Online Quickest Change Detection for Multiple Gaussian Sequences Using Stochastic Bandits

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Introduction

We consider the online multi-stream quickest change-point detection problem. An agent is given M independent data streams, one of which contains a change-point at an unknown time step ν shifting its mean by an unknown amount. The agent's goal is to minimize its detection delay while controlling for false alarms. Uninterrupted monitoring of every stream can be costly, so the agent only observes one stream at each time step. Our proposed algorithm combines an ϵ-greedy strategy with a detection procedure for unknown post-change means.

Background

- CUSUM is an asymptotically optimal detection procedure. The detection statistic at time t is
$$T_t^{\text{CUSUM}} = \max_{0\le k<t} \sum_{i=k+1}^{t} \log\left(\frac{f_1(X_i)}{f_0(X_i)}\right).$$
- The Generalized Likelihood Ratio is an extension of CUSUM which works when there is an unknown post-change parameter. The detection statistic is
$$T_t^{\text{GLR}} = \max_{0\le k<t} \sup_{\theta\in\Theta} \sum_{i=k+1}^{t} \log\left(\frac{f_\theta(X_i)}{f_0(X_i)}\right).$$
Romano et al. (2023) provide an implementation called FOCuS.

ϵ-FOCuS Algorithm

- At time t, each stream $m \in [M]$ has a local GLR statistic $T_t^{(m)}$.
- An exploration choice G_t is sampled from Bernoulli(ϵ), where ϵ is the probability of exploration. If $G_t = 1$, the agent randomly samples a stream. Otherwise,
$$A_t = \operatorname*{argmax}_{m\in[M]} T_{t-1}^{(m)}.$$
- The stopping procedure is
$$\tau = \inf\left\{ t \ge 1 : \max_{m\in[M]} T_t^{(m)} \ge \lambda \right\}.$$

Theoretical Results

- In the minimax formulation the goal is to minimize the worst-case delay
$$\text{CADD}(\tau) := \sup_{\nu\ge 0} \mathbb{E}[\tau - \nu | \tau > \nu],$$
subject to the average run length
$$\text{ARL}(\tau) := \mathbb{E}_\infty[\tau] \ge \gamma,$$
for $\gamma > 0$.
- It was shown in Theorem 1 of Lai (1998) that the asymptotic lower bound for the detection delay satisfies
$$\inf_{\tau:\text{ARL}(\tau)\ge\gamma} \text{CADD}(\tau) \ge \frac{(1+o(1))\log\gamma}{D(f_1||f_0)},$$
as $\gamma \to \infty$.

Theorem 1

Consider an agent using ϵ-FOCuS on M streams, and suppose that stream 1 contains a change-point at time $\nu = 0$ that shifts its distribution from $\mathcal{N}(0,1)$ to $\mathcal{N}(\mu_1, 1)$, where $\mu_1 \neq 0$. For a detection threshold $\lambda > 0$ and an exploration parameter $\epsilon \in (0,1)$, the expected time until detection τ is bounded via
$$\mathbb{E}_{M,0}[\tau] \le \frac{1}{1-\epsilon}\left(\frac{2\lambda(1+o(1))}{\mu_1^2} + C_{\epsilon,\mu_1,M}\right)$$
as $\lambda \to \infty$, where $C_{\epsilon,\mu_1,M} > 0$ is a constant determined by ϵ, μ_1, and M.

