

CS5002 Prof. Higger

Homework 6

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Problem 1

1 face 20%

2 face 15%

3 face 30%

4 face 15%

5 face 5%

6 face 15%

E: event that this die lands with an even number face

F: event that it lands with a number less than 5 face

i $S = \{1, 2, 3, 4, 5, 6\}$

ii

P(1)	1/5
P(2)	3/20
P(3)	3/10
P(4)	3/20
P(5)	1/20
P(6)	3/20

iii $E = \{2, 4, 6\}$

$F = \{1, 2, 3, 4\}$

iv $P(E^c) = 1 - P(E)$

$$= 1 - P(2) - P(4) - P(6)$$

$$= 1 - 3/20 - 3/20 - 3/20$$

$$= 11/20$$

v $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$$= P(1) + P(2) + P(3) + P(4) + P(6)$$

$$= 1 - P(5)$$

$$= 19/20$$

vi $P(E) = 9/20$

$$P(F) = 16/20$$

$$P(E^c \cap F^c) = P(\{1, 3, 5\} \cap \{5, 6\})$$

$$= P(5)$$

$$= 1/20$$

Problem 2

$$i \quad P(\text{num} = 4) = \frac{1}{C(6,1)} = \frac{1}{6}$$

$$ii \quad P(\text{num} = 4 \mid \text{color} = \text{black}) = \frac{0}{C(1,2)} = \frac{0}{2} = 0$$

$$iii \quad P(\text{num} = 4 \mid \text{color} = \text{red}) = \frac{1}{C(4,1)} = \frac{1}{4}$$

iv When uniformly selecting one unique cards from the set, we don't require a particular color, however, once required a particular color, we need to take the probability of 'given a color' in to account. In this case the sample size change from 6 to 2, and event size is 0.

v Given that the card is red, means you have to select from 4 red cards, so the sample space size becomes 4 (initially 6), and then there are 1 red four, so the probability is 1/4.

Problem 3

$$P = \frac{A(365,10)}{365^{10}}$$

Problem 4

i a 3-sided die

$$\begin{aligned} E(x) &= 1 \times 1/3 + 2 \times 1/3 + 3 \times 1/3 \\ &= 1/3 + 2/3 + 3/3 \\ &= 2 \end{aligned}$$

$$\begin{aligned} V(x) &= E(x^2) - (E(x))^2 \\ &= 1^2/3 + 2^2/3 + 3^2/3 - 2^2 \\ &= 14/3 - 12/3 \\ &= 2/3 \end{aligned}$$

ii an 8-sided die

$$\begin{aligned} E(x) &= 1 \times 1/8 + 2 \times 1/8 + 3 \times 1/8 + \dots + 8 \times 1/8 \\ &= 36/8 \\ &= 9/2 \end{aligned}$$

$$\begin{aligned} V(x) &= E(x^2) - (E(x))^2 \\ &= 1^2 \times 1/8 + 2^2 \times 1/8 + 3^2 \times 1/8 + \dots + 8^2 \times 1/8 - (9/2)^2 \\ &= 204/8 - 81/4 \\ &= 21/4 \end{aligned}$$

Problem 5

$$i \quad P(5) = (3/5)^5 = 243/3125 = 0.07776$$

$$ii \quad P(0) = (2/5)^5 = 32/3125 = 0.01024$$

$$iii \quad P = (3/5)^3 \times (2/5)^2 = 27/125 \times 4/25 = 108/3125 = 0.03456$$

$$iv \quad P = 0.6 \times 0.6 \times 0.6 \times 0.4 \times 0.4 \times C(5,3) = 0.3456$$

$$v \quad P(\text{at least } 4) = P(5) + P(4)$$

$$P(4) = 0.6 \times 0.6 \times 0.6 \times 0.6 \times 0.4 \times C(5,4) = 0.2592$$

$$P(5) = 0.07776$$

$$P(\text{at least } 4) = 0.2592 + 0.07776 = 0.33696$$

Problem 6

D: event that a parcel has a gold deposit

S: event that gold may be found in its streams

i $P(D)$: The probability that a parcel has a gold deposit

$P(D|S)$: The probability that a parcel has a gold deposit given that gold is found in its streams.

$P(S^c|D)$: The probability that gold isn't found in a parcel's stream but the parcel actually has a gold deposit.

ii $P(S) = 30\% \times 90\% + 70\% \times 40\% = 55\%$

iii $P = \frac{30\% \times 10\%}{30\% \times 10\% + 70\% \times 60\%} = \frac{1}{15}$