CS5002. Prof Higger

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Problem 1

i. $753 = 376 \times 2 + 1$
$376 = 188 \times 2 + 0$
$188 = 94 \times 2 + 0$
$94 = 47 \times 2 + 0$
$47 = 23 \times 2 + 1$
$23 = 11 \times 2 + 1$
$11 = 5 \times 2 + 1$
$5 = 2 \times 2 + 1$
$2 = 1 \times 2 + 0$
$1 = 0 \times 2 + 1$

$\Rightarrow 753_{10} = 1011110001_2$

ii. $(10100101)_2 = 1 \times 2^7 + 0 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$
$= 128 + 32 + 4 + 1$
$= 165$

iii. $45263 = 2828 \times 16 + 15$ (F)
$2828 = 176 \times 16 + 12$ (C)
$176 = 11 \times 16 + 0$
$11 = 0 \times 16 + 11$ (B)

$\Rightarrow 45263_{10} = B0CF_{16}$

iv. $(BFF)_{16} = 11 \times 16^2 + 15 \times 16^1 + 15 \times 16^0$
$= 2816 + 240 + 15$
$= 3071$

Problem 2.

i. $179_{10} = 2^7+2^5+2^4+2^1+2^0 = 10110011_2$

$55 = 2^5+2^4+2^2+2^1+2^0 = 0011\ 0111_2$

$223 = 2^7+2^6+2^4+2^3+2^2+2^1+2^0 = 11\ 01\ 1111_2$

$12 = 2^3+2^2 = 0000\ 1100$

the binary - hex convert table as follows:

0000	0	0100	4	1000	8	1100	C
0001	1	0101	5	1001	9	1101	D
0010	2	0110	6	1010	A	1110	E
0011	3	0111	7	1011	B	1111	F

$(10110011\,00110111\,1101\,1111\,00001100)_2 = (B337DF0C)_{16}$

$= 3006783244_{10}$

ii. $BAC2A78F = 1011\ 1010.\ 1100\,0010.\ 1010\ 0111.\ 1000\ 1111$

$= 186.194.167.143$ ······ dotted decimal

$= 3133319055$ ······ decimal

Problem 3

i. $10000000_2 \Rightarrow 01111111 \Rightarrow 10000000 = -2^7 = -128$

$11110011_2 \Rightarrow 00001100 \Rightarrow 00001101 = -(2^3+2^2+2^0) = -13$

$0111\ 1111_2 \Rightarrow 2^6+2^5+2^4+2^3+2^2+2^1+2^0 = 2^7-1 = 127$

ii. $55 \Rightarrow 2^5+2^4+2^2+2^1+2^0 = 0011\ 0111$

$83 \Rightarrow 2^6+2^4+2^1+2^0 = 0101\ 0011$

$79 = 2^6+2^3+2^2+2^1+2^0 = 01001111 \xRightarrow{flip} 1011\ 0000 \xRightarrow{+1} 10110001$

$\therefore -79 = 10110001_2$

$88 = 2^6+2^4+2^3 = 0101\ 1000 \Rightarrow 10100111 \Rightarrow 10101000$

$\therefore -88 = 1010\ 1000_2$

iii. $-79+55=-24$

$$\begin{array}{r} -79 \\ +\ 55 \end{array} \Rightarrow \begin{array}{r} 10110001 \\ +\,00110111 \\ \hline 11101000 \end{array}$$

$11101000 \Rightarrow 00010111 \Rightarrow 00011000 = -(2^4+2^3) = -24$

correct!

It's not an overflow

$-79-88=-167$

$$\begin{array}{r} -79 \\ +\ -88 \end{array} \Rightarrow \begin{array}{r} 10110001 \\ +\,10101000 \\ \hline 101011001 \end{array} = 101011001$$

It's an overflow

$101011001 \Rightarrow 010100110 \Rightarrow 010100111 = -(2^7+2^5+2^2+2^1+2^0) = -167$

$83+55=138$

$$\begin{array}{r} 83 \\ +55 \end{array} \Rightarrow \begin{array}{r} 01010011 \\ +\,00110111 \\ \hline 10001010 \end{array} = 10001010 = 2^7+2^3+2^1 = 138$$

It's an overflow

Problem 4.

1. When in 8-bit two's complement, the range is $[-128, 127]$.

   $\Rightarrow [-2^7, 2^7-1]$. $n \in Z$

   So we can figure in 10-bit two's complement,

   the range is $[-2^9, 2^9-1] \Rightarrow [-512, 511]$.

2. $\because\ 2^{11} < 2100 < 2^{12}$ $(2^{11}=2048,\ 2^{12}=4096)$

   We need the range $[-2^{12}, 2^{12}-1]$ to represent 2100

   $\therefore$ the minimum number of bits $= 13$

Problem 5.

i.

$$\begin{array}{r} 100111 \\ +\quad 1010 \\ \hline 110001 \end{array}$$

ii.

$$\begin{array}{r} 110111 \\ +\quad 1011 \\ \hline 1000010 \end{array}$$

iii.

$$\begin{array}{r} 101000 \\ \times\quad 1101 \\ \hline 101000 \\ 000000 \\ 101000 \\ 101000 \\ \hline 1000001000 \end{array}$$

iv.

$$\begin{array}{r} 110111 \\ \times\quad 1100 \\ \hline 000000 \\ 000000 \\ 110111 \\ 110111 \\ \hline 1010010100 \end{array}$$

Problem 6.

i. $8 \bmod 4 = 0 \Rightarrow x = 0$

ii. $x \bmod 5 = 3 \Rightarrow x = 5n+3$, $n \in N$. eg. $x \in \{3, 8, 13, 18 \cdots\}$

iii. $145 + 174 \bmod 17 = x$

$\Rightarrow 319 \bmod 17 = 13 \Rightarrow x = 13$

iv. $145 \bmod 17 = 9$

$174 \bmod 17 = 4 \Rightarrow x = 9+4 = 13$

V. $(x+y) \bmod n = x \bmod n + y \bmod n$, $x, y, n \in$ integer

It is always true.

Prove: assume $x \bmod n = k_1 \Rightarrow x = n \times a + k_1$ $\quad k_1, k_2 < n$

$y \bmod n = k_2 \Rightarrow y = n \times b + k_2$ $\quad \therefore a+b = A+B$

$\Rightarrow (x+y) \bmod n = k_3 = x+y = n \times (A+B) + k_3$ $\quad k_1 + k_2 = k_3$

$\Rightarrow n \times a + k_1 + n \times b + k_2 = n \times (A+B) + k_3$

To add some explanation here, the upper equation shows that n(a+b)+(k1+k2)=n(A+B)+k3, from linear algebra we know that only if the coefficients are the same can the two line be coincident.

Problem 7.

So the person is his/her own ancestor.

the total number of ancestor is $1+2+4+8 \cdots$

$$=(1+10+100+1000\cdots)_2$$

$$\Rightarrow 2^{n+1}-1=(\underbrace{111111\cdots1}_{n+1})_2$$

So each "1" in the binary format represents

the number added when generation n is added by 1.