CS5002 Prof. Higger
Midterm Exam
Due: Oct 14, 2020
Name: Shangjun Jiang
Start time 14:00
Finish Time 16:25

Problem 1
i

$74 = 37 \times 2 + 0$
$37 = 18 \times 2 + 1$
$18 = 9 \times 2 + 0$
$9 = 4 \times 2 + 1$
$4 = 2 \times 2 + 0$
$2 = 1 \times 2 + 0$
$1 = 0 \times 2 + 1$
$74 = (01001010)_2$

$12 = 6 \times 2 + 0$
$6 = 3 \times 2 + 0$
$3 = 1 \times 2 + 1$
$1 = 0 \times 2 + 1$
$12 = (00001100)_2$

$$(01001010)_2 + (00001100)_2 = \begin{array}{r} \color{red}{1} \\ 01001010 \\ +\quad 00001100 \\ \hline 01010110 \end{array}$$

$=$ **01010100** in 8 digit unsigned binary

There is not an overflow.

ii

$123 = 61 \times 2 + 1$

$61 = 30 \times 2 + 1$

$30 = 15 \times 2 + 0$

$15 = 7 \times 2 + 1$

$7 = 3 \times 2 + 1$

$3 = 1 \times 2 + 1$

$1 = 0 \times 2 + 1$

$123 = (01111011)_2$

$23 = 11 \times 2 + 1$

$11 = 5 \times 2 + 1$

$5 = 2 \times 2 + 1$

$2 = 1 \times 2 + 0$

$1 = 0 \times 2 + 1$

$23 = (00010111)_2$

$$(01111011)_2 + (00010111)_2 = \begin{array}{r} \color{red}{111111} \\ 01111011 \\ +\quad 00010111 \\ \hline 10010010 \end{array}$$

It is positive, thus the 8 digit two's complement binary is **10010010**.

146>127, An overflow occurred.

iii

$30 = 15 \times 2 + 0$

$15 = 7 \times 2 + 1$

$7 = 3 \times 2 + 1$

$3 = 1 \times 2 + 1$

$1 = 0 \times 2 + 1$

$30 = (00011110)_2$

In two's complement binary: -30 = 11100001+1=11100010

$42 = 21 \times 2 + 0$

$21 = 10 \times 2 + 1$

$10 = 5 \times 2 + 0$

$5 = 2 \times 2 + 1$

$2 = 1 \times 2 + 0$

$1 = 0 \times 2 + 1$

$42 = (00101010)_2$

$$(11100010)_2 + (00101010)_2 = \begin{array}{r} \color{red}{11\quad\ \ 1\ \ \ } \\ 11100010 \\ +\quad 00101010 \\ \hline 100001100 \end{array}$$

Discard the first '1', the 7 digit two's complement binary is **0001100**.

As it mentioned, it is 7 digit binary, 12<63, There is not an overflow.

iv

$240 = 15 \times 16 + 0$

$15 = 0 \times 16 + 15(\mathrm{F})$

$240 = (\mathrm{F}0)_{16}$

$20 = 1 \times 16 + 4$

$1 = 0 \times 16 + 1$

$20 = (14)_{16}$

$$(\mathrm{F}0)_{16} + (14)_{16} = \begin{array}{r} F0 \\ +\quad 14 \\ \hline 104 \end{array} = (104)_{16}$$

As it mentioned, it is 2 digit hexadecimal,

However, a 2-digit hexadecimal can repreasent up to 15×16+15×1=255

$255 < 260$

Thus, an overflow occurred

Problem 2

i True

ii True

iii False

iv True

v True

vi False

Problem 3

i

E	S	P	V	R
T	T	T	F	T
T	T	F	F	F
T	F	T	F	T
T	F	F	F	F
F	T	T	T	F
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

