1.

Base Case: N = 2k-1

2 - 1 = 1

## Assumption:

N = 2k-1

A tree with k leaves has N nodes.

## Induction:

Prove k+1:

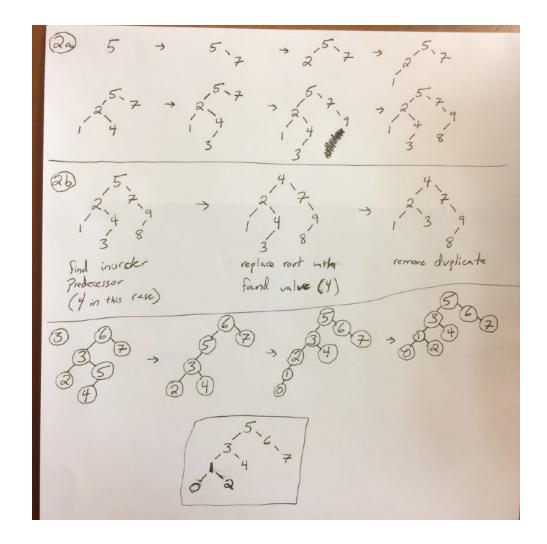
Plug in k+1 to N=2k-1:

N = 2(k+1)-1 = 2k+2-1 = 2k+1

If you add two children to one leaf, then you are adding one leaf and creating a tree of k+1, because you are eliminating the leaf you add the nodes to, but adding two children, resulting in a net increase of one node. The total nodes then increases from N to N+2:

$$N + 2 = (2k-1)+2 = 2k+1$$

This is equivalent to our previous answer, Q.E.D.



4.

Tree 1:



Tree 2:



The trees 1 and 2 both have the same preorder [A,B,C] and postorder [C,B,A] traversals, proving that only having those two traversals, and no inorder traversal is not enough to come to a unique tree.

(Tree 1: Root A, Right B, Right/Left C) (Tree 2: Root A, Left B, Left/Right C)