3.1

⇒ inner product 의 조건

① symmetric : $\langle x, y \rangle = \langle y, x \rangle$

② positive definite : $\langle x, x \rangle \geq 0$

↳ $\langle y, x \rangle = y_1 x_1 - (y_1 x_2 + y_2 x_1) + 2(y_2 x_2)$

$= \langle x, y \rangle$

↳ $\langle x, x \rangle = x_1^2 - (x_1 x_2 + x_2 x_1) + 2x_2^2$

$= x_1^2 - 2x_1 x_2 + 2x_2^2$

$= (x_1 - x_2)^2 + x_2^2 \geq 0$

$\therefore \langle x, y \rangle$는 inner product 이다.

3.2

$$\langle x, y \rangle = \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$$= \begin{bmatrix} 2x_1 + x_2 & 2x_2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$$= 2x_1 y_1 + x_2 y_1 + 2x_2 y_2$$

① symmetric

$$\langle y, x \rangle = 2y_1 x_1 + y_2 x_1 + 2y_2 x_2$$

$$\neq \langle x, y \rangle$$

$\therefore \langle \cdot, \cdot \rangle$는 inner product가 아니다.

3.3 : python으로 계산

$a : -3$, $b : -8$

3.4 : python

$a = -3$, $b = -11$

3.5

(a) $B'B\lambda = B'x \Leftrightarrow \lambda = (B'B)^{-1}B'x$

에서 $\lambda = \begin{bmatrix} -7.06 \\ 1.25 \\ 1.156 \\ 1.5 \end{bmatrix}$ 이고, $\pi_u(x) = B\lambda$ 에서

$$\pi_u(x) = \begin{bmatrix} -3.71875 \\ 3.4375 \\ -4.21875 \\ 1.0625 \\ 0.03125 \end{bmatrix}$$

(b) $d(x, U) = \| x - \pi_u(x) \|$

$$= \| x - (x_0 + \pi_v(x - x_0)) \|$$

$$= d(x - x_0, \pi_v(x - x_0))$$

$$= d(x - x_0, V)$$

에서

$d(x, U) = \| x - \pi_u(x) \|$ 를 이용하면,

$$= 13.49$$

3.6

a. $U = \begin{bmatrix} 2 & 0 \\ 1 & -1 \\ 0 & 2 \end{bmatrix}$, $e_2 = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$

에서 $B = U$ 이고,

$$\pi_u(e_2) = B\Lambda, \quad \Lambda = (B'B)^{-1}B'e_2$$

이서 $\pi_u(e_2) = B(B'B)^{-1}B'e_2$

이다.

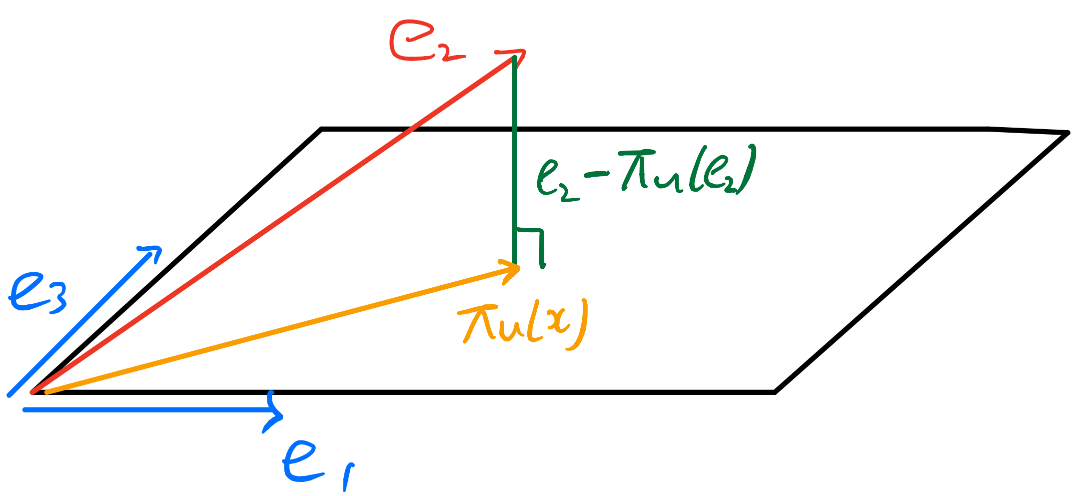
$B'B\Lambda = B'x$

$$\therefore \pi_u(e_2) = \begin{bmatrix} 1.33 \\ 1.33 \\ -1.33 \end{bmatrix}$$

b. $d(e_2, U) = \| e_2 - \pi_u(e_2) \|$

$= 0.8165$

c.

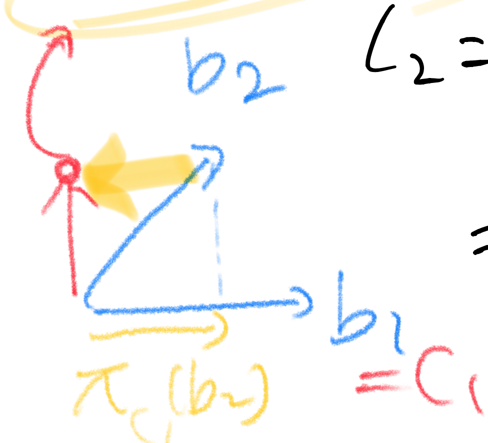


3.7 a ☆

b ☆

3.8. $c_1 = b_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$b_2 - \pi_{c_1}(b_2) = C_2$



$$c_2 = b_2 - \pi_{c_1}(b_2)$$

$$= b_2 - \frac{c_1 \cdot c_1'}{\|c_1\|} b_2$$

$$= \begin{bmatrix} -1.58 \\ 1.42 \\ -0.58 \end{bmatrix}$$

3.9 a.b

↳ python

3.10

$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$\sigma(k-1$

$$x_1 \longrightarrow \begin{bmatrix} 0.23 \\ 3.6 \end{bmatrix}$$

$$x_2 \longrightarrow \begin{bmatrix} 0.5 \\ -0.87 \end{bmatrix}$$