CS5002 Discrete and Data Structures	Prof Higger
Fall 2020	Sept 16, 2020

HW 2 Statement Representation (Logic)

Due: Sept 25, 2020

Instructions:

- This homework exists to strengthen your understanding of concepts so that you may apply them elsewhere
- To get full credit, show intermediate steps leading to your answers.
- You are welcome to work on problems with classmates though you may not directly view another student's solution to a given problem while working together. Include a brief statement at the beginning of your homework which lists your homework group members: "Homework group: person A, person B". If you did not work with other students on the assignment write "Homework group: none". A 5 point penalty will be applied to all work which does not include this statement.
- Questions whose points are labelled with an addition sign are extra credit (e.g. "+4 points"). These are designed to push you, so have fun and don't worry if you're not making headway immediately: they're supposed to take some time. Excellence will come with practice.

Problem 1 [20 points: (4 pt each)]: Arithmetic and Logical Operators Evaluate each of the following statements as True or False. Show intermediate steps for full credit. Try by simplifying only one operator ($\wedge, \vee, >, =, \ldots$) at a time if the whole expression is difficult to understand.

i $(4 \geq 4) \wedge (4 > 5)$

ii $(4 \geq 4) \vee (4 > 5)$

iii $(5 = 2 + 3) \wedge (5 < 0)$

iv $\neg(5 = 4)$

v $\neg((5 = 2 + 3) \vee (5 < 0))$

Problem 2 [12 points: (2 pt each)]: English to Logic (Lawn) Express each sentence using logical operations $\neg$, $\wedge$, $\vee$ and the propositional variables r, s, and l defined below. The use of the word "or" in the sentences below always means inclusive or.

r	it is raining
s	the sprinkler is on
l	the lawn is wet

i It is raining and the lawn is wet

ii It is not raining but the lawn is wet

iii The lawn is not wet and it is not raining

iv There is no way that it is raining

v It is raining or the sprinklers are on

vi Despite the fact that it is raining, the sprinklers are on

Problem 3 [6+4 points: (3, 3, +2, +2)]: English to Logic (Zoo) Consider the eclectic animal collection at the local zoo. Express each sentence using logical operations $\neg$, $\wedge$, $\vee$ and the propositional variables h, w, and d defined below. The use of the word "or" in the sentences below always means inclusive or.

h	They have hampsters
w	They have whales
d	They have dinosaurs

These are more challenging than the previous examples, you may find it helpful to go case by case through a truth table to ensure your expression is consistent with the sentence.

i The zoo doesn't have any hampsters or whales, but it does have dinosaurs.

ii The zoo has at least two of the three groups of animals.

iii The zoo has exactly[1] two of the three groups of animals.

iv The zoo has, at most, one of these groups of animals.

Problem 4 [8 points]: Truth Table Complete the truth table below for the statement

$$\neg[(x \wedge y) \vee z]$$

Use the intermediate columns to build the statement incrementally.

x	y	z	$x \wedge y$	$(x \wedge y) \vee z$	$\neg[(x \wedge y) \vee z]$
F	F	F			
F	F	T			
F	T	F			
F	T	T			
T	F	F			
T	F	T			
T	T	F			
T	T	T			

[1] no more or less

Problem 5 [12 points (3 points each)]: Tautology & Contradiction Simplify each of the following statements. Label each statement as a tautology[2], contradiction[3] or neither. For this problem, you need not label the boolean identities used to manipulate the expression.

i $(p \vee \neg p) \wedge (q \vee \neg q)$

ii $(p \wedge \neg p) \wedge (q \wedge \neg q)$

iii $\neg(p \vee q) \vee \neg(\neg p \wedge \neg q)$

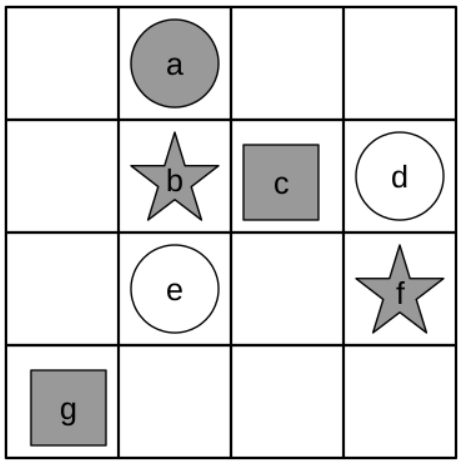
iv $(\neg p \vee \neg q) \wedge \neg(\neg p \vee \neg q)$

