Assignment 23 – Credit Risk Modeling and Simulation

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Introduction:

The purpose of this assignment is to model a credit-risky portfolio of corporate bonds. Consider a structural model for portfolio credit risk described in class. Using the data for 100 counterparties, simulate 1-year losses for each corporate bond. You will need to generate 3 sets of scenarios:

- Monte Carlo approximation 1: 5000 in-sample scenarios ($N = 1000 \cdot 5 = 5000$ (1000 systemic scenarios and 5 idiosyncratic scenarios for each systemic), non-Normal distribution of losses);
- Monte Carlo approximation 2: 5000 in-sample scenarios ($N = 5000$ (5000 systemic scenarios and 1 idiosyncratic scenario for each systemic), non-Normal distribution of losses);
- True distribution: 100000 out-of-sample scenarios ($N = 100000$ (100000 systemic scenarios and 1 idiosyncratic scenario for each systemic), non-Normal distribution of losses)

Evaluate VaR and CVaR at quantile levels 99% and 99.9% for the two portfolios:
(1) one unit invested in each of 100 bonds.
(2) equal value (dollar amount) is invested in each of 100 bonds.

Questions:

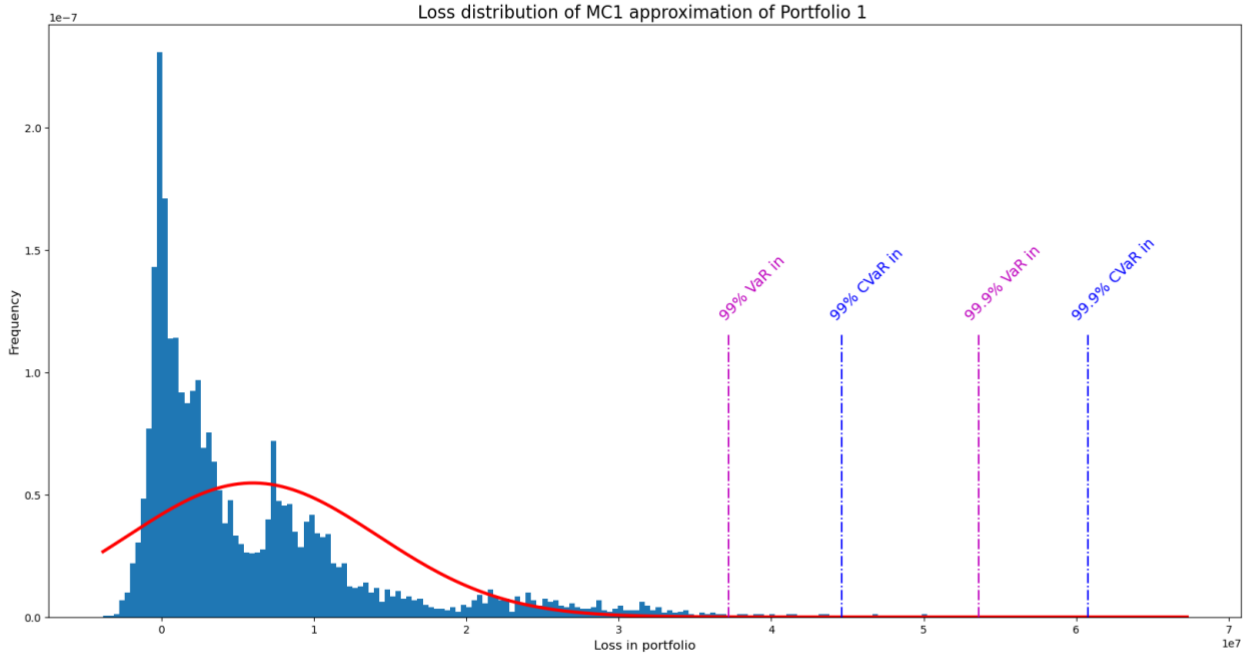
1. Implement portfolio credit risk simulation model in Python: (See jpynb file)

