[공분산 행렬 도출]

$$X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1m} \\ \vdots & & & \\ x_{n1} & x_{n2} & \cdots & x_{nm} \end{bmatrix} \qquad S = \begin{bmatrix} S_{x_1}^2 & S_{x_1x_2} & \cdots & S_{x_1x_m} \\ \vdots & & & \\ S_{x_mx_1} & & \cdots & S_{x_m}^2 \end{bmatrix}$$

n개 데이터, m개 feature

각 feature(m) 간 공분산!

S는 어떻게 생성하는가?

- 표본공분산 $= \underbrace{E[(x_{ij} - \bar{x}_j)(x_{ik} - \bar{x}_k)]}_{\text{j feature 와 k feature 의 공분산}}$

(i = 1 ~ N 데이터)

<S 도출>

① $\bar{x}_j$ 구하기 (j feature 평균)

$$\bar{x}_j = \frac{1}{N} \sum_{i=1}^{n} x_{ij}$$

② S_j^2 구하기

$$S_j^2 = \frac{1}{N} \sum_{i=1}^{n} (x_{ij} - \bar{x}_j)^2$$

③ S_{jk} 구하기

$$S_{jk} = \frac{1}{N} \sum_{i=1}^{n} (x_{ij} - \bar{x}_j)(x_{ik} - \bar{x}_k)$$

④ 공분산 행렬 구하기

1) 특징 벡터 정의

$$x_i = \begin{bmatrix} x_{i1} \\ \vdots \\ x_{im} \end{bmatrix}$$

2) 특징행렬 정의

$$X = \begin{bmatrix} - x_1^T - \\ \vdots \\ - x_n^T - \end{bmatrix}$$

3) 표본공분산 행렬 S ⇐ 각 feature 간 "편차 곱 평균"

$j-k$

평균: $\frac{1}{\text{데이터갯수}}$ (데이터 전체)

$$S = \sum_{i=1}^{n} (x_i - \bar{x})(x_i - \bar{x})^T = \sum_{i=1}^{n} (i\text{데이터 편차곱})$$

$$\bar{x} = \begin{bmatrix} \bar{x}_{01} \\ \vdots \\ \bar{x}_{0m} \end{bmatrix}$$

각 feature 평균만 모아놓은 평균벡터

1특징 평균

$$x_i = \begin{bmatrix} x_{i1} \\ \vdots \\ x_{im} \end{bmatrix} \qquad x_i - \bar{x} = \begin{bmatrix} x_{i1} - \bar{x}_{01} \\ \vdots \\ x_{im} - \bar{x}_{0m} \end{bmatrix}$$

$$S = \frac{1}{n} \sum_{i=1}^{n} \begin{bmatrix} x_{i1} - \bar{x}_{01} \\ \vdots \\ x_{im} - \bar{x}_{0m} \end{bmatrix} \begin{bmatrix} x_{i1} - \bar{x}_{01} & x_{i2} - \bar{x}_{02} & \cdots & x_{im} - \bar{x}_{0m} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{n} \sum_{i=1}^{n} (x_{i1} - \bar{x}_{01})(x_{i1} - \bar{x}_{01}) & \frac{1}{n} \sum_{i=1}^{n} (x_{i1} - \bar{x}_{01})(x_{i2} - \bar{x}_{02}) & \cdots \\ \vdots & & \end{bmatrix}$$

(모든 데이터의 1feature 편차) × (〃) → $S_{x_1}^2$

(모든 데이터의 1feature 편차) × (〃 2feature 편차) → $S_{x_1 x_2}$

$S = \frac{1}{n} X_0^T X_0$ 정리

$$x_i = \begin{bmatrix} x_1 \\ \vdots \\ x_m \end{bmatrix} \quad (i = 1 \sim n) \qquad \bar{x} = \begin{bmatrix} \bar{x}_{01} \\ \vdots \\ \bar{x}_{0m} \end{bmatrix} \Rightarrow \underset{(m \times 1)}{x_i - \bar{x}} = \begin{bmatrix} x_{i1} - \bar{x}_{01} \\ \vdots \\ x_{im} - \bar{x}_{0m} \end{bmatrix}$$

m차원 평균벡터
(feature 평균벡터)

