

CS5002 Prof. Higger

Homework2

Name: Shangjun Jiang

Homework group: Wei Han, Shangjun Jiang

Problem 1.

i. $P(20,8) = \frac{20!}{(20-8)!} = \frac{20!}{12!}$

ii. $C(20,9) = \frac{20!}{(20-9)! \times 9!} = \frac{20!}{11! \times 9!}$

iii. *This is a repetition allowed and order doesn't matter question*
 3^{20}

iv. $C(502,500) = \frac{502!}{(502-500)! \times 500!} = \frac{502!}{2! \times 500!}$

v. Here we can give each child 50\$ at first, and left 350\$

$$C(352,350) = \frac{352!}{(352-350)! \times 350!} = \frac{352!}{2! \times 350!}$$

Problem 2.

i. $C(20,11) = \frac{20!}{9! \times 11!}$

ii. $C(7,4) \times C(6,3) \times C(5,3) \times C(2,1) = \frac{7!}{3! \times 4!} \times \frac{6!}{3! \times 3!} \times \frac{5!}{2! \times 3!} \times \frac{2!}{1! \times 1!}$

iii. **case 1:** select the defender&attacker

1.1 the one play as defender

$$C(7,4) \times C(5,2) \times C(5,3) \times C(2,1) \quad \textcircled{1}$$

1.2 the one play as attacker

$$C(7,4) \times C(5,3) \times C(5,2) \times C(2,1) \quad \textcircled{2}$$

case 2: don't select the defender&attacker

$$C(7,4) \times C(5,3) \times C(5,3) \times C(2,1) \quad \textcircled{3}$$

$$\textcircled{1} + \textcircled{2} + \textcircled{3}$$

$$= \frac{7!}{3! \times 4!} \times \frac{5!}{3! \times 2!} \times \frac{5!}{2! \times 3!} \times \frac{2!}{1! \times 1!} + \frac{7!}{3! \times 4!} \times \frac{5!}{2! \times 3!} \times \frac{5!}{3! \times 2!} \times \frac{2!}{1! \times 1!} + \frac{7!}{3! \times 4!} \times \frac{5!}{2! \times 3!} \times \frac{5!}{2! \times 3!} \times \frac{2!}{1! \times 1!}$$

$$= \frac{7!}{3! \times 4!} \times \frac{5!}{3! \times 2!} \times \frac{5!}{2! \times 3!} \times \frac{2!}{1! \times 1!} + \frac{7!}{3! \times 4!} \times \frac{5!}{2! \times 3!} \times \frac{5!}{3! \times 2!} \times \frac{2!}{1! \times 1!} + \frac{7!}{3! \times 4!} \times \frac{5!}{2! \times 3!} \times \frac{5!}{2! \times 3!} \times \frac{2!}{1! \times 1!}$$

$$= 3 \times \frac{7!}{3! \times 4!} \times \frac{5!}{3! \times 2!} \times \frac{5!}{2! \times 3!} \times \frac{2!}{1! \times 1!} = 21000$$

Problem 3.

i. This is an order-matter question

$$P(12,3) = \frac{12!}{9!}$$

ii. We can decide the first rank and arrange the left 11 ranks

$$9 \times 11!$$

iii. case1: Grand Valor>Seabiscuit>Secretariat

12 ranks, determin 3 first, left 9

$$C(12,3) \times P(9,9) = 9! \times \frac{12!}{9! \times 3!}$$

case2: Grand Valor>Secretariat>Seabiscuit

$$C(12,3) \times P(9,9) = 9! \times \frac{12!}{9! \times 3!}$$

So, there are $2 \times 9! \times \frac{12!}{9! \times 3!} = 2 \times \frac{12!}{3!}$ different ways to satisfy that Grand Valor always beat Seabiscuit and Secretariat.