#4.1

$$A = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \\ 0 & 2 & 4 \end{bmatrix}$$

$$= (-1)^{1+1} \cdot 1 \cdot \begin{vmatrix} 4 & 6 \\ 2 & 4 \end{vmatrix} + (-1)^{1+2} \cdot 3 \cdot \begin{vmatrix} 2 & 6 \\ 0 & 4 \end{vmatrix} + (-1)^{1+3} \cdot 5 \cdot \begin{vmatrix} 2 & 4 \\ 0 & 2 \end{vmatrix}$$

$$= 1 \cdot (16-12) - 3 \cdot (8-0) + 5 \cdot (4-0)$$

$$= 1 \cdot 4 - 3 \cdot 8 + 5 \cdot 4 = 4 - 24 + 20$$

$$= 20$$

#4.2

$$\begin{bmatrix} 2 & 0 & 1 & 2 & 0 \\ 2 & -1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 1 & 2 \\ -2 & 0 & 2 & -1 & 2 \\ 2 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 1 & 2 & 0 \\ 0 & -1 & -1 & -1 & 1 \\ 0 & 1 & 2 & 1 & 2 \\ 0 & 0 & 3 & 1 & 2 \\ 0 & 0 & -1 & -1 & 1 \end{bmatrix}$$

$$= (-1)^{1+1} \cdot 2 \cdot \begin{vmatrix} -1 & -1 & -1 & 1 \\ 1 & 2 & 1 & 2 \\ 0 & 3 & 1 & 2 \\ 0 & -1 & -1 & 1 \end{vmatrix}$$

$$= 2 \cdot \begin{vmatrix} -1 & -1 & -1 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 3 & 1 & 2 \\ 0 & -1 & -1 & 1 \end{vmatrix}$$

$$= 2 \cdot (-1)^{1+1} \cdot (-1) \cdot \begin{vmatrix} 1 & 0 & 3 \\ 3 & 1 & 2 \\ -1 & -1 & 1 \end{vmatrix}$$

$$= -2 \cdot \{1 \cdot (1+2) - 0 \cdot (3+2) + 3 \cdot (-3+1)\}$$

$$= -2(3-6)$$

$$= 6$$

#4.3

a. $A := \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}$

$p_A(\lambda) = \lambda^2 - 2\lambda + 1$

$\lambda = 1$

$\therefore \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$

b. $B := \begin{vmatrix} -2 & 2 \\ 2 & 1 \end{vmatrix}$

$p_A(\lambda) = \lambda^2 - \lambda - 6$

$= (\lambda - 3)(\lambda + 2)$

$\lambda = 3, -2$

(1) $\lambda = 3 \rightarrow \begin{pmatrix} -5 & 2 \\ 2 & -2 \end{pmatrix}$

$= \begin{pmatrix} -5 & 2 \\ -3 & 0 \end{pmatrix}$

$\begin{pmatrix} t \\ \frac{5}{2}t \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{5}{2} \end{pmatrix} t$

$= \begin{pmatrix} 2 \\ 5 \end{pmatrix} t$

$\lambda = -2 \rightarrow \begin{pmatrix} 0 & 2 \\ 2 & -1 \end{pmatrix}$

$= \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$

#4.4

$A = \begin{bmatrix} 0 & -1 & 1 & 1 \\ -1 & 1 & -2 & 3 \\ 2 & -1 & 0 & 0 \\ 1 & -1 & 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} -\lambda & -1 & 1 & 1 \\ -1 & 1-\lambda & -2 & 3 \\ 2 & -1 & -\lambda & 0 \\ 1 & -1 & 1 & -\lambda \end{bmatrix}$

$=$

#4.5

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ → 대각 o, 역 o

↓

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ → 대각o, 역x

$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ → 대각x, 역o

↓

$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ → 대각x, 역o

↓

$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

#4.6

$$A = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 4 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P_A(\lambda) = -\lambda^3 + 7\lambda^2 - (4+2+5)\lambda + (2\cdot 4 - 3\cdot 1)$$

$$= -\lambda^3 + 7\lambda^2 - 11\lambda + 5$$

$$\begin{array}{r|rrrr} & -1 & 7 & -11 & 5 \\ 1 & & -1 & 6 & -5 \\ \hline & -1 & 6 & -5 & 0 \end{array}$$

$$= (\lambda-1)(-\lambda^2 + 6\lambda - 5)$$

$$= (\lambda-1)(-\lambda+1)(\lambda-5)$$

$$= -(\lambda-1)^2(\lambda-5)$$

(1) $\lambda = 5$

$$A - 5I = \begin{bmatrix} -3 & 3 & 0 \\ 1 & -1 & 3 \\ 0 & 0 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 3 & 0 \\ 0 & 0 & 3 \\ 0 & 0 & -4 \end{bmatrix}$$

$$x = \begin{bmatrix} \alpha \\ -\alpha \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \alpha$$