ii $V = \neg E \wedge S \wedge P$

iii $R = E \wedge P$

Problem 4

i

$((A \cap A^C) \cup (A \cap (A \cup B)))^C$

$= (\emptyset \cup (A \cap (A \cup B)))^C$ ....Complement Laws

$= (A \cap (A \cup B))^C$ ....Identity

$= A^C$ ....Absorption Laws

ii

$\neg((\neg P \vee \neg Q) \wedge Q)$

$= \neg((\neg P \wedge Q) \vee (\neg Q \wedge Q))$ ....Distributive Laws

$= \neg((\neg P \wedge Q) \vee F)$ ....Complement Laws

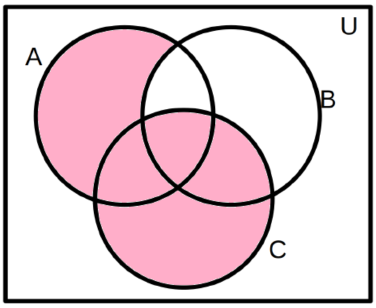
$= \neg(\neg P \wedge Q)$ ....Identity

$= \neg(\neg P) \vee \neg Q$ ....DeMorgan's Laws

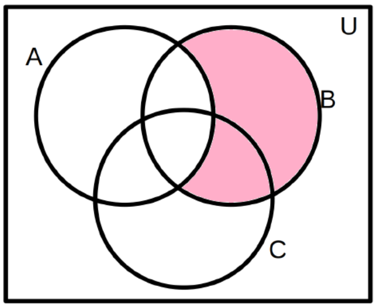
$= P \vee \neg Q$ ....Double Negation

Problem 5

$((A \cup C) - B) \cup C$



$A^C \cap (B \cup U^C)$



Problem 6

i $[160/24] = 7$ # In this document, [x] represents the cell of x.

The pigeonhole principle tells us that there are at least 7 students get the same grade.

ii This is not a pigeonhole problem.

0 other students must get the same grade as me, because I get a specific grade, I can get this grade alone, and others distributed in other grades.

Problem 7

i $A \cup B \cup C$

$= |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

$= 23 + 24 + 30 - 10 - 11 - 7 + 2$

$= 51$

ii Shaded area

$= |U| - |A \cup B|$

$= |U| - (|A| + |B| - |A \cap B|)$

$= 60 - (23 + 24 - 10)$

$= 60 - 37$

$= 23$

iii Shaded area

$= |A \cup B \cup C| - |C|$

$= 51 - 30$

$=21$

Problem 8

i select 6 packages from 10 unique packages:

$$C(10,6) = \frac{10!}{(10-6)! \times 6!} = \frac{10!}{4! \times 6!} = 210$$

The delivery driver has **210** ways to deliver.

ii discard 4 packages from 10 packages

$$C(10,4) = \frac{10!}{(10-4)! \times 4!} = \frac{10!}{6! \times 4!} = 210$$

The dilivery driver has **210** ways to discard 4 of these packages.
Literally, dicard 4 and deliver the other 6 is just equivalent to selecting 6 to deliver.
Mathmatically, $C(10,4)$ is just equal to $C(10,6)$.

iii $P(10,8) = \frac{10!}{(10-8)!} = \frac{10!}{2!} = 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 = \mathbf{1814400}$

iv It is an order-matter and repeatition-allow question.
10^{120}

v It is an order-not-matter and repetition-allow question.
n=10, k=120

$$\binom{k+n-1}{k} = \frac{129!}{(129-120)! \times 120!} = \frac{\mathbf{129}!}{\mathbf{9}! \times \mathbf{120}!}$$

vi Load each truck with 5 packages in advance.
Remain $120 - 5 \times 10 = 70$ packages
It is an order-not-matter and repetition-allow question.
n=10, k=70

$$\binom{k+n-1}{k} = \frac{79!}{(79-70)! \times 70!} = \frac{\mathbf{79}!}{\mathbf{9}! \times \mathbf{70}!}$$

vii Load 120 unique packages into 10 trucks evenly, which means each truck has 12 packages. So for the first truck, select 12 from 120 where order doesn't matter and repetition not allowed. For the second truck, select 12 from 108, for the third truck, select 12 from 96...
Thus, the number of different ways equals:
$C(120, 12) \times C(108, 12) \times C(96, 12) \times C(84, 12) \times C(72, 12) \times C(60, 12) \times C(48, 12) \times C(36, 12) \times C(24, 12) \times C(12, 12)$

$$= \frac{120!}{108! \times 12!} \times \frac{108!}{96! \times 12!} \times \frac{96!}{84! \times 12!} \times \dots \times \frac{24!}{12! \times 12!} \times \frac{12!}{0! \times 12!}$$

$$= \frac{\mathbf{120}!}{(\mathbf{12}!)^{\mathbf{10}}}$$