活性化関数、損失関数の導関数

Python 中級
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シグモイド関数 / Sigmoid Function

$$f(x) = \frac{1}{1+e^{-x}}$$

$$f'(x) = \frac{d}{dx}\frac{1}{1+e^{-x}} = \frac{-1}{(1+e^{-x})^2}\cdot -e^{-x} = \frac{e^{-x}}{(1+e^{-x})^2}$$

$$= \frac{1}{1+e^{-x}}\frac{e^{-x}}{1+e^{-x}} = \frac{1}{1+e^{-x}}\frac{1+e^{-x}-1}{1+e^{-x}} = \frac{1}{1+e^{-x}}\left(1-\frac{1}{1+e^{-x}}\right)$$

$$= f(x)\big(1-f(x)\big)$$

ReLU (Rectified Linear Unit)

$$f(x) = \begin{cases} x & \texttt{if } x > 0, \\ 0 & \texttt{otherwise.} \end{cases}$$

$$f'(x) = \begin{cases} 1 & \texttt{if } x > 0, \\ 0 & \texttt{otherwise.} \end{cases}$$

平均二乗誤差 / Mean Squared Error

$$L = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$

$$L_i' = -\frac{2}{n}(y_i - \hat{y}_i)$$

フーバー損失 / Huber Loss

$$L = \frac{1}{n}\sum_{i=1}^{n} L_\delta(y_i - \hat{y}_i) \quad , \qquad L_\delta(x) = \begin{cases} \frac{1}{2}x^2 & \texttt{if } |x| \le \delta, \\ \delta(|x| - \frac{1}{2}\delta) & \texttt{otherwise.} \end{cases}$$

$$L_i' = \frac{1}{n}L_\delta'(y_i - \hat{y}_i) \quad , \qquad L_\delta'(x) = \begin{cases} -x & \texttt{if } |x| \le \delta, \\ -\delta & \texttt{if } x > \delta, \\ \delta & \texttt{if } x < -\delta \end{cases}$$