Final Exam

Due: Dec 14, 2020

Instructions:

- **This exam must be uploaded to gradescope by 11:59 EST on the due date**. No work will be accepted beyond this time[1]. You may not use late days to extend this deadline.

- Unless instructed otherwise by a problem's instruction, you must show intermediate steps to justify each response for full credit.

- You may use notes and the textbook to help you complete the exam. You may also use online resources so long as they were not created for these exam questions (e.g. you may not post a question on any forum to receive support)

- **You may not receive math support from another person. This includes course staff.**

- You may, however, ask a clarifying question by posting a **private** question on piazza with the tag "exam". Course staff will be actively monitoring piazza throughout the day (10-11 AM, 12-1 PM, 2-6 PM, 7-8 PM, 9-10 PM all in EST).

- Don't forget that the exam will be curved upward so your grade will be at least as high as your percentage of points earned (see syllabus).

Problem 1 [21 points (3 pts each)]:
Indicate whether each of the statements below is True or False. No explanation is needed.

i If random variable X has a larger expected value than random variable Y then it must also have a larger variance.

ii Let random variable X be the height of a random CS5002 student in feet and X' be the height of a student from the same group in millimeters[2]. The variance of X' is greater than the variance of X.

iii For every positive $a, b \in \mathbb{R}$ there exists an n_0 such that:

$$n > n_0 \rightarrow an^3 > bn^2$$

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[1]Unless you have made arrangements with the Professor prior to the start of this exam

[2]Assume that not all students have the same height

Use the function definitions immediately below for the next 3 problems:

$$f(n) = 3n \log_{10} n + 5$$
$$g(n) = 99999 \log_3 n$$
$$h(n) = .00001 \log_3 n + .0000001n^2$$

iv $f(n) = O(h(n))$

v $g(n) = \Theta(h(n))$

vi $f(n) = \Omega(g(n))$

vii An undirected graph on n nodes with at least $\binom{n}{2}/2$ edges must be connected.

Problem 2 [20 points (10 pts each)]:
Modern cars provide 'blind-spot monitoring', a light which activates when an adjacent car is detected. Unfortunately, the system is not perfect. Given that another car is in the blind spot, the blind-spot light is activated 99.9% of the time. However, the system accidentally activates even when another car is not in the blind spot, this occurs 3% of the time. Assume that another car is in one's blind-spot 2% of the time.

i What is the probability that another car is actually in the blind-spot given the light is on?

ii Given the blind-spot light is off, how often is another car in the blind-spot?

Problem 3 [19 points (7 or 5 pts each)]:
For each of the following sequences:

- identify if the sequence is arithmetic, geometric or quadratic[3]. Justify your response.

- assuming the first item of each sequence is a_1, give an expression for a_i. (In other words, find a formula for the i-th term in the sequence).

- if the sequence is arithmetic or geometric, compute the sum of the first 10 terms in the sequence

i $2, -12, 72, -432, 2592, \ldots$

ii $9, 18, 31, 48, 69, 94, \ldots$

iii $14, 11.5, 9, 6.5, 4, 1.5, \ldots$

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[3]its possible a sequence can be neither arithmetic, geometric or quadratic, but each of these examples is of one of these three types
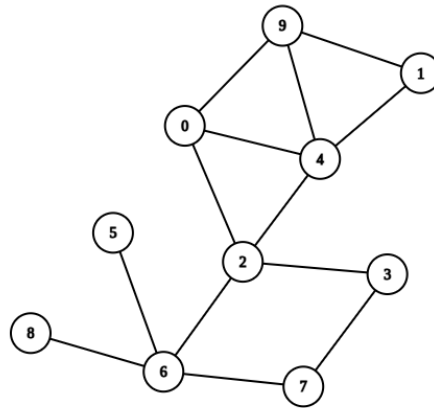
Problem 4 [12 points]:

Solve[4] the following recurrence equation:

$$T(n) = 9T(n-1) + 9 \quad \text{where } T(0) = 0$$

You may leave any series unsimplified in your solution (e.g. your answer can include terms like $\sum_{i=1}^{10} 2^i$)

Problem 5 [16 points (4 pts each)]:

Give the order of nodes in each of the searches below, starting at the indicated node. When a method may select from among many next candidate nodes, prefer the node which has a larger value.

i Breadth First Search starting at node 2

ii Depth First Search starting at node 2

iii Breadth First Search starting at node 4

iv Depth First Search starting at node 4

Problem 6 [12 points]:

The diagonal of a square matrix refers to all elements in a line from the top-left to the bottom-right of the matrix. For example, in the 3×3 matrix below, all diagonal entries are 1 where off-diagonal entries are 0:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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[4]Express without recurrence

Note that the only entry in a 1×1 matrix is on the diagonal.

The upper-diagonal of a matrix are all entries which are on the diagonal or above. For example, in the 3×3 matrix below, all upper-diagonal entries are 1 where non-upper-diagonal entries are 0:

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Use induction to show that an $n \times n$ matrix has $n(n+1)/2$ upper-diagonal entries.