CS5002 Prof. Higger
Homework 8
Name: Shangjun Jiang
Homework group: Wei Han, Shangjun Jiang

Problem 1

i {C,E}

ii {G}

iii {A,D}

iv {H,I,G,F}

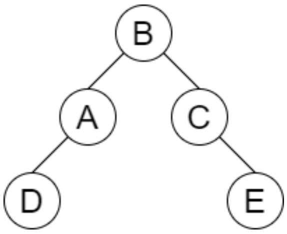
v {B,A,D,C,E}

vi {C,E,I,G}

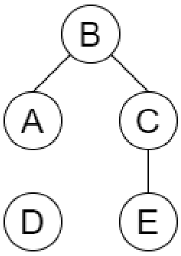
Problem 2

i If every pair of nodes has an edge, there should be a path of AB-BC-CD-DE-EF-FA, which contradicts with the statement of 'an acyclic graph'.

ii



iii



iv According to the definition of path, the length of a path can be infinite. e.g.(A-E-E-A-...-E) cite from textbook:

Here are some paths in the graph of example 16.1:

$\langle A, C, B, F\rangle$ is a path from vertex A to vertex F.

$\langle F, E, D\rangle$ is a path from vertex F to vertex D.

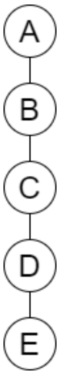
$\langle D, E, F\rangle$ is a path from vertex D to vertex F.

$\langle D, F\rangle$ is also a path from vertex D to vertex F.

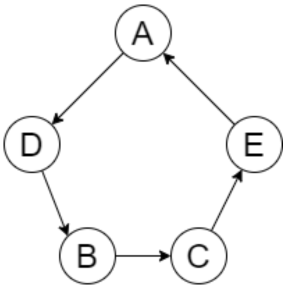
$\langle D, A, C, B, F\rangle$ is another path from vertex D to vertex F.

$\langle D, A, C, B, F, D, E, F, D, E, F\rangle$ is a long path from vertex D to vertex F.

v



vi



Problem 3

i ABFGCDEIHJK

ii ABCDFEHGIJK

iii EFHAIGBJKCD

iv EFABCDGHIJK

Problem 4

A→B→E→G weight:15

a	b	c	d	e	f	g
0	5(a)	10(a)	∞	∞	∞	∞
done	5	9(b)	∞	8(b)	∞	∞
done	done	9(b)	∞	8(b)	16(b)	15(e)
done	done	9(b)	17(c)	done	16(b)	15(e)
done	done	done		done		

At this step, the answer has come out. The path is A→B→E→G, and the list of nodes visited every step is shown as the table, which is abeg.

Problem 5

i $A \to B \to C$

$B \to A \to C \to D$

$C \to A \to B$

$D \to B$

ii

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

iii

$A \to B \to C$

$B \to D$

$C \to B$

D

iv

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Problem 6

Because every pair of cities have exactly one airline flies the route,

routes operated by the two airlines in total are

$$C(n,2)=n\times(n-1)/2$$

Now, assume that airline B can only connect (n-1) cities, which meet the requirement.

So, the most route it can operate will be

$$C(n-1,2)= (n-1)\times(n-2)/2$$

the least route the other airline operates will be

$$C(n,2)-C(n-1,2)=n-1$$

And obviously the left city must connect all other (n-1) cities, which means the other airline can connect all the n cities. It shows there exist an airline connect all cities.

k represent the route airline A operates

k=n-1, when airline A fly the least airlines, airline A have to connect all cities.

k=n, obviously airline A fly a new route which was operated by the airline B

k=n+1, airline A fly two new routes which was operated by the airline B

...

k=C(n,2)-1, when airline B have to connect the (n-1) cities with n-2 airlines

we have travelled all possiblities, all the cases shown that either A must connect all cities to assure only (n-1) vertax are covered by B