CS5002 Discrete and Data Structures	Prof Higger
Fall 2020	Oct 29, 2020

HW 07 Induction, Recursion, Sequences & Series

Due: Nov 06, 2020

Instructions:

- This homework exists to strengthen your understanding of concepts so that you may apply them elsewhere
- To get full credit, show intermediate steps leading to your answers.
- You are welcome to work on problems with classmates though you may not directly view another student's solution to a given problem while working together. Include a brief statement at the beginning of your homework which lists your homework group members: "Homework group: person A, person B". If you did not work with other students on the assignment write "Homework group: none". A 5 point penalty will be applied to all work which does not include this statement.
- Questions whose points are labelled with an addition sign are extra credit (e.g. "+4 points"). These are designed to push you, so have fun and don't worry if you're not making headway immediately: they're supposed to take some time. Excellence will come with practice.

Problem 1 [18 points]: Induction (Powerset Cardinality)
Using induction, show that the cardinality of the powerset of A is 2^n where n is the number of elements in A. In other words, given a set A with $|A| = n$ then $|\mathcal{P}(A)| = 2^n$. (You may find it helpful to see the practice problem on induction to see what is required of your proof).

Problem 2 [18 points]: Induction (Geometric Series)
Using induction, prove the geometric series formula:

$$\sum_{i=1}^{n} a_1 r^{i-1} = a_1 \frac{1-r^n}{1-r}$$

(You may find it helpful to see the practice problem on induction to see what is required of your proof).

Problem 3 [32 points]: Sequences and Series
For each of the following sequences:

- identify if the sequence is arithmetic, geometric or quadratic[1]. Justify your response.
- assuming the first item of each sequence is a_1, give an expression for a_i. (In other words, find a formula for the i-th term in the sequence).

[1] its possible a sequence can be neither arithmetic, geometric or quadratic, but each of these examples is of one of these three types

- if the sequence is arithmetic or geometric, compute the sum of the first 10 terms in the sequence

i $128, -64, 32, -16, 8, -4, \ldots$

ii $1, 4, 13, 28, 49, \ldots$

iii $-5, -2, 1, 4, 7, \ldots$

Problem 4 [32 points]: Recurrences

Solve eac of the following recurrences by substitution. Assume a base case of $T(1) = 1$. As part of your solution, you will need to establish a pattern for what the recurrence looks like after the k-th substitution. Check that this pattern is consistent with your substitutions, but you do not need to formally prove it is correct via induction.

i $T(n) = T(n-1) + 1$

ii $T(n) = T(n-3) + 4$

Problem 5 [+3 points]: A series

What is the sum of the series corresponding to sequence:

$$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}, \ldots$$

Be sure to sufficiently justify your answer, a sum without a clear and complete rationale will receive no credit.