

## HW 3: Sets

**Due:** Oct 2, 2020

**Instructions:**

- This homework exists to strengthen your understanding of concepts so that you may apply them elsewhere
- To get full credit, show intermediate steps leading to your answers.
- You are welcome to work on problems with classmates though you may not directly view another student's solution to a given problem while working together. Include a brief statement at the beginning of your homework which lists your homework group members: "Homework group: person A, person B". If you did not work with other students on the assignment write "Homework group: none". A 5 point penalty will be applied to all work which does not include this statement.
- Questions whose points are labelled with an addition sign are extra credit (e.g. "+4 points"). These are designed to push you, so have fun and don't worry if you're not making headway immediately: they're supposed to take some time. Excellence will come with practice.

**Problem 1 [15 points (5 points each)]: Set Algebra** Simplify each of the following expressions. Do not use the set difference operator in your simplifications.  $U$  indicates the universal set which contains all elements.

i  $((A^C \cap B) \cup (A^C \cap B^C))^C$

ii  $(A^C \cap B^C)^C \cap U$

iii  $(A \cup A) \cap (B \cup A^C)$

**Problem 2 [16 Points (4 points each)]: Function Properties** For each of the following functions, tell whether they are Injective, Surjective or Bijective. Justify your response for each of the three properties with a one sentence explanation.

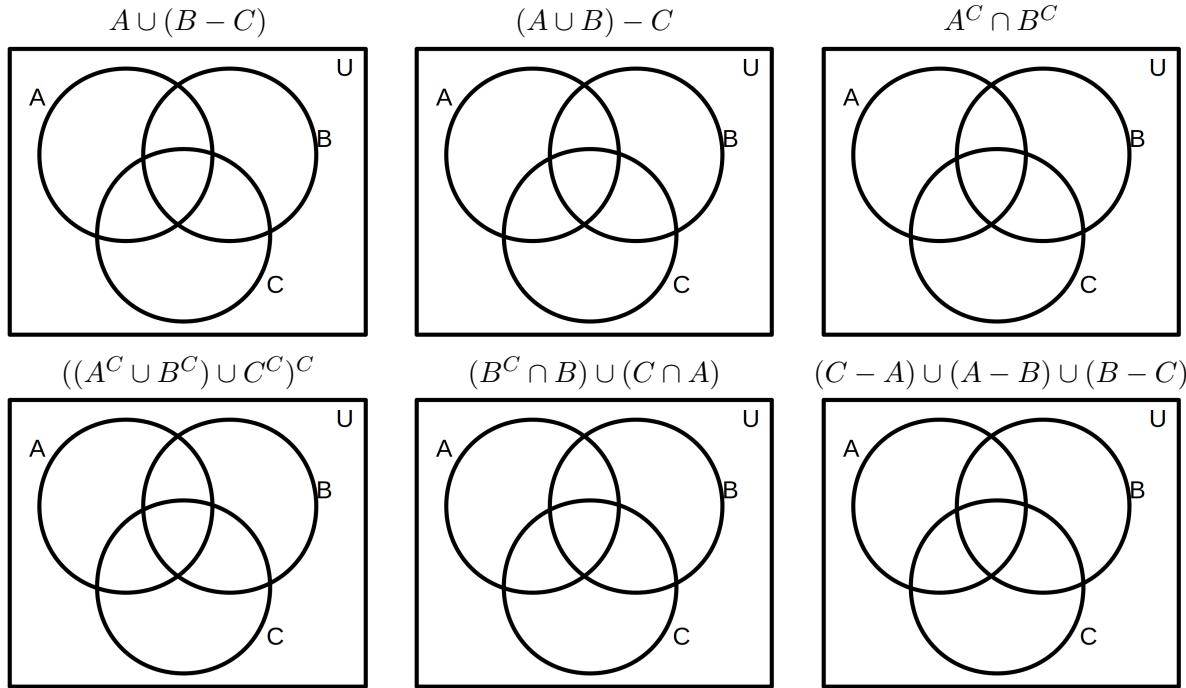
i  $e(n) = |n|$  where  $e : \mathbb{N} \rightarrow \mathbb{N}$  (where we define  $\mathbb{N}$  to exclude 0)

ii  $f(n) = n$  where  $f : \mathbb{Z} \rightarrow \mathbb{R}$

iii  $g(x) = \max(x, 100)$  where  $g : \mathbb{R} \rightarrow \mathbb{R}$ . The `max` function returns the maximum of its two input arguments so that  $g(1000) = 1000$  and  $g(10) = 100$ .

iv Let  $A = \{\text{apple, banana, apricot, cake, orange}\}$  and  $B = \{4, 5, 6, 7\}$ . Define the function  $h : A \rightarrow B$  which counts the number of letters in the input. For example,  $h(\text{apple}) = 5$  and  $h(\text{cake}) = 4$ .

**Problem 3 [18 points (3 points each)]: Set Operations** Shade the indicated regions of the following Venn diagrams.



**Problem 4 [15 points (5 points each)]: Computer Representation** Consider the bit string representation of sets  $A$  and  $B$ :

$$A = \{\text{paul, george}\}$$

$$B = \{\text{ringo, george}\}$$

$U$	john	paul	ringo	george
$A$	0	1	0	1
$B$	0	0	1	1

For each of sets below:

- add a row to the table which gives the bit string representation of the set
- tell which logical operator (AND, NOT, XOR, OR) of the bit string representations of  $A$ ,  $B$  yield the same bit string representation of the set

i  $A ∪ B$

ii  $A ∩ B$

iii  $A^C$

**Problem 5 [8 points (4 points each)]: Set Builder Notation**

i Express the set:

$$S = \{n \in \mathbb{N} \mid (-11 \leq n) \wedge (n \bmod 7 = 4) \wedge (n < 10)\}$$

by explicitly writing each item in a set (e.g.  $\{1, 2, 3\}$ ). We assume that the natural numbers includes 0 in the set above.

ii Express the set of all integers whose absolute value is less than 10 using set builder notation.

**Problem 6 [16 points (2, 3, 3, 3, 5)]: Power Sets** For each of the sets below, write out its powerset by listing the entire set.

i  $\emptyset$

ii  $A = \{1\}$

iii  $B = \{1, 2\}$

iv  $C = \{1, 2, 3\}$

v How many elements are in the powerset of  $N = \{1, 2, 3, \dots, n\}$  for some  $n \in \mathbb{N}$ ?

**Problem 7 [12 points (1 points each)]: Relations** Each of the operations below is a relation on  $\mathbb{N} \times \mathbb{N}$  where  $0 \in \mathbb{N}$ . Tell which relations has which property by writing true or false into each empty cell of the table below. No explanation is needed.

	Symmetric	Reflexive	Transitive	Equivilence Relation
less than				
greater than or equal to				
Absolute Value Function				

Remember, we say that "less than" relates 5 to 10 because  $5 < 10$ . Similarly, "greater than or equal to" relates 100 to 100 because  $100 \geq 100$ . Finally, the "Absolute Value Function" relates 10 to 10 because  $10 = |10|$