

CS5002 Prof. Higger

Homework 9

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Problem 1

I False

$1+1=2$, but 1 is an odd number

II False

$3^2 = (-3)^2$, but 3 is not equal to -3

III False

10 is even, and 4 is even, $10+4=14$. But $14/2=7$, 7 is an odd number

IV True

Proof: Let x and y be even, then we know $x = 2m$ for some m , and $y = 2n$ for some n , where m, n are integers. Then we have $x + y = 2(m + n)$, which implies $x + y$ is even.

Problem 2

Proof:

Case 1, $a \geq 0$ and $b \geq 0$, then $|a| + |b| = a + b$, $|a + b| = a + b$, then $|a| + |b| = |a + b|$

Case 2, $a \leq 0$ and $b \leq 0$, then $|a| + |b| = -a + (-b)$, $|a + b| = -a + (-b)$, then $|a| + |b| = |a + b|$

Case 3, $a \geq 0$ and $b \leq 0$, then $|a + b| = |a| - |b|$ or $|b| - |a|$, which all less than $|a| + |b|$

Case 4, $a \leq 0$ and $b \geq 0$, then $|a + b| = |a| - |b|$ or $|b| - |a|$, which all less than $|a| + |b|$

So we have $|a| + |b| \geq |a + b|$

Problem 3

Proof by contrapositive:

Consider the contrapositive, if n is even, then $2n^2 - 4n + 1$ is odd

Since n is even, $2n^2 - 4n = 2n(n-2)$, we know that a even number minus 2 is still even, and $2n$ is even too. Then we have $2n^2 - 4n$ is even, then $2n^2 - 4n + 1$ is odd because a even number plus 1 is odd.

So, if $2n^2 - 4n + 1$ is even, n is odd.

Problem 4

Proof by contradiction:

Let $n^2 + 11$ is odd and consider n is odd, we have $n = 2m + 1$ for some m , so $n^2 = 4m^2 + 4m + 1$, since $= 4m^2 + 4m$ is even, $n^2 = 4m^2 + 4m + 1$ is odd, then $n^2 + 11$ is even, so we get a contradiction.

So, we have If $n^2 + 11$ is odd, then n is even.