Problem 6 [10 points (5 points each)]: Boolean Simplification Simplify the expressions below. Your simplified expression should have the least possible number of operators ($\neg$, $\wedge$, $\vee$) possible. Show each step by applying and labelling the identity used (DeMorgan, indempotent law, etc).

i $\neg p \vee (p \wedge \neg q)$

ii $\neg[(\neg p \vee \neg q) \wedge p]$

Problem 7 [21 points (3 points each)]:Existential, Universal Qualifiers & Predicates



Use the following predicates:

- `circle`(x) is True when x is a circle
- `rect`(x) is True when x is a rectangle
- `star`(x) is True when x is a star

[2]a statement which is always True
[3]a statement which is always False

- $\texttt{shade}(x)$ is True when x is shaded in
- $\texttt{next\_to}(x, y)$ is True when the squares containing x and y touch (i.e. they're immediate neighbors left-right, top-bottom, or they share a corner).

to determine if each of the statements below is true or false. For statements which are true, justify your answer by describing all relevant elements (maybe all of them). For statements which are false, justify your answer by giving a single counter-example. For example, given the statement $\texttt{rect}(b)$ a correct response is, 'This is false as b is not a rectangle'.

i $\texttt{circle}(e) \wedge \texttt{rect}(g)$

ii $\forall x \, \texttt{rect}(x)$

iii $\exists x \, \neg\texttt{shade}(x) \wedge \texttt{star}(x)$

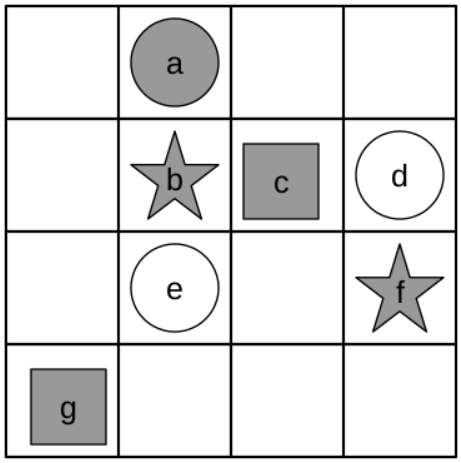
iv $\texttt{shade}(x) \rightarrow \texttt{star}(x)$

v $\text{star}(x) \rightarrow \texttt{shade}(x)$

vi $\forall x \exists y \, \texttt{next\_to}(x, y)$

vii $\exists x \forall y \, \texttt{next\_to}(x, y)$

Problem 8 [11 points (1, 2, 2, 2, 2, 2)]: Contrapositive, Converse, Inverse



Use the following predicates:

- $\texttt{circle}(x)$ is True when x is a circle
- $\texttt{rect}(x)$ is True when x is a rectangle
- $\texttt{star}(x)$ is True when x is a star
- $\texttt{shade}(x)$ is True when x is shaded in

- `next_to`(x, y) is True when the squares containing x and y touch (i.e. they're immediate neighbors left-right, top-bottom, or they share a corner).

Given the statement:

$$\neg\texttt{shade}(x) \rightarrow \texttt{circle}(x)$$

i Tell if the statement is true or false.

ii Write the **contrapositive** of this statement using logical symbols (not english). Tell if this new statement is true or false.

iii Write the **converse** of this statement using logical symbols (not english). Tell if this new statement is true or false.

iv Write the **inverse** of this statement using logical symbols (not english). Tell if this new statement is true or false.

v Putting this particular Tarski world aside ... Is it ever possible that a statement is True and its contrapositive is not? Explain your thinking in one sentence.

vi Is it ever possible that a statement's converse is True and the same statement's inverse is not? Explain your thinking in one sentence.