CS5002 Prof. Higger
Homework2
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Problem 1.

i. there are 26 lowercase letters in total
passwords must be exactly 5 characters long
so for each bit, there are 26 choices
total_num_password=26^5=26^5

ii. we need to calculate 5, 6, 7 character of password repectively and add them together
26^5+26^6+26^7=$26^5+26^6+26^7$
total_num_password=$26^5+26^6+26^7$

iii. since a, b, c is not allowed to start with, in first digit there are 23 choices
23*(26^4+26^5+26^6)=23*($26^4+26^5+26^6$)
total_num_password=23*($26^4+26^5+26^6$)

iv. 5 digits: 23*26^4-26-23=23*26^4-49
6 digits: 23*26^5-26^2-26*23*2
7 digits: 23*26^6-26^3-26^2*23*3
total_num_password
=23*26^4-49+23*26^5-26^2-26*23*2+23*26^6-26^3-26^2*23*3
=23($26^4+26^5+26^6$)-[(26+23)+(26^2+26*23*2)+(26^3+26^2*23*3)]
=23($26^4+26^5+26^6$)-66141

Problem 2.

i. [14/3]=5 #[x]means the ceil of x
There are 5 pigeons at least to share a single nest.

ii. 14*2+1=29
[29/14]=3, if there are 28 students, we got [29/14]=2, which cant satisfy the requirement. So the minimum number is 29

iii. There are 2^{10} boxes(kinds of answer) or 11 boxes(grades) in total, whereas there are only 123 students. So we can figure although there may not be two students with exactly the same answer, there must be at least [123/11]=12 students with the same grades.

Problem 3.

To ensure there are two cards identical we need to get at least [x/550]=2, which means the least x=551, furthermore, a package includes 15 cards, so 15*y>501, y=[551/15]=37, so we need to purchase at least 37 packages.

Problem 4.

To get the maximum number, we need to assume that the virus will only be sent to different mailboxes, and of course, each of the users have at least 100 mail address to send.

initial=1

first_around=100

second_around=100^2

third_aroubd=100^3

fourth_around=100^4

fifth_around=100^5

That is , there are $\sum_{n=1}^{5} 100^n$ different computers at most can be infected(exclude itself).

Problem 5.

i. A={A,B..., Y, Z}, |A|=26

|A_3|=|A*A*A|=26^3

|A_2|=|A*A|=26^2

B={x|x∈[0,9]}, |B|=10

|B_3|=10^3

|B_4|=10^4

total_num

=|A_3*B_3|+|A_2*B_4|

=|A_3|*|B_3|+|A_2|*|B_4|

=$26^3*10^3+26^2*10^4$

=$26^2*10^3*(26+10)$

=$36*26^2*10^3$

ii. L={8 left shoes}

R={12 right shoes}

total_num=|L*R|=|L|*|R|=8*12=96

Problem 6.

A=students who like Realistic Fiction

B=students who like Fantasy

C=students who like Mystery

$(A \cup B \cup C)=|A|+|B|+|C|-|A \cap B|-|A \cap C|-|B \cap C|+|A \cap B \cap C|$

=16+25+18-12-5-8+$|A \cap B \cap C|$

=39

$|A \cap B \cap C|$=39-34=5

Fantasy only=25-12-8+5=10

Realistic fiction only=16-12-5+5=4

Mystery only=18-5-8+5=10

1. either Realistic Fiction, Mystery or both?

39-10=29

2. like all three genres?

5

3. like exactly 2 genres?
   (8-5)+(12-5)+(5-5)=3+7+0=10
4. like exactly 1 genre?
   10+4+10=24

Problem 7.
{1,2,3,4,5,6,7,8,9,10,11,12,13,14}
we can pair them to 7 groups
{1,14}
{2,13}
{3,12}
{4,11}
{5,10}
{6,9}
{7,8}
where the sum of each group is 15
To avoid the sum of any two of the 8 numbers to be 15
We can not select the number from the same group, otherwise the sum will be 15
However, there are 7 boxes, but 8 objects to be put in
According to Pigeonhole, there are at least [8/7]=2 objects to be in the same boxes, which we say 'select the number from the same group'.

So, two of the selected numbers must sum to 15.