2. Result analysis:

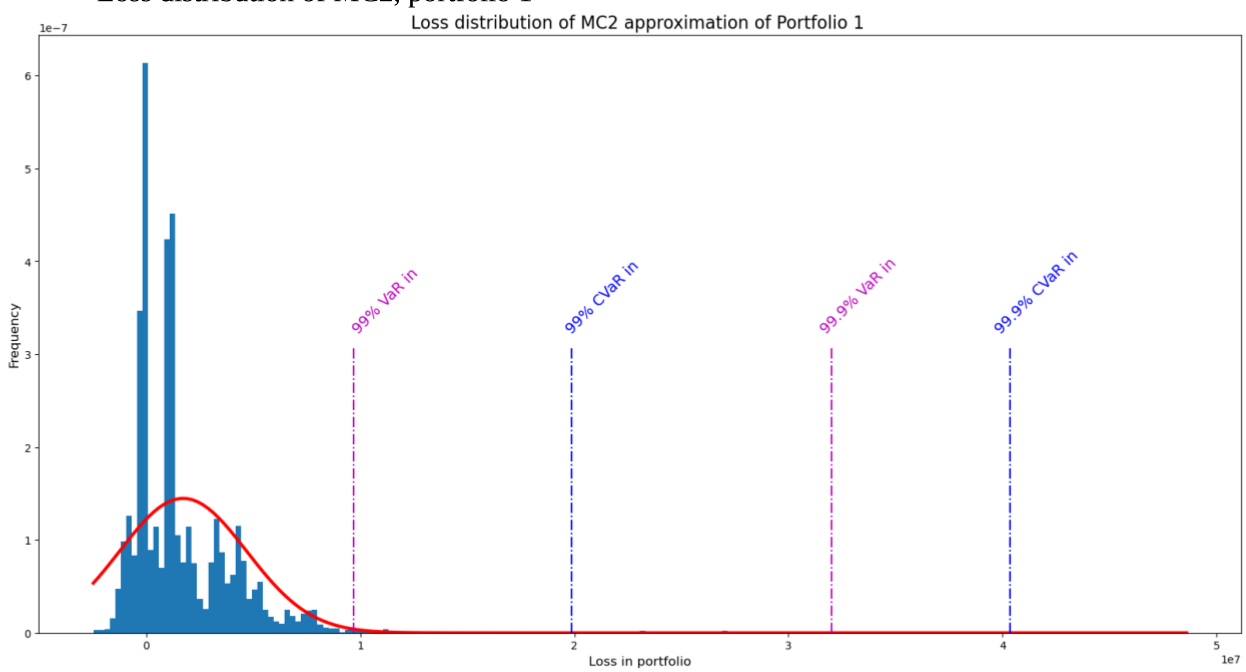
- Produce the following output from your Python code. (See Appendix I)
- Plot loss distributions in Python that illustrate both out-of-sample and in-sample results. Include plots that help illustrating your analysis in the report.

Portfolio1:

