CS5002 Discrete and Data Structures	Prof Higger
Fall 2020	Oct 14, 2020

Midterm Exam

Due: Oct 14, 2020

Instructions:

- To get full credit for a question show intermediate steps which justify your response.
- **This exam must be uploaded to gradescope by 11:59 EST on the due date**. No work will be accepted beyond this time[1]. You may not use late days to extend this deadline.
- You may use notes and the textbook to help you complete the exam. You may also use online resources so long as they were not created for these exam questions (e.g. you may not post a question on any forum to receive support)
- **You may not receive math support from another person. This includes course staff.**
- You may, however, ask a clarifying question by posting a **private** question on piazza with the tag "exam". Course staff will be actively monitoring piazza from 9AM - 2PM as well as 5-6PM, 7-8PM and 9-10PM.
- Don't forget that the exam will be curved upward so your grade will be at least as high as your percentage of points earned (see syllabus).

Problem 1 [16 Points (4pts x 4)]: Compute in other bases
For each of the operations below:

- convert all values to the given number system
- apply the operation within the given number system. (Just like the computer, you must return a value within the same number system.)
- indicate whether an overflow has occurred

i $74 + 12$ in 8 digit unsigned binary

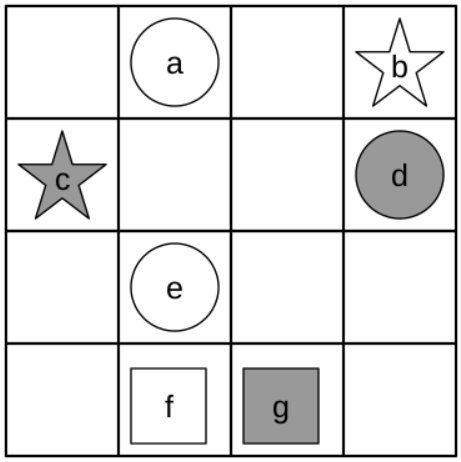
ii $123 + 23$ in 8 digit two's complement binary

iii $-30 + 42$ in 7 digit two's complement binary

iv $240 + 20$ in 2 digit hexadecimal

[1] Unless you have made arrangements with the Professor prior to the start of this exam

Problem 2 [12 points (2pts x 6)]: Tarski World



Using the following predicates:
`square(x)` is true if x is a square (otherwise it is false)
`star(x)` is true if x is a star (otherwise it is false)
`circ(x)` is true if x is a circle (otherwise it is false)
`shade(x)` is true if x is shaded (otherwise it is false)
`next_to`(x, y) is true if x and y are adjacent horizontally, vertically or diagonally. This relation is not reflexive for any object.

For each of the statements below, determine whether the statement is true or false. You need not justify your response.

i $\exists x\, \texttt{circ}(x) \wedge \texttt{shade}(x)$

ii $\forall x\, \texttt{shade}(x) \vee \neg\texttt{shade}(x)$

iii $\forall x\, \texttt{square}(x) \rightarrow \neg\texttt{shade}(x)$

iv $\forall x \forall y(\texttt{star}(x) \wedge \neg\texttt{shade}(x) \wedge \texttt{next\_to}(x, y) \rightarrow \texttt{shade}(y) \wedge \texttt{circ}(y))$

v $\forall y \exists x\, \texttt{next\_to}(x, y) \wedge \texttt{shade}(x)$

vi $\exists x \forall y\, \texttt{next\_to}(x, y) \wedge \texttt{shade}(x)$

Problem 3 [12 points (4pts x 3)]: Vending Machine
Consider the construction of a vending machine's logic. Assume that every soda costs a quarter, and that the machine accepts only quarters.

