Final
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Problem 1
i False
ii True
iii True
iv True
v False
vi True
vii False

Problem 2
A: light is on
B: car in blind-spot
i
P(B|A) = P(A∩B) / P(A)
= (0.02 × 0.999) / (0.02 × 0.999 + 0.98 × 0.03)
= 0.01998 / (0.01998 + 0.0294)
≈ 0.4046
So, given the light is on, the probability that another car is in the blind-spot is 0.4046

ii
P(B|¬A) = P(¬A∩B) / P(¬A)
= (0.02 × 0.001) / (0.02 × 0.001 + 0.98 × 0.97)
= 0.00002 / (0.00002 + 0.9506)
≈ 0.00002104
So, given the light is off, the probability that another car is in the blind-spot is 0.00002104

Problem 3
i 2, -12, 72, -432, 2592
It is geometric, because 2×(-6)=-12, -12×(-6)=72, 72×(-6)=-432, -432×(-6)=2592......
$a_i = 2 \times (-6)^{i-1}$

$$\sum a_n = 2 \times \frac{1-(-6)^n}{1-(-6)} = 2 \times \frac{1-(-6)^n}{7}$$

$$\sum_{i=1}^{10} 2 \times \frac{1-(-6)^{10}}{7} = -17256050$$

ii 9, 18, 31, 48, 69, 94

It is quadratic. Because 18-9=9, 31-18=13, 48-31=17, 69-48=21, 94-69=25, we get a new sequence 9, 13, 17, 21, 25, and 13-9=17-13=21-17=25-21=4, so it is quadratic.

$a_i = 2i^2 + 3i + 4$

iii 14, 11.5, 9, 6.5, 4, 1.5

It is arithmetic. Because 11.5-14=9-11.5=6.5-9=4-6.5=1.5-4=-2.5

$a_i = -2.5i + 16.5$

$\sum a_n = \frac{-2.5i^2+30.5i}{2}$, $\sum_{i=1}^{10} \frac{-2.5i^2+30.5i}{2} = 27.5$

Problem 4

T(n) = 9T(n-1) + 9

T(n-1) = 9T(n-2) + 9

T(n-2) = 9T(n-3) + 9

...

T(n) = 9T(n-1) + 9

= 9(9T(n-2) + 9) + 9 = 81T(n-2) + 90

= 9(9(9T(n-3) + 9) + 9) + 9= 729T(n-3)+819

= 9^kT(n-k) + $\frac{9}{8}$(9^k -1)

let n-k = 0, k=n

T(n) = 9^nT(0) + $\frac{9}{8}$(9^n -1)

since T(0) = 0

T(n) = $\frac{9}{8}$(9^n -1)

Problem 5

i 2, 6, 4, 3, 0, 8, 7, 5, 9, 1

ii 2, 6, 8, 7, 3, 5, 4, 9, 1, 0

iii 4, 9, 2, 1, 0, 6, 3, 8, 7, 5

iv 4, 9, 1, 0, 2, 6, 8, 7, 3, 5

Problem 6

Solution:

We use induction on a sequence of statements:

$n=1$, upper_diagonal entries = 1 ......statement 0

$n=2$, upper_diagonal entries = 3 ......statement 1

$n=3$, upper_diagonal entries = 6 ......statement 2

$n=4$, upper_diagonal entries = 10 ......statement 3

...

$n=n$, upper_diagonal entries = $n(n+1)/2$ ......statement 0

Base case:

when $n = 1$, there are 1 upper_diagonal entry

Inductive Step:

Assume statement n is true, that is when $n=n$, upper_diagonal entries = $n(n+1)/2$,

Consider $n=k$

For some k, we know that for some $k\times k$ metrix, there are $k(k+1)/2$ upper_diagonal entries.

Consider $n=k+1$, we know from metrix that we need to add a new colomn with $k+1$ entries to the former upper_diagonal entries, then we get $k(k+1)/2 + (k+1)$ when $n = k+1$, which is equal to $0.5k^2 + 1.5k + 1$, also by our hyphothesis, when $n = k+1$, we have $(k+1)(k+2)/2$ be the number of upper_diagonal entries, which is the same as $0.5k^2 + 1.5k + 1$

So, when $n = k+1$, we have $(k+1)(k+2)/2$ upper_diagonal entries, as we descired.

By induction, it is proved true for $n\times n$ metrix, it has $n(n+1)/2$ upper_diagonal entries.