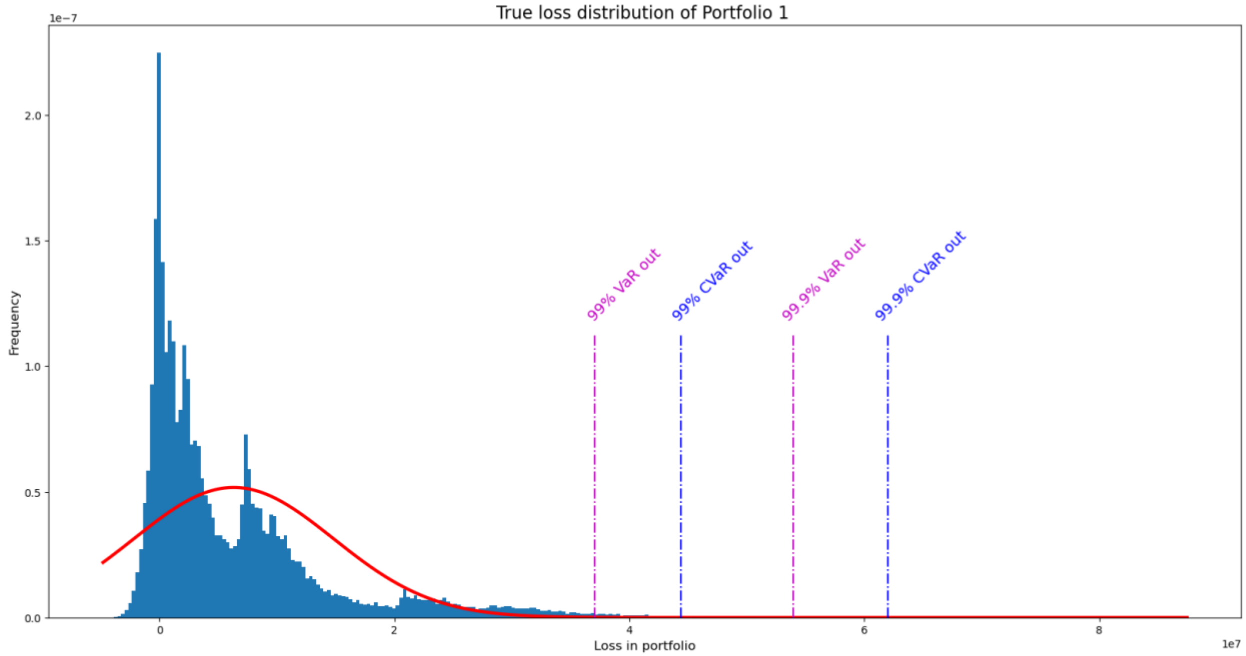
Loss distribution of MC1, portfolio 1



Loss distribution of MC2, portfolio 1