(2) $\lambda = 1$

$$A - 1I = \begin{bmatrix} 1 & 3 & 0 \\ 1 & 3 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

$$x = \begin{bmatrix} -3\alpha \\ \alpha \\ -\frac{2}{3}\alpha \end{bmatrix} \qquad 2\alpha + 3z = 0$$

$$z = -\frac{2}{3}\alpha$$

→ 고유벡터가 하나뿐

→ 대각화 x

b. $A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1-\lambda & 1 & 0 & 0 \\ 0 & -\lambda & 0 & 0 \\ 0 & 0 & -\lambda & 0 \\ 0 & 0 & 0 & -\lambda \end{bmatrix}$

$P_A(\lambda) = (1-\lambda) \cdot \begin{vmatrix} -\lambda & 0 & 0 \\ 0 & -\lambda & 0 \\ 0 & 0 & \lambda \end{vmatrix} - 1 \cdot \begin{vmatrix} 0 & 0 & 0 \\ 0 & -\lambda & 0 \\ 0 & 0 & -\lambda \end{vmatrix}$

$= (1-\lambda)(-\lambda) \cdot \lambda^2 = \lambda^3(\lambda-1)$

$\lambda = 0 \rightarrow \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ → 고유벡터 1개

→ 대각 X

#4.7

a. $A = \begin{bmatrix} 0 & 1 \\ -8 & 4 \end{bmatrix}$

$P_A(\lambda) = \lambda^2 - 4\lambda + 12$

$= (\lambda+2)(\lambda-6)$

$\lambda = -2 \rightarrow \begin{bmatrix} -2 & 1 \\ -8 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 0 & -2 \end{bmatrix}$

→ 고유벡터 X

b. $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$P_A(\lambda) = -\lambda^3 + 3\lambda^2$

$= -\lambda^2(\lambda-3)$

$\lambda = 0 \rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$x = \begin{bmatrix} -\alpha-\beta \\ \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}\alpha + \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}\beta$

$\lambda = 3 \rightarrow \begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 1 \\ -3 & 0 & 3 \\ -3 & 3 & 0 \end{bmatrix}$

$x = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

A는 대각화 가능

$X = \begin{bmatrix} -1 & -1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

c. $A = \begin{bmatrix} 5 & 4 & 2 & 1 \\ 0 & 1 & -1 & -1 \\ -1 & -1 & 3 & 0 \\ 1 & 1 & -1 & 2 \end{bmatrix}$

d $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$

$= -\lambda^3 + 5\lambda^2 - (-4-2-6)\lambda + (-20+12+36)$

$= -\lambda^3 + 5\lambda^2 + 12\lambda + 28$

$-1 \quad 5 \quad 12 \quad 28$

#4.8

$$A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{bmatrix}$$

$$A^TA = \begin{bmatrix} 3 & 2 \\ 2 & 3 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 13 & 12 & 2 \\ 12 & 13 & -2 \\ 2 & -2 & 8 \end{bmatrix}$$

$$-\lambda^3 + 34\lambda^2 - (104 - 4 + 108 + 4 - 24 - 26)$$

$$+ 13 \cdot 100 + 12 \cdot 112 + 2 \cdot (-50)$$

$$= -\lambda^3 + 34\lambda^2 - (100 + 112 - 50)$$

$$+$$

#4.9

$$A = \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix}$$

$$A^TA = \begin{bmatrix} 2 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix}$$

$$P_A(\lambda) = \lambda^2 - 10\lambda + 16$$

$$2$$
$$8$$

$$= (\lambda - 2)(\lambda - 8)$$

$$\lambda = 2, 8$$

$$\lambda = 2 \rightarrow \begin{bmatrix} 0 & 2 \\ -1 & -1 \end{bmatrix}$$

$$\lambda = 8 \rightarrow \begin{bmatrix} -6 & 2 \\ -1 & -7 \end{bmatrix}$$