

## HW 04 Counting

**Due:** Oct 09, 2020

**Instructions:**

- This homework exists to strengthen your understanding of concepts so that you may apply them elsewhere
- To get full credit, show intermediate steps leading to your answers.
- You are welcome to work on problems with classmates though you may not directly view another student's solution to a given problem while working together. Include a brief statement at the beginning of your homework which lists your homework group members: "Homework group: person A, person B". If you did not work with other students on the assignment write "Homework group: none". A 5 point penalty will be applied to all work which does not include this statement.
- Questions whose points are labelled with an addition sign are extra credit (e.g. "+4 points"). These are designed to push you, so have fun and don't worry if you're not making headway immediately: they're supposed to take some time. Excellence will come with practice.

**For this assignment and all future counting problems, please do not compute large exponents, you may leave answers unsimplified (e.g. write  $15^4$  instead of 50625).**

**Problem 1 [21 points (7, 7, 7, +2)]: Passwords** How many passwords can be created which meet each set of criteria below:

- passwords may only contain lowercase letters
  - passwords must be exactly 5 characters long
- passwords may only contain lowercase letters
  - passwords may be between 5 and 7 characters long
- passwords may only contain lowercase letters
  - passwords may be between 5 and 7 characters long
  - passwords may not start with the letters a, b or c
- passwords may only contain lowercase letters
  - passwords may be between 5 and 7 characters long
  - passwords may not start with the letters a, b or c
  - passwords may not contain the following 4-letter word: **sock**

**Problem 2 [21 points (7, 7, 7)]: Pigeonhole**

- i Suppose 14 pigeons land in 3 separate nests. How many pigeons, at least, are guaranteed to share a single nest?
- ii What is the minimum number of students that must be assigned to a classroom with 14 tables to guarantee that some table will have at least 3 students?
- iii Some exam consists of 10 TRUE or FALSE questions, each is worth an equal amount of points. Assume that 123 students take this exam. Write one sentence which tells what the Pigeonhole Principle implies about the exam results of all students. Your wording should be concise and unambiguous.

**Problem 3 [8 points]: Baseball Cards** A package of baseball cards contains 15 cards. How many packages must be purchased to ensure that two cards in these packages are identical if there are a total of 550 different cards?

**Problem 4 [10 points]: Computer Virus** Once a computer worm infects a personal computer via an infected e-mail message, it sends a copy of itself to 100 e-mail addresses it finds in the electronic message mailbox on this personal computer. What is the maximum number of different computers this one computer can infect in the time it takes for the infected message to be forwarded five times?

**Problem 5 [20 points (10, 10)]: Sum and Product Rules** Express your response via sets you define before computing a value below. For example, you might begin your response with ‘ $A$  = set of some objects’, ‘ $B$  = set of some other objects’ and then compute some expression for what you’re seeking to count:

$$|A \times B| = |A| \times |B| = 3 * 7 = 21$$

- i Regular MA license plates have 3 digits followed by 3 (uppercase) letters, e.g. ‘924 VXA’ or 4 digits followed by 2 (uppercase) letters, e.g. ‘2012 CS’. How many different license plates are possible?
- ii Henry owns 3 hiking shoes and 5 running shoes for his left foot. He owns 12 running shoes for his right foot. How many different pairs<sup>1</sup> of shoes, one left and one right, can he form?

**Problem 6 [20 points (5, 5, 5, 5)]: Principle of Inclusion Exclusion** There are 39 3rd graders in a school, each of whom likes at least one genre of books. There are 3 popular genres among the students: fiction, fantasy, and mystery:

- 16 like Realistic Fiction;
- 25 like Fantasy;
- 18 like Mystery;
- 12 like Realistic fiction and Fantasy;

---

<sup>1</sup>Henry requires only that he have one shoe for his right foot and another shoe for his left foot, they needn’t match

- 5 like Realistic Fiction and Mystery;
- 8 like fantasy and mystery.

Answer the following questions:

1. How many students like either Realistic Fiction, Mystery or both?
2. How many children like all 3 genres of books?
3. How many students like exactly 2 genres?
4. How many students like exactly 1 genre?

**Problem 7 [+3 points]: Challenge** Suppose a set of 8 numbers are selected from the set:

$$\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14\}$$

Show that two of the selected numbers must sum to 15. (Hint: Pigeonhole)