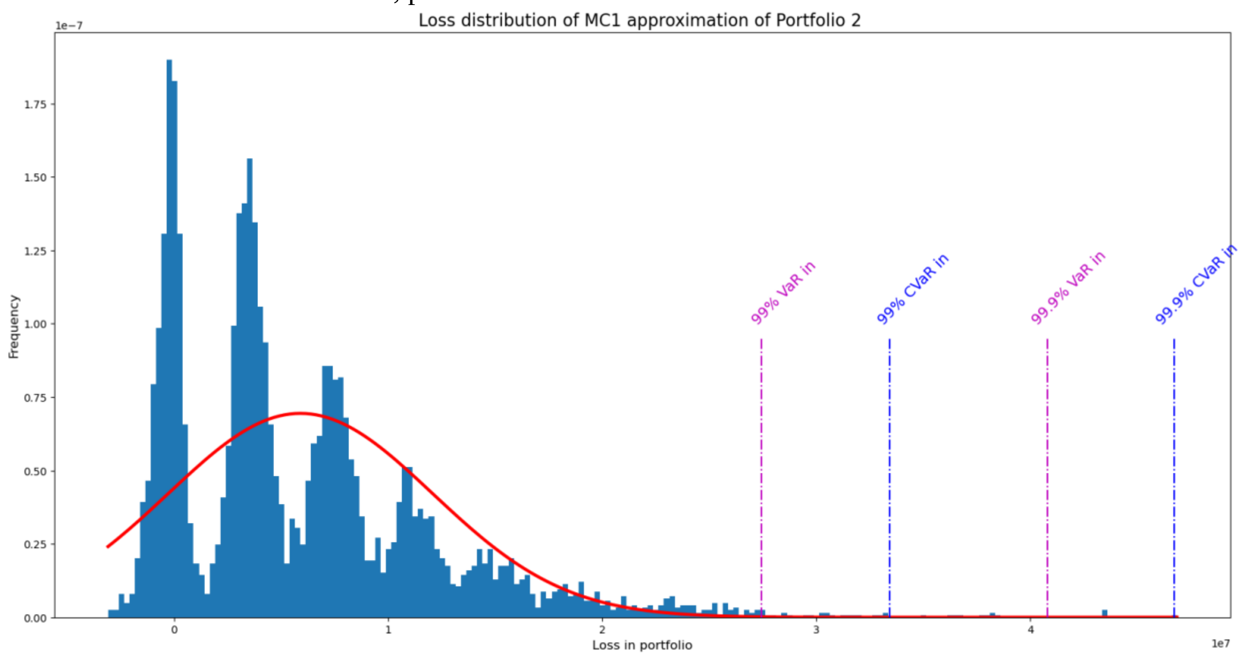


True loss distribution, portfolio 1

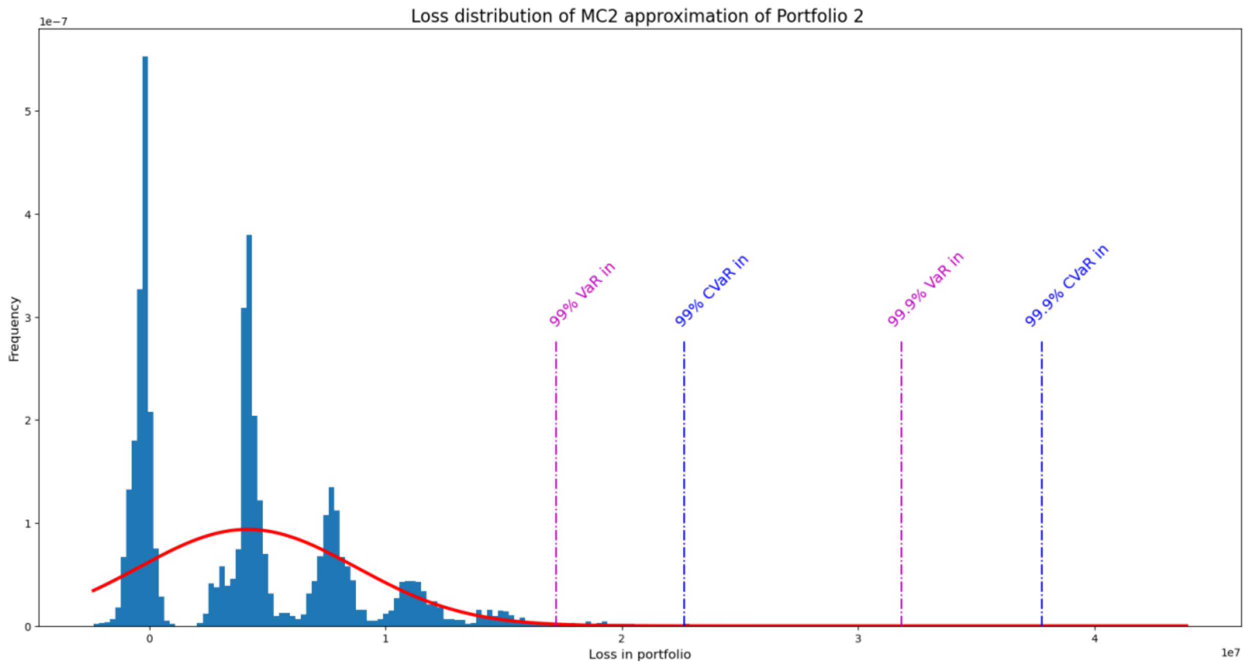


Portfolio 2:

