CS5002 Prof. Higger
Homework 9
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Problem 1

i True
ii True
iii True
iv False
v True
vi True
vii True
viii False
ix True
x False
xi False

Probelm 2

i

10000	$\log_2 n$, $3\log_3 n$	3n, 100n	$n\log_2 n$, $n\log_3 n$	n^2, $5n^2+3$	2^n	n!

ii

$c=200$, $n_0=1$

$f(n)=100n+5$, $g(n)=n$, there exists $c=200$, $n_0=1$, such that $100n+5 \leqslant 200n$, when $n \geqslant 1$.

iii

$c=1$, $n_o=1$

$f(n)=n$, $g(n)=2^n$, there exists $c=1$, $n_0=1$, such that $n \leqslant 2^n$, when $n \geqslant 1$.

iv

$3n+4n^2+3n!=O(f(n))$

as $n=O(n!)$, and $n^2=O(n!)$, we can figure out that

$3n \leqslant c_1 n!$

$4n^2 \leqslant c_2 n!$

and obviously, $3n! \leqslant c_3 n!$

so $3n+4n^2+3n! \leqslant (c_1+c_2+c_3)n!=cn!$,

so there exists, a non-negative real number c, when $n \geqslant n_0$ to meet the big O

$f(n)=n!$

Problem 3

18, 45, 23, 2, -5, 10, 99, 0

i

18, □, 45, 23, 2, -5, 10, 99, 0

18<45, next

18, 45, □, 23, 2, -5, 10, 99, 0

23<45, then 23>18, next

18, 23, 45, □, 2, -5, 10, 99, 0

2<45, then 2<23, then 2< 18, next

2, 18, 23, 45, □, -5, 10, 99, 0

-5<45, then -5<23, then -5<18, then -5<2, next

-5, 2, 18, 23, 45, □, 10, 99, 0

10<45, then 10<23, then 10<18, then 10>2, next

-5, 2, 10, 18, 23, 45, □, 99, 0

99>45, coming to the end of array, next

-5, 2, 10, 18, 23, 45, 99, □, 0

0<99, then 0<45 , then 0<23, then 0<18, then 0<10, then 0<2, then 0>-5, next

-5, 2, 0, 10, 18, 23, 45, 99

ii

Original List					18	45	23	2	-5	10	99	0				
Two Parts			18	45	23	2					-5	10	99	0		
Four Parts		18	45		23	2					-5	10		99	0	
Eight Parts	18		45		23		2			-5		10		99		0
Merge to Four Parts		18	45		2	23					-5	10		0	99	
Merge to Two Parts			2	18	23	45					-5	0	10	99		
Merge to Sorted Parts					-5	0	2	10	18	23	45	99				

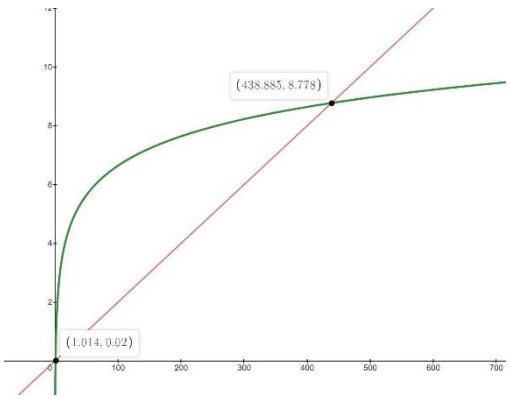
Problem 4

Time spent by each computer is:

Moe	$\frac{n}{50}$
Larry	$\frac{2\sqrt{n}}{5}$
Curly	$\log_2 n$

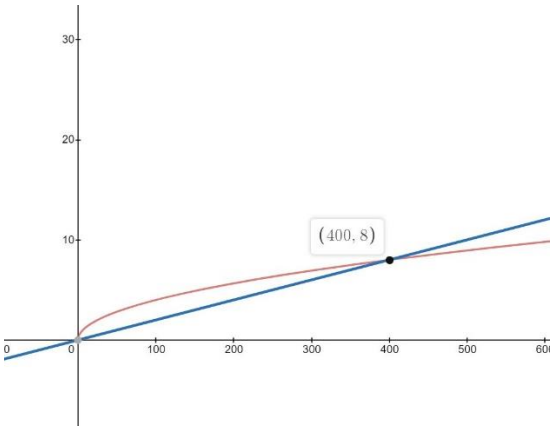
i From the graph of function we can see when n⩾439,

$\log_2 n < \frac{n}{50}$



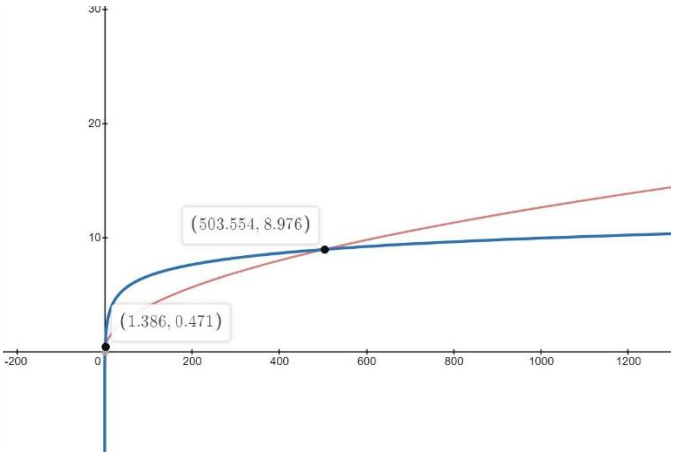
ii From the graph of function we can see when n⩾401,

$\frac{2\sqrt{n}}{5} < \frac{n}{50}$



iii From the graph of function we can see when n⩾504,

$\log_2 n < \frac{2\sqrt{n}}{5}$



Problem 5

i Bubble Sort is more efficient when using it to detect a sorted list, when the list is already sorted, the complexity of the bubble sort is only O(n), compared to $O(n^2)$ which is the worse case.

ii According to the definition of nearly-d sorted list, **any element in that list, if not sorted, should be no further than d spots from its sorted position**, which means we need to make d swaps to make sure the not-sorted element at its sorted place.

Now assume that Bubble sort need to iterate d+1 times to sort a nearly-d sorted list, which means at least one element is swaped d+1 times. It is contradicted with the hypothesis it is a nearly-d sorted list. So It is the case that Bubble Sort need only pass through a nearly-d sorted list d times to ensure the list is sorted.