CS5002 Discrete and Data Structures	Prof Higger
Fall 2020	Oct 22, 2020

HW 06 Probability

Due: Oct 30, 2020

Instructions:

- This homework exists to strengthen your understanding of concepts so that you may apply them elsewhere
- To get full credit, show intermediate steps leading to your answers.
- You are welcome to work on problems with classmates though you may not directly view another student's solution to a given problem while working together. Include a brief statement at the beginning of your homework which lists your homework group members: "Homework group: person A, person B". If you did not work with other students on the assignment write "Homework group: none". A 5 point penalty will be applied to all work which does not include this statement.
- Questions whose points are labelled with an addition sign are extra credit (e.g. "+4 points"). These are designed to push you, so have fun and don't worry if you're not making headway immediately: they're supposed to take some time. Excellence will come with practice.

Problem 1 [18 points (3 pts each)]: Loaded Die

A loaded die is observed and found to land with 1 face up 20% of the time, with 2 face up 15% of the time, with 3 face up 30% of the time, with 4 face up 15% of the time, with 5 face up 5% of the time, and with 6 face up 15% of the time. Let E be the event that this die lands with an even number face up, and let F be the event that it lands with a number less than 5 face up.

i What is the sample space S for this experiment?

ii Give the probability $P(x)$ for each outcome $x \in S$ as a table:

iii Write E and F using set notation.

iv What is $P(E^c)$?

v What is $P(E \cup F)$?

vi What is $P(E^c \cap F^c)$?

Problem 2 [20 points (4 pts each)]: Counting Cards

A set of 6 cards contains two black cards numbered 1 and 2 and four red cards numbered 1, 2, 3, 4. Each of the questions below refers to uniformly selecting two unique cards from the set of 6.

i What is the probability that you draw a card whose value is 4?

ii What is the probability that you draw a card whose value is 4 given the card is black?

iii What is the probability that you draw a card whose value is 4 given the card is red?

iv Give an intuition for how the probability of choosing a four changes between parts i and ii. In other words, why does the probability of choosing a four change this way when we require the chosen card be black?

v Give an intuition for how the probability of choosing a four changes between parts i and iii. In other words, why does the probability of choosing a four change this way when we require the chosen card be red?

Problem 3 [10 points]: Birthday Paradox
Assume there are 365 days of the year and people are equally likely to be born on any day. What is the probability that everyone in a room has a unique birthday[1] if there are 10 people in a room? Do not simplify, just write out your expression. (HINT: tree method)

Problem 4 [16 points (8 pts each)]: N-sided die
Compute the expected value and variance of each of experiments below. Assume that the die faces are numbered $1, 2, 3, \ldots, n$ for an n-sided die.

i a 3-sided die

ii an 8-sided die

Problem 5 [20 points (4 pt each)]: Soccer Penalty
When soccer matches end in a draw they may be decided by a penalty kick shootout. Each team takes 5 shots at the other goal and the team with the most goals, of the 5 opportunities, is the winner. Assume that 60% of these shots are successful.

i. What is the probability that one team makes 5 penalty kicks?

ii. What is the probability that one team makes no penalty kicks?

iii. What is the probability that one team makes their first 3 kicks and misses the last 2?

iv. What is the probability that one team makes exactly 3 penalty kicks, in any order?

v. What is the probability that one team makes at least 4 penalty kicks, in any order?

Problem 6 [16 points (6, 5, 5)]: Gold Prospecting
Gold prospecting is the search for new deposits of gold within the earth. If a parcel of land has a gold deposit, then there is a 90% chance that bits of gold will be found in one of its streams.

[1]For a fun follow-up, consider the following question:
If there are only 4 people in a room we'd expect the probability that two share a birthday to be near 0. However, if there are 300 people in a room we'd expect the probability that two share a birthday to be near 1. How many people are in the room when the probability that two people share a birthday is 50%? Use your answer to Problem 3 above to build an estimate and then see what google has in store for you if you search 'birthday paradox'. Enjoy!

However, gold can travel downstream into another parcel too! If a piece of land doesn't have a gold deposit, there is still a 40% chance that bits of gold are found in its streams. Only 30% of the parcels have gold deposits.
Let D be the event that a parcel has a gold deposit and S be the event that gold may be found in its streams.

i. Describe, in english, each of the probabilities below.

   (a) $P(D)$
   (b) $P(D|S)$
   (c) $P(S^C|D)$

ii. Compute $P(S)$

iii. What is the probability that a parcel of land has a gold deposit given that no gold may be found in its streams?