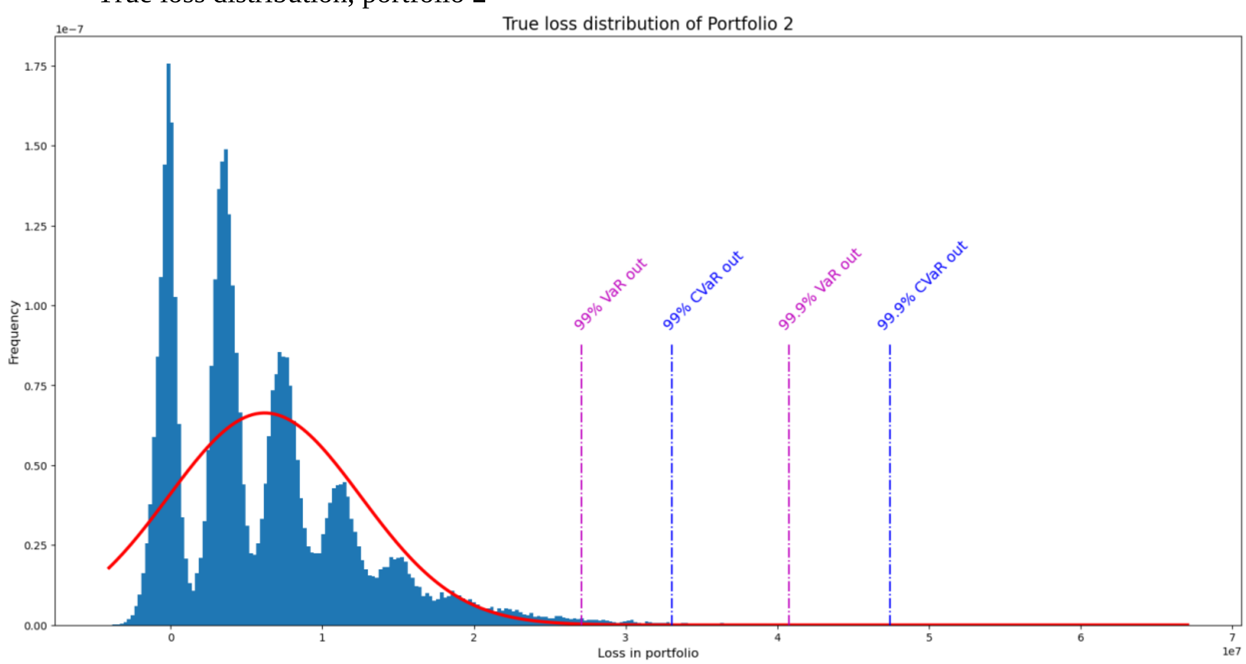
Loss distribution of MC1, portfolio 2



Loss distribution of MC2, portfolio 2



True loss distribution, portfolio 2



- Analyze sampling error when comparing non-Normal approximations to the true (out-of-sample) loss distribution. Analyze model error when comparing Normal approximations to the true (out-of-sample) loss distribution. Summarize the two types of errors in the tables.

Sampling error table

	distribution	sampling error
0	99% MC1 VaR(Portfolio 1)	0.436191
1	99% MC1 CVaR(Portfolio 1)	0.633688
2	99.9% MC1 VaR(Portfolio 1)	0.630509
3	99.9% MC1 CVaR(Portfolio 1)	2.284513
4	99% MC1 VaR(Portfolio 2)	1.329064
5	99% MC1 CVaR(Portfolio 2)	1.476920
6	99.9% MC1 VaR(Portfolio 2)	0.049306
7	99.9% MC1 CVaR(Portfolio 2)	1.724072
8	99% MC2 VaR(Portfolio 1)	73.837737
9	99% MC2 CVaR(Portfolio 1)	66.140069
10	99.9% MC2 VaR(Portfolio 1)	40.656633
11	99.9% MC2 CVaR(Portfolio 1)	40.190977
12	99% MC2 VaR(Portfolio 2)	36.372405
13	99% MC2 CVaR(Portfolio 2)	38.426593
14	99.9% MC2 VaR(Portfolio 2)	21.935234
15	99.9% MC2 CVaR(Portfolio 2)	23.750718

Monte Carlo approximation 2 have significantly larger sampling error comparing with Monte Carlo approximation 1 for both portfolios. In Monte Carlo approximation 1, portfolio 1 yields smaller sampling error. However, for Monte Carlo approximation 2, portfolio 2 yields smaller sampling error.

