CS5002 Prof. Higger
Homework2
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Problem 1.

i. $((A^C \cap B) \cup (A^C \cap B^C))^C$
$= (A^C \cap B)^C \cap (A^C \cap B^C)^C$
$= (A \cup B^C) \cap (A \cup B)$
$= A \cup (B^C \cap B)$
$= A \cup \emptyset$
$= A$

ii. $(A^C \cap B^C)^C \cap U$
$= (A^C \cap B^C)^C$
$= A \cup B$

iii. $(A \cup A) \cap (B \cup A^C)$
$= A \cap (B \cup A^C)$
$= (A \cap B) \cup (A \cap A^C)$
$= (A \cap B) \cup \emptyset$
$= (A \cap B)$

Problem 2.

i. $e(n) = |n|$ where $e : N \to N$ (where we define N to exclude 0)
Bijective. For x∈N(exclude 0), each x is positive, which means e(n)=n, where we can say that each element of the codomain is mapped to by exactly one element of the domain, and they are just the same value. So according to definition of bijective, it is bijective.

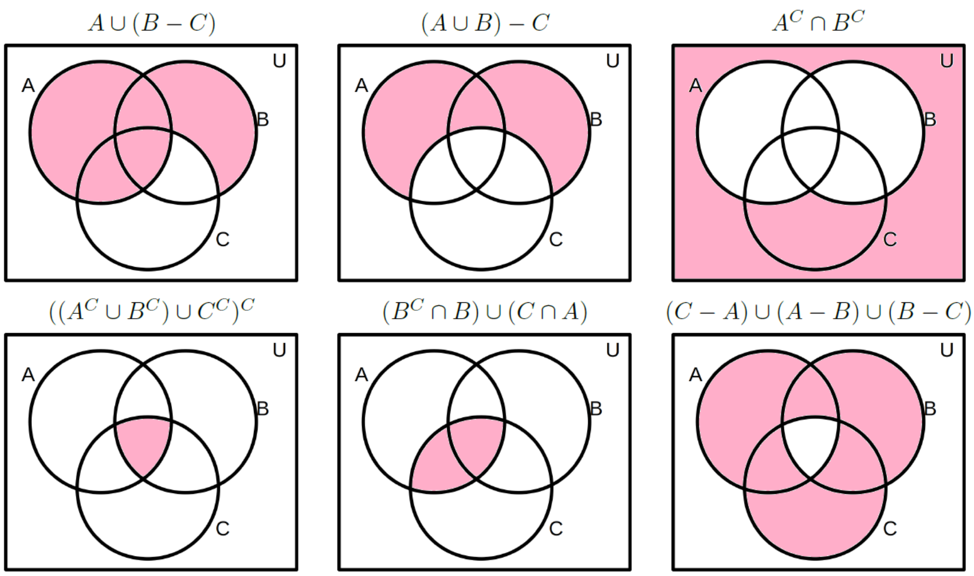
ii. $f(n) = n$ where $f : Z \to R$
Injective. This is injective however not surjective, that is because, for any f(a)=f(b), we can figure that a=b, which proves it is injective, however, the codomain is all real numbers, which means not all elememts in codomain is mapped to a an element in domain, so it is not surjective.

iii. $g(x) = \max(x, 100)$ where $g : R \to R$. The max function returns the maximum of its two input arguments so that $g(1000) = 1000$ and $g(10) = 100$
neither Injective nor Surjective. When x<100, we get g(x)=100, which means it is not an injection. For any x in R, g(x)⩾100, so elements in codomain which are less than 100 are not mapped to, so it is not a surjection.

iv. Let A = {apple, banana, apricot, cake, orange} and B = {4, 5, 6, 7}. Define the function $h : A \to B$ which counts the number of letters in the input. For example, h(apple) = 5 and h(cake) = 4.

Surjective. We can list that h(apple)=5, h(banana)=6, h(apricot)=7, h(cake)=4, h(orange)=6, so for each elements in B, it is mapped to by at least one element in A, which shows it is surjective, besides, as h(banana)=h(orange)=6, and orange≠banana, it is not injective.

Problem 3.



Problem 4.

i. A ∪ B 0111 OR

ii. A ∩ B 0001 AND

iii. A^C 1010 NOT

Problem 5.

i. {4}

ii. S = {x| -9⩽x⩽9 and x∈Z}

Problem 6.

i. {∅}

ii. {{1}, ∅}

iii. {∅, {1}, {2}, {1, 2}}

iv. {∅, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, {1, 2, 3}}

v. 2^n

Problem 7.

	Symmetric	Reflexive	Transitive	Equivilence Relation
less than	Flase	Flase	True	Flase
greater than or eaqual to	Flase	True	True	Flase
Absolute Value Function	True	True	True	True