- E = True indicates the machine is empty, it has no more sodas
- S = True indicates the user has made a selection

- P = True indicates the user has paid a quarter

The machine may send two control commands to its machinery:

- V = True indicates machine will give the user a soda (i.e. "vend")
- R = True indicates machine will return the user's quarter

The machine should return a user's quarter only when they have paid and the machine is empty. The machine should give a soda only when a user has paid, made a selection and the machine is not empty.

i Write a truth table for V and R in terms of E, S, P

ii Write an expression for V in terms of E, S, P using boolean operators $\vee, \wedge, \neg$

iii Write an expression for R in terms of E, S, P using boolean operators $\vee, \wedge, \neg$

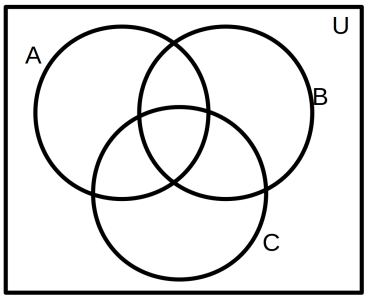
Problem 4 [10 points (5 pts x 2)]: Set and Boolean Algebra
Simplify each of the following expressions. Label each identity[2] used. Be mindful that you use the appropriate notation for set algebra or boolean algebra.

i $((A \cap A^C) \cup (A \cap (A \cup B)))^C$

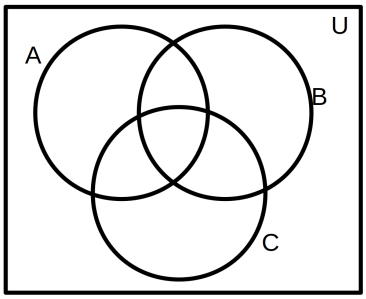
ii $\neg((\neg P \vee \neg Q) \wedge Q)$

Problem 5 [8 points (4 pts x 2)]: Venn Diagram Shading
Shade the indicated regions of the following Venn diagrams.

$((A \cup C) - B) \cup C$



$A^C \cap (B \cup U^C)$



Problem 6 [7 points]: Pigeonhole Principle (or not)
Assume that the 160 students who take this exam will receive one of 24 grades.

i What does the pigeonhole principle tell us about the exam results of these students?

[2] see `logic_set_identities.pdf`

ii Regardless of the scores assigned, how many other students must get the same grade as you?

Problem 7 [9 points (3 pts x 3)]: Three Set Principle Inclusion Exclusion

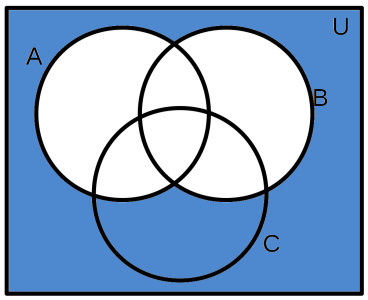
Given that:

- $|U| = 60$
- $|A| = 23$
- $|B| = 24$
- $|C| = 30$
- $|A \cap B| = 10$
- $|A \cap C| = 11$
- $|B \cap C| = 7$
- $|A \cap B \cap C| = 2$

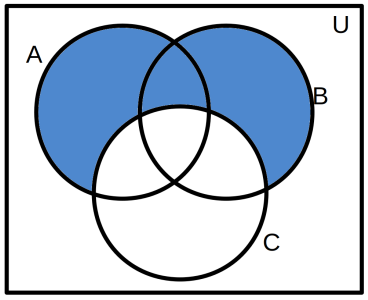
Count the number of elements in each set below (the last two sets are indicated as the blue shaded area):

i $A \cup B \cup C$

ii



iii



Problem 8 [26 points]: Package Delivery

A delivery driver has 10 **unique** packages to deliver.

i How many ways can the delivery driver select 6 of these packages to deliver?

ii How many ways can the delivery driver select 4 of these packages to discard while delivering the rest? Explain how this problem relates to the first subproblem.

iii How many different routes may the delivery truck drive to deliver 8 of the 10 packages? A route is an ordered sequence of destinations. You may assume each package may only be delivered to its unique destination.

iv Before leaving for the day, the delivery trucks are loaded with packages. How many ways can 120 **unique** packages be loaded into 10 delivery trucks where some trucks may have no packages?

v How many ways can 120 **identical** packages be loaded into 10 delivery trucks where some trucks may have no packages?

vi The delivery truck driver union is concerned that the workload is unequal. How many ways can 120 **identical** packages be loaded into 10 delivery trucks where each truck must have at least 5 packages?

vii How many ways can 120 **unique** packages be loaded into 10 delivery trucks where each truck must have the same number of packages?