Model error table

	distribution	sampling error
0	99% N1 VaR(Portfolio 1)	29.313825
1	99% N1 CVaR(Portfolio 1)	41.278841
2	99.9% N1 VaR(Portfolio 1)	39.389228
3	99.9% N1 CVaR(Portfolio 1)	49.962266
4	99% N1 VaR(Portfolio 2)	21.974888
5	99% N1 CVaR(Portfolio 2)	35.991893
6	99.9% N1 VaR(Portfolio 2)	36.208123
7	99.9% N1 CVaR(Portfolio 2)	48.206483
8	99% N2 VaR(Portfolio 1)	76.693477
9	99% N2 CVaR(Portfolio 1)	93.734180
10	99.9% N2 VaR(Portfolio 1)	79.786431
11	99.9% N2 CVaR(Portfolio 1)	93.207461
12	99% N2 VaR(Portfolio 2)	45.081724
13	99% N2 CVaR(Portfolio 2)	61.345076
14	99.9% N2 VaR(Portfolio 2)	54.913038
15	99.9% N2 CVaR(Portfolio 2)	68.130006

If we wrongly assumed that counterparty losses follow Normal distribution, we can still draw the conclusion that N2 has higher sampling error than N1 in scenarios.

3. Discuss possible strategies for minimizing impacts of sampling and model errors:

- If you report the in-sample VaR and CVaR to decision-makers in your bank, what consequences for the bank capital requirements it may have?
  In both VaR and CVaR analyses, MC1 and MC2 underestimate the loss in comparison to the true distribution. This means that the actual risk of portfolio 1 and portfolio 2 is higher than what is reported in the in-sample analysis. As a result, there may be an impact on increased capital requirements to compensate for the underestimated risk.

- Can you suggest techniques for minimizing impacts of sampling and model errors?
  Increasing the sample size is a potential strategy to minimize the impact of sampling error. Additionally, exploring different data generation methods to make sure the sample is representative for the population.

APPENDIX

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Portfolio 1:

Out-of-sample: VaR 99.0% = $37058388.88, CVaR 99.0% = $44380478.51
In-sample MC1: VaR 99.0% = $37220034.12, CVaR 99.0% = $44615312.95
In-sample MC2: VaR 99.0% = $9695313.06, CVaR 99.0% = $19870034.55
In-sample No: VaR 99.0% = $26063214.57, CVaR 99.0% = $28939551.48
In-sample N1: VaR 99.0% = $26195157.75, CVaR 99.0% = $29083205.15
In-sample N2: VaR 99.0% = $8637021.77, CVaR 99.0% = $9644101.50

Out-of-sample: VaR 99.9% = $53959872.96, CVaR 99.9% = $62024639.78
In-sample MC1: VaR 99.9% = $53619651.33, CVaR 99.9% = $60791919.56
In-sample MC2: VaR 99.9% = $32021605.40, CVaR 99.9% = $40337639.42
In-sample No: VaR 99.9% = $32547154.36, CVaR 99.9% = $34897155.28
In-sample N1: VaR 99.9% = $32705495.75, CVaR 99.9% = $35065064.28
In-sample N2: VaR 99.9% = $10907216.31, CVaR 99.9% = $11730012.24


Portfolio 2:

Out-of-sample: VaR 99.0% = $27070457.21, CVaR 99.0% = $33032372.21
In-sample MC1: VaR 99.0% = $27430240.99, CVaR 99.0% = $33432181.27
In-sample MC2: VaR 99.0% = $17224280.89, CVaR 99.0% = $22630117.72
In-sample No: VaR 99.0% = $20938574.10, CVaR 99.0% = $23085370.09
In-sample N1: VaR 99.0% = $21121754.63, CVaR 99.0% = $23289202.34
In-sample N2: VaR 99.0% = $14866628.38, CVaR 99.0% = $16425979.78

Out-of-sample: VaR 99.9% = $40769597.52, CVaR 99.9% = $47432110.79
In-sample MC1: VaR 99.9% = $40789699.35, CVaR 99.9% = $46729213.60
In-sample MC2: VaR 99.9% = $31826690.85, CVaR 99.9% = $37749038.57
In-sample No: VaR 99.9% = $25777957.11, CVaR 99.9% = $27531914.56
In-sample N1: VaR 99.9% = $26007691.45, CVaR 99.9% = $27778521.60
In-sample N2: VaR 99.9% = $18381773.12, CVaR 99.9% = $19655781.46
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