Neural Population Geometry

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Outline

Introduction

Problem Description & Motivation

- Advances in recording techniques: thousands to millions of neurons simultaneously
- Challenges in studying large neural populations:
  - Neurons respond to multiple variables simultaneously
  - Traditional tuning-based analyses have limitations for complex tasks
- Shift from single-neuron tuning to *geometric* approaches
- Neural computations emerge from structured, high-dimensional activity patterns
- Neural manifolds: low-dimensional geometric structures in high-dimensional neural state space

Neural Manifolds

- Key properties of neural manifolds:
  - Dimensionality: intrinsic dimensions of the representation
  - Curvature: how the manifold deviates from flat space
  - Separability: distinguishability between different manifolds
  - Capacity: number of distinct manifolds a system can support
- Examples of manifold structures in the brain:
  - Ring-shaped manifolds in hippocampus (head direction)
  - Ring-shaped manifolds in visual cortex (orientation)
- Direct mappings between manifold geometry and task variables provide mechanistic insights

Bridging Biological and Artificial Neural Networks

- Neural population geometry provides a unified framework for understanding:
  - Biological neural circuits
  - Artificial neural networks (ANNs)
- Neural computation as geometric transformations
- Population-level representations enable:
  - Robust information processing
  - Efficient encoding
  - Scalable computation
- Reveals fundamental principles that transcend specific implementations

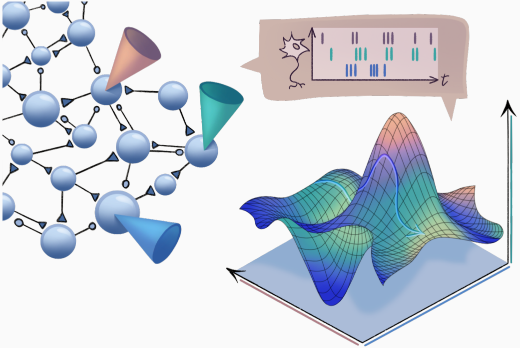
Paper Summary

- We demonstrate neural population geometry as a unified framework for understanding information encoding
- Focus on representation of circular variables—a fundamental computational challenge
- Parallel analyses of:
  - Orientation encoding in mouse visual cortex
  - Trained convolutional neural network (CNN)
- Show how similar ring-shaped manifolds emerge in both systems
- Circular manifolds arise from computational demands rather than architectural constraints
- Comparative approach reveals common principles across biological and artificial systems

Preliminaries

Neural State Space & Population Activity

- Neural state space: each axis represents a single neuron
- Population activity: $\mathbf{r} = (r_1, r_2, \ldots, r_n) \in \mathbb{R}^n$
- Repeated stimulus presentations create point clouds
- Neural variability induces fluctuations
- Manifolds emerge from stimulus responses



Mathematical Foundations of Neural Manifolds

- **Manifold**: Topological space that locally resembles Euclidean space
- **Neural manifold**: Population activity constrained to lower-dimensional subspace
- Formal definition: For stimulus condition s, the neural manifold $\mathcal{M}_s$ is:

$$\mathcal{M}_s = \{\mathbf{r}(t) \in \mathbb{R}^n : \text{condition } s \text{ is presented at time } t\} \tag{1}$$

- Real neural data extends beyond strict mathematical definition:
  - Neural variability creates point clouds rather than isolated points
  - Experimental limitations lead to sparse sampling of the manifold

Types of Neural Manifolds

- **Object/perceptual manifolds**: Arise from identity-preserving variations

$$\mathcal{M}_{\text{obj}} = \{f(T_\theta(\mathbf{x})) : \theta \in \Theta\} \tag{2}$$

  where f is neural encoding function, T_θ is transformation with parameter θ

- **Point-cloud manifolds**: Empirical manifestation in experimental data

$$\hat{\mathcal{M}} = \{\mathbf{r}_i : i = 1, 2, \ldots, k\} \tag{3}$$

  for stimuli $\{s_1, s_2, \ldots, s_k\}$ and responses $\{\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_k\}$

Manifold Estimation Techniques

- **Linear dimensionality reduction**:
  - Principal Component Analysis (PCA)
  - Efficient but misses nonlinear structure
- **Nonlinear dimensionality reduction**:
  - Isomap, t-SNE, MDS, UMAP, etc.
  - Preserve geometric relationships
  - May fail with complex topological structures
- **Topologically-motivated methods**:
  - SPUD (Spline Parameterization for Unsupervised Decoding)
  - Persistent homology
  - Essential for circular or toroidal structures

