

Registration No.: _____

PNR No.:114151MTH19371

Course Code: MTH107
Course Name: Elementary Mathematics for Engineers

Time Allowed: 03:00hrs

Max.Marks: 100

1. This paper contains 6 questions divided in two parts.
2. All question are compulsory.
3. The marks assigned to each question are shown at the end of each question.
4. Attempt either (a) OR (b) from each question of Part B
5. Answer all the questions in serial order.
6. Do not write anything on the question paper except your registration number at the designed space

Part A

Q1 (a) Find the basic solution of the following system of equation, identifying in each case of basic and non basic variables: $2x_1+x_2+4x_3=11$, $3x_1+x_2+5x_3=14$.

[2.5 Marks]

(b) Evaluate $\int \frac{x}{1+x^2} dx$

[2.5 Marks]

(c) if $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$, find $\frac{dy}{dx}$.

[2.5 Marks]

(d) If $z = f(x + ct) + g(x - ct)$, prove that : $\frac{\partial^2 z}{\partial t^2} = c^2 \frac{\partial^2 z}{\partial x^2}$

[2.5 Marks]

(e) Differentiate the Function $e^{\sin^2(\log x)}$ with respect to x.

[2.5 Marks]

(f) If $x = a(t - \sin t)$, $y = a(1 - \cos t)$. Find dy/dx .

[2.5 Marks]

(g) If $p = \text{Today is Sunday}$, $q = \text{It is sunny day}$, then translate $\sim p \wedge q$ into logical sentence

[2.5 Marks]

(h) IF $A = \{1, 3, 5, 9\}$, $B = \{1, 2, 5, 7, 9\}$, Show that
 $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

[2.5 Marks]

(i) Evaluate the integral $\int_0^{\frac{\pi}{4}} \frac{dx}{1 - \sin x}$

[2.5 Marks]

(j) Integrate $\frac{\sin \sqrt{x}}{\sqrt{x}}$ with respect to x.

[2.5 Marks]

PART B

Q2.(a) Solve $\int \frac{x^2+x+1}{(x-2)(x-1)^2}$

[15 Marks]

OR

(b)

(i) Solve $\int \frac{x^2 \tan^{-1} x dx}{(1+x^2)}$ [5]

(ii) Evaluate $\int \sin^{-1} \sqrt{\left(\frac{a}{a+x}\right)} dx$ [10]

[15 Marks]

Q3 (a) Differentiate $y = \left(\frac{x^2 - 2x}{3x - 4x^2}\right)^3 + \sin(\sqrt{x}) + \cos^{-1}\left(\frac{x^2 + 2x}{2x}\right)$

[15 Marks]

OR

(b) Find the maximum values of $x^5 - 5x^4 + 5x^3 - 1$.

[15 Marks]

Q4.(a) Evaluate $\int_0^{\frac{\pi}{2}} \frac{dx}{9 \cos x + 12 \sin x}$

OR

(b) Find by summation the value of $\int_a^b x^2 dx$

[15 Marks]

Q5. (a)(ii) Convert the following LPP to the standard form [5]

Min $Z = 3x + 2y + 5t$
 Subject to the constraints
 $-5x + 2y \leq 5$
 $2x + 3y + 4t \geq 7$
 $2x + 5t = -3$
 $x, y \geq 0$.

(ii) Solve the following graphically [10]

Max $Z = 5x_1 + 7x_2$
 subject to
 $x_1 + x_2 \leq 4$
 $5x_1 + 8x_2 \leq 24$
 $10x_1 + 7x_2 \leq 35$
 $x_1 \geq 0, x_2 \geq 0$

OR

b(i) Write the duals of the following problem

Max $Z = 10x_1 + 5x_2 + 13x_3 + 19x_4$
 subject to

$6x_1 + 5x_2 + 3x_3 \leq 26$
 $4x_1 + 2x_2 + 5x_3 \leq 7$
 $x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$

(ii) Find all basic solution to the following problem :

Max $Z = x_1 + 3x_2 + 3x_3$
 subject
 $x_1 + 2x_2 + 3 = 4, 2x_1 + 3x_2 + 5x_3 = 7, x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$

Which of the basic solution are (a) Non-degenerate basic feasible (b) Optimal basic feasible ?

[15 Marks]

Q6. (a)

(i). Evaluate $\lim_{x \rightarrow 0} \frac{e^{ax} - e^{bx}}{\sin ax - \sin bx}$

[10]

(ii) Show that function $f(x) = \begin{cases} \frac{|x-a|}{x-a}; & x \neq a \\ 1 & ; x = a \end{cases}$ is continuous at $x = a$. [5]

OR

[15 Marks]

(b) (i) Find the domain and range of function $f(x) = \sqrt{4 - x^2}$ [5]

(ii) If $f(x) = 3x^4 - 5x^2 + 9$. Find $f(-5)$, $f\left(\frac{1}{x}\right)$, $f\left(\frac{1}{x-1}\right)$, $f(x-3)$

-- End of Question Paper --