HW 05 Counting

Due: Oct 10, 2020 @ 11:59 EST (no late days may be used)

Instructions:

- This homework exists to strengthen your understanding of concepts so that you may apply them elsewhere

- To get full credit, show intermediate steps leading to your answers.

- You are welcome to work on problems with classmates though you may not directly view another student's solution to a given problem while working together. Include a brief statement at the beginning of your homework which lists your homework group members: "Homework group: person A, person B". If you did not work with other students on the assignment write "Homework group: none". A 5 point penalty will be applied to all work which does not include this statement.

- Questions whose points are labelled with an addition sign are extra credit (e.g. "+4 points"). These are designed to push you, so have fun and don't worry if you're not making headway immediately: they're supposed to take some time. Excellence will come with practice.

Simplify each binomial coefficient or permutations to factorial fractions:

$$\binom{n}{k} = \frac{n!}{(n-k)!k!} \quad P(n,k) = \frac{n!}{(n-k)!}$$

You need not simplify these expressions, such as 15^4 or $17!$, in your response.

Problem 1 [25 points (5 points each)]: Photos and Money

i Lee wants to hang 8 of his 20 photos in an ordered line on the wall above his desk. How many ways can he do this?

ii Lee wants to select 9 photos of his 20 to take on a trip. How many ways can he do this?

iii Lee has decided to give his collection of 20 unique photographs to his three children. In how many ways can he partition his photo collection among this three children where it may be that some children get no photographs?

iv Lee also has 500 dollars he wants to give his three children. In how many ways can he divide it among his three children? It may be the case that a child receives no money.

v Lee's children will be upset if he doesn't give each of them at least 50 dollars. How many ways can Lee partition his money so his children won't be upset?

Problem 2 [15 points (2, 4, 9)]: Soccer Lineups A local high school soccer team has 20 players.

i. In how many ways can the coach choose 11 players to play?

ii. A lineup describes the set of players which play together. A lineup consists of:

4 midfielders, 3 defenders, 3 attackers and 1 goalkeeper.

This team has:

7 midfielders, 6 defenders, 5 attackers and 2 goalkeepers

in total on their roster. In how many ways can the coach choose a lineup of players?

iii. Now assume that one of the defenders, after a summer or practice, may play the attack position in addition to defense. In how many ways can the coach choose a lineup?

Problem 3 [10 points (5, 5, +3)]: Horse Race A horse race has 12 horses.

i How many different ways can the podium be arranged? (The podium has a spot for only the 1st, 2nd and 3rd place horse)

ii How many different ways can the horses finish the race such that Grand Valor, Seabiscuit and Secretariat do not finish first?

iii How many different ways can the horses finish the race such that one horse (Grand Valor) always beats both Seabiscuit and Secretariat?