Neural Population Geometry

Key Concepts in Neural Population Geometry

- **Dimensionality**: Intrinsic dimensionality d of manifold $\mathcal{M}$
  - Estimated using PCA or nonlinear methods
- **Curvature**: How manifold deviates from flat space
  - High curvature: small stimulus changes $\rightarrow$ large neural response changes
- **Separability**: Distinguishability between manifolds

$$d(\mathcal{M}_i, \mathcal{M}_j) = \min_{\mathbf{r}_i \in \mathcal{M}_i, \mathbf{r}_j \in \mathcal{M}_j} \|\mathbf{r}_i - \mathbf{r}_j\| \tag{4}$$

- **Capacity**: Number of distinct manifolds a neural system can support

Simulation Study

Circular Variables Experiment

- Demonstration of geometric structures in neural representations
- Simulation experiment: encoding circular variables
- Illustrates how topological structures naturally emerge
- CNN trained to predict orientation of visual grating stimuli
- Analogous to orientation selectivity in visual cortex

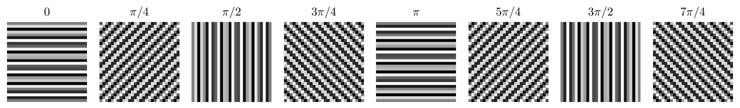


Figure 1: Sample grating stimuli at different orientations

CNN Architecture

- Convolutional layers followed by fully connected layers
- 32-dimensional latent space analyzed for geometric properties
- Network trained to predict sine and cosine components of orientation angle
- Sine-cosine encoding naturally handles circular topology
  - For small ϵ, angles ϵ and $2\pi - \epsilon$ are similar orientations
  - But numerically distant in raw angle representation
  - With sine-cosine: $(\cos(\epsilon), \sin(\epsilon)) \approx (\cos(2\pi - \epsilon), \sin(2\pi - \epsilon)) \approx (1, 0)$

Results: Circular Manifold

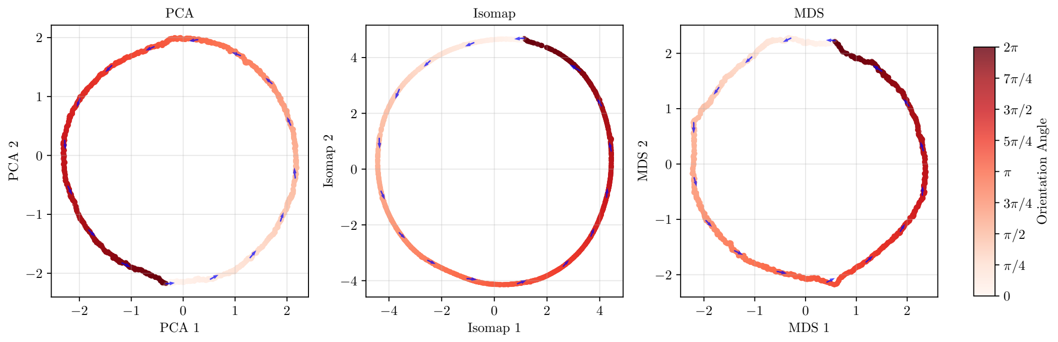


Figure 2: Dimensionality reduction of the CNN's latent space reveals a circular manifold. Each point represents a grating stimulus, colored by orientation angle. PCA, Isomap, and MDS all recover the circular topology.

- Circular manifold emerged in latent space
- Similar orientations positioned close together

Key Principles Demonstrated

- **Manifold structure**: Low-dimensional manifold (circle) embedded in high-dimensional space
- **Topological correspondence**: Manifold topology matches task space topology
- **Continuous representation**: Similar stimuli mapped to nearby points
- **Dimensionality reduction**: High-dimensional input compressed to essential variables

Insight
Circular topology emerged naturally through backpropagation—the network discovered that a circular representation is the most efficient way to encode a periodic variable

Biological Neural Networks

- Application of geometric principles to biological neural data
- Mouse visual cortex encodes orientation information
- Populations of neurons collectively form a ring-shaped manifold
- Manifold geometry reflects circular topology of orientation space
- Comparison with artificial neural network findings reveals common principles

Conclusion

Conclusion

- Neural population geometry provides a unified framework for understanding neural computation
- Circular manifolds arise naturally from computational demands of the task
- Similar geometric structures emerge in both biological and artificial systems
- Comparative approach reveals common principles:
    - Dimensionality
    - Curvature
    - Topological structure
- Insights into fundamental computational strategies that transcend specific neural implementations

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