CS5002 Prof. Higger
Homework2
Name: Shangjun Jiang
Homework group: Wei Han, Shangjun Jiang

Problem 1.

i. $(4 \geq 4) \wedge (4 > 5)$
$= T \wedge F$
$= F$

ii. $(4 \geq 4) \vee (4 > 5)$
$= T \vee F$
$= T$

iii. $(5 = 2 + 3) \wedge (5 < 0)$
$= T \wedge F$
$= F$

iv. $\neg(5 = 4)$
$= \neg F$
$= T$

v. $\neg((5 = 2 + 3) \vee (5 < 0))$
$= \neg(T \vee F)$
$= \neg T$
$= F$

Problem 2.

i. $r \wedge l$
ii. $\neg r \wedge l$
iii. $\neg l \wedge \neg r$
iv. $\neg r$
v. $r \vee s$
vi. $r \wedge s$

Problem 3.

i. $\neg(h \vee w) \wedge d$
ii. $(h \wedge w) \vee (h \wedge d) \vee (w \wedge d)$
iii. $(h \wedge w \wedge \neg d) \vee (h \wedge d \wedge \neg w) \vee (d \wedge w \wedge \neg h)$
iv. $(\neg w \wedge \neg d) \vee (\neg h \wedge \neg d) \vee (\neg h \wedge \neg w)$

Problem 4

x	y	z	$x \wedge y$	$(x \wedge y) \vee z$	$\neg[(x \wedge y) \vee z]$
F	F	F	F	F	T
F	F	T	F	T	F
F	T	F	F	F	T
F	T	T	F	T	F
T	F	F	F	F	T
T	F	T	F	T	F
T	T	F	T	T	F
T	T	T	T	T	F

Problem 5

i. $(p \vee \neg p) \wedge (q \vee \neg q)$

$= T \wedge T$

$= T$

ii. $(p \wedge \neg p) \wedge (q \wedge \neg q)$

$= F \wedge F$

$= F$

iii. $\neg(p \vee q) \vee \neg(\neg p \wedge \neg q)$

$= \neg(p \vee q) \vee (p \vee q)$

$= T$

iv. $(\neg p \vee \neg q) \wedge \neg(\neg p \vee \neg q)$

$= (\neg p \vee \neg q) \wedge (p \wedge q)$

$= \neg(p \wedge q) \wedge (p \wedge q)$

$= F$

Problem 6.

i. $\neg p \vee (p \wedge \neg q)$

$= (\neg p \vee p) \wedge (\neg p \vee \neg q)$…………Distributive laws

$= T \wedge (\neg p \vee \neg q)$…………Complement laws

$= \neg p \vee \neg q$…………Domination and Identity

$= \neg(p \wedge q)$…………DeMorgan's laws

ii. $\neg[(\neg p \vee \neg q) \wedge p]$

$= \neg[(p \wedge \neg p) \vee (p \wedge \neg q)]$…………Distributive laws

$= \neg[F \vee (p \wedge \neg q)]$…………Complement laws

$= \neg(p \wedge \neg q)$…………Domination and Identity

$= \neg p \vee q$…………Demorgan's laws

Problem 7.

i. T Because e is circle and g is rect.

ii. F When x=e, it is a cirvle, and rect(e)=F

iii. F When star(x)=T, x=b or f, which are all shade, thus ¬shade(x)=F

iv. F e.g. shade(gr) is not a star

v. T stars(x=b or f) are all shade

vi. T x=a, ∃y=b
x=b, ∃y=c
x=c, ∃y=d
x=d, ∃y=f
x=e, ∃y=b
x=f, ∃y=d
x=g, ∃y=e
So, we can conclude ∀x,∃y satisfy next_to(x, y)=T

vii. F next_to(a, e)=F
next_to(b, f)=F
next_to(c, g)=F
next_to(d, g)=F
next_to(e, a)=F
next_to(f, a)=F
next_to(g, a)=F
So, there does not exist such a 'x'.

Problem 8.

i. True. ¬shade(x) → circle(x) Because all not shade patterns are circle.

ii. contrapositive: ¬circle(x) → shade(x), it is True

iii. converse: circle(x) → ¬shade(x), it is False

iv. inverse: shade(x) → ¬circle(x), it is False

v. No. The Boolean value of a statement is the same as its contrapositive.

Explanation: Imagine a venn diagram, in which A is a subset of B, which means A can deduce B, hence ¬B is a subset of ¬A also, so we can conclude that ¬B can deduce ¬A too.

vi. No. This question is equal to question v., the relationship of converse and inverse statement is the same as initial and contrapositive statement, which has been explained above, it is not possible.