CS5002 Discrete and Data Structures	Prof Higger
Fall 2020	Sept 9, 2020

HW 01 Number Representation

Due: Sept 18, 2020

Instructions:

- This homework exists to strengthen your understanding of concepts so that you may apply them elsewhere
- To get full credit, show intermediate steps leading to your answers.
- You are welcome to work on problems with classmates though you may not directly view another student's solution to a given problem while working together. Include a brief statement at the beginning of your homework which lists your homework group members: "Homework group: person A, person B". If you did not work with other students on the assignment write "Homework group: none". A 5 point penalty will be applied to all work which does not include this statement.
- Questions whose points are labelled with an addition sign are extra credit (e.g. "+4 points"). These are designed to push you, so have fun and don't worry if you're not making headway immediately: they're supposed to take some time. Excellence will come with practice.

Problem 1 [20 points: (5, 5, 5, 5)]: Base conversions

i Convert the number 753 from decimal to binary using the Division Algorithm (the fast method).

ii Convert the binary number $(10100101)_2$ to decimal. Show each of the powers of two to be added to reproduce the value.

iii Convert the decimal number 45263 to base 16 (hexadecimal) using the division method:

iv Solve for x in the equation below:
$(BFF)_{16} = (x)_{10}$

Problem 2 [18 pts: (9, 9)]: IP Addresses Your home's address expresses where your house is in a network of roads. Similarly, an IP address expresses where a computer is in a network of computers. IP addresses are 32-bit binary numbers, commonly expressed in the dotted-decimal format in which the 32 bits are grouped into four bytes (each byte has 8 bits). So IP address 129.10.116.200 is equivalent to $(10000001.00001010.01110100.11001000)_2$ because $129 = (10000001)_2$, $10 = (00001010)_2$ and so on. Be careful in noting that $192.0.2.235 \neq 19202235$.

Notation	Value	Notes
Dotted decimal	192.0.2.235	$192.0.2.235 \neq 19202235$
Hexadecimal	0xC00002EB	'0x' implies remaining digits are hex
Decimal	3221226219	
Octal	$(030000001353)_8$	
Binary	$(11000000000000000000001011101011)_2$	smaller bases lengthen representation!

Table modified from Wikipedia on IPv4.

i Convert the IP address 179.55.223.12 from dotted decimal format to hexadecimal and decimal formats.

ii Convert the IP address $(BAC2A78F)_{16}$ to the dotted decimal and decimal formats.

Problem 3 [18 pts: (6, 6, 6)] 2's Complement & Overflow

i Give the decimal value which is represented by each of the following 8-bit two's complement numbers: $(10000000)_2, (11110011)_2, (01111111)_2$

ii Give the 8-bit two's complement representations of the following decimal values: 55, 83, -79, -88.

iii Compute the following operations as a computer would, directly from the 8-bit two's complement representations: $-79 + 55$, $-79 - 88$, and $83 + 55$. Check your work by converting values back to decimal form. Indicate whether the operation results in an overflow.

Note: Use the two's complement representations from part i above.

Problem 4 [14 pts: (7, 7)]: Expressiveness of a number representation

1. What range of numbers can be represented in 10-bit two's complement?
2. What are the *minimum* number of bits necessary to represent 2100 in two's complement?

Problem 5 [16 pts (4 pts each)]: Multiplication & Addition in Binary Perform the given arithmetic problems in the given base. All values are given in binary (not 2's complement). Add place values as needed to ensure correctness (don't consider overflow here). Show every step of your work, the carry operations in particular, to receive full credit. You are welcome to convert back to decimal, or use a calculator which works in these bases to check your work.
NOTE: Though swapping the order of operations for additions and multiplications ($a + b = b + a$ and $ab = ba$) will not change the final answer, it will make grading difficult. Please leave the values ordered as shown to simplify grading.

i

$$\begin{array}{ll} & (1\ 0\ 0\ 1\ 1\ 1)_2 \\ + & (1\ 0\ 1\ 0)_2 \\ \hline \end{array}$$

ii

$$\begin{array}{ll} & (1\ 1\ 0\ 1\ 1\ 1)_2 \\ + & (1\ 0\ 1\ 1)_2 \\ \hline \end{array}$$

iii

$$\begin{array}{ll} & (1\ 0\ 1\ 0\ 0\ 0)_2 \\ \times & (1\ 1\ 0\ 1)_2 \\ \hline \end{array}$$

iv

$$\begin{array}{ll} & (1\ 1\ 0\ 1\ 1\ 1)_2 \\ \times & (1\ 1\ 0\ 0)_2 \\ \hline \end{array}$$

Problem 6 [14+4 pts (3, 5, 3, 3, +4)]: Modular Arithmetic & Binary For each of the equations below, find **every** x which satisfies the equality. If there are infinitely many x which satisfy an equality, you may find it helpful to list them out in a sequence whose pattern is obvious (e.g. $\{4, 7, 10, 13, 16, \ldots\}$)

i $8 \mod 4 = x$

ii $x \mod 5 = 3$

iii $145 + 174 \mod 17 = x$

iv $(145 \mod 17) + (174 \mod 17) = x$

v (+4 points) The previous two problems suggest a rule. Write this rule and justify if it is always true or false.

Problem 7 [+5 pts]: Ancestors How many ancestors does a person have, in total, among the previous n generations of their relatives? (Assume every person has two parents, a person is their own ancestor and that one's parents comprise the $n = 1$ generation)

Explain the relationship between your response and an unsigned binary number, what does each digit represent in the context of counting ancestors?