Theorem 2

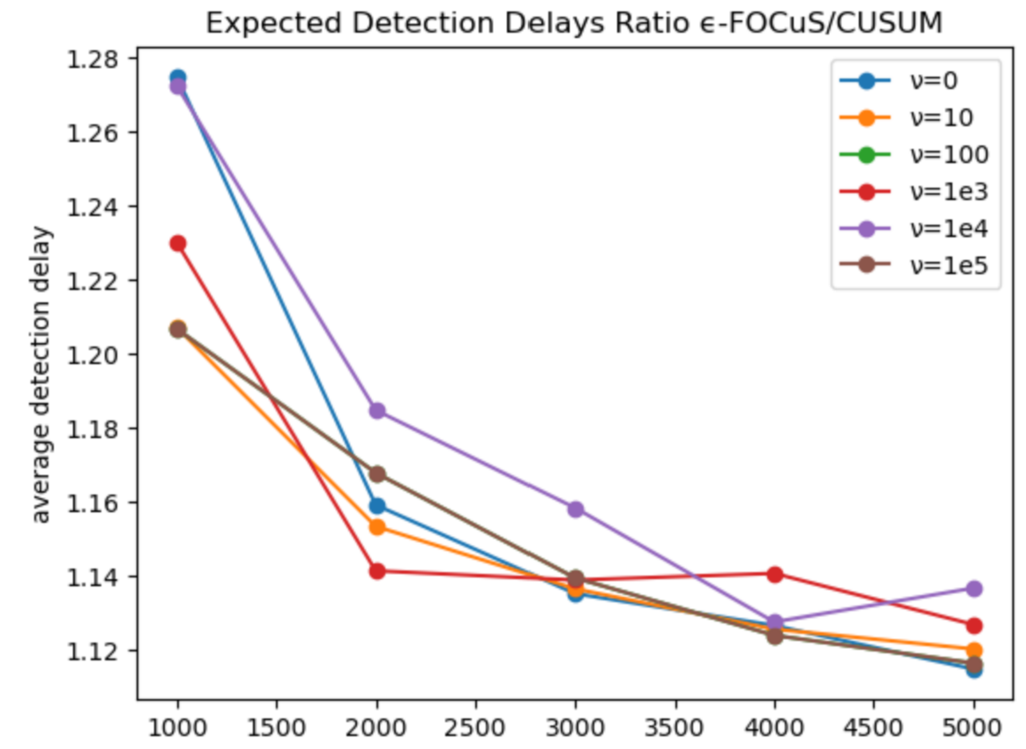
Consider an agent using ϵ-FOCuS on M streams, where all streams are distributed according to f_0, i.e., no change has occurred. Then given a detection threshold $\lambda > 0$ and an exploration parameter $\epsilon \in (0,1)$, the expected time until a false detection τ is bounded via
$$\mathbb{E}_{M,\infty}[\tau] \ge \frac{e^\lambda\sqrt{\pi}}{M\sqrt{\lambda}\int_0^\infty x g(x)^2 dx}$$
as $\lambda \to \infty$, where for $x > 0$, $g(x)$ is
$$g(x) = 2x^{-2}\exp\left[-2\sum_{n=1}^{\infty} n^{-1}\Phi\left(-xn^{1/2}/2\right)\right]$$

Discussion

- A consequence of the above theorems is that for a given ARL parameter $\gamma > 0$, we can derive a threshold such that $\mathbb{E}_{M,\infty}[\tau] \ge \gamma$ implies $\mathbb{E}_{M,0}[\tau] \le \left(\frac{1+o(1)}{1-\epsilon}\right)\frac{\log\gamma}{D(f_1||f_0)}$ as $\gamma \to \infty$.
- This is within a constant factor $\frac{1}{1-\epsilon}$ of Lai's (1998) lower bound on the single-stream detection delay. This implies our algorithm is within a constant factor of the asymptotic performance of the single-stream CUSUM algorithm, despite having to search for the change-point over M streams and having no prior knowledge of the post-change parameter.
- This is validated by our simulation results.

Simulations

- We compare the expected detection delay across a set of change-points with the asymptotic expected detection delay for CUSUM. We set $M = 10$ and $\epsilon = 0.1$. The unknown post-change parameter is $\mu_1 = 1$. The asymptotic ratio in our experiments matches the theoretical results and approaches $1/(1-\epsilon) \approx 1.11$.



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