldsymbol{w}_{new} = \begin{bmatrix}-0.5\\0\\-1.5\end{bmatrix}$ $b_{new} = b_{old} + e = 0.5 + 1 = 1.5$
$a = hardlim(\boldsymbol{w}^T\boldsymbol{p}_2 + b)$ $a = hardlim\left(\begin{bmatrix}-0.5 & 0 & -1.5\end{bmatrix}\begin{bmatrix}1\\1\\-1\end{bmatrix} + 1.5\right)$ $a = hardlim(-0.5 + 0 + 1.5 + 1.5)$ $a = hardlim(2.5)$ $a = 1$ $\boldsymbol{P}_2$ is misclassified, $t_2 = 0$ and $a = 1$ $e = t_2 - a = 0 - 1 = -1$	$\boldsymbol{w}_{new} = \boldsymbol{w}_{old} + e\boldsymbol{p}_2$ $\boldsymbol{w}_{new} = \begin{bmatrix}-0.5\\0\\-1.5\end{bmatrix} - \begin{bmatrix}1\\1\\-1\end{bmatrix}$ $\boldsymbol{w}_{new} = \begin{bmatrix}-1.5\\-1\\-0.5\end{bmatrix}$ $b_{new} = b_{old} + e = 1.5 - 1 = 0.5$
$a = hardlim(\boldsymbol{w}^T\boldsymbol{p}_1 + b)$ $a = hardlim\left(\begin{bmatrix}-1.5 & -1 & -0.5\end{bmatrix}\begin{bmatrix}-1\\1\\-1\end{bmatrix} + 0.5\right)$ $a = hardlim(1.5 - 1 + 0.5 + 0.5)$ $a = hardlim(1.5)$ $a = 1$ $\boldsymbol{P}_1$ is correctly classified, $t_1 = 1$ and $a = 1$	Linearly Discriminated $\boldsymbol{w}^T = \begin{bmatrix}-1.5 & -1 & -0.5\end{bmatrix}$ $b = 0.5$
$a = hardlim(\boldsymbol{w}^T\boldsymbol{p}_2 + b)$ $a = hardlim\left(\begin{bmatrix}-1.5 & -1 & -0.5\end{bmatrix}\begin{bmatrix}1\\1\\-1\end{bmatrix} + 0.5\right)$ $a = hardlim(-1.5 - 1 + 0.5 + 0.5)$ $a = hardlim(-1.5)$ $a = 0$ $\boldsymbol{P}_2$ is correctly classified, $t_2 = 0$ and $a = 0$	Decision Boundary: $\boldsymbol{w}^T\boldsymbol{p} + b = 0$ $\begin{bmatrix}-1.5 & -1 & -0.5\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} + 0.5 = 0$ $-\frac{3}{2}x_1 - x_2 - \frac{1}{2}x_3 = -\frac{1}{2}$

2. Calculate the distances of the points from the decision boundary.

$$d = \frac{|g(\boldsymbol{p}_1)|}{\|\boldsymbol{w}\|}$$

$$d = \frac{\left|\begin{bmatrix}-1.5 & -1 & -0.5\end{bmatrix}\begin{bmatrix}-1\\1\\-1\end{bmatrix} + 0.5\right|}{\sqrt{(1.5^2) + (-1^2) + (-0.5^2)}}$$

$$d = \frac{|1.5|}{\sqrt{2.25 + 1 + 0.25}}$$

$$d = \frac{1.5}{\sqrt{3.5}}$$

$$d \cong 0.80$$

$$d = \frac{|g(\boldsymbol{p}_2)|}{\|\boldsymbol{w}\|}$$

$$d = \frac{\left|\begin{bmatrix}-1.5 & -1 & -0.5\end{bmatrix}\begin{bmatrix}1\\1\\-1\end{bmatrix} + 0.5\right|}{\sqrt{(1.5^2) + (-1^2) + (-0.5^2)}}$$

$$d = \frac{|-1.5|}{\sqrt{2.25 + 1 + 0.25}}$$

$$d = \frac{1.5}{\sqrt{3.5}}$$

$$d \cong 0.80$$