m차원 평균제거벡터

$$\underset{(n \times m)}{X} = \begin{array}{c} \\ n \end{array}\begin{array}{c} m \\ \begin{bmatrix} - x_1^T - \\ \vdots \\ - x_n^T - \end{bmatrix} \end{array} \qquad \underset{(n \times m)}{X_0} = \begin{bmatrix} - (x_1 - \bar{x})^T - \\ \vdots \\ - (x_n - \bar{x})^T - \end{bmatrix}$$

$$S_{jk} = \frac{1}{n} \sum (x_{ij} - \bar{x}_j)(x_{ik} - \bar{x}_k)^T$$

임의의 j, k 편차의 곱 $\Rightarrow$ 벡터 내적
feature 행렬로 구현

$$\underset{m \times m}{S} = \frac{1}{n} \underset{m \times n}{X_0^T} \underset{n \times m}{X_0}$$

$$* \ \bar{x} = \begin{bmatrix} \frac{1}{n} \sum_{i=1}^{n} x_{i1} \\ \frac{1}{n} \sum^{n} x_{i2} \\ \vdots \\ \frac{1}{n} \sum^{n} x_{im} \end{bmatrix} = \frac{1}{n} \sum_{i=1}^{n} \begin{bmatrix} x_{i1} \\ \vdots \\ x_{im} \end{bmatrix} = \frac{1}{n} \sum^{n} \begin{bmatrix} x_i \end{bmatrix}$$

$$= \frac{1}{n} \underset{m \times n}{\overset{ⓐ}{\begin{bmatrix} | & & | \\ x_1 & \cdots & x_n \\ | & & | \end{bmatrix}}} \underset{n \times 1}{\begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}} = \begin{bmatrix} \frac{1}{n}(x_{11} + x_{21} + \cdots + x_{n1}) \\ \vdots \\ \frac{1}{n}(x_{n1} + \cdots + x_{nm}) \end{bmatrix} \quad \left(\because X = \begin{bmatrix} - x_1^T - \\ \vdots \\ - x_n^T - \end{bmatrix} \right)$$

문법상 1행 2열이니 x_{12}가 맞긴 맞지만,
ⓐ 행렬의 데이터 표기를 참고해, 2번데이터 1특징값
이라 x_{21}로 표기

$$= \bar{x}_{(m \times 1)}$$

$$X_0 = \begin{bmatrix} -(x_1-\bar{x})^T- \\ \vdots \\ -(x_n-\bar{x})^T- \end{bmatrix} = X - 1_N \bar{x}^T \ ? $$
$$= X - \tfrac{1}{N} 1_N 1_N^T X \ ?$$

$$\underset{(n\times m)}{X} = \begin{bmatrix} -x_1^T- \\ \vdots \\ -x_n^T- \end{bmatrix} \qquad \underset{(m\times 1)}{\bar{x}} = \begin{bmatrix} \bar{x}_{01} \\ \vdots \\ \bar{x}_{0m} \end{bmatrix} \qquad \underset{(1\times m)}{\bar{x}^T} = [\bar{x}_{01} \cdots \bar{x}_{0m}]$$

복제확장기능

- $\tfrac{1}{N} \underbrace{1_N}_{n\times 1} \underbrace{1_N^T X}_{1\times m}$ 각 feature별 합

"$\tfrac{1}{N}$ 합" = 평균

- $1_N^T X$: 각 feature별 합! $[\Sigma x_{i1} \ \ \Sigma x_{i2} \ \cdots \ \Sigma x_{im}]$

[——] 특징행렬

- $1_N \bar{x}^T$: $\begin{bmatrix} \bar{x}^T \\ \vdots \\ \bar{x}^T \end{bmatrix}$ $\quad$ × $\bar{x}^T$는 X_0 만드는 기전 $\left(X = \begin{bmatrix} -x_1^T- \\ \vdots \\ -x_n^T- \end{bmatrix} \right)$ 복제확장 기능

[ ]

- $1_N \bar{x}^T = \tfrac{1}{N} 1_N 1_N^T X = \begin{bmatrix} \bar{x}_{01} & \cdots & \bar{x}_{0m} \\ \vdots & & \vdots \\ \bar{x}_{01} & \cdots & \bar{x}_{0m} \end{bmatrix} = \begin{bmatrix} \bar{x}^T \\ \vdots \\ \bar{x}^T \end{bmatrix}$