

CS5002 Prof. Higger

Homework 9

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Problem 1

I False

$1+1=2$ , but 1 is an odd number

II False

$3^2 = (-3)^2$ , but 3 is not equal to -3

III False

10 is even, and 4 is even,  $10+4=14$ . But  $14/2=7$ , 7 is an odd number

IV True

Proof: Let  $x$  and  $y$  be even, then we know  $x = 2m$  for some  $m$ , and  $y = 2n$  for some  $n$ , where  $m, n$  are integers. Then we have  $x + y = 2(m + n)$ , which implies  $x + y$  is even.

Problem 2

Proof:

Case 1,  $a \geq 0$  and  $b \geq 0$ , then  $|a| + |b| = a + b$ ,  $|a + b| = a + b$ , then  $|a| + |b| = |a + b|$

Case 2,  $a \leq 0$  and  $b \leq 0$ , then  $|a| + |b| = -a + (-b)$ ,  $|a + b| = -a + (-b)$ , then  $|a| + |b| = |a + b|$

Case 3,  $a \geq 0$  and  $b \leq 0$ , then  $|a + b| = |a| - |b|$  or  $|b| - |a|$ , which all less than  $|a| + |b|$

Case 4,  $a \leq 0$  and  $b \geq 0$ , then  $|a + b| = |a| - |b|$  or  $|b| - |a|$ , which all less than  $|a| + |b|$

So we have  $|a| + |b| \geq |a + b|$

### Problem 3

Proof by contrapositive:

Consider the contrapositive, if  $n$  is even, then  $2n^2 - 4n + 1$  is odd

Since  $n$  is even,  $2n^2 - 4n = 2n(n-2)$ , we know that a even number minus 2 is still even, and  $2n$  is even too. Then we have  $2n^2 - 4n$  is even, then  $2n^2 - 4n + 1$  is odd because a even number plus 1 is odd.

So, if  $2n^2 - 4n + 1$  is even,  $n$  is odd.

### Problem 4

Proof by contradiction:

Let  $n^2 + 11$  is odd and consider  $n$  is odd, we have  $n = 2m + 1$  for some  $m$ , so  $n^2 = 4m^2 + 4m + 1$ , since  $4m^2 + 4m$  is even,  $n^2 = 4m^2 + 4m + 1$  is odd, then  $n^2 + 11$  is even, so we get a contradiction.

So, we have If  $n^2 + 11$  is odd, then  $n$  is even.